The missing dimension: effects of lateral variation on 1-D calculations of fluvial bedload transport

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Abstract

Most calculations of bedload transport in rivers, including those in numerical models of aggradation and degradation, are one-dimensional: all hydraulic and transport-rate calculations are averaged over the channel width. Because bedload transport laws are nonlinear, width-averaged calculations will underestimate the true bedload flux if there is any local spatial variation in either the bed or the flow. This paper analyses the effects on bedload transport capacity of spatial variation in applied (τ) and critical (τ_c) shear stress, separately and in combination. A simple but versatile statistical model is used to represent variability in τ, with allowance for differences between sand- and gravel-bed rivers and for below-bankfull flow. Bedload flux is shown to increase greatly with the variance of τ, especially in gravel-bed rivers. Variability in τ_c through bed patchiness may increase, reduce, or make little difference to bedload flux depending on the correlation between τ and τ_c. Simple width averaging leads to severe underestimation of bedload transport in most conditions; some alternatives are considered. The findings have implications for sediment routing models but further research is needed to explore the issue fully.

KEY WORDS: bedload transport; sediment routing; numerical models; spatial variation; patchiness.

Introduction

Many rivers undergo transient or long-term aggradation or degradation because local bedload transport capacity does not equal supply. Sediment imbalances are sometimes the result of human activity affecting either capacity (e.g. flow regulation or meander rectification) or supply (e.g. deforestation or mine waste disposal), but can also arise naturally for such reasons as climate change, sea-level change, overloading from hillsides during extreme storms, and reduced capacity as slope declines towards base level.

One of the ways to understand the behaviour of rivers in disequilibrium, and to predict the course of transient adjustment in specific circumstances, is to use a numerical sediment routing model (SRM hereafter). Such models compute at each of many time steps first the flow properties, then the transport capacity, at a series of nodes along the river. The channel long profile is then updated using the overall sediment continuity equation. Early SRMs (reviewed by Dawdy and Vanoni, 1986) represented the bed by a single grain size. This may be adequate for sand-bed rivers but is a severe simplification for gravel-bed rivers. In a newer generation of what may be called 'fractional' SRMs, designed for gravel-bed rivers, transport capacity is calculated separately for each size fraction of bed material. This allows simultaneous updating of both bed elevation and bed grain size distribution at each node, the latter using a fractional continuity equation. Much of the impetus for developing fractional SRMs came from geomorphology and sedimentology, especially through attempts to understand the evolution of downstream-fining aggradational sequences in gravel-bed rivers (Parker, 1991; Hoey and Ferguson, 1994) and the rock record (Robinson and Slingerland, 1998), but these models are now being applied to practical engineering and management problems of river response to disturbance (e.g. Talbot and Lapointe, 2002). Limitations in the accuracy of simple and fractional SRMs are therefore of potential significance to geomorphologists, sedimentologists, and engineers.

Almost all SRMs are one-dimensional in the sense that they treat flow and sediment transport on a width-averaged basis. This simplifies the calculations, allows better temporal and spatial resolution with available computing power, and minimises input-data requirements. But it may be a weakness if lateral variability in hydraulic conditions and bedload transport rates is important in the real rivers to which 1-D models are applied. Flow strength may vary across a river because parts of the channel are deeper than others, through the retarding effect of bank friction, or because of flow structures inherited from upstream. Bedload transport rates will vary accordingly, and will also be affected by any local sorting of the bed into coarser or finer patches. In applications to straight, narrow canals or flumes (e.g. Cui *et al*., 1996) it may be reasonable to approximate the channel as rectangular and assume that lateral variation in processes is negligibly small, but this may less realistic when modelling natural river channels with bar-pool-riffle morphology (e.g. Hoey and Ferguson, 1994). For example, Wathen *et al*. (1995) and Powell *et al*. (1999) measured substantial lateral variation in bedload flux even in straight reaches of such channels. Failure to allow for bed patchiness or local spatial variability in flow strength may lead to underestimation of bedload fluxes (Paola and Seal, 1995; Paola, 1996; Nicholas, 2000) and this could affect the results obtained by applying simple or fractional SRMs. The influence of lateral variability on a river's transport capacity is also an important consideration in the design of channelization and river-training projects.

This paper investigates how well a 1-D treatment represents bedload transport in non-uniform channels. An idealised mathematical model is developed. Results obtained from it suggest that the total bedload flux with given mean flow and bed conditions is highly dependent on the degree of lateral variation around the mean conditions, and that 1-D computations can give extremely biased estimates of flux. Preliminary consideration is also given to possible ways to reduce this bias without moving completely away from a 1-D approach.

Problem statement

The root of the lateral-variation issue is the averaging of a nonlinear process: bedload transport. There are many different predictive formulae for the transport capacity q [$L^3L^{-1}T^{-1}$] of bed material of a given diameter D under a specified fluid flow. The flow is usually quantified by the bed shear stress, τ. Some formulae use other flow properties, for example shear velocity or specific stream power, but the argument is essentially the same whichever is preferred and for definiteness I use shear stress below. There are many different τ-based bedload formulae, but they all predict that transport rate increases faster than linearly with τ, and is negligibly small when τ is less than some critical, quasi-threshold value τ_c which depends on D.

Even if D, and hence τ_c, is the same everywhere within a channel reach, if τ varies across the channel the true total flux Q [L^3T^{-1}] conveyed by the river has to be obtained by integrating across the width w of the channel:

$$Q = \int_0^w q(\tau) dx \qquad (1)$$

where x is lateral distance. The mean transport rate per unit width is then $q_{av} = Q/w$. In constrast, a 1-D (width-averaged) estimate of the flux makes the assumption that $q_{av} = q(\tau_{av})$ where τ_{av} is the mean shear stress averaged across the channel. This mean stress is generally computed as $\tau_{av} = \rho gRS$ where ρ is water density, g the acceleration of gravity, R the hydraulic radius (effectively the same as the mean depth in most rivers), and S the water-surface slope. The width-averaged estimate of the total flux is

$$Q_a = w\, q(\tau_{av}) \qquad (2)$$

It differs from the true flux Q given by eq.1 unless either the transport function $q(\tau)$ is linear, contrary to a huge mass of data, or the channel has laterally constant shear stress. With a nonlinear law, if τ varies across the channel the additional flux in high-τ parts of the channel outweighs the lower flux in low-τ parts, and the total flux Q is higher than Q_a.

Brownlie (1981) is reported as having recognised this as the likely reason why transport equations calibrated to flume data (laterally uniform conditions) under-predicted field measurements (laterally variable conditions). Paola (1996) analysed the problem in the context of braided rivers. By making assumptions about the spatial probability distribution of depth (used as a surrogate for τ) he showed the importance of confluence scour pools as conduits for bedload transport, and estimated that Q exceeded Q_a by a factor of ~3. Nicholas (2000) modified Paola's theory to allow for variable water discharge and applied it to a braided river in New Zealand. He showed that Q_a underestimated Q by a factor of 2 to 3 at low flows, but much less so in large floods.

Essentially the same argument applies to spatial variability in τ_c. With τ spatially uniform, but coarse and fine patches of bed material such that τ_c varies spatially, the nonlinearity of the transport law again means that the width-integrated bedload flux will exceed what would be calculated on a width-averaged basis using the mean value of τ_c. This was recognised by Paola and Seal (1995) as part of a wider discussion of the effects of bed patchiness on size selectivity in bedload transport. Their calculations suggested that the total flux can be several times greater than would be estimated using the grand-average grain size distribution.

If both τ and τ_c vary spatially the situation becomes more complicated, and it is clear that different outcomes are possible depending on the sign and magnitude of the correlation between τ_c and τ. Random patchiness in a channel with variable τ ought to give even higher bedload flux than variance in just τ or just τ_c, but how much higher is not obvious. In the unlikely event of a negative correlation between τ and τ_c the capacity would be further increased. Conversely a positive correlation (which seems the likeliest scenario, since patchiness is usually caused by flow nonuniformity) must reduce spatial differences in transport rate because high-τ parts of the channel have relatively armoured beds, and low-τ areas have more mobile fine sediment. But it is not clear whether the effect on total bedload flux of lateral variation in τ_c will completely cancel out that of variation in τ.

Analytical model

These issues are investigated below using a mathematical model which has been designed to represent a wide range of generic circumstances, whilst being sufficiently simple that general analytical solutions can be derived rather than relying on numerical calculations for just a few scenarios. The four components of the model are a bedload transport law; a statistical model for lateral variability in τ which allows different means and variances; a way of allowing for flow stage variation; and alternative representations of bed patchiness and thus variance in τ_c.

Bedload transport function

For definiteness in what follows a particular transport function is adopted. As in Paola (1996) and Nicholas (2000) the choice is the Meyer-Peter and Müller (MPM) equation, which is widely used as well as simple and mathematically tractable. Its performance in comparisons with field measurements has been uneven (Gomez and Church, 1989; Reid *et al*., 1996), perhaps according to degree of armouring, but it is no worse than some much more complicated functions. A different transport law would be expected to give qualitatively similar results.The MPM equation can be written in the form

$$q = k\,(\tau - \tau_c)^{1.5} \qquad (3)$$

for $\tau > \tau_c$, or $q = 0$ for lower stresses. The constant k depends on sediment density but not on grain size, which affects transport rate only through τ_c.

In fractional SRMs different size fractions of bed material are treated separately. To achieve this a transport function like (3) is modified in two ways: the transport rate for each size fraction is assumed proportional to its availability in the bed, and the value of τ_c is varied between sizes to allow for selective transport. For simplicity the first part of the analysis below ignores the existence of a spread of sizes at each point in the river bed, but later on I introduce a distribution of τ_c values which can represent either intra- or inter-patch variation.

Statistical model for spatial variation in shear stress

Although the spatially-averaged mean shear stress τ_{av} on the bed of a river can be calculated from the mean depth, the local shear stress τ does not depend directly on local depth d but on the vertical velocity gradient near the bed. Even in straight flat-bottomed laboratory channels and canals there is a reduction in τ from the centreline towards the edges where velocity is reduced by bank friction (e.g. Knight *et al*., 1994). In natural channels the spatial variability of τ is increased by three further factors: variation in d both laterally across bars and talwegs and longitudinally over pools and riffles; planimetric convergence and divergence of flow, with consequent acceleration and deceleration, for example in braid confluences and diffluences; and secondary circulation, with τ higher where there is downwelling of relatively fast surface water than where there is upwelling of slow near-bed fluid.

Just as τ_{av} depends on d_{av}, so it seems likely that the variance of τ increases with that of d. Previous investigators (Paola, 1996; Nicholas, 2000) have therefore used depth data, which are readily obtainable, as a surrogate for information on spatial patterns of τ which are extremely hard to measure. It is recognised that some areas (e.g. riffles and bar heads) may have low d but high τ, and other areas the reverse (e.g. in the lee of bars), partly because of local differences in water-surface slope. This was well demonstrated by Lisle *et al*. (2000) using a quasi-3-D flow model to estimate the spatial pattern of τ in several gravel-bed reaches with alternate bars. Field measurements by Bathurst *et al*. (1979) in gravel-bed meander bends also show that maximum τ need not coincide spatially with maximum d. But both studies showed a positive correlation between d and τ, and as Nicholas (2000) pointed out, local deviations from $\tau \propto d$ tend to cancel out so that the frequency distributions of τ and d are similar.

Paola (1996) and Nicholas (2000) represented the spatial variability of d by the two-parameter gamma distribution, which is versatile and gave good fits to the braided-river data they were considering. They found, however, that the shape parameter of the distribution varied systematically with flow level so that the distribution ought to be re-fitted for every flow level of interest. I have preferred to think in terms of a probability distribution of τ itself, and have developed a new model which makes it easier to consider the effects of change in flow level and is more amenable to an analytical treatment.

The starting point is a probability distribution for the spatial distribution of shear stress during bankfull flow, denoted hereafter by τ_b. An assumption about how the distribution alters as stage falls is added later. The bankfull distribution is represented by a compound rectangular distribution with two parameters, one fixing the mean and the other the variance of τ_b. Shear stress is assumed to be below its mean value *a* in a proportion *b* of the total channel width ($0 < b < 1$), and to vary randomly (i.e. follow a uniform or rectangular distribution) between 0 and *a* within that part of the channel. Shear stress is above average in the remaining proportion 1-*b* of the width, and is assumed to vary randomly between *a* and a maximum which has to be $a/(1-b)$ in order that the overall mean stress is *a*. Thus the probability density function of τ_b is

$$p(\tau_b) = \frac{b}{a} \qquad (0 \le \tau_b \le a) \qquad (4a)$$

$$= \frac{(1-b)^2}{ab} \qquad \left(a \le \tau_b \le \frac{a}{1-b}\right) \qquad (4b)$$

The variance of τ_b in this model can be found to be $a^2b/3(1-b)$, so the coefficient of variation (standard deviation divided by mean) in percentage form is

$$cv_\tau = 100 \sqrt{\frac{b}{3(1-b)}} \qquad (5)$$

This increases monotonically with *b*, which is therefore an index of spatial variability in bankfull shear stress. For example, the cv is 0, 29%, 58%, or 115% for *b* = 0, 0.2, 0.5, or 0.8.

Figure 1 illustrates this distribution model in two different ways. Figure 1a shows cumulative distributions of τ_b for selected values of *b*. The distributions have a dog-leg shape with two linear segments, except for *b* = 0.5 when they have the same slope and there is a single uniform distribution of τ_b between 0 and 2*a*. As *b* increases so does the maximum shear stress, from 25% above the mean for *b* = 0.2 to twice the mean for *b* = 0.5 and five times for *b* = 0.8. Figure 1b represents these distributions as asymmetric channel cross sections which would give the same distributions if τ_b varied exactly as local depth. When the model is visualised in this way the parameter *b* is an index of channel shape: progressively closer to rectangular as $b \rightarrow 0$, but progressively non-rectangular as $b \rightarrow 1$ and the contrast grows between extensive shallow, low-τ areas and a narrow, deep, high-τ talweg. Low values of *b* correspond to flume- or canal-like conditions, with a wide area in which τ_b slightly exceeds the mean and a small area with much lower stress (e.g. near the banks). Higher values of *b* represent the much less uniform conditions associated with unconstrained natural planforms, with extensive areas of below-average τ (e.g. bar tops and margins, backwaters) and local areas of much higher τ (e.g. meander pools, braid confluences, and other talwegs).

There appear to be no published data with which to check how representative this model is of natural rivers. However, two lines of evidence suggest it is not unrealistic. First, many of the depth distributions shown by Mosley (1982, 1983) and Nicholas (2000) have the dogleg character shown in Figure 1a. Also, in ongoing research I and colleagues are analysing acoustic doppler profiles of velocity at near-bankfull discharge in Fraser River, Canada. Distributions of shear stress estimated by fitting the law of the wall to the depth-averaged mean velocity, and also distributions of water depth, are fitted at least as well on average by the *b*-model as by a two-parameter gamma distribution. Values of *b*, estimated by matching moments, range from ~0.4 to ~0.7.

Allowance for sub-bankfull flow

The compound-rectangular statistical model just described is for the spatial distribution of shear stress at bankfull discharge. As discharge falls below bankfull, τ is expected to become lower everywhere in the channel, and to vanish in areas which become dry. The mean shear stress declines in direct proportion to the mean depth, and it seems reasonable in a generic model to suppose that the entire distribution of τ can be shifted. Thus it is assumed that the stress distribution continues to follow (4), but with τ_b replaced by

$$\tau = \tau_b - fa \qquad (6)$$

where *f* is the fractional reduction in mean τ (and also mean depth) below its bankfull value. Note that this does not represent an assumption that the distribution retains the same shape at lower stages. Rather, for values $f < 1$ a fraction *fb* of the width is now dry so that the distribution is progressively truncated from one end, becoming a single rectangular distribution for $f \geq 1$. This treatment would be exactly correct if local shear stress was proportional to local depth.

The relationship between *f* and water discharge can be established using Manning's equation with constant values of *n* and slope. Eq. 6 then implies that the ratio *R* of water discharge at stage *f* to water discharge at bankfull is

$$R = (1 - fb)(1 - f + f^2 b/2)^{5/3} \qquad (7)$$

This depends far more on *f* than on *b*, in a slightly nonlinear manner. Values of $f = 0.4$ and 0.8 are used for illustrative purposes below; they correspond to discharges ~60% and ~90% below bankfull respectively.

Representation of threshold for transport

The simplest assumption about the critical shear stress τ_c is that it is the same everywhere in the channel. This case is considered first, but is relaxed later on. To keep the model nondimensional the constant value is specified as a fraction *c* of the mean bankfull shear stress; that is,

$$\tau_c = ca \qquad (8)$$

In alternative scenarios which allow for bed patchiness the mean value of τ_c continues to follow (8). Values of *c* are assumed to lie between 0 and 1 in the subsequent analysis, so that in bankfull conditions all areas with above-average shear stress experience some bedload transport as do some areas of lower τ to the extent that $c < 1$.

The ratio $1/c = \tau_{av}/\tau_c$ is a dimensionless transport stage in bankfull conditions. In gravel-bed rivers it has a typical value of 1.2-1.4 (e.g. Parker, 1978; Andrews, 1984; Paola, 1996), implying $c \approx 0.7$-0.8. A value of 0.8 is used below as typical of gravel-bed rivers in the bankfull or near-bankfull conditions at which most bedload is conveyed. With increasing amounts of sand in the bed the value of *c* falls, tending to zero in large sand-bed rivers.

Analysing the effect of lateral variation in τ_c requires making assumptions about the nature of spatial variability in grain size, and in particular to what extent it is correlated with shear stress. Field evidence on this is virtually nonexistent. Limited data do exist on the correlation of grain size and depth (e.g. Mosley, 1982, 1983; Seal and Paola, 1995), but no clear generalisations have emerged. Two limiting scenarios are therefore considered below: purely random patchiness, and perfect fining upwards as in classical models of meander point bars. These cases correspond to correlations of 0 and 1 between τ_c and τ_b.

Results

I describe the implications of my model in stages, starting with the effect of lateral variation in shear stress on total bedload flux in channels with uniform beds and then moving on to the effect of bed patchiness.

The effects are quantified using a general analytical solution for the width-integrated bedload flux in a channel with spatially variable τ and/or τ_c. This involves integrating the MPM equation (eq.3) across the channel. The distribution of τ-τ_c is given by eq. 4 with τ_b replaced by $\tau_b - (c+f)a$ (from eqs. 6 and 8). Then $Q = w\, q_{av}$ with the mean transport rate obtained as

$$q_{av} = k\int_0^{y\max} y^{1.5}\, p(y)\,dy$$

where y denotes τ-τ_c and $y_{\max} = a/(1\text{-}b) - (c+f)a$. For the case $c + f < 1$ this integration yields

$$Q = 0.4kwa^{1.5}\left\{ b(1-c-f)^{2.5} + \frac{[1-(1-b)(c+f)]^{2.5}}{b(1-b)^{0.5}} - \frac{(1-b)^2(1-c-f)^{2.5}}{b} \right\} \qquad (9)$$

The first term inside the curly brackets quantifies the flux in parts of the channel which have below-average τ in bankfull conditions; the other two terms relate to areas of higher τ. If $c+f$ >1, as can happen in gravel-bed rivers at sub-bankfull discharge, the first and third terms disappear. As $b \to 0$ eq. 9 converges on the width-averaged estimate of bedload flux, which is given as in eq. 2 by

$$Q_a = kw(\tau_{av} - \tau_c)^{1.5} = kwa^{1.5}(1 - c - f)^{1.5} \qquad (10)$$

if $c + f < 1$, or zero for higher values of $c + f$.

According to the model, then, the total bedload flux in a river depends on the channel width w and mean bankfull shear stress a; the transport-rate coefficient k in the MPM equation; and the dimensionless parameters which characterise stress variance or channel shape (b), critical shear stress (c), and flow reduction below bankfull (f). The dimensional variables w, a and constant k determine the absolute levels of bedload flux, irrespective of channel character, and can be fixed when investigating the effects of b, c, f. For illustrative purposes below I set $kwa^{1.5} = 1$, so that all results are normalised relative to the bedload conveyance of bankfull flow in a sand-bed canal (eq.10 with $c = f = 0$).

Effect of shear stress variance on bedload flux

The width-integrated flux Q in a channel with no patchiness is plotted in Figure 2 as a function of the stress-variance index b. Although Q depends on both c and f as well as b, these parameters do not have unique effects since they occur only in the combination $c+f$. The full range of conditions is therefore covered by plotting curves of Q for different values of $c+f$, each of which could be made up of different combinations of c and f.

Figure 2 shows that, whatever the value of $c+f$, Q increases considerably with b. The bedload conveyance is greater under non-uniform flow because the increase in specific flux q in parts of the channel with above-average shear stress is bigger than the decrease in q in parts with below-average stress. For bankfull sand-bed conditions ($c+f = 0$) the total flux Q increases by only about 40% from b = 0 to 0.8, but for bankfull gravel-bed conditions ($c+f$ = 0.8) it increases by a factor of 8. The mid-range value b = 0.5 of the channel-shape or stress-variance index gives about a threefold increase in bedload conveyance compared to a uniform channel, which is about the same as Paola (1996) and Nicholas (2000) calculated for specific gravel-bed situations. The curve for $c+f = 0.8$ also describes a sand-bed river at low flow. When $c+f > 1$, which corresponds to a gravel-bed river at below-bankfull flow, there is no bedload transport in near-uniform channels (*b* close to 0) because $\tau < \tau_c$ everywhere, but in channels with higher *b* there is transport in the parts of the channel with highest τ so that the flux increases rapidly with *b*. The value of $c+f$ makes rather less difference in high-*b* channels than in those which are more rectangular.

Lateral variability in shear stress therefore makes a big difference to bedload transport capacity, but one which varies according to the precise conditions. The finding that the difference between Q and Q_a depends on both flow stage and cross-section geometry is consistent with the calculations of Nicholas (2000) for specific field sites in New Zealand. It is noteworthy that the flux in a highly non-uniform gravel-bed channel can attain ~80% of that in a uniform sand-bed channel of the same width and mean τ, despite the much greater value of τ_c for gravel than sand.

Eq.9 may also be used to assess the flow level at which significant bedload transport commences in channels with different degrees of lateral variation in τ. Figure 2 indicates that Q increases with increasing *b* for given $c+f$, and with decreasing $c+f$ for given *b*. This implies that, for a given value of *c* (i.e. of τ_{av}/τ_c), as the channel becomes less uniform (higher *b*) a given flux can be conveyed by a lower flow (higher *f*). To illustrate this, let 'significant' transport be defined as 10% of the flux conveyed at bankfull discharge in a channel with the same values of τ_{av} and τ_c but no variability in τ. For a sand-bed river ($c = 0$) significant transport begins at $f \approx 0.7$ (~15% of bankfull discharge) for $b = 0$, but at progressively lower levels ($f \approx 0.9$, 1.3, and >3) as *b* increases from 0.2 through 0.5 to 0.8. In a gravel-bed river ($c = 0.8$) significant transport requires near-bankfull flow ($f \approx 0.1$) for $b = 0$, but again occurs at progressively lower levels ($f \approx 0.4$, 0.5, >3) for $b = 0.2$, 0.5, and 0.8. The same total flux Q is conveyed in less and less width through an increase in the intensity of transport. This accords with the observation that in irregular natural channels, bedload continues to move at discharges well below bankfull.

Effects of bed sorting and patchiness on bedload flux

The results in Figure 2 are for channels with constant τ_c, implying spatially uniform grain size. For sand-bed rivers this may be a reasonable assumption if bedforms are fairly uniform, but the more diverse bed material in gravel-bed rivers is often sorted spatially into coarser or finer patches. There is also a size distribution at each point, with lower τ_c for the fine fractions than the coarse ones.

The effects of intra-patch variation in τ_c, and of inter-patch variation that is spatially random, can be represented in my analysis by averaging solutions to eq.9 across a distribution of values of *c*. For illustrative purposes I have used an approximately normal distribution of values: c-2e, c-e, c, c+e, c+2e with weights 0.1, 0.2, 0.4, 0.2, 0.1 respectively. With $e = 0$ this reduces to eq.9; higher values of *e*, up to the limit $e = c/2$, represent more poorly-sorted sediment at a point and/or a greater degree of random patchiness across the channel.

Figure 3 shows the results of this calculation for bankfull or near-bankfull flow in gravel-bed rivers ($c = 0.8$). Two curves are shown, one for $e = 0.2$ which implies that τ_c varies by $\pm$50% around its mean value and the other for $e = 0.4$ which implies the maximum possible variation of $\pm$100% in τ_c. In both cases the bedload conveyance of the channel is increased over that for uniform τ_c and the same variance in τ as indexed by *b*. The reduction in flux over coarser patches (or in the coarse tail of the size distribution at a point) is less than the increase in flux over finer patches (or in the fine tail of the distribution). The increase is greater for higher *e*, as expected, and is substantial in fairly uniform flow conditions (low values of *b*). In the limiting case $b = 0$ the bedload conveyance is higher by a factor of ~1.5 for $e = 0.2$ and ~2.5 for $e = 0.4$. But as b increases, the effect of bed variability becomes less and less important compared to the effect of flow variability, so that by $b = 0.8$ the increase in conveyance is only a few percent even for $e = 0.4$. It should be noted that 'increase' is used here relative to the true flux for the given degree of hydraulic variability (Q in Figure 3), not the width-averaged estimate of the flux calculated using a single grain size (Q_a). Figure 3 shows that Q_a is always considerably lower than the true flux in a channel with random patches and/or grain-size distribution.

Random patchiness is one end-member in the likely range of scenarios with different degrees of positive correlation between τ and τ_c. The other extreme, with a correlation of +1 instead of 0, is perfect fining upwards of the kind predicted in the classical meander point-bar model in which both shear stress and grain size decrease systematically from the outer-bank pool to the top of the inner-bank point bar. This scenario can be represented in the present model by assuming that $\tau_c = c\tau$ at all points, thus increasing from 0 to a maximum of $ac/(1-b)$ while retaining the same mean value ca as in other scenarios. It follows that $\tau - \tau_c = \tau_b(1-c)-fa$. The bedload conveyance of a channel with perfect fining upwards can therefore be calculated using eq.9 with the parameters a, c, f replaced by $a' = a(1-c)$, $c' = 0$, and $f' = f/(1-c)$.

Figure 3 shows that this scenario gives a very different result from random patchiness. The conveyance of the channel is drastically reduced, and varies much less with stress variance than in the no-patch or random-patch cases. It is therefore much closer to the width-averaged estimate Q_a than in other scenarios. The physical explanation for this is that high-stress parts of the channel are armoured by coarse bed material. The reduction in flux in these areas far outweighs the gain in flux in low-stress areas with a fine bed. Put another way, the variance of the excess stress τ-τ_c is greatly reduced and so therefore is the bias in the width-averaged estimate of total conveyance.

The range of possibilities between these two end-member scenarios is evidently very wide. A simple or weighted average of the random and fining-upwards cases would give a higher bedload flux than with no patches in near-uniform flow conditions (low b in Figure 3), but lower than with no patches in more variable flow conditions (high b). Calculations for scenarios with less-than-perfect fining upwards, or with a quasi-random range of below-average values of τ_c where τ is below average and a range of above-average values of τ_c where τ is above average, yield a wide variety of curves depending on the precise assumptions. It seems safe to say that bedload conveyance in situations where both τ and τ_c vary spatially in a partly-correlated way will be higher than a width-averaged calculation would suggest; but by how much, and whether by as much as in the no-patch model, will depend on the circumstances.

Alternatives to simple width-averaged estimation of bedload flux

Researchers modelling downstream fining using fractional SRMs have generally treated channel cross-sections as rectangular, and used a single shear stress value and bed grain-size distribution (Hoey and Ferguson, 1994; Cui *et al*., 1996; Ferguson *et al*., 2001; Talbot and Lapointe, 2002). The width-averaged calculation is then on the lines of (10) above (Q_a, the special case $b = 0$ of the general model) except that fluxes are calculated separately for each size fraction of the bed. Clearly there will be some underestimation of total flux (and also competence) if in reality the channel has above-average shear stress in some places. The bias is likely to be greatest in relatively wide sections where the mean stress estimated from the depth-slope product is low but there is substantial transport in one or more narrow talwegs. Ferguson *et al*. (2001) found that agreement between observed and simulated downstream fining and aggradation rate was better on the whole when the same width was used for every section, on the argument that only part of a wide section is hydraulically effective. But the present model may be helpful in checking the merits of more sophisticated alternatives to simple width-averaging over the measured bankfull width.

Non-fractional SRM applications have often used the actual channel cross-section at each node, calculated the water surface elevation and the corresponding wetted width, then applied the width-averaged transport calculation only over the wetted width. This makes no difference in bankfull conditions, but should be a step in the right direction when estimating bedload transport at lower stages. It can be treated analytically using the model of Figure 1, and yields a flux estimator

$$Q_w = \frac{kwa^{1.5}}{(1-bf)^{0.5}}\left(1-c-f+bcf+\frac{bf^2}{2}\right)^{1.5} \qquad (11)$$

This reduces to (10) when $f = 0$. For $f > 0$ it is a function of c and f separately, so cannot be plotted as a function only of b and $c+f$ as in Figures 2 and 3. It gives a higher estimate than Q_a, increasingly so for higher values of b. For gravel-bed conditions it always falls well short of the true flux as calculated using eq.9, but for sand-bed conditions it comes closer.

It has been observed several times that bedload transport in braided gravel-bed rivers is concentrated in confluences and other talwegs, sometimes to the extent that transport elsewhere is negligibly small in comparison (Mosley, 1982; Carson and Griffiths, 1987; Paola, 1996). Transport in meandering channels is also concentrated in high-τ areas, not always in the deepest part of the pool because of the delayed crossover of flow, but at least near the talweg on the margin of the point bar (e.g. Dietrich and Smith, 1984). This suggests there may be merit in making use of the maximum depth, as well as the mean depth, in the calculation of τ_{av} for use in a width-averaged treatment of bedload flux. Talbot and Lapointe (2002), in an application of Hoey and Ferguson's (1994) model, experimented with estimating shear stress from mean depth, maximum depth, or an average of these two, and found that the average of the two gave the best results. The equivalent in the present context is to use the mean of τ_{av} and τ_{max} in the 1-D calculation. For $c+f \leq 1$ this turns out to overestimate the true flux Q progressively with higher b. For $c+f > 1$ it predicts zero flux for low b, underestimates Q for intermediate b, and overestimates for high b.

Another possibility which turns out to work better is to ignore completely the shallow, low-τ parts of the channel and do a width-averaged calculation over the deeper areas only. Nicholas (2000) describes one such procedure that was developed by engineers in New Zealand. In the present model an obvious way to do such a calculation is to consider only the proportion 1-b of the width in which $\tau > \tau_{av}$. This gives an estimator

$$Q_d = \frac{awh^{1.5}}{(1-b)^{0.5}}\left[1-0.5b-(1-b)(c+f)\right]^{1.5} \qquad (12)$$

For $b = 0$ this reduces to the standard width-averaged estimator Q_0, but for non-rectangular channels it gives a higher estimate than Q_a for all but very low values of $c+f$.

Figure 4 compares the simple (Q_a, eq.10), wetted-width (Q_w, eq.11), and deep-areas-only (Q_d, eq.12) width-averaged estimators of bedload flux with the exact width-integrated flux (Q, eq.9) for different degrees of variability in τ as indexed by b. The plot shows fluxes for $c = f = 0$, which represents a sand-bed river at bankfull discharge, and $c+f = 0.8$ which represents either a gravel-bed river at bankfull flow ($c = 0.8$, $f = 0$) or a sandier river at lower flow ($c = 0.4$ and $f = 0.4$, or $c = 0$ and $f = 0.8$). In all cases the width-averaged estimators under-predict the actual flux for all non-rectangular channels ($b > 0$), but there are clear differences in the amount of bias both between methods and according to parameter values in the model. Q_w increases with b for below-bankfull flows, so is more accurate than Q_a which ignores the effect of channel shape. The degree to which Q_w underestimates the true flux is less for higher reductions in flow level (compare the curves for c = 0.4, f = 0.4 and c = 0, f = 0.8). However, in all circumstances with f >0 Q_d is higher than Q_w and is closer to the exact flux Q, while still underestimating it. The one circumstance in which Q_d is not the best of the estimators compared here is $c+f$ <0.2, i.e. a sand-bed river at or close to bankfull level. As seen in the top part of Figure 4, in these conditions Q_d dips slightly below Q_a for intermediate shape factors, though neither estimator is seriously in error.

For all gravel-bed cases, therefore, and also for sand-bed rivers below bankfull, it appears that ignoring the parts of the channel with low shear stress gives a more accurate indication of true bedload flux than does averaging over the bankfull width or the actual wetted width at the given discharge. This finding has implications for operational computations using SRMs.

Which width-averaged estimator of bedload flux performs best on a patchy bed depends on the degree of correlation between the spatial variation in applied shear stress and that in critical shear stress. For random or near-random patchiness, it is clear from a comparison of Figures 3 and 4 that the deeper-areas average Q_d remains much less biased than the bankfull or wetted-width averages Q_a and Q_w. However, in a channel with strong fining upwards, Q_d will overestimate and Q_w may get closer to the true flux.

Discussion and conclusions

Because bedload transport rate varies nonlinearly with excess shear stress τ-τ_c, the total bedload flux in a river depends not only on the mean values of τ and τ_c but also on their spatial variances within the domain of the calculation. The existence of spatial variation in flux has obvious implications for the design of bedload sampling programmes, but attention is restricted here to the implications for numerical calculations using bedload formulae. In some applications the calculation may be for a single cross section and only lateral variation matters, but in sediment routing models applied to long stretches of river the effective domain of the bedload calculation for section i is the channel area extending from midway between sections i-1 and i to midway between i and i+1. Local longitudinal variability, for example between pools and riffles, may then become relevant.

Other things being equal, greater variability in either τ or τ_c leads to greater flux. But although the existence of this effect is undeniable, its magnitude is uncertain. There is virtually no empirical evidence; to the best of my knowledge nobody has measured bedload flux in either natural channels or flumes with the same mean values of τ and τ_c but different variances, and indeed there is very little data of any kind on within-channel variation in τ. In the absence of experimental data one has to fall back on calculations. Paola (1996) and Nicholas (2000) did so using general models fitted to specific flume experiments and field sites respectively, and using flow depth as an imperfect surrogate for shear stress. But to arrive at a wider assessment of the phenomenon one must either amass more and more such case studies, covering a wide range of conditions, or resort to a purely theoretical approach in which parameters indexing the amount and nature of spatial variability in τ and τ_c are varied while other river properties are kept constant. The latter approach has been taken here.

By devising a model with a general analytical solution I have been able to investigate how total bedload flux depends on four key aspects of channel configuration: the degree of lateral variation in τ, represented by the parameter b in the model; the extent to which mean τ_c approaches mean τ, represented by c; the reduction in flow stage below bankfull, represented by f; and the nature and degree of bed patchiness, which determine the lateral variance of τ_c.

In turn, this allows an evaluation of the performance of width-averaged estimators of bedload flux as used in 1-D sediment routing models. There are three main conclusions.

(1) With all else equal, bedload flux increases with the variance of τ. The effect is progressively stronger as $c+f$ increases, which corresponds to moving from sand-bed rivers at bankfull flow to lower flows in sand-bed rivers, then bankfull flows in gravel-bed rivers, and finally lower flows in gravel-bed rivers. In the gravel-bed cases there can be at least a 5-fold difference in flux according to the variance of τ. This result is not surprising. The root of the phenomenon is the nonlinearity of the bedload transport law, and this is greatest when $\tau \approx \tau_c$ so that a small increase in τ makes the difference between no transport and some, or a low rate and a much higher rate. In terms of geometry rather than flow, a cross-section with one or more deep talwegs will have locally high transport rates per unit width, and this will have most effect on the total bedload conveyance when there is little or no transport over the rest of the width.

(2) The effect of bed patchiness depends less on the variance of τ_c than on the degree of correlation between τ_c and τ. Random patchiness increases bedload flux, especially in channels without much lateral variation in τ where the flux may be doubled if there is a high degree of random variability; in more variable hydraulic conditions the effect is relatively small. In contrast, perfect fining upwards of bed material (and its corollary, armouring of talwegs) reduces bedload flux, more so in channels where there is substantial variation in τ. Real-world situations will fall somewhere between these end-member scenarios. In most situations this probably means the river will convey less bedload than it would without patches, but in near-uniform hydraulic conditions the opposite might apply. The limited field evidence in the literature (Mosley, 1982, 1983; Seal and Paola, 1995) suggests correlations between bed grain size and flow depth are weak, but these studies were all for braided rivers and may not generalise to other morphologies. There is a need for further field research on this issue.

(3) Simple width-averaging over the full channel width severely underestimates bedload flux in channels with substantial lateral variation in τ, except when it is accompanied by a matching variation in τ_c on the lines of the classic point-bar fining-upwards model. Averaging only over the wetted width helps when flow is below the bankfull level, but merely reduces the degree of underestimation; estimates remain biased downwards to an appreciable extent. Averaging only over parts of the channel with above-average depth or τ does better than either of the more obvious approaches in almost all circumstances.

Conclusions 1 and 2 are consistent with the results of Paola (1996) and Nicholas (2000), but qualify and extend them. Paola used a different, though equally general, analytical model which did not allow for variation in flow level. Since he only fitted his model to one data set (the braided-stream flume experiments of Ashmore, 1985) he was unable to generalise about the extent to which τ variance inflates bedload flux, though he proposed a factor of ~3 for Ashmore's data. The present analysis is very much in the spirit of Paola's work but the results show that lateral variation does not increase bedload conveyance by any fixed factor; it depends on all four aspects of the channel configuration that were identified above. Nicholas (2000) allowed for sub-bankfull flows but this necessitated re-fitting his statistical model for each water discharge of interest. His main conclusions were that higher variance of τ allowed significant bedload transport to begin at a lower water discharge and gave a higher flux than for a rectangular section at the same water discharge, but with progressive convergence of fluxes at extremely high flow levels. All three findings are supported by the present analysis, but with the qualification that the effects would be less pronounced for sand-bed rivers than for gravel-bed channels such as the one Nicholas was studying.

Conclusion 3 has implications for developers or users of both simple and fractional sediment routing models. The present analysis did not extend to fractional transport calculations, though work is in progress on this using numerical simulations rather than an analytical model. Preliminary results (Louise Sime, personal communication, 2002) suggest the effects on flux are qualitatively the same as in the single-size analysis: it increases with variability in τ, and may either increase or decrease with bed patchiness depending on the correlation with τ. This implies that standard 1-D calculations will tend to underestimate bedload flux. Underestimation by a constant factor would not be a great problem, since it would affect only the pace of aggradation or degradation and not the longitudinal pattern of change in bed elevation or of associated surface fining or coarsening. Moreover, in longer-term studies of downstream fining (e.g. Hoey and Ferguson, 1994; Robinson and Slingerland, 1998) the timescale of development may not be known precisely and the main interest is in spatial patterns. But the lateral variance of τ is likely to alter from section to section in most rivers, in which case patterns of aggradation/degradation could be mis-represented by width-averaged computations. The effects of spatial variability in τ and τ_c on the size selectivity of bedload transport also need to be investigated. Paola and Seal (1995) showed theoretically that bed patchiness can enhance size selectivity, but hydraulic variability could have the opposite effect since locally high shear stresses will maintain coarser sediment in transport than would be the case in a rectangular channel.

There is obvious scope for investigating how the output from 1-D SRMs is affected by using alternatives to simple width-averaged transport calculations, and whether the changes are beneficial. In effect what is needed is a way of allowing for what in a 1-D model is sub-grid-scale spatial variation in τ and τ_c. The present, single-size, analysis suggests that a width-averaged transport calculation which ignores all parts of the channel that are shallower than the mean bankfull depth gives a good approximation to the true width-integrated bedload flux in most circumstances. It remains to be discovered whether it still does when applied to a local grain size distribution, as required in a fractional SRM, and whether the bedload size distribution is then simulated sufficiently well to give acceptable accuracy in predictions of surface fining or coarsening. There are also operational details to consider about just how such an approach would be implemented, including how to update the bed elevations defining the cross-section: is aggradation/degradation evenly distributed, or concentrated in the talweg? Likewise, how is patchiness to be updated without a fully 2-D model? Yet again further research is needed. Here as throughout, questions of whether and how to allow for lateral variability are seen to be central to a better understanding of the imperfections of 1-D sediment routing models.

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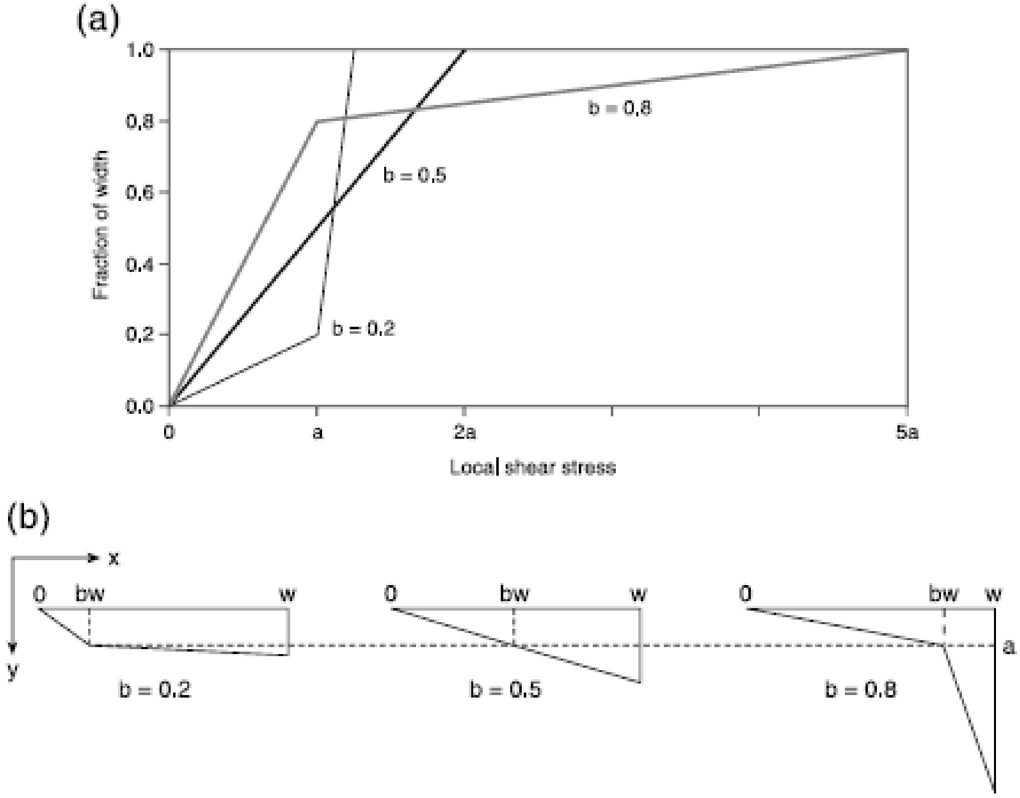

Fig. 1. Compound-rectangular probability model of Eqs. (4a) and (4b) for variance in bankfull shear stress τ_b: (a) cumulative distribution functions for different values of the parameter b; (b) visualisations in the form of channel cross sections in which τ is assumed proportional to depth.

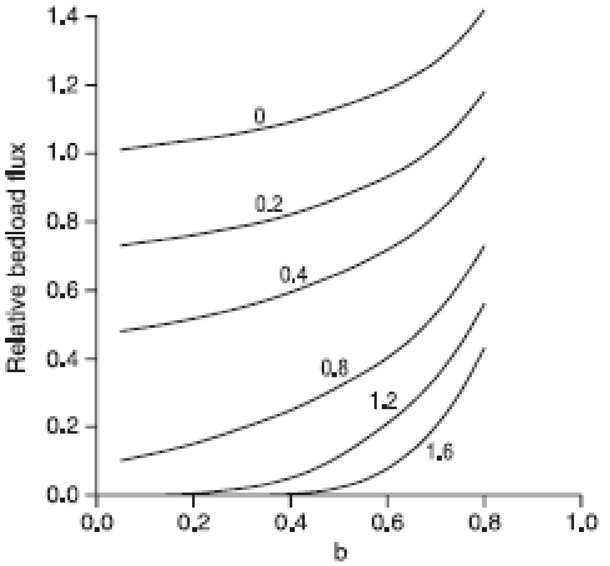

Fig. 2. Increase in bedload flux Q, calculated using Eq. (9), with stress variance as indexed by b. Separate curves are plotted for different threshold and flow-level combinations, indicated by values of $c+f$. The width-averaged estimate Q_a is the value of Q at $b=0$. Flux is in arbitrary units such that $Q=1$ for $b=0$ and $c+f=0$, representing bankfull flow in a rectangular sand-bed channel. Progressively higher values of $c+f$ correspond to lower flows in sand-bed rivers, bankfull flow in gravel-bed rivers, and lower flows in gravel-bed rivers.

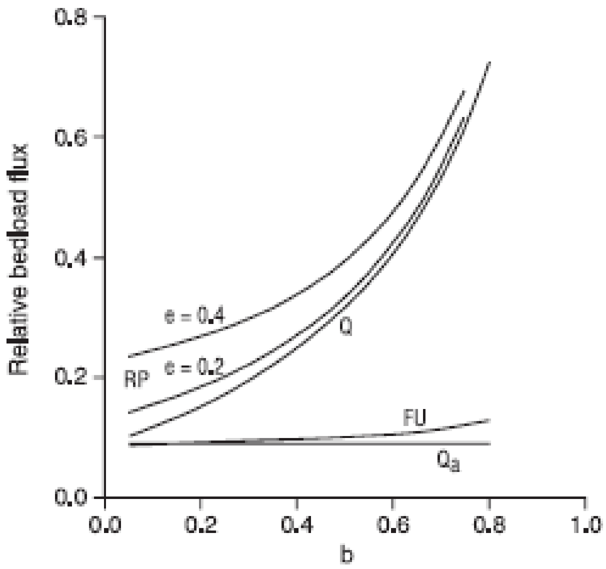

Fig. 3. Effect of bed patchiness on bedload flux in channels of different stress variance as indexed by b. Curves are for gravel-bed rivers ($c=0.8$) at bankfull flow ($f=0$). Those labelled Q, RP, and FU are for no patches, random patchiness with variance indexed by e, and perfect fining upwards. Line labelled Q_a is the simple width-averaged estimate.

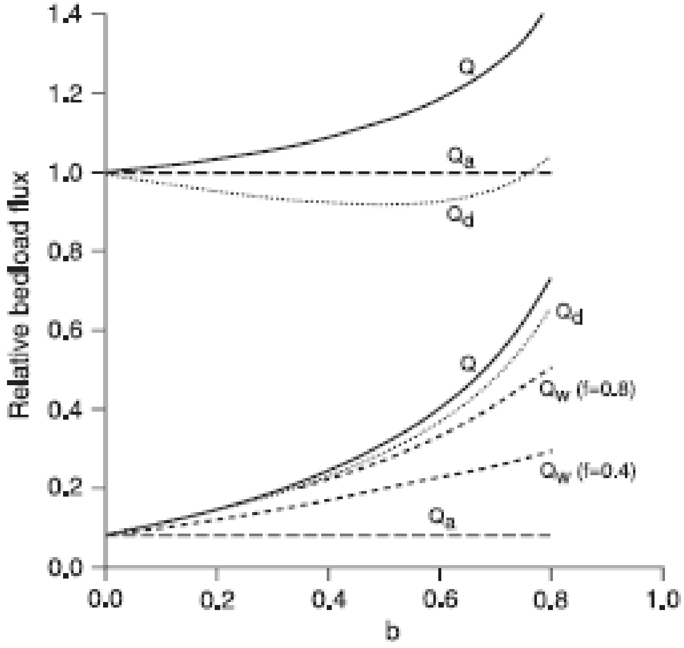

Fig. 4. Performance of simple (Q_a), wetted-width (Q_w), and deeper-areas-only (Q_d) width-averaged estimators of bedload flux in channels with different stress variance as indexed by *b*. True flux *Q* is reproduced from Fig. 2 for comparison. Curves of *Q*, Q_a, and Q_d are shown for $c+f=0$ and 0.8, representing bankfull flow in sand- and gravel-bed rivers, respectively. Q_w is plotted for $c=0.4$, $f=0.4$ and $c=0$, $f=0.8$ which represent below-bankfull flows in sandy rivers.