The SM prediction of $g - 2$ of the muon

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Abstract

We calculate $(g - 2)/2$ of the muon, by improving the determination of the hadronic vacuum polarisation contribution, $a_\mu^{\mathrm{had,LO}}$, and its uncertainties. The different e^+e^- data sets for each exclusive (and the inclusive) channel are combined in order to obtain the optimum estimate of the cross sections and their uncertainties. QCD sum rules are evaluated in order to resolve an apparent discrepancy between the inclusive data and the sum of the exclusive channels. We conclude $a_\mu^{\mathrm{had,LO}} = (683.1 \pm 5.9_{\mathrm{exp}} \pm 2.0_{\mathrm{rad}}) \times 10^{-10}$ which, when combined with the other contributions to $(g - 2)/2$, is about 3σ below the present world average measurement.

1 Introduction

The muon anomalous magnetic moment, $a_\mu \equiv (g_\mu - 2)/2$, is one of the most precisely measured quantities in contemporary particle physics. The world average of the existing measurements is

$$a_\mu^{\mathrm{exp}} = 11659203(8) \times 10^{-10}, \tag{1}$$

which is dominated by the recent value obtained by the E821 collaboration at BNL[1]. It is so precisely measured that it is very useful in probing and constraining New Physics beyond the Standard Model (SM). It is therefore important to evaluate the SM prediction of a_μ as accurately as possible.

The SM contribution to a_μ may be written as the sum of three terms,

$$a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{had}}. \tag{2}$$

The QED contribution, a_μ^{QED}, has been calculated up to and including estimates of the 5-loop contribution, see reviews [2, 3],

$$a_\mu^{\mathrm{QED}} = 116\ 584\ 705.6(2.9) \times 10^{-11}. \tag{3}$$

In comparison with the experimental error in eq. (1), and with the hadronic contribution error discussed later, the uncertainty in a_μ^{QED} is much less important than other sources of uncertainty. The electroweak contribution a_μ^{EW} is calculated through second order to be [4, 5, 6]

$$a_\mu^{\mathrm{EW}} = 152(1) \times 10^{-11}. \tag{4}$$

Here again the error is negligibly small.

Less accurately known is the hadronic contribution a_μ^{had}. It can be divided into three pieces,

$$a_\mu^{\mathrm{had}} = a_\mu^{\mathrm{had,LO}} + a_\mu^{\mathrm{had,NLO}} + a_\mu^{\mathrm{had,l\text{-}b\text{-}l}}. \tag{5}$$

The lowest-order (vacuum polarisation) hadronic contribution, $a_\mu^{\mathrm{had,LO}}$, has been calculated by a number of groups. The value

$$a_\mu^{\mathrm{had,LO}} = 6\ 924(62) \times 10^{-11} \tag{6}$$

taken from Ref. [7] has been frequently used in making comparisons with the data. The next-to-leading order hadronic contribution, $a_\mu^{\mathrm{had,NLO}}$, is evaluated to be [8, 9]

$$a_\mu^{\mathrm{had,NLO}} = -100(6) \times 10^{-11}. \tag{7}$$

The hadronic light-by-light scattering contribution $a_\mu^{\mathrm{had,l\text{-}b\text{-}l}}$ has been recently reevaluated [10]–[15], and it is found to be

$$a_\mu^{\mathrm{had,l\text{-}b\text{-}l}} = 80(40) \times 10^{-11}, \tag{8}$$

where we quote the estimate of the full hadronic light-by-light contributions given in Ref. [16]. From eqs. (6), (7) and (8), we can see that $a_\mu^{\mathrm{had,LO}}$ has the largest uncertainty, although the uncertainty in the light-by-light contribution $a_\mu^{\mathrm{had,l\text{-}b\text{-}l}}$ is also large.

In this letter we update the evaluation of $a_\mu^{\mathrm{had,LO}}$, which is given by the dispersion relation

$$a_\mu^{\mathrm{had,LO}} = \frac{1}{4\pi^3} \int_{s_{\mathrm{th}}}^{\infty} ds\ \sigma_{\mathrm{had}}(s) \left(\frac{m_\mu^2}{3s}\ K(s) \right), \tag{9}$$

where $\sigma_{\text{had}}(s)$ is the total cross section for $e^+e^- \to$ hadrons $(+\gamma)$ at centre-of-mass energy $\sqrt{s}$. The kernel function $K(s)$ is given by

$$K(s > 4m_\mu^2) = \frac{3s}{m_\mu^2}\left\{\frac{x^2}{2}(2-x^2) + \frac{(1+x^2)(1+x)^2}{x^2}\left(\ln(1+x) - x + \frac{x^2}{2}\right) + \frac{1+x}{1-x}x^2\ln x\right\}, \quad (10)$$

with $x \equiv (1 - \beta_\mu)/(1 + \beta_\mu)$ where $\beta_\mu \equiv \sqrt{1 - 4m_\mu^2/s}$. $K(s > 4m_\mu^2)$ increases monotonically from 0.63 to 1 in the range $4m_\pi^2 < s < \infty$. The form of K for $s < 4m_\pi^2$ is given in [17], and is used to evaluate the small $\pi^0\gamma$ contribution to $a_\mu^{\text{had,LO}}$.

To evaluate $\sigma_{\text{had}}(s)$, we use experimental data up to 11.09 GeV and perturbative QCD thereafter. The most important contribution to $\sigma_{\text{had}}(s)$ comes from the $e^+e^- \to \pi^+\pi^-$ channel; the channels $\pi^+\pi^-\pi^0$, K^+K^-, $K^0_S K^0_L$, $\pi^+\pi^-\pi^+\pi^-$, $\pi^+\pi^-\pi^0\pi^0$, etc. give subleading contributions. For example, if we evaluate $\sigma_{\text{had}}(s)$ using the sum of the data for the above 6 exclusive channels up to $\sqrt{s} = 1.43\text{GeV}$, we obtain 87% of the total $a_\mu^{\text{had,LO}}$, with the $\pi^+\pi^-$ contributing 72%. Then if from 1.43 to 11.09 GeV we were to use the measurements $R(s) \equiv \sigma_{\text{had}}(s)/\sigma(e^+e^- \to \mu^+\mu^-)$ to determine $\sigma_{\text{had}}(s)$, we would obtain another 11% of the total.

As mentioned, our study to update $a_\mu^{\text{had,LO}}$ is motivated by the increasing accuracy of the experimental value for a_μ^{exp}, see eq. (1). The special features of our analysis are (i) that it is data-driven, based on all available data, including the new data on exclusive channels from Novosibirsk, particularly $\pi^+\pi^-$ [18], and the BES data on the R ratio [19], (ii) the careful application of a clustering method, so that data of differing precision can be combined consistently, and (iii) the use of QCD sum rules to resolve an apparent discrepancy between the inclusive and exclusive determination of $\sigma_{\text{had}}(s)$ for $1.4 \lesssim \sqrt{s} \lesssim 2$ GeV.

We chose not to use data on hadronic τ decays to further constrain the $e^+e^- \to 2\pi, 4\pi$ channels for $\sqrt{s} \lesssim m_\tau$, because of the possible uncertainties connected with isospin-breaking effects. A careful study of the effects of including τ data has been made in Ref. [20].

2 Processing the hadronic data

We apply the hadronic vacuum polarisation corrections given by Swartz [21]. Moreover, we calculated the final state radiative effects for all $e^+e^- \to \pi^+\pi^-$ data, except for the new CMD–2 data [18], based on eq. (45) of Ref. [22]. These two corrections increase the $\pi^+\pi^-$ contribution by about 1.1×10^{-10}, to which we assign a 50% error. For the dominant CMD–2 data the radiative corrections are already included by simply taking the cross section numbers for $\sigma^0_{\pi\pi(\gamma)}$. Since the recent accurate data [23] in the ω and ϕ resonance regions have not been corrected for any vacuum polarisation effects (see the comment in [20]), we apply the full (including the lepton vacuum polarisation) corrections to these data. For the data on the other exclusive channels, and the inclusive data not discussed in [21], insufficient information is available to make reliable radiative corrections. We therefore assign an additional $\pm1\%$ uncertainty to their contribution to $a_\mu^{\text{had,LO}}$. The net effect is an error of about $\pm2 \times 10^{-10}$ due to radiative corrections; a further discussion will be given in [24].

We now come to the important problem of 'clustering' data from different experiments (for the same hadronic channel). To combine all data points for the same channel which fall in suitably chosen (narrow) energy bins, we determine the mean R values and their errors for all clusters by minimising the non-linear χ^2 function

$$\chi^2(R_m, f_k) = \sum_{k=1}^{N_{\text{exp}}} \left[(1 - f_k)/\text{d}f_k\right]^2 + \sum_{m=1}^{N_{\text{clust}}} \sum_{i=1}^{N_{\{k,m\}}} \left[\left(R_i^{\{k,m\}} - f_k R_m\right)/\text{d}R_i^{\{k,m\}}\right]^2. \quad (11)$$

Here R_m and f_k are the fit parameters for the mean R value of the mth cluster and the overall normalization factor of the kth experiment, respectively. $R_i^{\{k,m\}}$ and $\mathrm{d}R_i^{\{k,m\}}$ are the R values and errors from experiment k contributing to cluster m. For $\mathrm{d}R_i^{\{k,m\}}$ the statistical and point-to-point systematic errors are added in quadrature, whereas $\mathrm{d}f_k$ is the overall systematic error of the kth experiment. Minimization of (11) with respect to the $(N_{\text{exp}} + N_{\text{clust}})$ parameters, f_k and R_m, gives our best estimates for these parameters together with their error correlations.

Our definition (11) implies piecewise constant R values but imposes no further constraints on the form of the hadronic cross section ('minimum bias'). Due to use of the overall normalization factors it also results in an adjustment of the different sets within their systematic uncertainties. This means, for example, that sparse but precise data will dominate the normalization. Still, the information on the shape of R from sets with larger systematic uncertainties is preserved, and all data contribute weighted according to their significance.

The error estimate for each hadronic channel is then done using the complete covariance matrix returned by our χ^2 minimization. Therefore statistical and systematic (point-to-point as well as overall) errors from the different sets are taken into account including correlations between different energies (clusters). The minimum χ^2 directly reflects the quality of the fit and the consistency of the data. We have checked that for all hadronic channels we find a stable value and error for $a_\mu^{\text{had,LO}}$, together with a good[1] χ^2 if we vary the minimal cluster size around our chosen default values (which are typically about 0.2 MeV for a narrow resonance and about 30 MeV for the continuum).

The dispersion integral (9) is performed integrating (using the trapezoidal rule) over the clustered data directly for all hadronic channels, including the ω and ϕ resonances. Thus we avoid possible problems due to missing or double-counting of non-resonant backgrounds, and interference effects are taken into account automatically. As an example we display in Fig. 1 the most important $\pi^+\pi^-$ channel, together with an enlargement of the region of ρ–ω interference.

In the region between 1.43 and ~ 2 GeV we have the choice between summing up the exclusive channels or relying on the inclusive measurements from the $\gamma\gamma 2$, MEA, M3N and ADONE experiments [25]. Surprisingly, the sum of the exclusive channels overshoots the inclusive data, even after having corrected the latter for missing two-body and (some) purely neutral modes. The discrepancy is shown in Fig. 2, where we display data points with errors after application of our clustering algorithm.

3   Results

In Table 1 we list the contributions to $a_\mu^{\text{had,LO}}$ from different energy regimes: From the two-pion threshold up to 0.32 GeV, chiral perturbation theory is applied (see e.g. [26]–[28]) and for the high energy tail above 11.09 GeV, R is calculated using perturbative QCD. The J/ψ, $\psi(2S)$ and $\Upsilon(1S-6S)$ resonance contributions are evaluated in narrow-width-approximation. Apart from those contributions we use the direct integration of clustered data as described above. For the controversial region from 1.43 to 2 GeV we present two results: if we use the lower lying inclusive data the corresponding contribution is considerably smaller than the one resulting from the use of the sum over the exclusive channels.

A more detailed breakdown of the contributions, and a full discussion of the data used, will be presented elsewhere [24]. There, we will also present an updated value of the QED coupling at the Z pole, $\alpha(M_Z^2)$.

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[1]In the channels $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$, $\pi^+\pi^-\pi^0$, in which data sets are mutually incompatible, χ^2/degree of freedom = 2.1, 1.6. For both cases the error is enlarged by a factor of $\sqrt{\chi^2/\text{dof}}$.
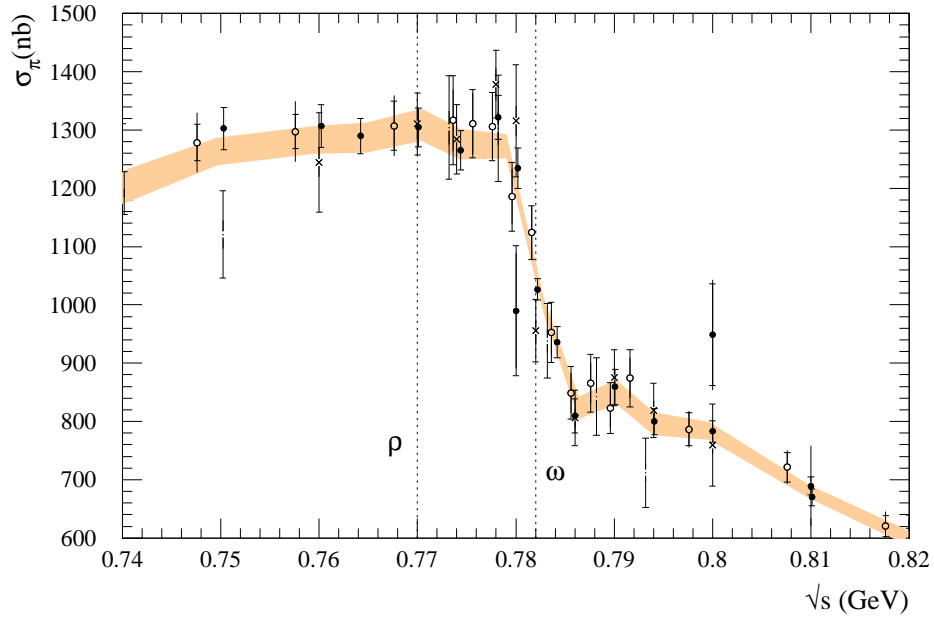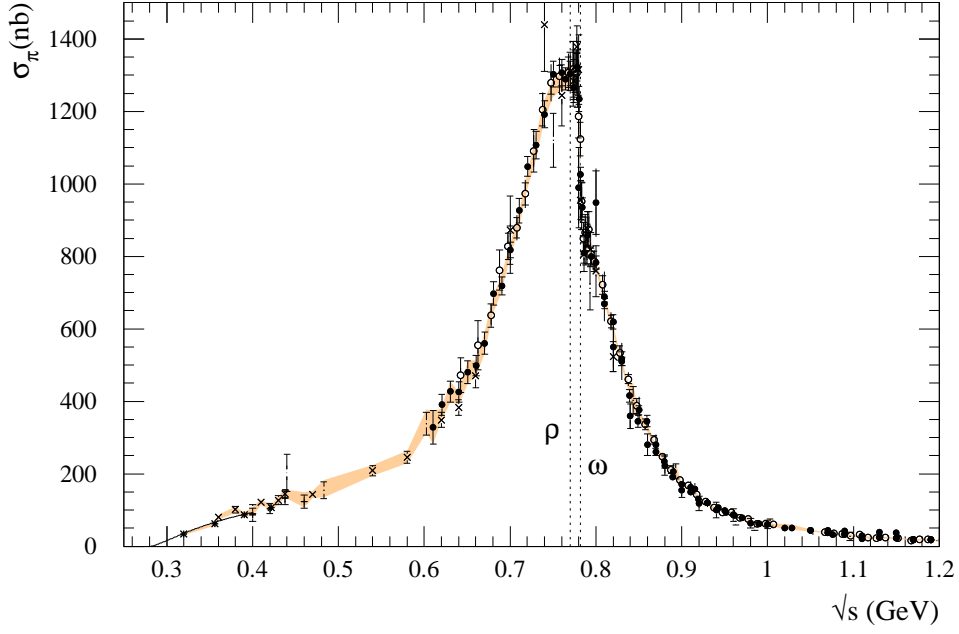
Figure 1: $e^+e^- \to \pi^+\pi^-$ data up to 1.2 GeV, where the shaded band shows the result of clustering. The second plot is an enlargement of the ρ-ω interference region.
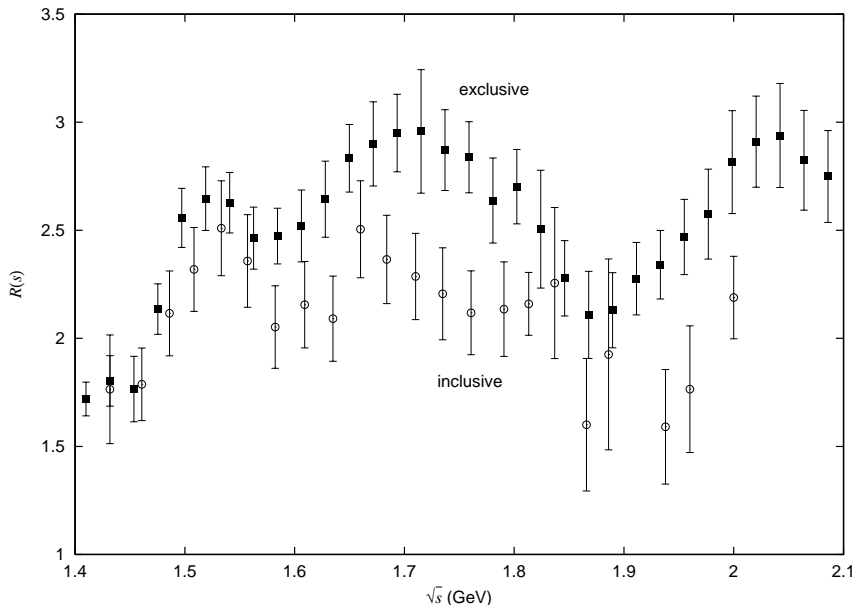
Figure 2: The inclusive and the sum of exclusive channel values of R, *after* the data have been clustered.

4 Resolution of the ambiguity: QCD sum rules

To resolve the ambiguity between the exclusive and inclusive data values for $R(s)$ in the range $1.43 < \sqrt{s} < 2$ GeV (see Fig. 2), we evaluate QCD sum rules of the form

$$\int_{s_{\mathrm{th}}}^{s_0} \mathrm{d}s \, R(s)f(s) \;=\; \int_C \mathrm{d}s \, D(s)g(s) \tag{12}$$

where s_0 is chosen just below the open charm threshold and C is a circular contour of radius s_0. $D(s)$ is the Adler D function,

$$D(s) \;\equiv\; -12\pi^2 s \frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{\Pi(s)}{s}\right) \qquad \text{where} \quad R(s) \;=\; \frac{12\pi}{s}\,\mathrm{Im}\,\Pi(s). \tag{13}$$

We use the experimental data[2] for $R(s)$ (or equivalently $\sigma_{\mathrm{had}}(s)$) to evaluate the left-hand-side, while QCD is used to determine [29]

$$D(s) \;=\; D_0(s) + D_{\mathrm{m}}(s) + D_{\mathrm{np}}(s), \tag{14}$$

where D_0 is the $O(\alpha_S^3)$ massless, three-flavour QCD prediction, D_{m} is the (small) quark mass correction and D_{np} is the (very small) contribution of the condensates. We take $f(s)$ to be of the form $(1 - s/s_0)^n(s/s_0)^m$, with $n + m = 0, 1$ or 2. Once $f(s)$ is chosen, the functional form of $g(s)$ is readily evaluated. For example, the $n = 1, m = 0$ sum rule is

$$\int_{s_{\mathrm{th}}}^{s_0} \mathrm{d}s \, R(s)\left(1 - \frac{s}{s_0}\right) \;=\; \frac{i}{2\pi}\int_C \mathrm{d}s \left(-\frac{s}{2s_0} + 1 - \frac{s_0}{2s}\right) D(s). \tag{15}$$

The sum rules with $n = 1$ or 2 and $m = 0$ are found to maximize the fractional contribution of the left-hand-side of (12) coming from the relevant $1.43 < \sqrt{s} < 2$ GeV interval. The evaluations of these two sum rules are shown in Table 2. Consistency clearly selects the inclusive, as opposed to the exclusive, determination of $R(s)$. A more detailed discussion of the QCD sum rules, and their

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[2]The J/ψ and $\psi(2S)$ resonance contributions are, of course, omitted.

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energy range (GeV)	comments	$a_\mu^{\mathrm{had,LO}} \times 10^{10}$
$2m_\pi \ldots 0.32$	chiral PT	2.30 ± 0.05
$0.32 \ldots 1.43$	excl. only	596.73 ± 5.18
$1.43 \ldots 2.00$	excl. only	38.14 ± 1.72
	incl. only	32.43 ± 2.46
$2.00 \ldots 11.09$	incl. only	42.09 ± 1.25
J/ψ and $\psi(2S)$	nar. width	7.31 ± 0.43
$\Upsilon(1S - 6S)$	nar. width	0.10 ± 0.00
$11.09 \ldots \infty$	pQCD	2.14 ± 0.01
$\sum$ of all	ex-ex-in	688.81 ± 6.17
	ex-in-in	683.11 ± 5.89

Table 1: A breakdown of the contributions to different intervals of the integration (9) for $a_\mu^{\mathrm{had,LO}}$. The alternative numbers for the interval $1.43 < \sqrt{s} < 2$ GeV correspond to using data for either the sum of the exclusive channels or the inclusive measurements, see Fig. 2. The total also includes a small 0.13×10^{-10} contribution from the $\pi^0 \gamma$ channel near its threshold (also included in the second line above).

sum rule	l.h.s. (data)	r.h.s. (QCD)
$n = 1, m = 0$	15.34 ± 0.39 (incl) 15.99 ± 0.35 (excl)	15.34 ± 0.08
$n = 2, m = 0$	10.40 ± 0.25 (incl) 10.90 ± 0.22 (excl)	10.30 ± 0.06

Table 2: The results of evaluating sum rule (15) and the corresponding one with $f(s) = (1 - s/s_0)^2$, where $\sqrt{s_0} = 3.7$ GeV. The main QCD error comes from $\alpha_S(M_Z^2) = 0.117 \pm 0.002$ [30]. The 'incl' and 'excl' alternatives refer to using the inclusive or exclusive $e^+ e^-$ data in the region $1.43 < \sqrt{s} < 2$ GeV, see Fig. 2.

evaluation, will be given in [24].

The same conclusion with regard to the resolution of the inclusive/exclusive ambiguity in the $1.43 < \sqrt{s} < 2$ GeV interval was reached in an independent analysis [31].

5 Conclusions

We have undertaken a data-driven determination of the hadronic vacuum polarisation contribution to $a_\mu^{\mathrm{had,LO}}$. We have used all available $e^+ e^-$ data and a non-linear χ^2 approach to cluster data for the same channel in narrow bins. In particular, the fit allows the normalizations of the individual data sets to be collectively optimized within their uncertainties. We found that there was a discrepancy between the inclusive value for $\sigma(e^+ e^- \to \text{hadrons})$ and the sum of the exclusive channels in the region $1.4 \lesssim \sqrt{s} \lesssim 2$ GeV, which gave an uncertainty of about 6×10^{-10} in $a_\mu^{\mathrm{had,LO}}$. We used a QCD sum rule analysis to resolve the discrepancy in favour of the inclusive data. Thus finally we find that the SM predicts

$$a_\mu^{\mathrm{had,LO}} = (683.1 \pm 5.9_{\mathrm{exp}} \pm 2.0_{\mathrm{rad}}) \times 10^{-10}. \tag{16}$$

Summing up all SM contributions to $a_\mu^{\rm SM}$ as given in eqs. (2)–(8), with (6) replaced by (16), we conclude that

$$a_\mu^{\rm SM} = (11659166.9 \pm 7.4) \times 10^{-10}, \tag{17}$$

which is 36.1×10^{-10} (3.3σ) below the world average experimental measurement. If, on the other hand, we were to take the value of $a_\mu^{\rm had,LO}$ obtained using the sum of the exclusive data in the interval $1.43 < \sqrt{s} < 2$ GeV then we would find $a_\mu^{\rm SM} = (11659172.6 \pm 7.7) \times 10^{-10}$, which is 30.4×10^{-10} (2.7σ) below $a_\mu^{\rm exp}$.

An independent SM prediction has very recently been made [20]. Their final e^+e^--based result, $(684.7 \pm 6.0_{\rm exp} \pm 3.6_{\rm rad}) \times 10^{-10}$, is very similar to ours. However, the overall agreement hides larger differences in individual contributions (but within the quoted uncertainties). Our result (16) agrees also fairly well with a recent reevaluation of the leading hadronic contribution from F. Jegerlehner, who also used the recent CMD-2 data [18] and obtained $(688.9 \pm 5.8) \times 10^{-10}$, see [32]. In order to facilitate a comparison with these two predictions, we will present a detailed breakdown of our result elsewhere [24].

For the future, we can expect further improvement in the accuracy of the experimental $(g-2)/2$ measurement. As far as the SM prediction is concerned, we may anticipate low energy data for a variety of e^+e^- channels, produced via initial state radiation, at the ϕ-factory DAΦNE [33] and at the B-factories, BaBar and Belle, see, for example, [34]. For instance, by detecting the $\pi^+\pi^-\gamma$ channel, it may be possible to measure $e^+e^- \to \pi^+\pi^-$ as low as about $\sqrt{s} = 400$ MeV. Moreover, CMD–2 measurements of $e^+e^- \to \pi^+\pi^-$ have already been made in this low energy region [35]. When these latter data are final, we anticipate that they would improve the error on $a_\mu^{\rm had,LO}$ by about 1×10^{-10}.

Acknowledgements

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