THE ASTROPHYSICAL JOURNAL, 330: L1–L4, 1988 July 1

ENERGETIC CONSTRAINTS ON SPECTRAL DISTORTIONS OF THE MICROWAVE BACKGROUND

CEDRIC G. LACEY AND GEORGE B. FIELD
Harvard-Smithsonian Center for Astrophysics
Received 1988 March 7; accepted 1988 April 6

ABSTRACT

We investigate the energetic constraints on mechanisms for producing the spectral distortion of the microwave background claimed by Matsumoto *et al.*, in the light of the new upper bounds on the cosmological baryon density derived by Kawano *et al.* from primordial nucleosynthesis. We consider the thermal emission from dust heated by stars at large redshift and Compton scattering by a hot intergalactic medium heated by supernova explosions, and we conclude that both mechanisms fail on energetic grounds if they rely on stars formed with the same mass function as is observed in the solar neighborhood.

Subject headings: cosmic background radiation — nucleosynthesis — radiation processes

I. INTRODUCTION

Matsumoto *et al.* (1988) have claimed detection of a distortion from blackbody form in the spectrum of the microwave background. Relative to the $T = 2.74$ K blackbody which fits the observations at long wavelengths, there is an excess in the background which peaks at $\lambda \approx 600\ \mu$m and has energy density $U_r \approx 5 \times 10^{-14}$ ergs cm^{-3}, i.e., about 10% of the total. [We use the symbols u for the proper energy density and U for the energy per unit comoving volume respectively; they are related by $U(z) = u(z)/(1+z)^3$, z being the redshift.] Matsumoto *et al.* suggest three possible mechanisms to explain the excess: thermal emission from dust at large redshifts, Compton scattering of microwave background photons by a hot intergalactic medium, and radiative decay of particles. These ideas are discussed in more detail by Hayakawa *et al.* (1987). In this *Letter* we consider the energetic constraints on the production of the spectral distortion in the light of a new upper limit on the cosmological baryon density derived from primordial nucleosynthesis. In particular, we investigate whether the necessary energy can be supplied by a stellar population formed with the same mass function as applies in the solar neighborhood.

Kawano, Schramm, and Steigman (1988) derive new bounds on the cosmological density of baryons from calculations of light element production in the standard model of primordial nucleosynthesis. In terms of η_{10}, the baryon-to-photon ratio in units of 10^{-10}, they find a best fit value $\eta_{10} = 3 \pm 1$ assuming that the ^{7}Li/H ratio in Population II stars has its primordial value, and an upper limit $\eta_{10} < 6$ allowing for possible astration, derived from the observed ^{7}Li/D ratio. Assuming $T_{r0} = 2.74$ K for the present temperature of the microwave background (Matsumoto *et al.*), this upper bound on η_{10} translates to a bound on Ω_b, the density parameter for baryons, of $\Omega_b h^2 < 2.2 \times 10^{-2}$, where $h = H_0/100$ km s^{-1} Mpc^{-1}. Observations indicate that $0.4 < h < 1$.

Suppose that the energy required to produce the spectral distortion is derived from baryons. Then, since the observed excess radiation energy density U_r is a lower bound on the energy injected per unit comoving volume, we obtain a lower limit on the efficiency ϵ with which baryonic rest mass needs to be converted into energy: $\epsilon > U_r/\rho_{b0} c^2 > 1.4 \times 10^{-4}$. In fact, other physical constraints in the specific mechanisms we consider force one to a considerably larger value of ϵ, mainly because the energy injection is required to occur at large redshift.

II. EMISSION BY DUST

We consider first the model in which the excess radiation is produced by emission from dust grains heated by radiation from stars in galaxies at large redshifts. We assume that the dust grains have absorption cross sections σ_λ with wavelength dependence $\sigma_\lambda = \sigma_0(\lambda_0/\lambda)^\alpha$, with $\alpha = 1$ or 2. Then the power emitted by a dust grain at temperature T_d is $P_d = 4\pi A_\alpha \sigma_0 (k\lambda_0/hc)^\alpha \sigma_R T_d^{4+\alpha}$, where $A_\alpha = (15/\pi^5)\Gamma(4+\alpha)\zeta(4+\alpha)$ and σ_R is the Stefan-Boltzmann constant. If the dust grains emit at rate $P_d(z)$ and have number density $n_d(z)$, the total energy emitted per unit comoving volume is

$$U_{\rm em} = \int_0^\infty \frac{n_d(z)P_d(z)}{(1+z)^3}\left|\frac{dt}{dz}\right| dz\ , \qquad (2.1)$$

while the energy density at $z = 0$ of the radiation produced, U_r, is the same integral except with $(1+z)^4$ replacing $(1+z)^3$. For an $\Omega = 1$ cosmology, $|dt/dz| = 1/H_0(1+z)^{5/2}$. According to Matsumoto *et al.*, the spectrum of the excess radiation can be fitted by a single-temperature dust emission model, with observed temperature T_{d0}. If the emission occurs over a range of redshift $z_1 < z < z_2$, the dust temperature over this range is then required to vary as $T_d(z) = T_{d0}(1+z)$. With this constraint on the dust temperature, $U_{\rm em}$ is minimized for given U_r by setting $z_1 = 0$, $z_2 = z_h$, with z_h to be determined. We assume that the dust density varies as $n_d(z) = n_{d0}(1+z)^3$, corresponding to constant density per unit comoving volume. Substituting these dependences into equation (2.1), and assuming that $(1+z_h)^{\alpha+3/2} \gg 1$, we find

$$(1+z_h) \approx \left[\frac{(\alpha+3/2)H_0 U_r}{n_{d0} P_{d0}}\right]^{1/(\alpha+3/2)} \qquad (2.2a)$$

and

$$U_{\rm em} \approx [(\alpha+3/2)/(\alpha+5/2)]U_r(1+z_h)\ . \qquad (2.2b)$$

For $\alpha = 1$ or 2, most of the energy is emitted near redshift z_h. Assuming a constant dust-to-gas ratio, $n_{d0} \propto \Omega_g$. If this energy is to be produced by nuclear burning in stars, we require a conversion efficiency $\epsilon_s = U_{em}/\rho_s c^2 \propto [\Omega_s \Omega_g^{1/(\alpha+3/2)}]^{-1}$. Since $\Omega_g + \Omega_s \le \Omega_b$, the required efficiency is minimized if all the baryons are either in gas or stars, with $\Omega_g/\Omega_b = 1/(\alpha + 5/2)$ and $\Omega_s/\Omega_b = (\alpha + 3/2)/(\alpha + 5/2)$.

We assume that the dust has the same properties as that in the local interstellar medium. We consider two models for the form of the dust cross section σ_λ at $\lambda \gtrsim 10^{-3}$ cm. One is the theoretical model of Draine and Lee (1984), which has $\alpha = 2$ in this wavelength range, with the normalization that the dust cross section per hydrogen atom is $\sigma_0' = 7 \times 10^{-23}$ cm^2 at $\lambda_0 = 10^{-3}$ cm. The other is the empirical model proposed by Rowan-Robinson (1986), which has $\alpha = 1$ and $\sigma_0' = 2 \times 10^{-23}$ cm^2 at the same λ_0. Hayakawa *et al.* find that the spectrum is fitted by $T_{d0} = 3.7$ K for $\alpha = 2$, and $T_{d0} = 4.4$ K for $\alpha = 1$, assuming $T_{r0} = 2.74$ K. Substituting these values in equations (2.2a) and (2.2b), and assuming the optimum values for Ω_g/Ω_b and Ω_s/Ω_b derived above, we find

$$\begin{aligned}(1 + z_h) &\approx 2.3(f_d \Omega_b h)^{-2/5} \gtrsim 11 f_d^{-2/5} h^{2/5} \quad (\alpha = 1)\\ (1 + z_h) &\approx 6.8(f_d \Omega_b h)^{-2/7} \gtrsim 20 f_d^{-2/7} h^{2/7} \quad (\alpha = 2)\end{aligned} \tag{2.3}$$

and

$$\begin{aligned}\epsilon_s &\approx 6.9 \times 10^{-6} f_d^{-2/5} \Omega_b^{-7/5} h^{-12/5}\\ &\gtrsim 1.5 \times 10^{-3} f_d^{-2/5} h^{2/5} \quad (\alpha = 1)\\ \epsilon_s &\approx 2.0 \times 10^{-5} f_d^{-2/7} \Omega_b^{-9/7} h^{-16/7}\\ &\gtrsim 2.7 \times 10^{-3} f_d^{-2/7} h^{2/7}\,, \quad (\alpha = 2)\end{aligned} \tag{2.4}$$

where f_d is defined to be the value of σ_0' relative to that in the solar neighborhood, and the last part of each equation is obtained on substituting $\Omega_b h^2 < 2.2 \times 10^{-2}$.

We now compare the required ϵ_s with the energy obtainable from nuclear burning in stars having a normal initial mass function (IMF). We define $\phi(m)\,dm$ to be the number of stars in the mass range $(m, m + dm)$, with the normalization $\int_0^\infty m\phi(m)\,dm = 1$. The solar neighborhood IMF derived by Scalo (1986) is well fitted by a power law for $m \gtrsim 2\ M_\odot$. It is convenient to express it in the form $\phi(m) = (x - 1)\xi_1 m_1^{-2}(m/m_1)^{-(x+1)}$ for $m_1 \le m \le m_U$. For $m_U/m_1 \gg 1$, ξ_1 is the total mass fraction in stars with $m \ge m_1$. We take $m_1 = 2\ M_\odot$ and assume $m_U = 100\ M_\odot$. The IMF slope is $x \approx 1.7$. The parameter ξ_1 is less certain, depending on assumptions made about the past time-dependence of the star-formation rate. Scalo's results correspond to values $\xi_1 \approx 0.2$ or $\xi_1 \approx 0.4$ for constant or declining star formation rates respectively. One can try to estimate ξ_1 indirectly by using nucleosynthetic constraints. The yield of heavy elements y_z and returned fraction R are defined as integrals over $\phi(m)$, following Tinsley (1980, eqs. [3.11] and [3.12]). Assuming that the stellar remnant mass is $m_R \approx 0.7\ M_\odot$, and that the mass of heavy elements ejected by each Type II supernova is $m_{Zej} \approx 0.4(m - m_S)$ for $m \ge m_S = 10\ M_\odot$ (Woosley and Weaver 1986), we find $(1 - R)y_Z \approx 0.05\ \xi_1$, and $R \approx \xi_1$. Chemical evolution models of the solar neighborhood imply $y_Z \approx Z_\odot \approx 0.02$ (Tinsley 1980), from which one derives $\xi_1 \approx 0.3$, intermediate between the two previous estimates.

We now consider the energy production by a single generation of stars formed with the mass function described above. The total energy produced per unit mass by time τ after formation is

$$E_s(\tau) = \int_0^{m_b} L_{MS}(m)\tau\phi(m)\,dm + \int_{m_b}^\infty \epsilon_m(m)mc^2\phi(m)\,dm\,, \tag{2.5}$$

where the first term gives the contribution of stars still on the main sequence and having luminosities L_{MS}, and the second term the contribution of stars which have burned out, after emitting a total energy (including post-main-sequence stages) $\epsilon_m mc^2$. The burn-out mass m_b is defined in terms of the main-sequence lifetime $\tau_{MS}(m)$ by the relation $\tau_{MS}(m_b) = \tau$. From the data compiled by Tinsley (1980), we find that the main-sequence lifetime is $\tau_{MS} \approx 7.1$ Gyr$(m/M_\odot)^{-3.0}$ for $2 \lesssim m/M_\odot \lesssim 6$, while the main-sequence luminosity is $L_{MS} \approx L_\odot(m/M_\odot)^{4.35}$ for $0.8 \lesssim m/M_\odot \lesssim 6$. For the burned-out stars, we have $\epsilon_m(m) = \epsilon_H X f_H$, where $\epsilon_H = 6.9 \times 10^{-3}$ is the efficiency of energy generation in burning of hydrogen to helium, $X \approx 0.7$ is the hydrogen mass fraction, and f_H is the mass fraction of the star in which complete H → He burning occurs. (Later stages of nuclear burning contribute little to the total radiative energy output.) From the stellar evolution calculations of Iben (1985), Alcock and Paczyński (1978), Woosley and Weaver (1986), and Nomoto and Hashimoto (1987), we find $f_H \approx 0.25$ for $2 \lesssim m/M_\odot \lesssim 10$, and $f_H \approx 0.25(m/10\ M_\odot)^{0.35}$ for $10 \lesssim m/M_\odot \lesssim 100$. Carrying out the integrals in equation (2.5), we obtain for the net efficiency $\epsilon_s = E_s/c^2$ as a function of burn-out mass

$$\epsilon_s(m_b) \approx 1.2 \times 10^{-3}\xi_1[0.14(m_b/2\ M_\odot)^{-0.35} + (m_b/2\ M_\odot)^{-0.7}] \tag{2.6}$$

in the range $2 \lesssim m_b/M_\odot \lesssim 6$.

For an $\Omega = 1$ cosmology, the age of the universe at redshift z is $t = (2/3H_0)(1 + z)^{-3/2} \approx 6.5$ Gyr $h^{-1}(1 + z)^{-3/2}$. Combining this with the previous expression for the main-sequence lifetime gives the minimum possible burn-out mass at redshift z: $m_b \approx 1.0\ M_\odot\ h^{1/3}(1 + z)^{1/2}$ for $4.3\ h^{-2/3} \lesssim (1 + z) \lesssim 38\ h^{-2/3}$. Consider the case $\alpha = 2$. Combining with equation (2.3) for z_h (with $f_d \lesssim 1$) gives $m_b \gtrsim 4.5\ M_\odot\ h^{0.48}$. Substituting this into equation (2.6), with $\xi_1 = 0.4$, we find that the maximum efficiency obtainable from nuclear burning in stars is smaller than the efficiency we require, given by equation (2.4), by factors of 5 and 8 for $h = 0.4$ and $h = 1$, respectively. For $\alpha = 1$, we find $m_b \gtrsim 3.3\ M_\odot\ h^{0.53}$, and nuclear burning provides insufficient energy by a factor between 2 and 4 for $0.4 \le h \le 1$.

III. COMPTON SCATTERING BY HOT IGM

Now consider Compton scattering of the microwave background by a hot intergalactic medium. A hydrogen-helium plasma with electron density $n_e(z) = n_{e0}(1 + z)^3$ and temperature $T_e(z)$ has energy density $u = (3/2)(\mu_e/\mu)n_e kT_e$, assuming that electrons and ions are in thermal equilibrium, where μ and μ_e are, respectively, the mean molecular weights per particle and per electron. Suppose the plasma is instantaneously heated to temperature T_h at redshift z_h. The energy density subsequently evolves according to the equation

$$\frac{du}{dt} = -\frac{8}{3}\left(\frac{\mu}{\mu_e}\right)\frac{\sigma_T a T_r^4}{m_e c}u + \frac{5}{3}\left(\frac{1}{n_e}\frac{dn_e}{dt}\right)u\,, \tag{3.1}$$

where the first term on the right-hand side represents the effect of Compton cooling by the microwave background with temperature $T_r = T_{r0}(1 + z)$ (we assume $T_e \gg T_r$), and the second term represents the effect of adiabatic expansion; including other energy losses would only strengthen the limit we derive. Integrating this equation gives

$$\left(\frac{u}{n_e^{5/3}}\right) = \left(\frac{u}{n_e^{5/3}}\right)_h \exp\left[-\int_z^{z_h}\frac{8}{3}\left(\frac{\mu}{\mu_e}\right)\frac{\sigma_T a T_r^4}{m_e c}\left|\frac{dt}{dz}\right| dz\right]. \tag{3.2}$$

Taking $|dt/dz|$ for an $\Omega = 1$ cosmology, we obtain

$$T(z)/T_h = [(1+z)/(1+z_h)]^2 \times \exp\{-\beta[(1+z_h)^{5/2} - (1+z)^{5/2}]\}\,, \quad (3.3)$$

where $\beta = 16\mu\sigma_T a T_{r0}{}^4/15\mu_e m_e c H_0 \approx 1.76 \times 10^{-3}\ h^{-1}$, assuming a helium abundance $Y = 0.24$ by mass. The Compton y-parameter for nonrelativistic scattering is $y = \int_0^{z_h} (kT_e/m_e c^2) n_e \sigma_T c\,|dt/dz|\,dz$. Substituting the expression (3.3), and assuming $(1+z_h)^{7/2} \gg 1$, we obtain

$$y \approx [(4\mu U_h \sigma_T)/(15\mu_e m_e c H_0)]\beta^{-3/5} F(\xi)\,, \quad (3.4a)$$

where $\xi = \beta(1+z_h)^{5/2}$ and

$$F(\xi) = \xi^{-4/5} e^{-\xi} \int_0^{\xi} x^{2/5} e^x\, dx\,. \quad (3.4b)$$

Here $U_h = (3/2)(\mu_e/\mu) n_{e0} k T_h$ is the injected energy density per unit comoving volume. The function $F(\xi)$ has the limiting behaviors $F(\xi) \approx \frac{5}{7}\xi^{3/5} \propto (1+z_h)^{3/2}$ for $\xi \ll 1$, and $F(\xi) \approx \xi^{-2/5} \propto (1+z_h)^{-1}$ for $\xi \gg 1$, achieving a maximum value $F_{max} \approx 0.54$ at $\xi_{max} \approx 2.1$. Physically, the maximum occurs because for small z_h, the Comptonization efficiency is small, while for large z_h, Comptonization is efficient, but the injected energy is redshifted. Substituting numerical values, we find that the maximum possible spectral distortion for a given energy input U_h is $y_{max} = 2.5 \times 10^{10}$ (U_h/ergs cm^{-3}) $h^{-2/5}$, corresponding to energy injection at redshift $(1+z_h)_{max} = 17\ h^{2/5}$. Note that although this limit on y for given U_h was derived for energy injection at a single time, it applies for any distribution of the heating over time, since all the equations are linear in the energy density u.

Hayakawa *et al.* (1987) find that to explain the spectral distortion by Compton scattering, they need $y = 0.028$. This therefore requires the injection of energy per unit comoving volume $U_h \gtrsim 1.1 \times 10^{-12}\ h^{2/5}$ ergs cm^{-3}. This corresponds to an efficiency for converting baryonic mass to energy of

$$\epsilon_C = U_h/\rho_{b0} c^2 \gtrsim 6.6 \times 10^{-5} (\Omega_b h^2)^{-1} h^{2/5} > 3.0 \times 10^{-3} h^{2/5} > 2.1 \times 10^{-3}\,, \quad (3.5)$$

where the last two inequalities are obtained by substituting $\Omega_b h^2 < 2.2 \times 10^{-2}$ and then $h > 0.4$. A similar constraint on the required efficiency is derived by Yoshioka and Ikeuchi (1987), in the context of a specific model of large-scale explosions.

The only plausible mechanism for energizing a hot IGM that involves stars uses the kinetic energy of supernova ejecta (Bookbinder *et al.* 1980). We therefore compare the limit (3.5) with the total energy obtainable from supernova explosions in a stellar population with a normal IMF. To maximize this energy, we assume that all baryons are formed into stars. We assume that each star with $m \geq 10\ M_\odot$ explodes as a supernova with energy $E_{SN} = 10^{51}$ ergs. Integrating over the mass function given above, one obtains an efficiency $\epsilon_{SN} \approx 7.3 \times 10^{-6}\ \xi_1 = 2.9 \times 10^{-6}$, where the last follows on assuming $\xi_1 = 0.4$. We see that supernovae fail to provide sufficient energy by a factor of at least 700.

A weaker constraint, which is however independent of the heating mechanism, follows from the fact (Hayakawa *et al.*) that the Compton scattering must be nonrelativistic in order to give the correct shape for the spectral distortion. According to Rybicki and Lightman (1979, eqs. [7.36] and [7.38]), the average frequency shift in a single scattering is $\Delta\nu/\nu \approx (4kT/m_e c^2)$ when the electrons are nonrelativistic, and $\Delta\nu/\nu \approx (4kT/m_e c^2)^2$ when they are ultrarelativistic. Thus the critical temperature at which $\Delta\nu/\nu \approx 1$ is $T \approx m_e c^2/4k \approx 1.5 \times 10^9$ K. When $\Delta\nu/\nu \gtrsim 1$ in a single scattering, the Kompaneets equation for the spectral distortion breaks down, and the distortion no longer fits the observations, as shown for instance by Figure 1 of Hayakawa *et al.* In the integral for y, most of the contribution comes from the range where $T \approx T_h$. If we combine the upper limit $T_h \lesssim 1.5 \times 10^9$ K with the previously derived lower limit on U_h and assume that the heating of the plasma occurs at a single epoch, we find that the plasma must have density parameter $\Omega_h h^2 \gtrsim 0.19\ h^{2/5} > 0.13$ in order to produce the observed distortion, where the last follows on assuming $h > 0.4$. This exceeds the limit on the total baryonic density $\Omega_b h^2 < 2.2 \times 10^{-2}$ by at least a factor of 6. However, this limit can be circumvented (at the cost of increasing the required value of U_h) by injecting energy over a range of redshift such that T always remains below T_h.

IV. CONCLUSIONS

We have seen that even in the favorable case $h = 0.4$, emission by dust fails by a factor >2–5 to account for the observed spectral distortion, while Compton scattering by hot gas fails by a factor >700, assuming that the energy is supplied by stars formed with the same mass function as in the solar neighborhood, and, in the case of dust, that the dust has the same properties and the same abundance relative to the gas as in the solar neighborhood. We remind the reader that these numbers are obtained by assuming the optimum redshift for energy injection, and, in the case of dust, the optimum division of the baryons between stars and gas, a case which might be thought unlikely to occur in practice. How then can one reconcile theory with observations? There are various possibilities:

1. Limits on Ω_b from primordial nucleosynthesis may be too low, either because the effects of astration (in particular, of ^{7}Li) have been underestimated, or because primordial nucleosynthesis differs from the standard model (e.g., Applegate, Hogan, and Scherrer 1987; Dimopoulos *et al.* 1988).

2. Energy generation can be more efficient if the initial mass function differs from its solar neighborhood form. If the shape of the IMF is assumed fixed, then ξ_1 cannot be increased much above the value $\xi_1 = 0.4$ assumed here without overproducing heavy elements compared to what is observed in our galaxy and other galaxies. However, if the upper mass limit m_U is reduced to below the minimum mass $m_s \approx 10\ M_\odot$ for supernovae, the mass fraction in intermediate-mass stars ($2 \lesssim m/M_\odot \lesssim 10$) could be increased without violating the metallicity constraint.

3. A more extreme version of this idea is to assume that the energy is produced by a first generation of stars which are Very Massive Objects (VMOs), with $10^2 \lesssim m/M_\odot \lesssim 10^5$ (Carr, Bond, and Arnett 1984). VMOs in the range $100 \lesssim m/M_\odot \lesssim 200$ explode, with explosive efficiency up to $\epsilon \approx 2 \times 10^{-4}$. However, at the same time they eject back a fraction $Z_{ej} \approx 0.5$ of their initial mass as heavy elements, so at most a fraction $f_{max} \approx Z_{min}/Z_{ej}$ of the baryons can be processed through such objects without producing a metallicity larger than the minimum value observed in the oldest stars ($Z_{min} \approx 10^{-5}$ in Population II, and $Z_{min} \approx 10^{-3}$ in Population I), as discussed by Carr *et al.* VMOs with $m \gtrsim 200\ M_\odot$ avoid this constraint by collapsing to black holes after nuclear burning. They burn a fraction $f_H \approx 0.8$ of their mass, releasing energy with efficiency $\epsilon \approx 4 \times 10^{-3}$, and so might be candidates for heating dust.

4. In the dust mechanism, the efficiency ϵ_s required can be reduced (mainly by reducing the redshift z_h) if the dust cross section per unit gas mass is increased relative to its local value. This could be achieved for instance by making the grains needle-shaped (Wright 1982).

5. In the Compton mechanism, the IGM might be heated by active galactic nuclei. Suppose that each L_* galaxy contains a central black hole of mass $M_{\rm H}$, which releases total energy $\epsilon_{\rm H} M_{\rm H} c^2$. Then, since the effective density of L_* galaxies is $n_* \approx 1.7 \times 10^{-2}\ h^3\ {\rm Mpc}^{-3}$ (Efstathiou and Silk 1983), we require $M_{\rm H} \gtrsim 1.1 \times 10^{10}\ M_\odot\ h^{-13/5}(\epsilon_{\rm H}/0.1)^{-1}$, which seems unreasonably large.

6. Some more exotic mechanism may be involved, such as heating of the IGM by superconducting cosmic strings (Ostriker and Thompson 1987), or radiation from decaying particles (e.g., Silk and Stebbins 1983).

We thank B. Wang, F. Thielemann, and D. Wilkinson for useful discussions, and C. Hogan and B. Ryden for comments on a draft of this *Letter*. This research was supported by NASA grant NAGW-931.

REFERENCES

Alcock, C., and Paczyński, B. 1978, *Ap. J.*, **223**, 244.
Applegate, J. H., Hogan, C. J., and Scherrer, R. J. 1987, *Phys. Rev. D*, **35**, 1151.
Bookbinder, J., Cowie, L. L., Krolik, J. H., Ostriker, J. P., and Rees, M. J. 1980, *Ap. J.*, **237**, 647.
Carr, B. J., Bond, J. R., and Arnett, W. D. 1984, *Ap. J.*, **277**, 445.
Dimopoulos, S., Esmailzadeh, R., Hall, L. J., and Starkman, G. D. 1988, *Phys. Rev. Letters*, **60**, 7.
Draine, B. T., and Lee, H. M. 1984, *Ap. J.*, **285**, 89.
Efstathiou, G., and Silk, J. 1983, *Fund. Cosmic Phys.*, **9**, 1.
Hayakawa, S., Matsumoto, T., Matsuo, H., Murakami, H., Sato, S., Lange, A. E., and Richards, P. L. 1987, *Pub. Astr. Soc. Japan*, **39**, 941.
Iben, I. 1985, *Quart. J.R.A.S.*, **26**, 1.
Kawano, L., Schramm, D., and Steigman, G. 1988, *Ap. J.*, **327**, 750.
Matsumoto, T., Hayakawa, S., Matsuo, H., Murakami, H., Sato, S., Lange, A. E., and Richards, P. L. 1988, *Ap. J.*, **329**, 567.
Nomoto, K., and Hashimoto, M. 1987, preprint.
Ostriker, J.P., and Thompson, C. 1987, *Ap. J. Letters*, **323**, L97.
Rowan-Robinson, M. 1986, *M.N.R.A.S.*, **219**, 737.
Rybicki, G. B., and Lightman, A. P. 1979, *Radiative Processes in Astrophysics* New York: Wiley).
Scalo, J. M. 1986, *Fund. Cosmic Phys.*, **11**, 1.
Silk, J., and Stebbins, A. 1983, *Ap. J.*, **269**, 1.
Tinsley, B. M. 1980, *Fund. Cosmic Phys.*, **5**, 287.
Woosley, S. E., and Weaver, T. A. 1986, *Ann. Rev. Astr. Ap.*, **24**, 205.
Wright, E. L. 1982, *Ap. J.*, **255**, 401.
Yoshioka, S., and Ikeuchi, S. 1987, *Ap. J. (Letters)*, **323**, L7.

George B. Field and Cedric G. Lacey: Center for Astrophysics, MS 51, 60 Garden Street, Cambridge, MA 02138