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Causally pathological spacetimes are physically relevant

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Abstract

We argue that in the context of string theory, the usual restriction to globally hyperbolic spacetimes should be considerably relaxed. We exhibit an example of a spacetime which only satisfies the causal condition, and so is arbitrarily close to admitting closed causal curves, but which has a well-behaved dual description, free of paradoxes.

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A striking feature of general relativity is that it has solutions with nontrivial, and sometimes even pathological, causal properties. Although the local light cone is that of special relativity, the global causal properties can be completely different; spacetimes can have closed timelike curves. Observers in such a spacetime can return to the same point in spacetime, leading to the famous "grandfather paradox".

Time machines have continued to fascinate the science-fiction writer and the student of general relativity alike. Although many solutions with closed timelike curves have been found, it is much more difficult to construct examples where a time machine develops from causally regular initial conditions. In the Kerr solution, for example, there is a region of closed timelike curves inside the inner horizon, but this horizon is believed to be unstable due to an infinite blueshift. This has led to the celebrated chronology protection conjecture [1], which asserts the impossibility of forming a time machine. This has remained an open question, and a full resolution may require a deeper understanding of quantum gravity. In practical investigations, however (for example, in numerical relativity, and canonical approaches to quantum gravity), attention is commonly restricted to globally hyperbolic spacetimes, which admit a global Cauchy surface.

String theory is a candidate theory of quantum gravity, and it has now been developed to the point where we have a well-studied proposal for a nonperturbative and background independent description of a certain class of spacetimes, via the AdS/CFT dualities [2]. It is interesting to ask what this proposal has to say about the question of causality violations. The first thing to note is that the usual restriction to globally hyperbolic spacetimes must be relaxed, as Anti-de Sitter space, the paradigmatic example of the duality, is not globally hyperbolic.

How far do we need to relax? A rich hierarchy of causality properties, weaker than global hyperbolicity, has been defined in the past. In order of increasing strength, the most common conditions are: causal $<$ weakly distinguishing $<$ (future and past) distinguishing $<$ strongly causal $<$ stably causal $<$ globally hyperbolic. (For a review, see [3].) The most natural criterion would be to consider stably causal spacetimes. A spacetime is *stably causal* if it remains causal under any sufficiently small deformation of the metric. In a theory of quantum gravity, where the metric is subject to quantum fluctuations, one would therefore expect that stable causality should constitute the minimal requirement needed to avoid possible pathologies. Also, spacetimes which are causal but not stably causal must, by definition, form a subset of measure zero in the space of continuous metrics, and until recently, few physically interesting examples of such spacetimes were known. In standard texts such as [3], examples of such spacetimes are constructed by the rather unphysical method of excising points from the spacetime manifold.

Indeed, Anti-de Sitter space is stably causal. The failure of global hyperbolicity is a result of the asymptotic behaviour: it has a timelike conformal boundary. This is important, as it implies that once we specify appropriate boundary conditions on this conformal boundary, the spacetime will have a well-defined initial value problem, defining 'Cauchy' evolution from an initial slice. There is a Cauchy horizon associated with each Cauchy surface in this spacetime, but unlike in the example of the Kerr spacetime, it is not a surface of infinite blueshift. Hence there is no natural mechanism to render the physical part of the spacetime globally hyperbolic. The failure of global hyperbolicity is thereby closely tied to the dual CFT, whose definition is linked to the asymptotic boundary conditions in spacetime.

Another interesting class of backgrounds which are stably causal but not globally hyperbolic are plane waves. The story is similar for these solutions. It was observed by Penrose [4] that they are not globally hyperbolic spacetimes, and more recently, they were shown to be stably causal [5]. The failure of global hyperbolicity is again associated with the asymptotic behaviour of the spacetime. A family of plane wave spacetimes has been shown to have a one-dimensional null boundary [6, 7], which is both to the past and future of points in the interior of the spacetime. Given boundary conditions on this conformal boundary, a well-defined initial data problem should exist. Certain plane waves have also been related to dual field theory descriptions through the BMN correspondence [8], which is obtained by considering the plane wave that arises in the Penrose limit of an asymptotically AdS spacetime.

The previous two examples suggest a picture where we relax the condition of global hyperbolicity only slightly, considering stably causal spacetimes which, once we specify appropriate asymptotic boundary conditions, still have a well-defined initial value problem.

However, we now wish to argue that we actually need to relax it much more radically, and extend our criterion to include causal spacetimes which are not even distinguishing. (A spacetime is *causal* if it does not contain any closed causal curves, and it is *distinguishing* if distinct points have distinct causal past and future sets.)

The first evidence that we should allow ourselves to consider spacetimes with such weak causal properties comes from the recent construction of simple uncontrived examples. It was pointed out in [5, 9] that for pp-waves[1], whose metric can be written as

$$ds^2 = -2\,du\,dv - F(u, x^i)\,du^2 + dx^i\,dx^i,$$

if the function $F(u, x^i)$ grows faster than quadratically in x^i for some x^i, the spacetime is actually *not* stably causal; in fact, it is not even distinguishing! Indeed, such pp-waves are rather spectacularly non-distinguishing, as *all* spacetime points with the same u coordinate (say $u = u_0$) have the same past and future, consisting of all points with $u < u_0$ and $u > u_0$, respectively. In [10] it was shown that some examples, where the function $F(u, x^i)$ grew exponentially at large x^i for some x^i, could be described in string theory, via a world-sheet conformal field theory with no obvious pathologies.

However, lacking a full non-perturbative description, one might still question whether such spacetimes can be physically relevant, as they are arbitrarily close to developing closed timelike

[1]pp-waves are generalizations of plane waves, which still have a covariantly constant null Killing field, but are less symmetric.

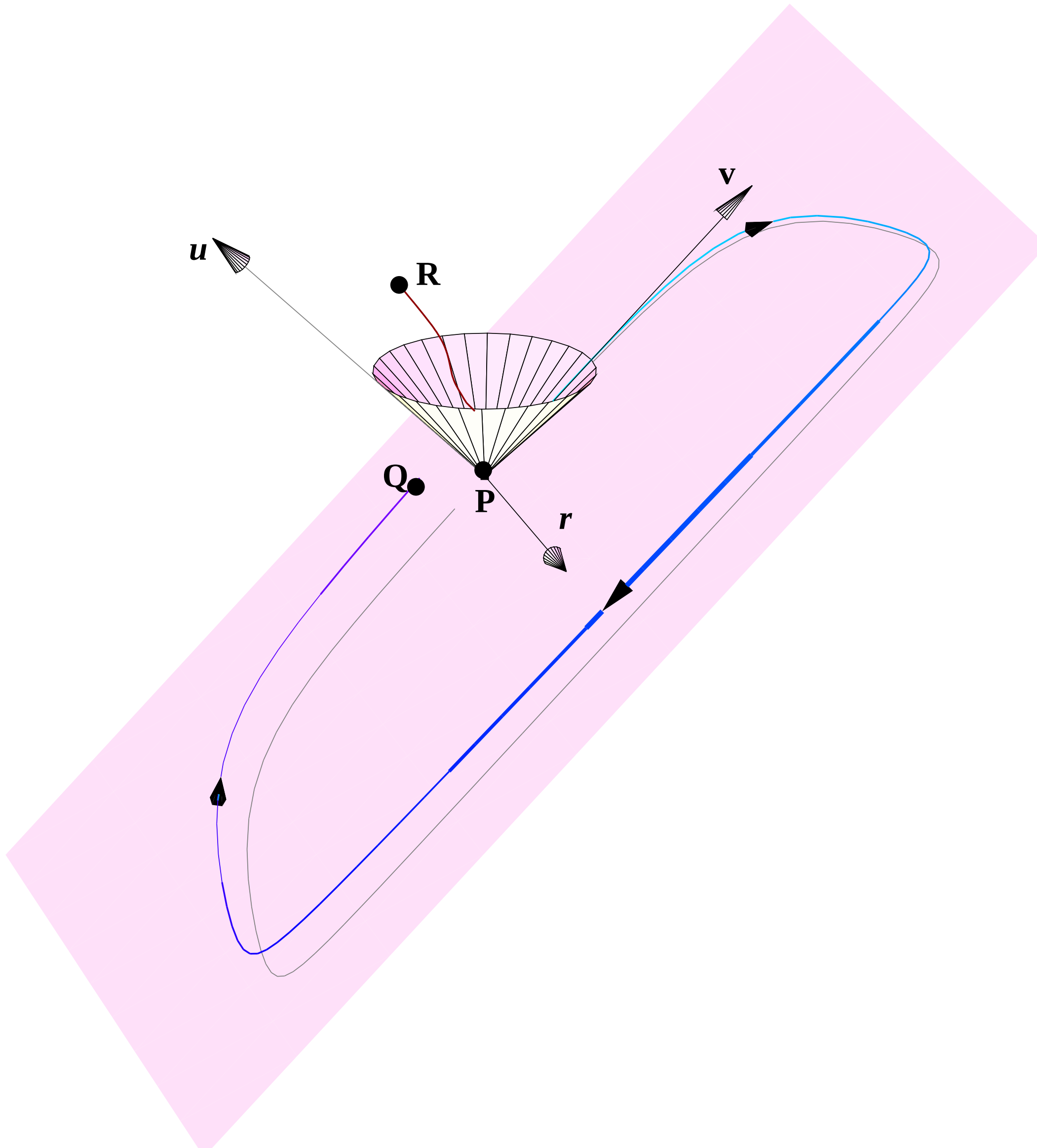

Figure 1: Light cone structure in the non-distinguishing spacetime (1). From a point $\mathbf{P}$ in the spacetime, it is possible to reach point $\mathbf{R}$ by a usual causal curve. To reach point $\mathbf{Q}$ (which is "close" to $\mathbf{P}$) we need to go out towards the asymptotic region and then loop back. Such a curve is drawn above along with its projection onto a constant $u = u_{\mathbf{P}}$ plane. By going out further in r we can engineer curves to return arbitrarily close to our starting point. Existence of these causal curves implies that the causal future of $\mathbf{P}$ is the part of the spacetime above the plane of constant u drawn in the figure. Contrast this with the flat space light cone of the point.

curves. A new example, which is related to a field theory description via an AdS/CFT type duality, provides strong evidence that in fact they are physically relevant. The metric may be written as

$$\begin{aligned} ds^2 &= r^2 \left(-2\,du\,dv - \beta^2\, r^4\, du^2 + dx^2 + dy^2 + \frac{dr^2}{r^4} + \frac{1}{r^2}\, d\Omega_5^2 \right) \\ &= \frac{1}{z^2} \left(-2\,du\,dv - \frac{\beta^2}{z^4}\, du^2 + dx^2 + dy^2 + dz^2 + z^2\, d\Omega_5^2 \right) \ . \end{aligned} \tag{1}$$

The second line, included to explicitly demonstrate the spacetime being conformal to a pp-wave, is obtained by $z = 1/r$. The asymptotic behaviour of the metric as $r \to \infty$ is responsible for its non-distinguishingness. Note that for $\beta = 0$ the metric is that of $\mathrm{AdS}_5 \times \mathbf{S}^5$ in Poincaré coordinates. In Fig. 1 we illustrate some causal properties of the spacetime (1).

This geometry can be obtained by a solution generating transformation from the Anti-de Sitter spacetime involved in the AdS/CFT correspondence (for details, see [11]). One can consider the action of this transformation in the field theory variables, to show that this geometry is also related to a field theory "living on the boundary" — in the present case the dual is a lightlike non-commutative field theory. This is a non-local theory, but it nevertheless obeys all the usual properties of a quantum field theory and hence serves as a non-perturbative description of quantum gravity in the background (1). We take the view that any spacetime which is dual to a well-defined quantum field theory must itself be sensible (physically relevant). The field theory description guarantees that the causal pathologies cannot lead to paradoxes. One can in fact show that the non-distinguishing nature of the spacetime is a necessary consequence of the non-local interactions of the lightlike non-commutative field theory; the micro-causality conditions in the field theory force this unusual causal behaviour on the dual spacetime!

Note that in (1), the violation of stronger causality conditions comes again from the asymptotic region of spacetime, at $r \to \infty$. Since the relation to the field theory fixes the asymptotic boundary conditions, the metric should not be thought of as freely fluctuating in this $r \to \infty$ region. Hence, it will not suffer from any problems associated with closed timelike curves, even though the field theory really corresponds to treating the metric quantum mechanically. (As a result, we do not regard this example as evidence that we should consider spacetimes with closed timelike curves as physical.)

Thus, in the AdS/CFT context, we are driven to relax the causality requirement as far as possible: not only is global hyperbolicity too stringent a requirement, but even the weak distinguishing property appears to be too constraining! There exist spacetimes which are arbitrarily close to admitting closed causal curves but which are still dual to sensible quantum field theories, free of paradoxes. We therefore conclude that spacetimes wherein the asymptotic behaviour leads to violations of the stronger causality conditions may play an important role.

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