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Cracking and deformation of axially reinforced members subjected to pure tension

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This paper describes work carried out as part of a major experimental and theoretical investigation into tension stiffening. This work was carried out at the universities of Leeds and Durham. The experimental work enabled a detailed study to be undertaken of the behaviour of concrete in tension surrounding reinforcing bars. This study showed that a more-or-less linear variation in concrete and steel stress occurred in the region affected by a crack and also that behaviour of the concrete in the region of a crack remained substantially elastic. The experimental study permitted equations to be developed to predict the deformations and crack widths in axially reinforced tension members. Tests of the formulae for crack widths are reported and the formulae are shown to be satisfactory. A more detailed model of the behaviour in the region of a crack is proposed and shown to be reasonable.

Notation

A_c	area of concrete
A_s	area of reinforcement
c	cover
E_c	modulus of elasticity of the concrete
E_s	modulus of elasticity of the reinforcement
f_c	compressive strength of concrete
f_{ct}	tensile strength of the concrete
L	the length of the specimen (or the length over which measurements are taken)
N	axial tension force
n	number of cracks
S_o	the transfer length within which there is a transfer of force by bond between the reinforcement and the concrete
S_{rm}	average crack spacing
s	the slip at the point considered
s_1	the slip at the maximum bond stress
w	the crack width
w_{av}	the average crack width
α	a coefficient
α_e	modular ratio $= E_s/E_c$
Δ_a	the extension of a specimen prior to cracking
Δ_b	the extension of the reinforcement over the length $2S_o$ centred on a crack
Δ_c	the shortening of the concrete over the distance $2S_o$ on formation of a crack
β	a constant of integration such that the shaded area in Fig. 5(b) is given by $2S_o\beta(\varepsilon_{s2}-\varepsilon_o)$
ε_{cm}	average strain in the concrete (used in MC 90)
ε_{cs}	strain caused by shrinkage (used in MC 90)
ε_o	the strain in an uncracked tension member
ε_{sm}	average strain in the reinforcement = the average strain in a tension member
ε_{s2}	the strain in the reinforcement at a crack (concrete carries no tension)
ρ	A_s/A_c
σ_{ct}	stress in the concrete
τ	the bond stress.
τ_{max}	the maximum bond stress which occurs at a slip s_1
ϕ	bar diameter

Introduction

The interaction of reinforcement and the surrounding concrete is fundamental to the understanding of the behaviour of reinforced concrete. There are a number of aspects to this problem which include an understanding of bond behaviour, an understanding of cracking behaviour and the related problem of tension stiffening. Any study of the literature will show that these are interrelated. Although much research has been carried

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out into these problems, issues that require resolution still remain.

Traditionally, it is assumed in the design of reinforced concrete that concrete has no tensile strength. On this assumption, and assuming elastic behaviour of the steel and the concrete in compression, it is possible to calculate the stresses and strains in the concrete and the reinforcement and hence the deformations of the member. In practice, it is found that this procedure overestimates the deformations because the concrete in tension surrounding the reinforcement does, on average, carry some stress, even after cracking. This reduction in deformation or increase in stiffness relative to that calculated assuming the concrete supports no tension is referred to as *tension stiffening*.

Recently, two linked Engineering and Physical Research Council (EPSRC) grants were awarded: one to the University of Durham and the other to the University of Leeds to investigate tension stiffening and, in particular, the reduction in tension stiffening with time. In addition to the grants from the EPSRC, industrial contributions to the project, both financial and in-kind, were made by Arup Research and Development, Giffords, Cadogan Tietz and the Concrete Society.

The experimental work carried out under these grants has enabled a very thorough study to be carried out into the behaviour of concrete in tension surrounding reinforcement. In this paper, the results from the experimental work will be used to explore the behaviour of tension zones under service levels of short-term loading. It is believed that this work has led to a much clearer idea to be developed of the nature of the interactions between steel and concrete. Details of the testing and instrumentation and some results from this project have been published elsewhere[1–4] so these will not be described in detail in this paper. A future paper will deal with the detail of tension stiffening behaviour over time.

Basic behaviour of tension elements

Before attempting to develop further the theory of the behaviour of members subjected to pure tension it would be useful to gain a picture of the behaviour of such members. This will be done by looking at test data from tension tests carried out by Scott and Gill.[5]

A procedure for fixing strain gauges at very close spacings along the length of reinforcing bars has been developed at the University of Durham. This has been used over a period of some 20 years in a large number of projects in which detailed measurements of the variations of strain or stress along the reinforcement are necessary. This procedure has been described in detail.[5] Scott and Gill's tests on specimens with deformed bars are detailed in Table 1.

Initially, strain data for specimen 100T12 will be presented as this gives a convenient illustration of a number of aspects of behaviour. Data from other specimens will be considered later to illustrate particular points. Fig. 1 shows the load–average reinforcement strain response for this specimen.

It will be seen that the response is not a continuous smooth curve as is commonly plotted, but is made up of a series of linear segments separated by a sudden increase in strain on the occurrence of each crack. Up to a load of about 35 kN, these linear segments, extrapolated backwards, can be seen to pass through the origin. The behaviour of the tension specimen with a given number of cracks is thus elastic. Using the computer to produce 'best fit' lines for each segment enables the stiffness of the specimen to be established for each crack configuration. Fig. 2 shows this stiffness plotted against the number of cracks. It will be seen that there is a linear relationship between stiffness and

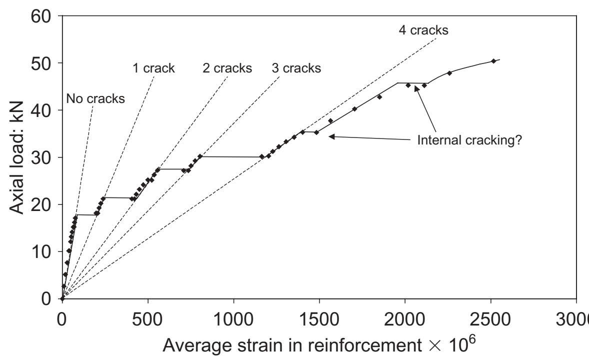

Fig. 1. Detailed load–strain response of specimen 100T12[5]

Table 1. Details of Scott and Gill's tests[5]

Specimen	Cross-section: mm	Nominal bar size: mm	Bar area: mm^2	% reinforce-ment	Cube strength: N/mm^2	Tensile strength: N/mm^2
70T12	70 × 70	12	81	1·61	47·0	3·0
100T12	100 × 100	12	86	0·83	45·0	2·7
100T20	100 × 100	20	260	2·48	47·0	3·1
140T12	140 × 140	12	86	0·43	54·0	3·1
140T20	140 × 140	20	266	1·33	55·0	2·8
200T20	200 × 200	20	258	0·65	60·0	3·1
300/100T20	300 × 100	20	262	0·87	48·0	3·2

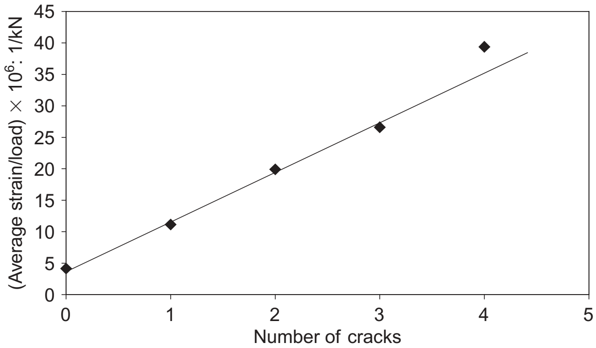

Fig. 2. Variation in stiffness of specimen 100T12[5] as a function of number of cracks

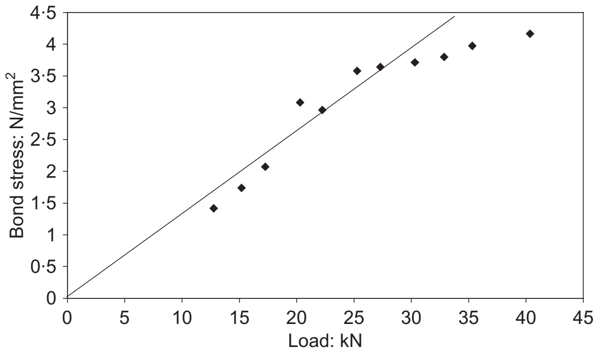

Fig. 4. Variation in bond stress with axial load for specimen 100T20[5]

number of cracks. This implies that the formation of each crack reduces the stiffness of the element by a constant amount. The final point for four cracks does not quite fit the linear relationship. This point is obtained from the behaviour immediately after formation of the fourth crack. Fig. 1 shows that at higher loads there are two further sudden increases in strain. These increases were not related to the formation of visible surface cracks and it may be speculated that they arise from some form of internal failure. It should be noted that both events occurred only at very high levels of stress in the reinforcement (> 400 N/mm^2).

A further point to note from Scott and Gill's tests is that the variation in steel strain on either side of a crack is close to linear. A typical result is shown in Fig. 3, which shows the variation in steel strain between two cracks at two levels of load in specimen 100T20. This close approximation to a linear strain variation is typical of all specimens in work by Scott and Gill[5], Beeby and Scott[1] and in other research programmes in which the Durham system of instrumenting bars has been used. This linear variation of stress implies a constant bond stress, which initially suggests some form of plastic behaviour. In fact, this is not the case—as can be seen from Fig. 4. This figure shows the bond stresses, calculated from the average change in stress over the transfer length, S_o, for specimen 100T20. It will be seen that there is an almost linear increase in bond stress with increase in applied axial force up to an axial force of approximately 30 kN or a bond stress of about 4 N/mm^2. Above this, the rate of increase in bond stress with increase in load reduces substantially to close to zero. Other specimens behave similarly, although the change from the linear response occurs at different loading states. For example, the change from the linear for specimen 140T20 is at a bond stress of close to 6 N/mm^2 and an axial force of approximately 65 kN. From the limited data available, however, it seems likely that the kink in the response is not related to any intrinsic property of bond.

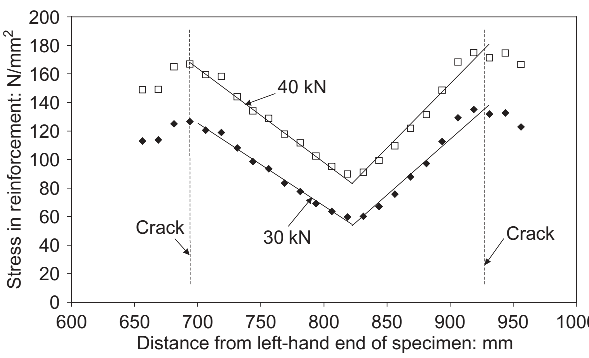

Fig. 3. Variation in stress in reinforcement between two cracks (specimen 100T20[5])

Derivation of equations for short-term loading

The results from Scott and Gill's tests have provided the following information on the behaviour of axially reinforced tension prisms.

(*a*) The formation of a crack results in a reduction in the stiffness of the element. For a particular element, the reduction in stiffness caused by the formation of a crack is close to constant.
(*b*) The load–deformation response during load increments between the formation of one crack and the formation of the next crack is linear. Extrapolation of the straight-line relationship back to the axes shows that the lines pass very close to the origin, suggesting that the behaviour is effectively elastic.
(*c*) The distribution of the stresses in the reinforcement and the concrete over the length S_o on either side of a crack closely approximates to linearity.

These findings may be used to develop equations for the prediction of both the deformation and the crack width in axially reinforced prisms subjected to tension.

Figure 5 shows a schematic picture of the variation of strain in the region of a crack. Before a crack forms,

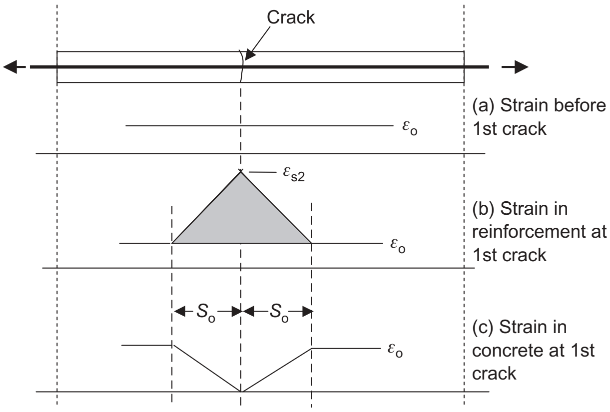

Fig. 5. Strain conditions in the region of the first crack

the strain in both the reinforcement and the concrete may be written as

$$\varepsilon_o = N/[E_c A_c(1 + \alpha_e \rho)] \tag{1}$$

From equation (1), the extension of the member without any cracks is given by

$$\Delta_a = NL/[E_c A_c(1 + \alpha_e \rho)]$$

N can, however, be written as

$$N = \varepsilon_{s2} A_s E_s \tag{2}$$

Substitution for N thus gives

$$\Delta_a = \varepsilon_{s2} A_s E_s L/[E_c A_c(1 + \alpha_e \rho)] = \varepsilon_{s2} \alpha_e \rho L/(1 + \alpha_e \rho) \tag{3}$$

The formation of a crack results in an extension of the reinforcement in the region of the crack equal to the shaded area in Fig. 5 divided by the modulus of elasticity of the steel. This area may conveniently be written as

$$\Delta_b = 2S_o \beta(\varepsilon_{s2} - \varepsilon_o)$$

Taking ε_o from equation (1) and N from equation (2), this can be simplified to give

$$\Delta_b = 2S_o \beta \varepsilon_{s2}/(1 + \alpha_e \rho) \tag{4}$$

The average strain can now be calculated from the total extension divided by the overall length of the specimen, L. This extension is given by

$$\text{Extension} = \Delta_a + n\Delta_b$$

and hence the average strain is given by

$$\varepsilon_{sm} = \varepsilon_{s2}(\alpha_e \rho + 2\beta S_o n/L)/(1 + \alpha_e \rho)$$

As it appears that the variation in stress in the reinforcement and concrete is linear over the length S_o, it can be assumed that $\beta = 0{\cdot}5$, giving

$$\varepsilon_{sm} = \varepsilon_{s2}(\alpha_e \rho + S_o n/L)/(1 + \alpha_e \rho) \tag{5}$$

Equation (5) can be seen to have exactly the same form as the line in Fig. 2. Since the crack spacing, S, can be expressed as $L/(n+1)$, n/L can be more conveniently expressed in terms of S, if desired.

These equations only apply rigorously in situations where the average crack spacing is greater than $2S_o$. It is possible to derive similar equations for situations where the spacing is less than $2S_o$. In this case, the stress in the reinforcement mid-way between cracks does not reduce to $\varepsilon_{s2}\alpha_e\rho/(1 + \alpha_e\rho)$ but, if the bond stress is assumed to remain constant, the resulting equation is

$$\varepsilon_{sm} = \varepsilon_{s2}[1 + \alpha_e \rho - S_{rm}/(4S_o)]/(1 + \alpha_e \rho) \tag{6}$$

A recent paper by Beeby and Scott[2] develops and tests equations (5) and (6) in detail. A formula is developed for the prediction of the crack spacing, S, which is needed in equations (5) and (6). Using this, it is shown that equation (5) predicts the deformation of a tension prism with considerable precision in cases where the spacing is greater than S_o. This is a larger range of applicability than is theoretically correct but, practically, a very useful conclusion.

The basic model developed from the test data can also be used to establish the expected crack width.

The extension of the reinforcement in the region of the crack, provided the crack spacing exceeds $2S_o$, is Δ_b, given by equation (4) above. This is not quite the complete crack width since, as well as an extension of the reinforcement, there is a contraction of the concrete in the region of the crack since the stress in the concrete has been reduced. The stress in the concrete at the end of the length S_o is given by

$$\sigma_{ct} = \varepsilon_{s2} A_s E_s / A_c(1 + \alpha_e \rho) = \varepsilon_{s2} \rho E_s/(1 + \alpha_e \rho)$$

From this, the increment in crack width caused by change in length of the concrete can be written as

$$\Delta_c = 2S_o \beta \alpha_e \rho \varepsilon_{s2}/(1 + \alpha_e \rho)$$

The total crack width is now given by

$$w = \Delta_b + \Delta_c = 2S_o \beta \varepsilon_{s2}(1 + \alpha_e \rho)/(1 + \alpha_e \rho) = 2S_o \beta \varepsilon_{s2}$$

If, as before, it is assumed that $\beta = 0{\cdot}5$ then

$$w = S_o \varepsilon_{s2} \tag{7}$$

ε_{s2} can be obtained directly from equation (2).

One point should be noted about this equation. Since it is assumed that the cracks are more than $2S_o$ apart, all cracks will depend on the deformations of the reinforcement and concrete over the full $2S_o$. All the cracks on the element will therefore be predicted to be the same width. In practice, this is clearly unlikely to be true and there will inevitably be a variability owing to the inherent variability in material properties from section to section along the member. This is likely to appear as a random variability in S_o along the element.

A different formula would be expected to apply when the average spacing became less than $2S_o$ and this can easily be derived as

$$w_{av} = S_{rm}\varepsilon_{s2}(1 - S_{rm}/4S_o) \quad (8)$$

This equation gives an average crack width as a function of the average crack spacing. It can be argued that the maximum crack width will still be given by equation (7) since the maximum width will correspond to the situation where the crack spacing on either side of the crack is just $2S_o$. Furthermore, equation (8) is difficult to apply. It requires calculation of the crack spacing under any particular loading. An equation for this has been presented[2] but this, in turn, depends upon the average strain. In practice, therefore, it may be appropriate to ignore equation (8) and use equation (7) for all situations. Equation (8) will therefore not be considered further in this paper.

A set of equations has now been derived for both the deformation and the crack widths in reinforced concrete prisms derived from the same model of behaviour of tension zones. This seems to be a valuable step in the development of rational equations for the prediction of serviceability behaviour. The remaining variable that needs to be established in order to produce a complete set of formulae for deflections and crack widths in tension members is the distance, S_o, over which the stresses are affected by a crack

Development of an expression for S_o

There are a number of ways by which values of S_o can be obtained from the available test data. The most obvious is that, where the distribution of strain or stress in the reinforcing bars has been measured, then S_o can be obtained directly from the distribution. This requires some care and is really only possible at early load stages before the cracks are sufficiently closely spaced to interfere with each other. Nevertheless, values of S_o have been obtained directly for most of Scott and Gill's specimens reinforced with deformed bars.

The second approach is to use the equation for deformations derived earlier (equation (5)). This can be rearranged to give

$$S_o = [\varepsilon_{sm}/\varepsilon_{s2}(1 + \alpha_e\rho) - \alpha_e\rho]L/n$$

Considering Fig. 1, it will be seen that $\varepsilon_{sm}/\varepsilon_{s2}$ is the gradient of the line drawn through the data points corresponding to a particular number of cracks. All other variables in the equation are known and hence S_o can be obtained. This has been done for all the square-sectioned prisms tested by Scott and Gill[5] and also a selection of the results obtained by Farra and Jaccoud.[6] Farra and Jaccoud tested a series of more than 130 prisms, all of which had a 100 mm square cross-section and were reinforced with either a 10, 14 or 20 mm axially placed bar. The main variable considered was the concrete mix, and a wide range of concrete strengths and cement types were used. For each mix and bar size, a set of three nominally identical specimens were cast and tested. For the purposes of this exercise, it seemed excessively time consuming to calculate S_o for all the specimens and so only a few determinations have been made for typical specimens for each bar diameter and a range of concrete strengths. Since S_o is predicted to vary largely as a function of geometric variables, carrying out determinations of S_o for a much greater number of specimens would simply have given a better picture of the scatter of results. The scatter can be investigated with much less effort when the crack width predicted by equation (7) is evaluated for a much larger population of Farra and Jaccoud's specimens in the next section of the report. Farra and Jaccoud tested their specimens using increments in strain rather than increments in load since they were primarily interested in cracking caused by restraint. They present detailed graphs and tables of stresses, strains and crack widths for all load stages for all specimens. In general, only one determination has been made for each specimen for the case where $n = 1$. This was difficult to do for specimens with low concrete strengths and the highest reinforcement ratio because only one or two strain readings were taken for these specimens when just one crack was present. To enable a result to be obtained in this case, the stiffnesses were calculated for the situations in which there were one, two and three cracks and the resulting assessment of S_o averaged.

Finally, Farra and Jaccoud measured extensive crack width data and this can be used to establish S_o directly from equation (7). Fig. 6 shows the average crack widths measured by Farra and Jaccoud for a series of three specimens reinforced with a 14 mm bar plotted against the strain in the reinforcement at a crack (σ_{s2}/E_s). A good linear relationship is obtained between crack width and steel strain and, according to equation (7), the gradient of this line is S_o. This approach has been used to establish S_o for the same specimens as had been used previously.

The values of S_o found by the three methods described above have been tabulated in Table 2. There is a certain amount of subjectivity in the measurement of

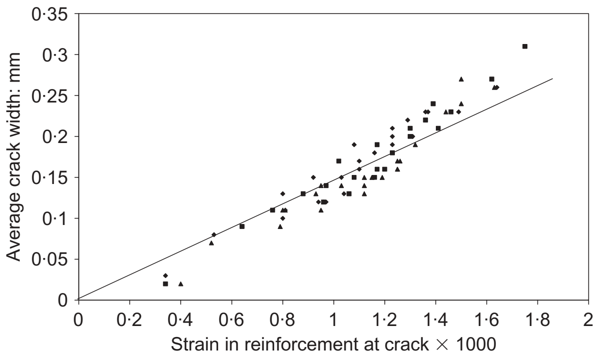

Fig. 6. Average crack width plotted against strain in reinforcement at a crack for all specimens of series H52-14[6]

Table 2. S_o obtained by various methods

Specimen No.	S_o measured from stress distribution	S_o calculated from stiffness	S_o calculated from crack widths.
Scott and Gill tests			
70T12	92	121	
100T12	140	115	
140T12	200	–	
100T20	135	128	
140T20	144	140	
200T20	262	282	
Farra tests			
N10-10		126	133
N10-20		102	113
H52-14		124	150
H52-20		144	133

S_o by the first method and in the establishment of the best-fit lines in the other two methods. Nevertheless, agreement between the methods is sufficiently good to give reasonable confidence that each method is measuring the same thing. It now remains to establish the factors that influence S_o. The most well-established theory for the prediction of cracking predicts that S_o will be proportional to ϕ/ρ. In a recent paper[7] Beeby showed, from an analysis of extensive cracking data from many sources, that ϕ/ρ has no discernable influence on cracking, and hence on S_o. It is shown that the critical parameter is the cover, c. This confirms the conclusions of earlier work in the UK[8] and USA.[9] Thus, S_o is plotted against cover in Fig. 7, which shows that a reasonable expression for S_o is:

$$S_o = 3{\cdot}05c \tag{9}$$

where c is the cover.

Validation of the equations

The validation of equations (1) and (2) has been covered fully[2] and will not be considered in detail here. It has been shown[3] that equation (1) will predict the average stress in the concrete in cracked prisms tested by Farra and Jaccoud[6] with a mean error of 0·08 N/mm^2 and a standard deviation of 0·29 N/mm^2. This prediction is considerably superior to other formulae considered. This paper focuses on consideration of the crack width prediction equations (equations (7) and (8)).

Figure 8 compares the calculated crack widths given by equation (7) with the experimental average crack widths obtained by Ramos and Serre.[10] Generally, agreement is very good. The slight tendency for equation (7) to overestimate the crack width may arise from the comparison of the predictions with the average crack widths rather than the maximum values, which were not reported. Ramos and Serre's data cover a wide variation in covers and bar diameters, but little variation in reinforcement ratio.

Figure 9 shows similar comparisons using data produced by Haqqi.[11] Haqqi tested larger tension specimens with multiple reinforcing bars with the objective of modelling more practical situations. The data used are for crack widths measured on the surface perpendicularly away from a reinforcing bar. The data cover a wide range of covers, reinforcement ratios and bar diameters. Agreement is not as good as that achieved with the data from Ramos and Serre, but it is still reasonable. The differences may result from the more

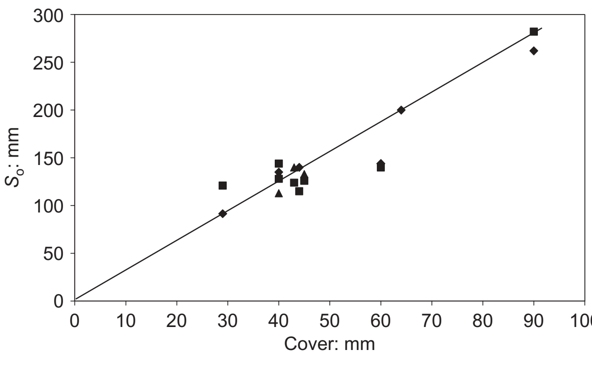

Fig. 7. Relationship between S_o and cover

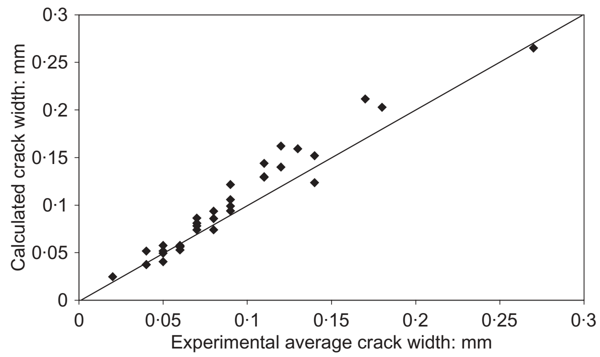

Fig. 8. Comparison of experimental average crack widths[10] with crack widths calculated using equation (7)

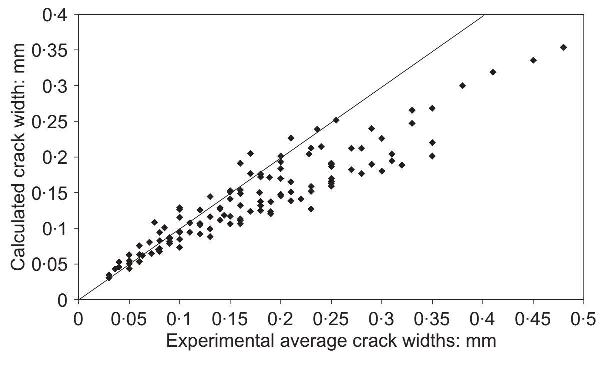

Fig. 9. Comparison of experimental average crack widths[11] and calculated crack widths

complex form of Haqqi's specimens and interaction between adjacent bars: a factor not taken into account in the derivation of the equations.

Figure 10 compares calculated widths with some experimental results from Farra and Jaccoud.[6] These tests consider three sections: 100 × 100 mm prisms reinforced with either 10, 14 or 20 mm diameter bars. This gives a good range of bar diameters and reinforcement ratios but little variation in cover. Some care needs to be taken with the selection of results from Farra and Jaccoud. Their tests were carried out by incrementing the strain rather than increasing the load, as was done with all the other data considered. The effect of this difference in test procedure is that, when a crack forms, the stiffness of the prism is reduced and the loads, stresses and crack widths reduce. Data have been selected which, as closely as possible, correspond to the maximum crack widths just before formation of a further crack. Furthermore, the major variable considered was the concrete mix design. It was found when using these tests to validate the equations for the prediction of the deformations that the crack formation varied quite considerably with variations in cement type. The reason for this variability is unclear, although it has been suggested[2] that it might be the result of changes in the coefficient of variation of the concrete tensile strength. As a result of this problem, crack widths have been used for only one cement type: Portland cement without the addition of silica fume. Fig. 10 shows that, using this limited set of data, good agreement is obtained between calculated and experimental crack widths. Including all the cements used significantly increases the scatter.

To gain an overall picture of the accuracy of the proposed formula, the data from the three sets of tests considered above have been combined. This gives 280 individual results. The average value of the ratio of the experimental crack width to the calculated width is 1·10 and the standard deviation of the ratio is 0·23. The results from Farra and Jaccoud[6] and Haqqi[11] have also been compared with widths calculated using the equation in the Comité Euro-International du Béton (CEB) model code.[12] In applying this formula, the value of $(\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs})$ has been taken as the measured average strain. The 36 results from Ramos and Serre[10] have not been used in this comparison because the average strain was not reported. The results from 244 comparisons of the CEB calculation with the experimental values gives an average ratio of the experimental crack width to the calculated width of 0·81 and the standard deviation of the ratio of 0·49, more than twice the value obtained for the proposed formula.

Overall, the comparisons carried out and presented in this section suggest that equation (7) gives a good estimate of the crack width in tension members, which are either centrally reinforced prisms or where the crack widths are measured directly over the bars. A previous paper by the current authors[2] shows that equation (4) predicts the deformation well. The basic approach to the prediction of the behaviour of tension members thus seems justified.

Mechanisms to accommodate high strains in reinforcement

A full understanding of the behaviour of concrete in tension around reinforcing bars requires that the mechanisms for accommodating strains in the reinforcement, which are substantially greater than the ultimate tensile strain of the concrete, are understood. Fig. 11 illustrates the issue. This shows the reinforcement strains at four stages during the loading of specimen T16B1.[1] It will be seen that at every stage above 8 kN, there is a major area adjacent to a crack or a free end of the specimen where the strain substantially exceeds the strain capacity of the concrete (assuming this to be in the region of $100 - 150 \times 10^{-6}$). Even at a load of 20 kN, which is below the cracking load, there is a length of approximately 200 mm at each end of the specimen where the strain exceeds the strain capacity of the concrete. Once the cracking has developed, the strain in the reinforcement exceeds the strain capacity

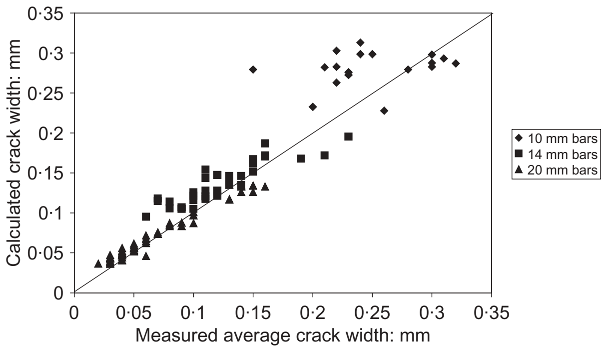

Fig. 10. Comparison of calculated and measured average crack widths[6] for prisms made from ordinary Portland cement concrete without silica fume

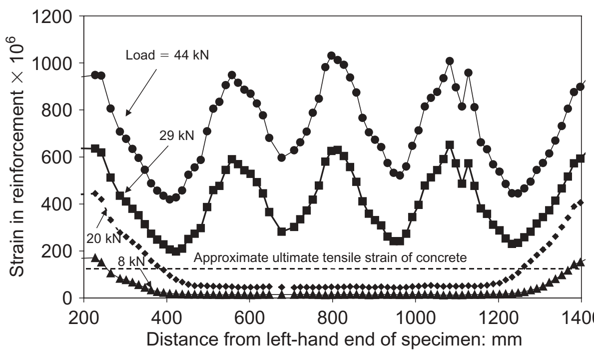

Fig. 11. Variation in reinforcement strain along the length of specimen T16B1[1] at various levels of axial load

of the concrete at all points along the specimen. Clearly, some mechanism must exist to accommodate this strain incompatibility.

Two mechanisms have been proposed. The first is bond–slip. This seems a rational assumption when the bars are smooth, but less so for ribbed bars. Nevertheless, this assumption of bond failure followed by slip between the bar and the concrete has remained the most commonly invoked concept and one which lies at the root of many models of behaviour. In particular, it lies at the root of most derivations of formulae for the prediction of crack widths and is enshrined in the CEB model code.[12] The alternative mechanism is internal cracking of the form demonstrated by Goto.[13] The two mechanisms are illustrated in Fig. 12. In the slip mechanism, a relationship has to be postulated between the slip at any point along the bar and the bond stress. Many forms of bond–slip relationship have been proposed. The CEB/FIP model code 90[12] provides what must be the closest to a 'consensus' view on bond–slip relationships and, for the low levels of slip obtaining under service conditions, this is given by the relationship:

$$\tau = \tau_{\max}(s/s_1)^{\alpha} \quad (10)$$

In the model code, $\tau_{\max}$ is taken as $2\sqrt{f_c}$, s_1 as 0·6 mm and α as 0·4 for the conditions appropriate to the tension tests.

Most classical derivations of crack width formulae assume that bond failure occurs and that the average bond stress can be uniquely related to the ultimate bond strength, independently of the amount of slip. These methods might be considered to assume a constant bond stress.

In the internal cracking model, the deformation of the bar is accommodated by the deformation of the teeth between the internal cracks. These teeth are in reality cones of concrete caused by the axi-symmetric nature of the bar and the surrounding concrete. No real attempt has been made to analyse this situation in the context of the development of crack prediction formulae. However, this mechanism has been invoked in the development of bond failure theories (e.g. see Tepfers[14]). A possible approach might reasonably be to consider the cones as conical springs. By analogy with the deformation of circular plates, the deformation at the inner edge of the cones could be expected to be proportional to the load applied by the interaction between the concrete and the rib. An attempt must be made to establish which of these mechanisms is most consistent with the behaviour revealed by the tests.

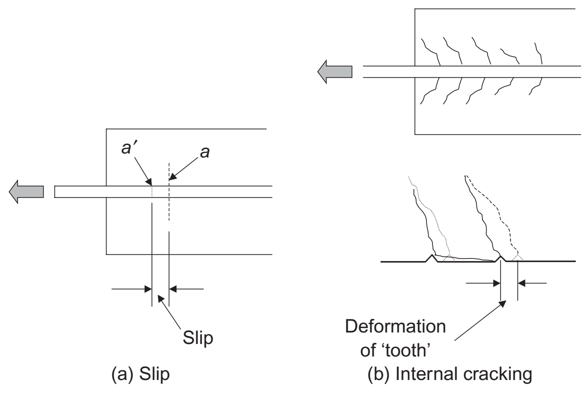

Fig. 12. Mechanisms for accommodating the strain incompatibility between the reinforcing bar and the concrete

Classical theories generally implicitly assume that the crack width is simply the sum of the slip on either side of the crack and that the width is constant between the bar and the concrete surface. A considerable number of studies have been carried out to investigate this and the resulting information on the shape of cracks has been considered in some detail[1] and has been summarised by Farra and Jaccoud.[6] These studies clearly show that the deformation of the crack faces is more consistent with the internal cracking model than with a bond–slip model. Also, the problem with the bond–slip model is the necessity for the concrete to be able to slip past the ribs on the bar. If this occurs, there should be clear and widespread evidence of disruption of the concrete around the ribs. Inspection of the concrete interface revealed that, after breaking the concrete off, the bar rarely shows any sign of this at the level of load relevant to serviceability considerations.

The difference between the two mechanisms may not be too revolutionary: bond stress is simply a shear stress at the interface between the steel and concrete. Failure could either occur by failure along the interface or as cracks in the concrete which form under action of the principal tensile stress.[13] This would be expected to be at some angle to the axis of the bar. In other forms of reinforced concrete behaviour, such as punching shear, it is clearly the diagonal cracking which occurs rather than a vertical failure along the slab–column interface.

It may help further discussion to set down some experimental results which will show what a viable model needs to be able to predict. The experimental results show the following.

(*a*) The variation in the stress in the reinforcement over the length S_o is close to linear. This implies a constant bond stress over this length.
(*b*) There is little evidence of general slip.
(*c*) The bond stress increases with increasing load.
(*d*) Equations (5) and (7) appear to predict behaviour well and have been derived on the assumption of elastic behaviour punctuated by discrete cracking events.
(*e*) It has been shown by Beeby[7] and in the present paper that S_o and crack widths are proportional to cover and independent of ϕ or ρ.

The bond–slip model can now be considered to see how well it fits these requirements. No detailed model for cracking has been developed for an internal cracking mechanism so a study will have to be initiated to establish whether such a model could, potentially incorporate these requirements.

There are, in fact, two mutually incompatible bond–slip models. The classical model, on which the CEB model code crack formula is based, assumes that bond failure has taken place at some point along S_o and so the average bond stress can be related directly to the ultimate bond strength. It is not essential to assume a uniform bond stress along the length S_o, although this is often done. Since the bond stress has reached its ultimate value, however, there can be no further increase in bond stress with increase in load. This violates point (*c*) above. Point (*d*) is also probably violated and the lack of evidence for bond failure and slip at the bar–concrete interface also counts against this model. Finally, the model predicts that cracking will be a function of ϕ/ρ, which has been shown to be untrue.[7]

The second bond–slip model is to assume a bond–slip relationship obtained from tests such as the CEB relationship given by equation (10). This is incompatible with the model used for the CEB crack prediction formula because bond failure is not predicted to occur until a slip of 0·6 mm, corresponding to a crack width at the bar surface of 1·2 mm and a width at the concrete surface of possibly 2 mm or more. In the range of slips appropriate to serviceability behaviour, the bond stress is predicted to increase with increasing load. This model could therefore satisfy requirements (*b*), (*c*) and (*d*) above. There are, however, difficulties with requirement (*a*). Any slip model requires that there is zero slip at the end of S_o remote from the crack and then increases up to about half the crack width at the crack face. The bond–slip model would thus require that the bond stress increased from zero at the end of S_o remote from the crack and reached a maximum at the crack. This contradicts the clear experimental evidence of a more-or-less constant bond stress over the whole of S_o. Bond–slip models thus appear unable to explain the actual behaviour, and a viable model must be found elsewhere.

Development of an internal cracking model

The problem which needs to be addressed is to formulate a model in which the transfer of stress between the reinforcing bar and the concrete can be predicted, allowing for the presence of internal cracks. The model selected is that of a bar restrained by compression springs at intervals along the bar. This is illustrated in Fig. 13. Each pair of springs models the stiffness of a compression strut in the concrete between two internal cracks. These compression struts are actually akin to conical springs, bearing in mind the axial symmetry of the system. The chain dotted line to the right of Fig. 13 indicates either the point at which there is zero stress in the bar or the section mid-way between cracks where, by symmetry, there is no horizontal movement of the bar. If stiffnesses are ascribed to the springs and the bar and the distance ΔL is defined, then the variation of stress along the bar can easily be calculated.

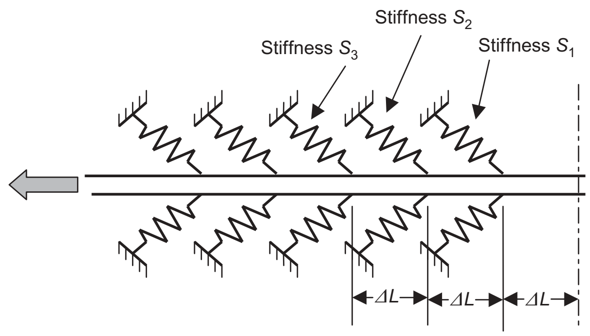

Fig. 13. Schematic diagram of proposed bar–concrete interaction

Inspection of standard textbooks on elasticity shows that the deformation at the centre of a circular plate, regardless of the boundary conditions, can be expressed by the relation:

$$\delta = kN(1 - \nu^2)R^2/(Et^3)$$

where δ = the deflection at the centre

k = a constant depending on the boundary conditions and form of the loading
N = the applied load
R = the radius of the plate
E = the modulus of elasticity of the material
t = the plate thickness
ν = Poisson's ratio.

For a given configuration of conical spring and a given material, k, t, E and ν will be constants so that the equation reduces to:

$$\delta_i = k' N_i R_i^2 \text{ or } k'/R_i^2 = N_i/\delta_i \qquad (11)$$

where the subscript 'i' indicates the ith tooth.

If a linear variation in steel stress is to be modelled then, for a given overall axial load, the load N_i carried by each tooth should be constant along the length S_o. If the model is to give a linear increase in stress in the reinforcement with distance from the end of S_o remote from a crack, the displacement of the tooth, δ_i, at the bar surface can be written as:

$$\delta_i = \sigma_s x/2E_s$$

but $\sigma_s = kx$ and hence $\delta_i = kx^2/2E_s$

Substituting for δ in equation (11) and rearranging gives:

$$R_i = k'x/\sqrt{(2E_s N_i)} \qquad (12)$$

Thus, the model suggests that, for a linear change in steel strain or stress, the height of the cracks, R_i should be proportional to distance from the end of S_o remote from the crack, x. There is some experimental evidence to support this. Fig. 14 shows the internal cracks around bars anchored in blocks of concrete and then pulled. The figure is drawn from a photograph in a paper by Otsuka and Ozaka.[15] The figure shows a more-or-less linear decrease in crack length with increasing distance from the face of the concrete.

The possibility of change in length of the cracks with increase in load on the prisms has not yet been considered. Intuitively, the crack length and hence the stiffness of the interface could be expected to decrease with increase in loading as the deformation of each tooth must increase with increase in load. The model can be studied to establish what might happen. If it is assumed that bond is entirely elastic and that the bond stress increases linearly with increase in applied load, then the internal cracking would have to develop immediately on formation of the crack and would then have to stay constant. If, on the other hand, it was assumed that the bond stress remained constant with increase in loading, then equation (11) suggests that R at any point along S_o should vary proportionally to the square root of the load applied to the bar at a crack. The test results suggest that, at low levels of loading above the cracking load, the bond is elastic while at higher loads, the bond stress tends to approach a constant value. This suggests that the internal crack pattern forms immediately on formation of the crack and that the length of these cracks remains sensibly constant for some considerable increase in loading. Beyond a certain point, however, the lengths of the internal cracks start to increase. At high loads, the rate of increase approaches proportionality with the square root of the applied load. The effectiveness of the equations derived earlier for crack width and deformation, which assume elastic bond, shows that, in practice, the assumption of elastic bond appears practically reasonable.

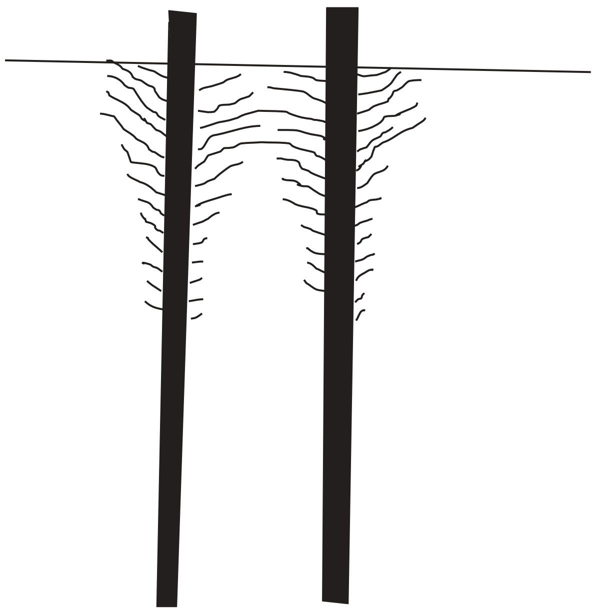

Fig. 14. Internal cracks around anchored bars[15]

Discussion

The object of this brief discussion is simply to clarify what the work reported in this paper claims to have achieved. The study has been concerned almost entirely with the behaviour of axially reinforced prisms. Such elements have been used frequently in the past as the basis for deriving formulae for the prediction of cracking and the results from prisms have been assumed to apply to any other tension zone; comparative behaviour being obtained simply by the empirical definition of an appropriate effective area of concrete surrounding the bar. It is not intended to claim that this can necessarily be done with the results reported here and it is not claimed that the formulae can reliably be applied at present to any elements other than axially reinforced prisms, although further work may enable a more general application. Particular issues which remain to be considered are (*a*) the effect of multiple bars and hence the influence of bar spacing and (*b*) the rather different stress conditions which occur in members subjected to bending where there is a strain gradient across the tension zone.

What is claimed is that the research has shed much light on the actual behaviour of tension zones under conditions where this can be established unambiguously. In the present authors' view, the behavioural mechanisms revealed or postulated are likely to hold generally true in all cracked tension zones, although there may be further mechanisms acting in more complex situations. While these further mechanisms may complicate the situation, the present authors believe that the fundamental mechanisms will remain valid. The equations derived for the prediction of crack widths and deformations perform a useful purpose in that they enable the reasonableness of the proposed mechanism to be tested for this simple experimental arrangement. The results of this testing have been highly satisfactory. The current authors are therefore reasonably confident that the basic aspects of the behaviour of reinforced tension zones have been correctly identified. Further work will, however, be required to develop generally applicable practical formulae. The formulae derived in this paper should turn out to be limiting cases of more general formulae. How this greater understanding will eventually influence practice remains to be seen and will depend upon the further research outlined above. However, tension stiffening is a major factor in the prediction of the deformation of lightly reinforced members such as slabs. In such members, deformation is frequently a major defining parameter in the design and any improvements in prediction methods will be of practical significance. Worldwide, there is remarkably little consensus on the cracking behaviour of reinforced concrete, despite the derivation of very many formulae. Hopefully, this research will help to develop a clearer consensus on the behaviour of concrete in tension surrounding reinforcement and will lead to more rational formulae in due course.

Conclusions

(*a*) Study of experimental results where the variation in the strains in the reinforcement can be measured in detail shows that the stiffness of an axially reinforced tension member is directly related to the number of cracks. For a given number of cracks, the behaviour is elastic.

(*b*) The experimental evidence also shows that the variation in reinforcement strain is linear in the region where it is affected by a crack.

(*c*) The length of the specimen over which the crack influences the stress distribution, S_o, is shown to be proportional to the cover.

(*d*) On the basis of these findings, formulae for the prediction of deformations and crack widths can be derived. These equations are shown to give good predictions of behaviour.

(*e*) Consideration is given to the mechanism by which the incompatibility between the deformation of the reinforcement and the strain capacity of the surrounding concrete is accommodated. It is concluded that the most likely mechanism is internal cracking rather than bond failure. Initial analysis of an internal cracking model suggests that it is capable of modelling all aspects of the observed behaviour.

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References

1. Beeby A. W. and Scott R. H. *Influence of Tension Stiffening on Deflection of Reinforced Concrete Structures*. The Concrete Society, Camberley, 2003, Technical Report 59.
2. Beeby A. W. and Scott R. H. Insights into the cracking and tension stiffening behaviour of reinforced concrete tension members revealed by computer modelling. *Magazine of Concrete Research*, 2004, **56**, No. 3, 179–190.
3. Beeby A. W., Scott R. H. and Jones A. E. K. Revised code provisions for long term deflection calculations. *Proceedings of Institution of Civil Engineers—Structures and Buildings*, 2005, **158**, No. 1, 71–75.
4. Beeby A. W. and Scott R. H. and Antonopoulos A. Preliminary results for long term tension stiffening effects in reinforced concrete members. *Concrete Floors and Slabs* (Dhir R. K., Newlands M. D. and Harrison T. A. (eds)), Thomas Telford Publishing, London, 2002, pp. 327–335.
5. Scott R. H. and Gill P. A. T. Short term distributions of strain and bond stress along tension reinforcement. *The Structural Engineer*, 1987, **65B**, No. 2, 39–43.
6. Farra B. and Jaccoud J.-P. *Influence du Beton et de l'armature sur la fissuration des structures en Beton*. Rapport des essays de tirants sous deformation impose de court duree. Department de Genie Civil, Ecole Polytechnique Federale de Lausanne, November 1993, Publication No. 140.
7. Beeby A. W. The influence of the parameter ϕ/ρ_{eff} on crack widths. *Structural Concete*, 2004, **5**, No. 2, 71–83.
8. Base G. D., Read J. B., Beeby A. W. and Taylor H. P. J. *An investigation of the crack control characteristics of various types of bar in reinforced concrete beams*. Cement and Concrete Association Research, December 1966, Report 18.
9. Broms B. B. and Lutz L. A. Effects of arrangement of reinforcement on crack widths and spacings of reinforced concrete members. *ACI Journal*, 1965, **62**, No. 11, 1395–1410.
10. Ramos C. R. and Serre J. P. *Determinacion Experimental del Coeficiente de Fisuracion en Barras de Acero utilizadas en Concreto Armado*. Instituto de Materials y Modelos Estructurales. Universidad Central de Venezuela. Boletin Tecnico, July–Sept 1969, pp. 5–37.
11. Haqqi I. S. *Serviceability of reinforced concrete subjected to tension*. PhD Thesis, Council for National Academic Awards, London, 1983.
12. Comité Euro-International du Béton. CEB-*FIP Model Code 1990*. Bulletin d'Information No. 213/214, Lausanne, May 1993.
13. Goto Y. Cracks formed in concrete around deformed tension bars. *Journal of the American Concrete Institute,* 1972, **68**, No. 4, 244–251.
14. Tepfers R. Tensile lap splices with confining reinforcement. *Bond in Concrete* (Bartos P. (ed.)). Applied Science Publishers, London, 1982, pp. 318–330.
15. Otsuka K. and Ozaka Y. Group effects on anchorage strength of deformed bars embedded in massive concrete block. *Proceedings of International Conference on Bond in Concrete—From Research to Practice*, Riga Technical University, Riga, October 1992, Vol. 1. pp. 1·38–1·47.

Discussion contributions on this paper should reach the editor by 1 June 2006