Magazine of Concrete Research, 2005, **57**, No. 1, February, 9–20

Moment redistribution effects in beams

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Moment redistribution in beams has traditionally been considered as an ultimate limit state (ULS) phenomenon closely associated with considerations of reinforcement ductility. This paper demonstrates that a significant proportion of this redistribution will almost always occur at the serviceability limit state (SLS) because of the mismatch between the flexural stiffnesses assumed when calculating moments for the ULS and those actually occurring at the SLS due to variations in the reinforcement layout along the member and the influence of cracking. This is demonstrated analytically in the paper and comments are made concerning the current recommendations in BS 8110 concerning member stiffness. Tests on 33 two-span beams are then presented, parameters investigated being values of redistribution, beam depth, reinforcement arrangements, concrete strength and the effect of brittle reinforcement. The results confirm that there is significant redistribution at the SLS and that there is scope for increasing the permissible limits for redistribution beyond those currently prescribed in design codes. This is supported by further modelling although it is shown that considerations of crack width may become a limiting factor.

Introduction

Normal practice is to use a linear elastic analysis for calculating the bending moment and shear force distributions in a reinforced concrete structure. This has the virtue of simplicity as well as permitting results from a series of analyses to be combined using the principle of superposition. It is endorsed by major design codes such as BS 8110,[1] EC 2[2] and ACI 318.[3]

The assumption of linear elastic behaviour is reasonable at low levels of loading but it becomes increasingly invalid at higher loads due to cracking and the development of plastic deformations. Once an element cracks the behaviour becomes non-linear but it is still reasonable to assume that the tension reinforcement and the concrete in compression both behave elastically up to yield of the reinforcement.

Design codes permit elastic analysis to be used at the ultimate limit state (ULS) but acknowledge this non-linear behaviour by allowing a limited amount of moment redistribution from one part of the structure to another. The permissible moment redistribution is linked to the ductility of the reinforcement at the ULS.[4]

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(MCR 1208) Paper received 1 December 2003; last revised 2 March 2004; accepted 5 May 2004

Moment redistribution is useful for practical design as it allows some flexibility in the arrangement of reinforcement. It can be used to transfer moment away from congested areas (e.g. beam–column connections) into less congested areas (e.g. mid-spans of beams) or help to allow standard reinforcement layouts where small differences occur in the bending moment distributions for a series of beams, thus avoiding the need to detail each beam separately. In addition, useful economies can be achieved when moment redistribution is applied to different load combinations, resulting in a smaller bending moment envelope which still satisfies equilibrium.

Implicit in the current use of moment redistribution is the assumption that sections possess sufficient ductility for the requisite plastic deformations to occur. Design codes achieve this by specifying rules which ensure that the tension steel must have yielded, explicitly in the case of ACI 318 (which specifies a minimum reinforcement strain of 7500 microstrain) and implicitly in the case of BS 8110 and EC 2 (which link percentage redistribution to neutral axis depth). This leads to the question of whether an upper strain limit should be specified in order to avoid rupturing the reinforcement since, with small neutral axis depths, very high reinforcement strains can be expected. BS 8110[1] imposes a minimum neutral axis depth of $0{\cdot}11d$ (where d is the effective depth to the tension steel) although this was introduced for the practical reason that the top surfaces of beams and slabs are often quite rough and, as a consequence, it was deemed sensible to restrict the lever arm used in design calculations. However, BS 8110's limit has the effect of restricting reinforcement strains to a maximum of 28 000 microstrain, when making the usual assumption of linear strain distribution across the section. In reality, this value is largely meaningless since gross yield of the reinforcement will have occurred by the time this neutral axis depth has been reached, leading to strains greatly in excess of this nominal value. Consequently, designers have effectively worked on the assumption that the reinforcement will be able to develop whatever level of strain is actually required by a specified neutral axis depth and that failure of a section would always be initiated by crushing of the concrete in compression.

The above view was challenged in 1987 by the work of Eligehausen and Langer[5] who investigated the consequences of a finite limit to reinforcement ductility on the failure mode of a section. They demonstrated that reinforcement strain at the ULS could be the controlling parameter in the more lightly reinforced types of members, such as slabs. This prompted considerable discussion concerning the ductility requirements for steel reinforcement. It also raised the question of what upper limit to moment redistribution should be permitted in design codes and whether more stringent limitations should be imposed on the level of reinforcement strains developed at the ULS.

This paper presents the results of an investigation which aimed to explore the nature of moment redistribution as load is increased on a structure and thus provide some design guidance on the issues outlined above. The first stage in this process is to examine some of the assumptions made when using the code method of elastic analysis to determine bending moment distributions in a reinforced concrete structure.

Flexural stiffness considerations

Moment redistribution involves adjustment of the bending moment distribution obtained from a linear elastic analysis with the difference between the redistributed and initial bending moment diagrams being an indicator of the amount of redistribution which has occurred. The *percentage of moment redistribution* at a section along a beam is calculated as follows:-

$$\% \text{ redistribution} = \frac{\left(\begin{array}{l}\text{moment after redistribution}-\\ \text{moment before redistribution}\end{array}\right)}{\text{moment before redistribution}} \times 100 \qquad (1)$$

The initial elastic bending moment diagram thus forms the baseline for the redistribution calculation and any assumptions or approximations made in its determination will directly affect the level of redistribution calculated using the above expression.

An elastic analysis is controlled by assumptions concerning the value of flexural stiffness (EI) along the member. Recommendations in design codes vary from, at one extreme, EC 2[2] which is non-specific, to BS 8110[1] which gives three options for calculating the EI value at any particular section (clause 2·5·2). These are the *concrete section* (the entire concrete cross section, ignoring the reinforcement), the *gross section* (the entire concrete cross section with the reinforcement included on the basis of modular ratio) and the *transformed section* (the compression area of the concrete cross section, ignoring the concrete in tension, combined with the reinforcement on the basis of modular ratio). A further option would be to take the cracked section properties including tension stiffening effects of the concrete. Since the reinforcement is not known until the end of the design process, the most frequently used, by far, in practical design is the *concrete section*. This has the practical virtue of simplicity (for example, a rectangular continuous beam will have constant flexural stiffness along its entire length) but it also has implications for moment redistribution which, perhaps, may not be fully appreciated.

Consider the propped cantilever shown in Fig. 1(a), which has a central point load, and compare the effect of altering the EI value along its length. First, assume that the EI for sagging and hogging moments is the same throughout the member (BS 8110's *concrete section* approach). Then analyse for a range of situations

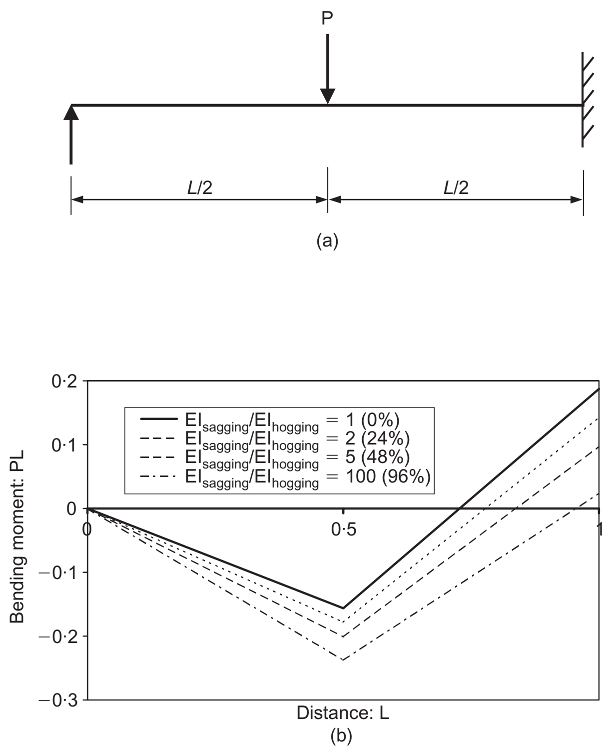


Fig 1. Influence of flexural stiffness on moment redistribution. (a) propped cantilever; (b) bending moment distributions

where the EI for sagging moments differs from that for hogging moments. To illustrate the point, ratios of $EI_{sagging}/EI_{hogging}$ of 1, 2, 5 and 100 have been selected and the resulting bending moment distributions are plotted in Fig. 1(b). The bending moment distribution for $EI_{sagging}/EI_{hogging} = 1$ corresponds, of course, to the situation of constant EI all along the beam and thus acts as a reference line for the other distributions.

Increasing the sagging stiffness (applicable in the span) relative to the hogging stiffness (applicable at the support) has the expected effect of shifting moments away from the support and into the span. In the extreme case when a stiffness ratio of 100 is used the behaviour becomes very close to that for a simply supported beam. Thus, relative to the bending moment distribution obtained using the *concrete section* approach, moment redistribution has occurred in all the other bending moment distributions *even though all sections along the member are always behaving in a linearly elastic fashion*. The percentages of redistribution obtained at the support, when calculated relative to the constant stiffness distribution, are also shown in Fig. 1(b) in order to illustrate this point. Fig. 1(b) emphasises that it is the *relative* values of EI that control the bending moment distribution, not the absolute values at any particular section. *Any* change in the relative EI values will cause moment redistribution to occur.

The above has consequences for understanding the behaviour of reinforced concrete beams. If, as is usual, the *concrete section* is used for the analysis, then the bending moment distribution is that for EI constant all along the beam and this is the situation for which the reinforcement is normally designed at the ULS. It is also the bending moment distribution which will actually be achieved at ultimate. However, in the vast majority of beams, the resulting reinforcement layout will vary along the span. This means that *actual* EI values based on either the *gross* or *transformed section* will change along the span giving *relative* EI values which are no longer unity. Thus the bending moment distributions actually developed along the beam, even at the serviceability limit state (SLS), will differ from those obtained from analyses that use the *concrete section* approach. Consequently, moment redistribution will occur even though none was anticipated in the calculations. This redistribution will be very small prior to cracking but after cracking the ratio of $EI_{span} : EI_{support}$ will normally increase with a corresponding increase in the level of redistribution, even though the reinforcement behaves elastically. The magnitude of this redistribution can be quite considerable, as will be demonstrated later. A further complication occurs once the section cracks as tension stiffening then causes the EI to vary with the applied moment.

The redistribution which occurs as a result of this mismatch between the actual and assumed EI values will be termed *elastic redistribution* in this paper. It should be noted that this redistribution is occurring under SLS conditions and that the reinforcement is behaving elastically. After the reinforcement yields further redistribution will occur owing to further changes in the relative values of EI. This additional, post-yield, redistribution will be termed *plastic redistribution* in this paper. Plastic redistribution is the redistribution with which design engineers have traditionally been familiar and which they implicitly make use of during traditional moment redistribution calculations. Total redistribution at the ULS will actually be the sum of the elastic and plastic components. It will not be all plastic as is commonly assumed at present. Elastic redistribution can nearly always be expected even though none may have been anticipated in the design calculations. This is likely to be inconsistent with the amount of redistribution for which the beam has actually been designed and thus the plastic redistribution that actually occurs will be different from the design value. Consequently, a beam designed for *no* moment redistribution at the ULS will actually have to undergo plastic redistribution in order to offset the elastic redistribution caused by the supports being more heavily reinforced than the spans and the effects of cracking.

An apparently attractive alternative is to use for analysis a distribution of EI which is consistent with the actual stiffness distribution of the reinforcement (BS 8110's *transformed section* option). To be successful this requires that the reinforcement layout be known prior to undertaking the analysis so that 'correct' EI values can be determined and thus ensure that the calculated bending moment diagram is consistent with the beam's own moments of resistance. This is very difficult to achieve so an alternative is to try an iterative approach. For instance, the beam could first be analysed assuming constant EI (the *concrete section* approach). The reinforcement is then calculated but this will change the actual distribution of EI. The beam is then re-analysed but the bending moment distribution will now have changed because of the changes in EI and the reinforcement layout must now be adjusted to accommodate the new bending moment distribution. This again changes the EI distribution prompting another analysis and another adjustment of the reinforcement layout. This causes yet another change in the distribution of EI leading to yet another analysis and so on. The procedure can become unstable and is thus unsuitable for practical design purposes. Similar comments can be made concerning use of the gross section.

From the above it is suggested that, in spite of the recommendations of clause 2·5·2 in BS 8110, both the *gross section* and *transformed section* approaches are unsuitable for design calculation of bending moment distributions at the ULS. Furthermore, it is suggested that the only practical approach is to use EI values for the *concrete section* when calculating these moments although it is important to recognise the simplifications and approximations which result from this. The above is, of course, not true for the analysis of existing sections where the reinforcement layout is known.

A programme of laboratory tests was undertaken to investigate the above points in greater detail. The effects of the following parameters were investigated:

(*a*) depth of section
(*b*) different values of design moment redistribution
(*c*) different arrangements of reinforcement (e.g. large bars versus small bars)
(*d*) different concrete strengths (i.e. use of high-strength concrete for some specimens)
(*e*) the effect of brittle reinforcement [i.e. glass fibre reinforced polymer (GFRP) bars].

Further details are given in the next section.

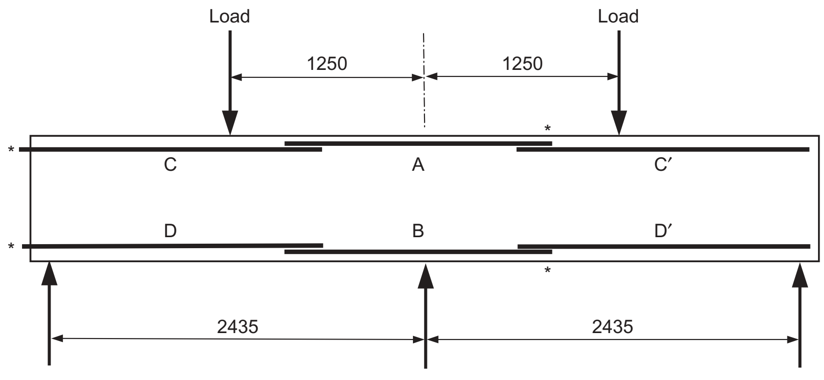
Specimen details and test procedure

A total of 33 two-span beams, 5·2 m long and 300 mm wide, were tested to investigate moment redistribution effects between the centre support and adjacent spans. Depths of 400 mm (series A), 250 mm (series B) and 150 mm (series C, D and E) were used. Each span was 2435 mm long and loaded approximately at its mid-point, as indicated in Fig. 2. The main set of 23 specimens, shown in Table 1, was designed for 30% redistribution from the centre support into the adjacent spans and will be described first. The remaining ten specimens, which investigated other aspects of moment redistribution behaviour, are described later.

A consistent coding system was used to reference the specimens. A typical example is B2T12D where B indicates *high yield bar reinforcement*, 2T12 is the tension reinforcement provided over the centre support and D shows the specimen belongs to series D. An H suffix would indicate that the specimen was made with high-strength concrete e.g. B2T12DH. Other suffices (typically X) are used when specimens were repeated.

Each series investigated a different percentage of tension (top) steel at the centre support, values ranging from about 0·3% (series E) up to around 1·7% (series C). However, the several specimens within each series (with the exception of series A which comprised one specimen only) achieved this percentage by using different combinations of bar diameters such as 2T20 or 3T16 or 5T12. This enabled bar diameter effects to be examined since the combinations gave approximately the same total area of tension steel. All other reinforcement (bottom steel at the centre support and top and bottom steel in the spans) was the same for all beams within each series. Some tests were repeated and nominally identical specimens are block shaded in Table 1.

Seventeen specimens were cast using normal strength concrete and six with high-strength concrete. The normal strength concrete had a maximum aggregate size of 10 mm, an aggregate : cement ratio of 5·5 and an overall water : cement ratio of 0·6. The average compressive cube strength was 51·8 MPa (standard deviation 7·9 MPa) and the average indirect cylinder strength was 3·0 MPa (standard deviation 0·2 MPa). The high-strength concrete used 10 mm aggregate and microsilica (in slurry form) in addition to Ordinary Portland Cement (CEM 1) and sand. The overall water : cement ratio was 0·25 with adequate workability being achieved by the addition of admixtures. It was commercially batched and delivered by truck. The average compres-



All dimensions in mm.
All beams 5200 mm long overall × 300 mm wide. Depths vary – see Tables 1 and 2.
For bar diameters see Tables 1 and 2.
Bars at locations A and B all 1500 mm long. Laps all 300 mm.
Bars at A, C, C′ and B, D, D′ all in the same horizontal plane. Bars at C, C′, D, D′ cranked on plan at lap positions to achieve this.
30 mm cover to centre of longitudinal bars.
Stirrups: Series A and B: T10s at 200 centres, all other specimens R6s at 100 mm centres.
* indicates normal exit point for wires in gauged bars – see text and Tables 1 and 2.

Fig 2. Diagrammatic layout of test specimens

Table 1. Specimen details: 30% moment redistribution

Series	Specimen code	Overall depth: mm	Support reinforcement		Span reinforcement	
			Top (A)	Bottom (B)	Top (C, C′)	Bottom (D, D′)
A	B3T16A**	400	3T16 (0·53%)	3T12	3T12	3T20
B	B2T20B**	250	2T20 (0·93%)	3T12	3T12	3T20
	B3T16B**		3T16 (0·89%)			
	B3T16BL**		3T16 (0·89%)			
	B3T16BLX		3T16 (0·91%)			
	B5T12B**		5T12 (0·83%)			
	B2T20BH*		***2T20 (0·93%)***			
	B2T20BHX		***2T20 (0·95%)***			
	B5T12BH*		***5T12 (0·83%)***			
	B5T12BHX		***5T12 (0·86%)***			
C	B2T20C**	150	2T20 (1·70%)	3T12	3T12	3T20
	B3T16C**		3T16 (1·63%)			
	B5T12C**		5T12 (1·53%)			
D	B2T12D**	150	2T12 (0·58%)	3T10	3T10	3T12
	B2T12DX		2T12 (0·63%)			
	B2T12DXX		2T12 (0·63%)			
	B3T10D**		3T10 (0·63%)			
	B5T8D**		5T8 (0·68%)			
	B2T12DH*		***2T12 (0·58%)***			
	B2T12DHX		***2T12 (0·63%)***			
E	B2T8E**	150	2T8 (0·26%)	2T8	2T8	3T8
	B2T8EX		2T8 (0·28%)			
	B4T6E**		4T6 (0·30%)			

This table should be read in conjunction with Fig. 2. ** indicates four strain-gauged bars. * indicates two strain gauged bars. Details in italics are for high-strength specimens. Shaded groups of specimens are nominally identical.

sive cube strength was 120·4 MPa (standard deviation 10·0 MPa) and the average indirect cylinder strength was 4·1 MPa (standard deviation 0·5 MPa). High-strength specimens have the H suffix, as mentioned previously, and are also shown in *italics* in Table 1.

As indicated in Table 1, 15 of the 23 specimens contained strain-gauged reinforcing bars. Specimens with normal strength concrete contained four gauged bars, one each at the locations A, B, C, D shown in Fig. 1 and referred to as rods 1–4, respectively. Specimens with high-strength concrete contained two gauged bars, one each at locations A and D and referred to as rods 1 and 4. Gauged bars had electric resistance strain gauges installed in a central longitudinal duct having a cross-section of 4 mm × 4 mm in bars of 12 mm dia. and above but which was reduced to 3·2 mm × 3·2 mm in 10 mm dia. bars and to 2·5 mm × 2·5 mm in 6 mm and 8 mm dia. bars. Typically, 51 strain gauges were installed in each bar of 10 mm dia. or above but this number was reduced to 30 gauges in the 6 mm and 8 mm dia. bars. Gauge spacing varied from 12·5 mm in zones where rapid changes in strain were expected to 50 mm in less critical areas. Gauges generally had a strain limit of 3% but high elongation gauges having a strain limit of 10–15% were used in areas where high strain levels were anticipated. The wiring normally exited from one end of each bar (see Fig. 2) except for the 8 mm and 10 mm dia. bars where both ends were used. Gauged bars formed part of the main reinforcement in each beam and, wherever possible, the centre bar in a group of three was gauged to maintain symmetry of stiffness across the beam width. Obviously this was not possible in locations where there were only two bars but the effects of the resulting asymmetry were believed to be small. Further details of the gauging technique, which ensures that the bond characteristic between the reinforcement and surrounding concrete is undisturbed, have been published elsewhere.[6]

A further ten specimens, shown in Table 2, were tested to investigate other aspects of moment redistribution. Three specimens (prefixed W) used ribbed high-yield wires instead of bar reinforcement but were otherwise a repeat of the strain-gauged specimens in series D. Three specimens with bar reinforcement were reinforced for zero redistribution, one for 55% redistribution and two specimens examined the effects of

Table 2. Details of additional specimens

Specimen code	% Redistribution	Overall depth: mm	Support reinforcement		Span reinforcement	
			Top (A)	Bottom (B)	Top (C, C′)	Bottom (D, D′)
W2T12D**	30	150	2T12 (0·58%)	3T10	3T10	3T12
W3T10D**	30		3T10 (0·63%)			
W5T8D**	30		5T8 (0·68%)			
B3T20B	0	250	3T20 (1·43%)	3T12	3T12	2T20+1T16
B8T12B	0	250	8T12 (1·37%)	3T12	3T12	2T20+1T16
B3T12D	0	150	3T12 (0·94%)	3T10	3T10	2T12+1T8
B2T10D	55	150	2T10 (0·44%)	3T10	3T10	2T12+2T10
B4T20B	−30	250	4T20 (1·90%)	3T12	3T12	2T16+2T12
B5T20B	−60	250	5T20 (2·38%)	3T12	3T12	2T16+1T12
F5F13B	30|	250	5 No. 13 mm diameter GFRP bars (1·01%)	3T12	3T12	2 No. 25 mm diameter GFRP bars

This table should be read in conjunction with Fig. 2. ** indicates four strain-gauged bars.

redistributing moments from the spans back to the centre support by 30 and 60%, respectively (shown as negative values in Table 2). The final specimen (F5F13B) used GFRP bars for the main tension reinforcement with five 13 mm dia. bars over the centre support and two 25 mm dia. bars in each span (the only two diameters readily obtainable by the authors at the time). This gave the closest possible match to specimen B5T12B with the available materials.

Typical stress–strain relationships for all the reinforcement types for strains up to 14 000 microstrain are shown in Fig. 3.

The test procedure was to load the beams incrementally until failure occurred. Applied loads, support reactions and reinforcement strain gauge readings (when appropriate) were recorded at every load stage. Surface strains were measured on one side face of all specimens, at selected load stages, using a Demec gauge and a grillage of Demec points.

Moment redistribution behaviour

Results from the test programme are summarised in Table 3, for specimens designed for 30% redistribution, and Table 4 for the additional specimens. All quoted values of redistribution are calculated relative to the uniform EI case (i.e. an analysis based on the stiffness of the *concrete section*). The specimens in Table 3 will be discussed first.

Specimens designed for 30% moment redistribution

The test results confirmed that a number of discrete stages could be identified in moment redistribution

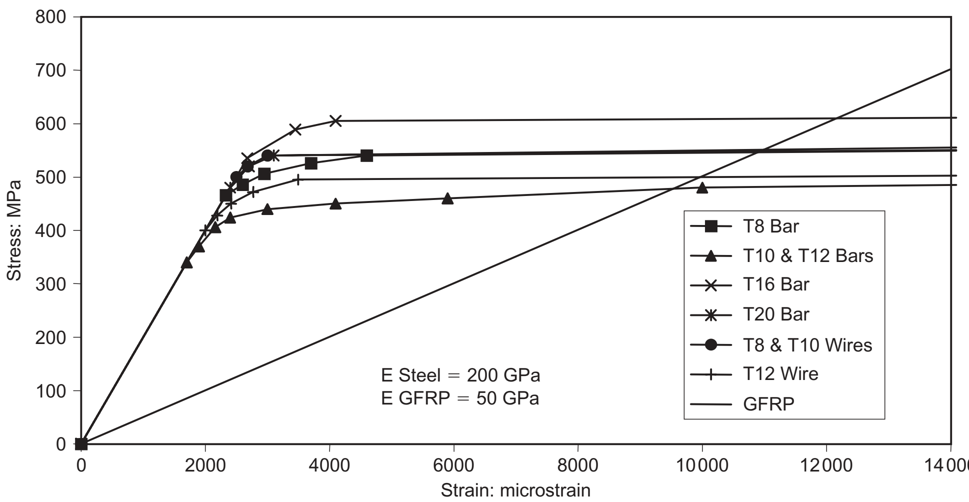


Fig. 3. Reinforcement stress–strain relationships

Table 3. Results for specimens designed for 30% moment redistribution

Series	Specimen code	Flexural failure	Measured % redistribution		Max. rod 1 strain: microstrain
			Elastic	Total	
A	B3T16A		27–32	–	Ungauged
B	B2T20B		25	–	2250
	B3T16B		24–30	37	18 300
	B3T16BL		10–20	21	4100
	B3T16BLX		20	–	Ungauged
	B5T12B		10–18	32	4000
	B2T20BH	***Yes***	***25–33***	***38***	***27 000***
	B2T20BHX	***Yes***	***27***	***33***	***Ungauged***
	B5T12BH		***15–20***	***24***	***3700***
	B5T12BHX		***12–28***	***34***	***Ungauged***
C	B2T20C		12–18	23	10 800
	B3T16C		20	32	16 400
	B5T12C		5–10	27	12 700
D	B2T12D	Yes	15	27	31 000
	B2T12DX	Yes	34	38	Ungauged
	B2T12DXX	Yes	25	34	Ungauged
	B3T10D	Yes	16	26	37 500
	B5T8D	Yes	10–15	22	28 200
	B2T12DH	***Yes***	***30–25***	***40***	***42 000***
	B2T12DHX	***Yes***	***28–30***	***45***	***Ungauged***
E	B2T8E	Yes	90–38	40	35 000
	B2T8EX	Yes	85–55	55	Ungauged
	B4T6E	Yes	20–28	39	53 000

Details in italics are for high-strength specimens. Shaded groups of specimens are nominally identical.

Table 4. Results for additional specimens

Specimen code	% Designed redistribution	Flexural failure	Measured % redistribution		Max. rod 1 strain: microstrain
			Elastic	Total	
W2T12D	30	Yes	22–15	25	32 600
W3T10D	30	Yes	10–16	32	33 000
W5T8D	30	Yes	14	27	20 000
B3T20B	0		10	13	Ungauged
B8T12B	0		3	20	Ungauged
B3T12D	0	Yes	18	0	Ungauged
B2T10D	55	Yes	46–35	50	Ungauged
B4T20B	−30		−5	0	Ungauged
B5T20B	−260		−18	15	Ungauged
F5F13B	30		5–25	25	Ungauged

behaviour, as illustrated in Fig. 4 for the strain-gauged specimen B2T12DH. This figure shows the relationship between percentage redistribution at the centre support and experimental bending moment at the centre support over the full load history of this specimen.

From the start of the test until point A (see Fig. 4) the specimen was uncracked, the small values of redistribution being due to the influence of the different percentages of tension steel along the beam on the values for the uncracked EI. The load increment after

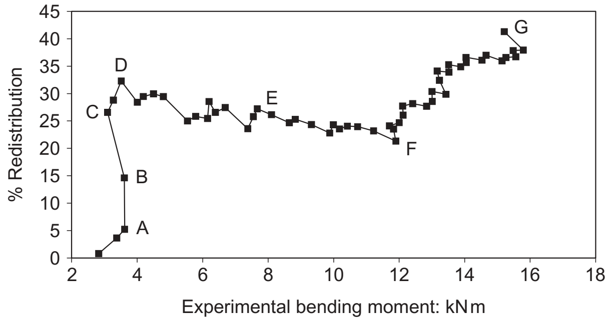


Fig. 4. Development of moment redistribution in specimen B2T12DH

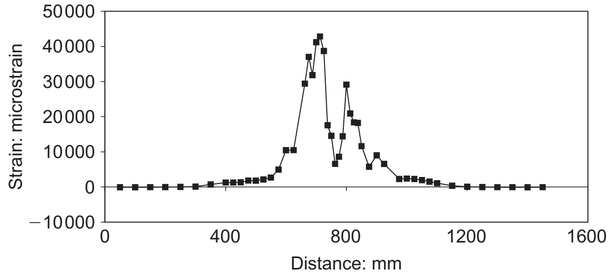


Fig. 6. Specimen B2T12DH: ultimate strain distribution along rod 1

point A caused a crack to form at the centre support and the strains in the top tension bars at this location then increased with succeeding load stages until point D was reached. The measured strain distributions in rod 1 over the centre support (location A in Fig. 2) for these load stages are shown in Fig. 5, with curves A to D in Fig. 5 corresponding to points A to D in Fig. 4. Fig. 5 clearly shows how the crack developed between load stages A and D, causing a significant reduction in the support EI relative to that in the span and hence the step change in redistribution behaviour observable in Fig. 4.

The span cracked at point D which reduced the span EI of the beam. Between D and E a stable crack pattern developed, with all reinforcement strains elastic, and this zone corresponded to the SLS. Point E marks the onset of yield (i.e. initial deviance from linear behaviour) in the reinforcement at the centre support and at point F a similar situation occurred in the span, by which time the strain in rod 1 had reached around 5000 microstrain. F to G was a region of rapid strain development in both locations leading to plastic hinges being formed. The resulting flexural failure was characterised by an inability to maintain applied loads due to the excessive deflections which occurred as a result of the mechanism action. Strains had reached around 42 000 microstrain and 18 000 microstrain at the support and span locations respectively (rods 1 and 4) by the end of the test. The final strain distribution in rod 1 at the centre support is shown in Fig. 6. This indicates the very localised nature of the gross yield associated with plastic hinge development.

Specimen B2T12DH achieved around 40% redistribution by the end of the test. However, it is important to note that some 25% redistribution was achieved when the reinforcement was still behaving elastically; that is, of the 40% total, 25% was elastic redistribution and only 15% was plastic redistribution.

A similar pattern of behaviour was observed with the five specimens of series D designed for 30% redistribution, which are all plotted in Fig. 7. These specimens all achieved the designed 30% redistribution at the ULS (albeit only just with B2T12D but comfortably so with the others) with the elastic contribution ranging from around 15% up to about 35%, this spread of values being caused by variations in the nature of the crack patterns in the individual specimens. These specimens all exhibited flexural failures.

Ductility limitations were not an issue at any stage during the test programme. None of the reinforcing bars, which were all of UK origin, failed in spite of the very high strain levels reached in some instances.

High levels of redistribution were also achieved even when the reinforcement was still behaving elastically at failure. B2T20B failed in shear with a peak strain in rod 1 of only 2250 microstrain yet it achieved 25% redistribution at failure, all of which was elastic. The strain distributions for rod 1 in this specimen, shown in Fig. 8, indicate the flatter spread of strains resulting from the absence of gross yield. As Beeby has pointed out,[4] rotations associated with strain distributions such

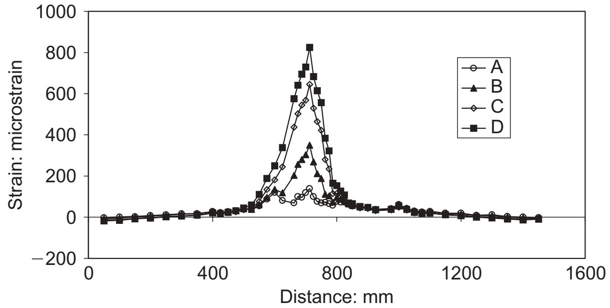


Fig. 5. Specimen B2T12DH: early strain distributions along rod 1

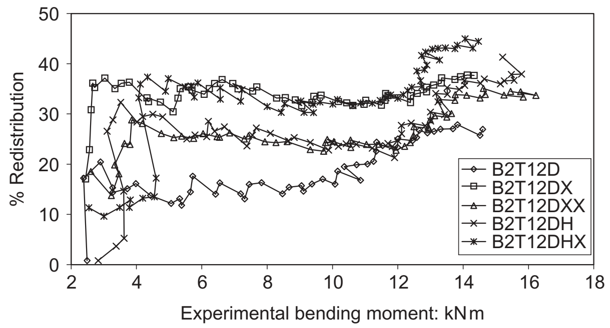


Fig. 7. Moment redistributions for beams of type B2T12D

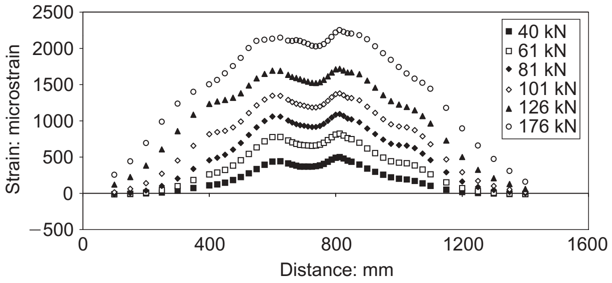


Fig. 8. Specimen B2T20B: strain distributions for rod 1

as those shown in Fig. 8 can be comparable with those due to the much more localised effects of gross yield, as shown in Fig. 6.

Figure 9 shows redistribution plots for a range of B series specimens including B2T20B and the FRP specimen F5F13B (Table 4). The two high-strength specimens B2T20BH and B2T20BHX had flexural failures but the others all failed in shear. Specimens B2T20B, B2T20BH and B2T20BHX (which all had two T20 bars as the main support tension steel) behaved very similarly over the elastic range, achieving 25–33% redistribution. These tests, supported by others in the test programme, indicated that concrete strength was not a significant parameter as far as elastic behaviour was concerned (although specimens made with high-strength concrete achieved a higher failure moment due to their larger moment of resistance).

Bar diameter may have had an influence on behaviour in the elastic range, as shown by B5T12B in Fig. 9, due to variations in crack pattern resulting from using several small diameter bars rather than a few bars with larger diameter. However, this was not conclusive and there were no discernible differences in redistribution percentages achieved at the ULS between specimens having similar reinforcement percentages but different bar combinations (e.g. 2T20s, 3T16s or 5T12s). An interesting result is that for the FRP specimen F5F13B. This first cracked in the span, leading to high early levels of redistribution (see Fig. 9) but then, as further cracks developed along the specimen, it closely followed the behaviour of B5T12B. F5F13B had achieved 25% redistribution when it failed even though its main tension reinforcement had stress–strain characteristics which were entirely linearly elastic.

Specimens of series E, which had a low percentage of tension steel over the centre support (typically 0·28%), showed very high levels of redistribution very early in the tests. This is illustrated in Fig. 10 for specimen B2T8E. When the support cracked (point A in Fig. 10), the resulting 90% redistribution meant that the beam effectively behaved as two simply supported spans until cracks in the span (initiated at point B) caused moment to be transferred back to the centre support. Both support and span tension steel were elastic until point C was reached.

To summarise, specimens which failed in flexure virtually all achieved the designed 30% total redistribution by the end of the test. Levels of elastic redistribution varied from 10% up to around 30% with the 10–15% range being typical even in those specimens that failed in shear.

Additional specimens

The results for the additional specimens shown in Table 4 will now be discussed (except for F5F13B already dealt with above).

Specimens designed for zero percentage of redistribution (B3T20B, B8T12B, B3T12D) had slightly more reinforcement over the centre support than in the span (see Table 2). On first loading, these specimens cracked at the centre support (the location of the largest bending moment) which caused elastic redistribution into the spans of up to 16%, as illustrated in Fig. 11 for specimen B3T12D. With B3T20B the reinforcement remained elastic until a shear failure occurred and there was no reduction in the 10% elastic redistribution achieved in this specimen. Plastic redistribution was needed to offset the elastic redistribution if zero redistribution at the ULS was to be achieved. This was the case with B3T12D (Fig. 11), which failed in flexure, but not so with B3T20B and B8T12B which both failed in shear. This supports the point made earlier that plastic redistribution must occur if specimens designed

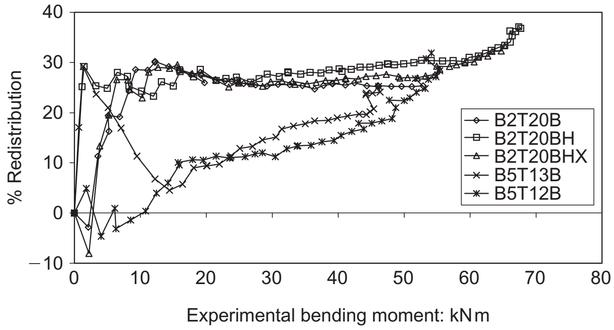


Fig. 9. Moment redistributions for beams of type B2T20B

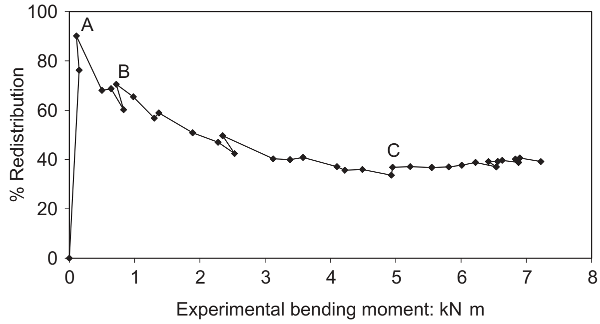


Fig. 10. Moment redistribution for specimenB2T8E

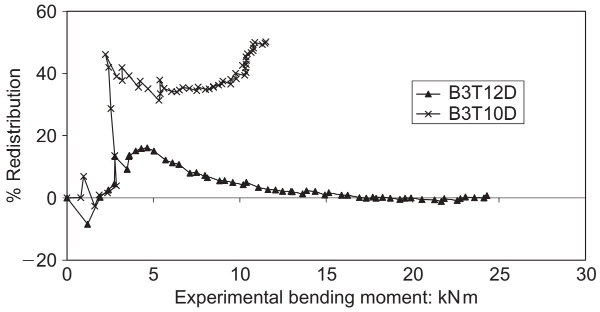


Fig. 11. Beams with zero and 55% redistribution

for zero redistribution are actually to achieve this situation in practice.

Results for the single specimen designed for 55% redistribution are also shown in Fig. 11. This specimen actually achieved 50% redistribution at the ULS plus the quite remarkable value of 35% at the SLS.

Specimens B4T20B and B5T20B were designed to redistribute moments from the span to the support, by 30 and 60%, respectively. The results for these two specimens are shown in Fig. 12 with negative values indicating that support moments were being increased.

Early cracking over the support meant that moments were initially being redistributed into the adjacent spans. As span cracking developed, moments were transferred back to the supports, as intended, but neither specimen achieved anywhere near its designed level of redistribution at the ULS. Both specimens failed in shear, which probably accounts both for the rapid reduction in redistribution at the end of the tests and for the tendency to transfer moment back into the spans, but neither specimen ever seemed likely to achieve its design value, which is a potential cause for concern. However, the results from these two tests are certainly not conclusive and further work in this area would be helpful.

The specimens reinforced with ribbed wire (W2T12D, W3T10D, W5T8D) behaved very similarly to the equivalent bar specimens of series D.

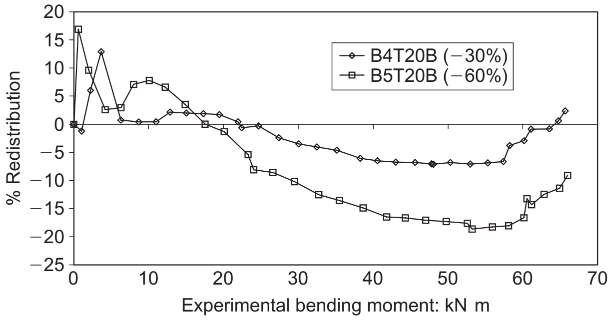


Fig. 12. Beams designed for −30 and −60% redistribution

Design considerations

The test results confirmed that total moment redistribution has both elastic and plastic components with the former making a significant contribution to the whole. The results also suggested that total redistribution was not limited by considerations of (UK) reinforcement ductility. Consequently, there is a case for increasing the current 30% design limit on redistribution prescribed in BS 8110[1] and EC 2[2] and the 20% limit in ACI 318[3] by recognising the contribution made by elastic redistribution which, at present, is a matter not addressed by the design codes.

Elastic redistribution occurs at the SLS due to the mismatch between the assumption, for purposes of analysis, of a constant EI (flexural stiffness) value along the member and the actual EI values which are a consequence of the designed reinforcement layout. To explore this further, a programme of analysis was undertaken to find how much elastic redistribution might be expected (from supports to adjacent spans – no attempt was made to model the effects of redistribution from spans to supports) for a range of $EI_{span} : EI_{support}$ stiffness ratios. (The actual value of this ratio for a particular beam will be dependent on the chosen reinforcement layout). The following geometry/loading combinations were analysed to cover situations that approximated to the end span and interior span conditions of a continuous beam.

(*a*) Propped cantilever, central point load (simulating the test beams);
(*b*) propped cantilever, uniformly distributed load over whole span;
(*c*) fixed-ended beam, central point load;
(*d*) fixed-ended beam, uniformly distributed load over whole span.

The resulting relationships are shown in Fig. 13. Positive values confirm that the elastic contribution can be quite considerable. Negative values indicate that, at the SLS, redistribution will occur from the *span to the*

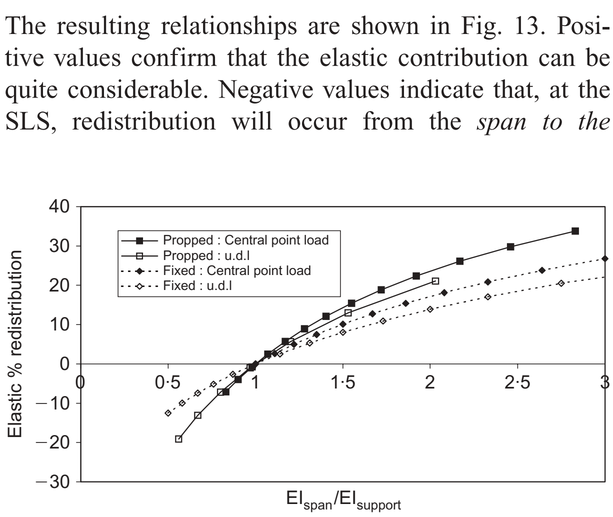


Fig. 13. Relationship between elastic percentage redistribution and flexural stiffness ratio. u.d.l., Uniformly distributed load

supports even though the beam was designed for redistribution at the ULS to be in the opposite direction—a sort of 'reverse redistribution' effect. As discussed earlier, even beams designed for *zero* redistribution will exhibit elastic redistribution from the spans to the supports at the SLS and thus must undergo *plastic* redistribution to achieve the designed 0% at the ULS. This 'reverse redistribution' at the SLS could be considered undesirable since the beam is being forced to behave in different ways at the SLS and the ULS. 'Reverse redistribution' can be avoided, or at least reduced, by the level of designed redistribution which is selected, should this be considered desirable.

The lower and upper ends of each curve in Fig. 13 correspond approximately to 0% (i.e. support(s) fully fixed) and 60% *total* redistribution respectively. Thus the figure indicates that, for a propped cantilever, designing for 60% *total* redistribution will give about 35% *elastic* redistribution for the case of central point load or about 20% *elastic* redistribution in the case of a uniformly distributed load. Rather more comprehensive relationships between elastic redistribution and total redistribution were also developed as part of the modelling exercise and these are shown in Fig. 14. The curve for the propped cantilever/central point load case is consistent with the test results but, overall, the curves should be treated as approximate and further work is needed were they to be used in design charts. Fig. 14 does, however, give a good and realistic overview of the elastic/total redistribution relationships for the four conditions considered and reinforces the case that elastic redistribution is often a significant proportion of the whole. It also reinforces the points made above concerning 'reverse redistribution'. Commonly, of course, support moments, such as those in a continuous beam, are less than those pertaining to the fixed end case. This will lead to greater span moments, an increase in the $EI_{span} : EI_{support}$ ratio and hence more elastic redistribution. Thus the curves in Fig. 14 are likely to be conservative in this situation.

It is important to ensure that, whatever upper limit is adopted, the reinforcement remains elastic under service loads and that crack widths at service loads are kept within acceptable limits. Consequently, a further series of analyses was undertaken to investigate these two issues.

The beam geometry and loading arrangement modelled were the same as those used in the test beams. The procedure was to choose a reinforcement layout for the midspan sections of the beam and then, for a given level of design redistribution, calculate the reinforcement at the central support. The moment–curvature relationships for the span and support locations were then obtained for concretes having compressive cube strengths of 35 and 100 MPa. The applied loads on the beam were then increased from zero through to the ULS and the bending moment distribution and hence percentage redistribution determined at each load stage. Crack widths were also calculated at the central support and at midspan.

Two SLS measures were considered. The first was that the structure must not have reached first yield at service load, since the consequent deformations would be unacceptable. A ratio of service load to yield load of 0·9 was assumed to be acceptable. Second, the BS 8110 crack width limit of 0·3 mm was adopted.

Figure 15 shows the relationship between service load/yield load and % redistribution for beam depths and concrete strengths consistent with those used in the test programme. Test results for beams with gauged reinforcement are also included and the agreement is good. Fig. 15 indicates that over 50% redistribution is possible before yield at the SLS occurs, except for high-strength beams 150 mm deep for which a limit of 45% would seem appropriate. Midspan crack widths at the SLS were less than 0·3 mm in all cases but, over the supports, they rose sharply to unacceptable limits as the yield load was approached for redistribution levels greater than 40%. Overall, for the geometry and loading of the test beams a limit of 40% redistribution would seem acceptable and this could be increased to 45% if crack widths were not an issue.

More work is required to investigate crack width considerations in greater detail before specific design recommendations can be made covering a range of geometries and load combinations. At this stage it is still necessary to consider each situation on its individual merits.

The work described in this paper indicates that there

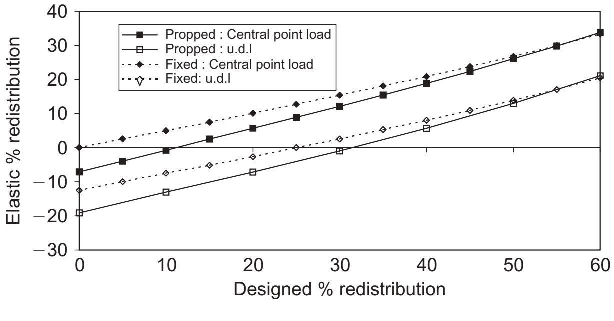


Fig. 14. Relationships between elastic and designed percentage redistribution. u.d.l., Uniformly distributed load

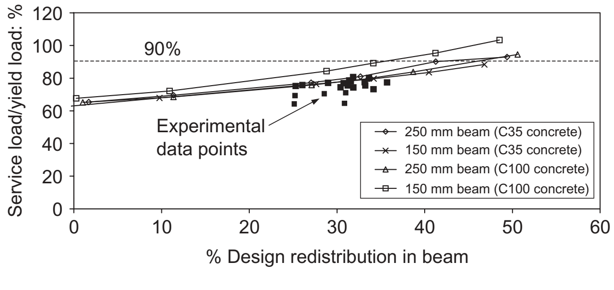


Fig. 15. Ratio of service load to yield load versus percentage design redistribution

is no case for reducing the maximum permitted 30% limit to moment redistribution currently allowed in BS 8110 and EC 2. Furthermore, the limit of 20% given in ACI 318 would appear to be over-conservative. It seems reasonable that the upper limit can be increased beyond 30% by taking into consideration the contribution of elastic redistribution and, with total redistribution being the sum of the elastic and plastic components, it is suggested that the 30% limit (together with the current rules limiting neutral axis depth) now be applied just to the plastic component. The absolute upper limit to total redistribution will depend on beam geometry and loading arrangement but could now be as high as 55%. However, crack width considerations and avoidance of reinforcement yield at the SLS are likely to become limiting criteria. Nevertheless, there will frequently be scope for design engineers safely to exceed the current 30% limit if they so wish.

Conclusions

(*a*) An investigation of moment redistribution effects was undertaken comprising laboratory tests on 33 two-span beams supported by programmes of numerical modelling.

(*b*) When carrying out elastic analysis at the ULS the most sensible practical approach is to use the EI values for the *concrete section*, as defined in BS 8110.

(*c*) Total moment redistribution has two components. These are *elastic redistribution*, which occurs because of the non-uniformity of EI and *plastic redistribution* which occurs after yield of the tension steel. *Elastic redistribution* is caused by a mismatch between the uniform flexural stiffness assumed and the stiffness values which actually occur due to variations in the reinforcement layout along the member and the influence of cracking. The contribution of elastic redistribution is significant even in members which fail in shear.

(*d*) A consequence of elastic redistribution is that beams designed for *zero* redistribution will, in fact, undergo plastic redistribution before the ULS is reached.

(*e*) Reinforcement arrangement (e.g. large bars versus small bars) has little effect on total redistribution. However, the percentage of elastic redistribution may be affected owing to variations in the distributions of cracks.

(*f*) Concrete strength can influence total redistribution at the ULS since the moment of resistance of the section is increased. However, at the SLS, the influence seems minimal.

(*g*) No failure of the reinforcing bars occurred in any of the tests which indicates that there was always sufficient ductility in the UK reinforcement.

(*h*) There was no evidence that the maximum permissible redistribution limits given in BS 8110, EC 2 and ACI 318 should be reduced. The limit given in BS 8110 relating to the neutral axis depth is only related to plastic redistribution and 30% total redistribution should always be possible. However, there is now a strong case for increasing this value although it is not now possible to specify a unique upper limit since the contribution of elastic redistribution is dependent on support conditions and loading arrangement. It should also be noted that, when current limits are exceeded, crack widths and the possibility of yield at the SLS should be checked.

Acknowledgements

Funding for this project was provided by the Engineering and Physical Sciences Research Council with additional support from Arup Research and Development. The strain-gauged beams were tested by Dr A. R. Azizi and the crack width modelling was undertaken by Mr C. Pope of Arup R&D. In addition, the assistance of the following is also gratefully acknowledged: Tarmac Topmix Ltd, Professor A. W. Beeby FREng, the technical staff of the University of Durham and several generations of project students.

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Discussion contributions on this paper should reach the editor by 1 August 2005