On the Connection of Leptogenesis with Low Energy CP Violation and LFV Charged Lepton Decays

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Abstract

Assuming only a hierarchical structure of the heavy Majorana neutrino masses and of the neutrino Dirac mass matrix m_D of the see–saw mechanism, we find that in order to produce the observed baryon asymmetry of the Universe via leptogenesis, the scale of m_D should be given by the up-quark masses. Lepton flavor violating decays $\mu \to e + \gamma$, $\tau \to \mu + \gamma$ and $\tau \to e + \gamma$ are considered and a characteristic relation between their decay rates is predicted. The effective Majorana mass in neutrinoless double beta decay depends on the CP violating phase controlling the leptogenesis if one of the heavy Majorana neutrinos is much heavier than the other two. Successful leptogenesis requires a rather mild mass hierarchy between the latter. The indicated hierarchical relations are also compatible with the low-energy neutrino mixing phenomenology. The scenario under study is compatible with the low-energy neutrino mixing phenomenology. The CP violation effects in neutrino oscillations can be observable. In general, there is no direct connection between the latter and the CP violation in leptogenesis. If the CPviolating phases of the see-saw model satisfy certain relations, the baryon asymmetry of the Universe and the rephasing invariant J_{CP} which determines the magnitude of the CP violation effects in neutrino oscillations, depend on the same CP violating phase and their signs are correlated.

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1 Introduction

Explaining both the observed baryon asymmetry of the Universe (Y_B) and the smallness of the neutrino masses, can be done successfully by combining the see-saw [1] and the leptogenesis [2] mechanisms. The first predicts the neutrinos to be Majorana particles and introduces additional right-handed heavy Majorana neutrinos, whose out-of-equilibrium decay in the early Universe generates Y_B via the second. The mounting evidence in favor of neutrino oscillations (see, e.g., [3, 4, 5, 6]) gave analyzes of these two mechanisms a firmer phenomenological basis. Establishing a connection between the low energy (neutrino mixing) and high energy (leptogenesis) parameters has gathered much attention in recent years [7, 8, 9, 10, 11, 12, 13, 14, 15]. However, the number of parameters of the see-saw mechanism is significantly larger than the number of quantities measurable in the "low energy" neutrino experiments. It is the neutrino Dirac mass matrix m_D that contains most of the unknown parameters and on whose knowledge any statement about a possible connection of low and high energy phenomena relies. While, in general, it is impossible to establish a direct link between the phenomena related to neutrino mixing and to leptogenesis, most specific models usually allow very well for such a connection.

In the present article we investigate the possible link between the leptogenesis, taking place at "high energy", and the "low energy" phenomena associated with the existence of neutrino mixing. Our analysis is based on the assumption of a hierarchical structure of the heavy Majorana neutrino masses and of the Dirac mass matrix m_D , both of which are part of the see–saw mechanism. As specific examples of low energy phenomena related to neutrino mixing we consider the lepton flavour violating (LFV) charged lepton decays, $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$ and $\tau \rightarrow e + \gamma$, the effective Majorana mass in neutrinoless double beta $((\beta\beta)_{0\nu})$ – decay, $|\langle m \rangle|$, and the *CP* violating asymmetry in neutrino oscillations. We analyze, in particular, the effects of the high energy *CP* violating phases, which control the generation of the baryon asymmetry in the leptogenesis scenario, in $|\langle m \rangle|$ and in the leptonic *CP* violation rephasing invariant J_{CP} [16] which determines the magnitude of the *CP* violation in neutrino oscillations ¹.

In the next Section we briefly review the formalisms associated with the see-saw mechanism and with the leptogenesis scenario, and that of LFV charged lepton decays. Different parameterizations of m_D used in the literature and the corresponding connection they imply between the high and low energy physical parameters are discussed in Section 3. In Section 4 we formulate the assumptions of a hierarchical structure of the heavy Majorana neutrino masses and of the Dirac mass matrix m_D on which our investigation is based, and analyze the leptogenesis in the so-called "bi–unitary" parametrization of m_D . Predictions for the branching rations of the LFV charged lepton decays are also given. In Section 5 the effects of the high energy leptogenesis CP violating phases on $|\langle m \rangle|$ and on the leptonic CP violation rephasing invariant J_{CP} . Finally, we conclude in Section 6.

¹For similar earlier attempts (not taking into account the questions of CP violation and LFV decays), see, e.g., [17, 18].

2 Framework

The neutrino oscillation data can consistently be described within a 3-neutrino mixing scheme with massive Majorana neutrinos, in which the light neutrino Majorana mass matrix m_{ν} is given by:

$$m_{\nu} = U_{\rm PMNS} \ m_{\nu}^{\rm diag} \ U_{\rm PMNS}^T \ . \tag{1}$$

Here m_{ν}^{diag} is a diagonal matrix containing the masses $m_{1,2,3}$ of the three massive Majorana neutrinos and U_{PMNS} is the unitary Pontecorvo–Maki–Nagakawa–Sakata [19] lepton mixing matrix, which can be parametrized as

$$U_{\rm PMNS} = V \operatorname{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)})$$

$$= \begin{pmatrix} c_1 c_3 & s_1 c_3 & s_3 e^{-i\delta} \\ -s_1 c_2 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 - s_1 s_2 s_3 e^{i\delta} & s_2 c_3 \\ s_1 s_2 - c_1 c_2 s_3 e^{i\delta} & -c_1 s_2 - s_1 c_2 s_3 e^{i\delta} & c_2 c_3 \end{pmatrix} \operatorname{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)}), \qquad (2)$$

where δ is a Dirac *CP* violating phase, α and β are Majorana *CP* violating phases [20, 21], $c_i = \cos \theta_i$ and $s_i = \sin \theta_i$. The angles θ_1 and θ_2 control the oscillations of solar and atmospheric neutrinos, respectively. The angle θ_3 is limited by the CHOOZ and Palo Verde reactor $\bar{\nu}_e$ experiments: one has $\sin^2 \theta_3 < 0.05$ [22, 5]. The Dirac phase δ can be measured, in principle, in long base–line neutrino oscillation experiments (see, e.g., [23, 24]). The flavour neutrino oscillations are insensitive to the Majorana phases α and β [20, 25]. Information about these phases can be obtained by studying processes in which the total lepton charge *L* is not conserved and changes by two units [26, 27, 28, 29, 30]: $(\beta\beta)_{0\nu}$ -decay, $K^+ \to \pi^- + \mu^+ + \mu^+$, etc.

Introducing a Dirac neutrino mass term and a Majorana mass term for the right–handed neutrinos via the Lagrangian

$$-\mathcal{L} = \overline{\nu_{Li}} (m_D)_{ij} N_{Rj} + \frac{1}{2} \overline{(N_{Ri})^c} (M_R)_{ij} N_{Rj} , \qquad (3)$$

leads for sufficiently large M_R to the well know see-saw [1] formula

$$m_{\nu} \simeq -m_D \, M_R^{-1} \, m_D^T \;,$$
 (4)

where terms of order $\mathcal{O}(M_R^{-2})$ are neglected. In order to explain the smallness of neutrino masses, the see–saw mechanism requires the existence of heavy right–handed Majorana neutrinos. The latter can create a lepton asymmetry via their *CP* violating out–of–equilibrium decays induced by the see–saw related Yukawa couplings. At later epoch the lepton asymmetry is converted into the baryon asymmetry of the Universe through sphaleron mediated processes [2]. Thus, leptogenesis is naturally incorporated in the see–saw model, which makes the model particularly attractive.

2.1 Leptogenesis

Leptogenesis fulfills all of Sakharov's three conditions [31] for generation of a non-vanishing baryon asymmetry Y_B . The requisite CP violating asymmetry is caused by the interference

of the tree level contribution and the one-loop corrections in the decay rate of the lightest of the three heavy Majorana neutrinos, $N_1 \to \Phi^- \ell^+$ and $N_1 \to \Phi^+ \ell^-$:

$$\varepsilon_{1} = \frac{\Gamma(N_{1} \to \Phi^{-} \ell^{+}) - \Gamma(N_{1} \to \Phi^{+} \ell^{-})}{\Gamma(N_{1} \to \Phi^{-} \ell^{+}) + \Gamma(N_{1} \to \Phi^{+} \ell^{-})}$$

$$\simeq \frac{1}{8 \pi v^{2}} \frac{1}{(m_{D}^{\dagger} m_{D})_{11}} \sum_{j=2,3} \operatorname{Im}(m_{D}^{\dagger} m_{D})_{1j}^{2} \left(f(M_{j}^{2}/M_{1}^{2}) + g(M_{j}^{2}/M_{1}^{2})\right) .$$
(5)

Here $v \simeq 174$ GeV is the electroweak symmetry breaking scale. The function f stems from vertex [2, 32] and g from self-energy [33] contributions:

$$f(x) = \sqrt{x} \left(1 - (1+x) \ln\left(\frac{1+x}{x}\right) \right) , \quad g(x) = \frac{\sqrt{x}}{1-x}$$
 (6)

For $x \gg 1$, i.e., for hierarchical heavy Majorana neutrinos, one has $f(x) + g(x) \simeq -\frac{3}{2\sqrt{x}}$. The baryon asymmetry is obtained via

$$Y_B = a \frac{\kappa}{g^*} \varepsilon_1 , \qquad (7)$$

where $a \simeq -1/2$ is the fraction of the lepton asymmetry converted into a baryon asymmetry [34], $g^* \simeq 100$ is the number of massless degrees of freedom at the time of the decay, and κ is a dilution factor that is obtained by solving the Boltzmann equations. Typically, one gets $Y_B \sim 10^{-10}$ when $\varepsilon_1 \sim (10^{-6} - 10^{-7})$ and $\kappa \sim (10^{-3} - 10^{-2})$. We note that this estimate of Y_B is valid in the supersymmetric (SUSY) theories as well since in the latter both g^* and ε_1 are larger approximately by the same factor of 2.

2.2 LFV Charged Lepton Decays

In extensions of the Standard Theory including massive neutrinos, lepton flavour nonconserving (LFV) processes such as $\mu \to e + \gamma$, $\mu \to 3e$, $\tau \to \mu + \gamma$, $\tau \to e + \gamma$, etc. are predicted to take place (see, e.g., [35]). However, in the non–SUSY versions of the see– saw model, the decay rates and cross sections of the LFV processes are strongly suppressed and are practically unobservable [36, 37]. The most stringent experimental limit on the $\mu \to e + \gamma$ decay branching ratio reads [38]

$$BR(\mu \to e + \gamma) < 1.2 \times 10^{-11}$$
 (8)

There are prospects to improve this limit by ~ 3 orders of magnitude in the future (see, e.g., [39]). The existing bounds on the LFV decays of the τ -lepton are considerably less stringent: one has, e.g., $BR(\tau \to \mu + \gamma) < 6.0 \times 10^{-7}$ [40]. There are possibilities to reach a sensitivity to values of $BR(\tau \to \mu + \gamma) \sim (10^{-8} - 10^{-9})$ at B-factory and LHC experiments [41].

In SUSY theories incorporating the see-saw mechanism, new sources of lepton flavour violation arise. In SUSY GUT theories these are typically related to the soft SUSY-breaking terms of the Lagrangian. Under the assumption of flavour universality of the SUSY breaking sector (scalar masses and trilinear couplings) at the GUT scale M_X , new lepton flavour non-conserving couplings are induced at low energy by the matrix of neutrino Yukawa couplings

 m_D/v through renormalization group running of the scalar lepton masses. The latter generates non-zero flavour non-diagonal entries in the slepton mass matrices and in the trilinear scalar couplings. These were shown to be proportional to $m_D m_D^{\dagger}$ [42]. More specifically, using the leading-log approximation, one can estimate [42, 43, 44] the branching ratio of the charged lepton decays $\ell_i \to \ell_j + \gamma$, $\ell_i(\ell_j) = \tau, \mu, e$ for i(j) = 3, 2, 1, i > j,

$$BR(\ell_i \to \ell_j + \gamma) \simeq \alpha^3 \left(\frac{(3+a_0) m_0^2}{8 \pi^2 m_S^4 G_F v^2} \right)^2 \left| (m_D L m_D^{\dagger})_{ij} \right|^2 \tan^2 \beta , \qquad (9)$$

where m_S denotes a slepton mass, m_0 is the universal mass scale of the sparticles and a_0 is a trilinear coupling (all at M_X). The diagonal matrix L reads: $L = \text{diag}(L_1, L_2, L_3)$, where $L_i = \log M_X/M_i$. To get a feeling for the numbers involved, for $a_0 = \mathcal{O}(1)$, $m_S \sim m_0 \sim 10^2$ GeV, $M_X \sim 10^{16}$ GeV and $M_i \sim 10^{10}$ GeV, one finds

$$BR(\ell_i \to \ell_j + \gamma) \sim 10^{-15} \tan^2 \beta \left| \frac{(m_D m_D^{\dagger})_{ij}}{\text{GeV}^2} \right|^2 .$$
(10)

This is within the planned sensitivity of the next generation experiments. The predicted values of $BR(\ell_i \rightarrow \ell_j + \gamma)$ are strongly dependent on the SUSY parameters, a topic beyond the scope of the present article.

Since the rates of LFV charged lepton decays depend on $m_D m_D^{\dagger}$, the decay asymmetry ε_1 depends on $m_D^{\dagger} m_D$ and leptonic CP violating effects originate from $m_{\nu} \sim m_D m_D^T$, one might in a given model expect some interplay between these three phenomena. We shall investigate this in Sections 4 and 5.

3 How it Works

3.1 Parameterizations

At low energy and in the case of N flavour neutrinos, we can measure, in principle, the $N \times N$ light neutrino mass matrix m_{ν} . It contains N mass eigenstates, $\frac{1}{2}N(N-1)$ mixing angles and $\frac{1}{2}N(N-1)$ phases, thus altogether N^2 measurable quantities. Of these $\frac{1}{2}N(N+1)$ are real and $\frac{1}{2}N(N-1)$ are CP violating phases. One can easily count the number of all see–saw parameters in the weak basis, in which both, M_R and the charged lepton mass matrix are real and diagonal. In this basis the Dirac mass matrix m_D contains all information about CP violation. We shall discuss now three possible parameterizations of m_D .

1. Bi–unitary parametrization

With N light and N' heavy neutrinos, denoting this particle content as the " $N \times N'$ see–saw model", one can write the complex $N \times N'$ Dirac mass matrix as

$$m_D = U_L^{\dagger} \, m_D^{\text{diag}} \, U_R \;, \tag{11}$$

where $U_L(U_R)$ is a unitary $N \times N(N' \times N')$ matrix and m_D^{diag} is a real matrix with non-zero elements only at its *ii* entries. Thus, m_D^{diag} contains $\min(N, N')$ real parameters we shall denote by m_{Di} . Any unitary $N \times N$ matrix can be written as

$$U = e^{i\Phi} P \tilde{U} Q , \qquad (12)$$

where $P \equiv \text{diag}(1, e^{i\phi_1}, e^{i\phi_2}, \ldots)$ and $Q \equiv \text{diag}(1, e^{i\alpha}, e^{i\beta}, \ldots)$ are diagonal phase matrices with (N-1) phases, and \tilde{U} is a unitary "CKM–like" matrix containing $\frac{1}{2}(N-2)(N-1)$ phases and $\frac{1}{2}N(N-1)$ angles. In total U contains $\frac{1}{2}N(N+1)$ phases. Parameterizing U_L^{\dagger} and U_R in the same way, we get for the Dirac mass matrix

$$m_D = e^{i(\Phi_R + \Phi_L)} P_L \tilde{U}_L Q_L m_D^{\text{diag}} P_R \tilde{U}_R Q_R , \qquad (13)$$

where the index L(R) indicates that the angles and phases in the respective matrices also carry this index. The common phase $(\Phi_R + \Phi_L)$ and the (N - 1) phases in P_L can be absorbed by a redefinition of the charged lepton fields. From the original (N + N' - 2)phases in the matrices Q_L and P_R , only $(N + N' - 2) - (\max(N, N') - 1)$ appear in the product $Q_L m_D^{\text{diag}} P_R$. Thus, we can write

$$m_D = \tilde{U}_L W \, m_D^{\text{diag}} \, \tilde{U}_R \, Q_R \, \,, \tag{14}$$

where W is a diagonal matrix containing N' - 1 (N - 1) phases if N > N' $(N \le N')$. The number of physical CP violating phases is therefore

$$\frac{1}{2}N(N+1) + \frac{1}{2}N'(N'+1) - 2 - (N-1) - (\max(N,N') - 1)$$

= $\frac{1}{2}N(N-1) + \frac{1}{2}N'(N'+1) - \max(N,N')$. (15)
= $\frac{1}{2}N(N-3) + \frac{1}{2}N'(N'-3) + N' + \min(N,N')$

Adding the number of the real parameters m_{Di} , $\min(N, N')$, of the masses M_i , N', and of the angles in \tilde{U}_L (\tilde{U}_R), $\frac{1}{2}N(N-1)$ ($\frac{1}{2}N'(N'-1)$), we obtain the total number of parameters in the $N \times N'$ see–saw model. The result is $N(N-1) + N'(N'+1) - \max(N, N') + \min(N, N')$. For N = N' this reduces to $2N^2$ parameters, N(N-1) phases and N(N+1) real one. In general, the difference between the number of real parameters and phases is N + N'.

Comparing the number of low and high energy parameters, we see that integrating out the heavy Majorana neutrinos leave us short of a total of $N'(N'+1) - N - \max(N, N') + \min(N, N')$ parameters, of which $\frac{1}{2}N'(N'+1) - N + \min(N, N')$ are real and $\frac{1}{2}N'(N'+1) - \max(N, N')$ are phases. For N = N' half of the parameters "get lost", $\frac{1}{2}N(N-1)$ phases and $\frac{1}{2}N(N-1)$ real ones. The counting of the independent physical parameters in m_D through the independent parameters in $U_{L,R}$ made above, is valid for |N - N'| = 0, 1. For arbitrary |N - N'| > 1, the total number of independent physical parameters in m_D is obviously N(2N'-1), of which NN' are moduli and N(N'-1) are phases. Their number is smaller than the number our counting through $U_{L,R}$ gives because when |N - N'| > 1 there are more unphysical parameters in $U_{L,R}$.

The usual 3×3 see—saw model has therefore 18 parameters, which are composed of 12 real parameters and 6 phases. Integrating out the heavy Majorana neutrinos leaves us with the observable mass matrix m_{ν} , which contains 9 observable parameters — 6 real and 3 phases. Hence, half of the parameters of the model get "lost" at low energy. However, in approaches to reconstruct the high energy physics from low energy data, one usually assumes the existence of relations between mass matrices (e.g., $m_D = m_{\rm up}$), which reduces the number of unknown parameters. If in addition specific textures in the matrices are implemented, the situation improves further. It is worth noting that "reduced" models allow for a simpler connection between the high and low energy physics phenomena. Consider the minimal 3×2 see–saw model [12, 13], which contains only 2 heavy Majorana neutrinos. This model has altogether 11 parameters, 8 real and 3 *CP* violating phases. Comparing with the 9 observable parameters in m_{ν} , we see that only 2 real parameters are "missing" at low energy. Interestingly, this makes the model superior to the even more reduced "too minimal" 2×2 see–saw model with 2 heavy and 2 light Majorana neutrinos ². The latter has 8 parameters, two of which are phases, and this has to be compared with the 4 parameters (including 1 phase) a 2×2 light Majorana neutrino mass matrix effectively contains.

We shall use in the analysis that follows the bi–unitary parametrization. Before proceeding further, however, we will describe briefly two alternative parameterizations.

2. Triangular parametrization

This parametrization relies on the property that any complex matrix can be written as a product of a unitary and a lower triangular matrix [14]:

$$m_D = v \, U \, Y \ . \tag{16}$$

The triangular matrix Y has zeros as entries above the diagonal axis and contains 3 phases in the three off-diagonal entries. The unitary matrix U can be parametrized in analogy to the PMNS matrix, i.e., in the form of a CKM-like matrix with one phase times a diagonal matrix containing the other two phases. An analysis of leptogenesis using this parametrization was performed in Ref. [15].

3. Orthogonal parametrization

The following parametrization shows clearly that without any assumptions there is no connection between the low and high energy parameters governing respectively neutrino mixing and leptogenesis. One can write the Dirac matrix as [43]

$$m_D = i U_{\rm PMNS} \sqrt{m_{\nu}^{\rm diag}} R \sqrt{M_R} , \qquad (17)$$

where R is a complex orthogonal matrix. It contains 3 real parameters and 3 phases, which, together with the 3 M_i and the 9 parameters from m_{ν} , sums up again to a total of 18 parameters. In Eq. (17), m_D seems to contain all 18 parameters. However, the product $\sqrt{m_{\nu}^{\text{diag}}} R \sqrt{M_R}$ includes only 9 independent parameters.

3.2 Leptogenesis, m_{ν} and LFV Charged Lepton Decays

The decay asymmetry ε_1 depends on the three mass eigenvalues M_i and on the hermitian matrix $m_D^{\dagger}m_D$. Due to its hermiticity, the latter will have a reduced number of parameters with respect to m_D : for every row (or column) we get rid of one complex number, or equivalently of one real parameter and one phase. Indeed,

$$m_D^{\dagger} m_D = \begin{cases} U_R^{\dagger} (m_D^{\text{diag}})^2 U_R , & \text{bi-unitary;} \\ v^2 Y^{\dagger} Y , & \text{triangular;} \\ \sqrt{M_R} R^{\dagger} m_{\nu}^{\text{diag}} R \sqrt{M_R} , & \text{orthogonal,} \end{cases}$$
(18)

²This model can be motivated by a vanishing mixing angle θ_3 . The ν_e oscillates then into the flavor state $(\nu_{\mu} + \nu_{\tau})/\sqrt{2}$ with an amplitude $\sin^2 2\theta_{\odot}$.

depends on 15-6 = 9 parameters, which for the bi–unitary parametrization are the 3 angles and 3 phases in U_R and the 3 mass eigenvalues m_{Di} ; for the triangular parametrization, these are the 6 real entries and 3 phases in Y. In the orthogonal parametrization we have the 9 parameters from the combination $\sqrt{m_{\nu}^{\text{diag}}} R \sqrt{M_R}$. Of course, there are three leptogenesis phases in each case.

The form of $m_D^{\dagger}m_D$ in the orthogonal parametrization shows that the PMNS matrix U_{PMNS} does not enter into the expression for ε_1 and that there is, in general, no connection between the low energy observables in U_{PMNS} and leptogenesis. In particular, there is no connection between the low and high energy CP violation. Figure 1 shows a sketch of the situation: in order to connect low energy lepton charge non-conservation and CP violation effects (making an assumption about the currently unknown values of the phases and the lowest neutrino mass eigenstate) with the baryon asymmetry of the Universe Y_B , there must exist a connection between the light left-handed neutrinos ν_L and the heavy right-handed Majorana neutrinos N_R . This typically involves some kind of see-saw mechanism within the framework of a GUT theory. The N_R produce Y_B via the leptogenesis mechanism. Without the crucial GUT or see-saw link there is no connection.

Inserting the form of m_D from Eqs. (11) and (16) in Eq. (4) shows ³ that all six phases in m_D contribute to the phases in U_{PMNS} :

$$m_{\nu} = \begin{cases} -U_L^{\dagger} m_D^{\text{diag}} U_R M_R^{-1} U_R^T m_D^{\text{diag}} U_L^* , & \text{bi-unitary;} \\ -v^2 U Y M_R^{-1} Y^T U^T , & \text{triangular.} \end{cases}$$
(19)

One may expect that a connection between the high and low energy CP violation would exist if three of the phases in m_D are negligible, so that the numbers of the CP violating phases at high and low energy coincide. In order to have successful leptogenesis, the negligible phases should not be those appearing in Y or U_R . Thus, one can assume that, e.g., U in Eq. (16) is real, as has been done in [15]. We shall pursue a different approach and consider the bi–unitary parametrization keeping all the phases.

In supersymmetric versions of the see–saw mechanism, the rates of LFV charged lepton decays, such as $\mu \to e + \gamma$, and the *T* violating asymmetries in, e.g., $\mu \to 3e$ decay, depend approximately ⁴ on [42, 43, 44]

$$m_D m_D^{\dagger} = \begin{cases} U_L^{\dagger} (m_D^{\text{diag}})^2 U_L , & \text{bi-unitary;} \\ v^2 U Y Y^{\dagger} U^{\dagger} , & \text{triangular;} \\ U_{\text{PMNS}} \sqrt{m_{\nu}^{\text{diag}}} R M_R R^{\dagger} \sqrt{m_{\nu}^{\text{diag}}} U_{\text{PMNS}}^{\dagger} , & \text{orthogonal.} \end{cases}$$
(20)

In principle, taking also a possible data on the electric dipole moments of charged leptons into account, there are enough observables for a full reconstruction of the see-saw model. The bi-unitary parametrization seems to be especially convenient for this purpose.

³Note that inserting Eq. (17) in (4) yields an identity.

⁴The actual dependence is a bit more evolved, see Section 2.2.

4 The bi–Unitary Parametrization for m_D

As we saw, the 3×3 see–saw model has 6 phases as possible sources of leptonic CP violation. The 3 low energy CP violating phases depend, in general, on all 6 see–saw phases. The unitary matrices U_L and U_R have a simple interpretation: U_L diagonalizes $m_D m_D^{\dagger}$, thus being responsible for lepton flavor violation, and U_R diagonalizes $m_D^{\dagger} m_D$, therefore being responsible for the total lepton number non–conservation. Since the latter is a necessary ingredient of leptogenesis and leads also to $(\beta\beta)_{0\nu}$ –decay, one might assume naively that there will be a certain connection between the $(\beta\beta)_{0\nu}$ –decay rate and Y_B . We shall see that within certain plausible assumptions this is indeed the case.

The first step is to determine the form of U_L and U_R . Since in any GUT theory m_D is related to the known charged fermion masses, we can assume a hierarchical structure of m_D , i.e., $m_{D3} \gg m_{D2} \gg m_{D1}$, such that

$$m_D \simeq \operatorname{diag}(m_{D1}, m_{D2}, m_{D3}) + \mathcal{O}(\operatorname{few}\%) ,$$
 (21)

where the second term indicates that there are corrections not exceeding 10 % on both the diagonal and off-diagonal entries of m_D . It is helpful to parametrize U_L^{\dagger} and U_R in analogy to U_{PMNS} in Eq. (2), with the angles θ_i replaced by $\theta_{L(R)i}$. In what regards the *CP* violating phases, we shall denote the phases in \tilde{U}_R and \tilde{U}_L^{\dagger} with δ_R and δ_L respectively, the two "Majorana phases" in Q_R by α_R , $\beta_R + \delta_R$ and the two phases in W by α_W and $\beta_W + \delta_L$. In order to obtain a hierarchical m_D we shall assume that $s_{L(R)1} \sim 10^{-1} > s_{L(R)2} \sim 10^{-2} > s_{L(R)3}$ with $s_{L(R)3} \leq 10^{-3}$, as well as that $m_{D3} \gg m_{D2} \gg m_{D1}$. These assumptions are inspired by observed mixing in the quark sector and by the known hierarchies between the masses of the up-type quarks, between the masses of the down-type quarks and between the charged lepton masses. In the numerical estimates we give in what follows we will always use the values $m_{D1} \sim 100$ MeV, $m_{D2} \sim 1$ GeV and $m_{D3} \sim 100$ GeV, which to a certain degree are suggested by the up-quark mass values.

Once $U_L^{\dagger}(U_R)$ is chosen to contain hierarchical mixing angles, the requirement that m_D takes the form (21) implies hierarchical mixing angles also for $U_R(U_L^{\dagger})$. The structure of the Dirac matrix is then found to be

$$m_D \simeq \begin{pmatrix} m_{D1} - \tilde{m}_{D2} \, s_{1L} \, s_{1R} & e^{i\alpha_R} \, \tilde{m}_{D2} \, s_{1L} & e^{i(\beta_R + \delta_R - \delta_L)} \, \tilde{m}_{D3} \, s_{3L} \\ - \tilde{m}_{D2} \, s_{1R} & e^{i\alpha_R} \, \tilde{m}_{D2} & e^{i(\beta_R + \delta_R)} \, \tilde{m}_{D3} \, s_{2L} \\ \tilde{m}_{D3} \left(s_{1R} \, s_{2R} - e^{i\delta_R} \, s_{3R} \right) & -e^{i\alpha_R} \, \tilde{m}_{D3} \, s_{2R} & e^{i(\beta_R + \delta_R)} \, \tilde{m}_{D3} \end{pmatrix} , \quad (22)$$

where $\tilde{m}_{D2} \equiv m_{D2} e^{i\alpha_W}$, $\tilde{m}_{D3} \equiv m_{D3} e^{i(\beta_W + \delta_L)}$ and we have set all the $\cos \theta_{L(R)i}$ to one. Products of the $\sin \theta_{L(R)i}$ which are smaller than 10^{-3} are neglected. We have neglected also terms of order $\mathcal{O}(m_{D3} s_{2L} s_{2R})$ with respect to m_{D2} . Not surprisingly, the off-diagonal entries of m_D are suppressed with respect to the diagonal terms by the small quantities $s_{iL(R)}$, i = 1, 2, 3. A hierarchical structure of m_D is most naturally described using the bi–unitary parametrization.

Rather than tuning the parameters involved in the bi–unitary parametrization to reproduce precisely the neutrino mass squared differences and mixing angles determined from the available neutrino oscillation data, we shall focus only on the dependence of the quantities of interest on the parameters in m_D and M_R : we are primarily interested to find out under which circumstances a connection between the neutrino mixing related phenomena and leptogenesis is possible. We made nevertheless extensive consistency checks regarding the form of m_{ν} as obtained with the matrix m_D in Eq. (22) and with M_R . Using the form of m_D given in Eq. (22) and assuming a hierarchy for M_i , $M_1 \ll M_2 \ll M_3$, from Eq. (4) one can obtain a theoretically predicted expression for the low energy neutrino mass matrix, m_{ν}^{th} . The form of m_{ν}^{th} should be confronted with the neutrino mass matrix which is obtained from Eq. (1), m_{ν}^{exp} . For the angles $\theta_1 \equiv \theta_{\odot}$ and $\theta_2 \equiv \theta_{\text{atm}}$, we use the best fit values found in the analyzes [5, 6] of the solar and atmospheric neutrino data, respectively: $\tan^2 \theta_{\odot} = 0.46$ and $\sin^2 2\theta_{\text{atm}} = 1$. The angle θ_3 is varied within its allowed 3σ range, $\sin^2 \theta_3 < 0.05$, while the three CP violating phases δ , α and β are treated as free parameters. For the normal hierarchical neutrino mass spectrum one has: $m_1 \ll m_2 \simeq \sqrt{\Delta m_{\odot}^2} \ll m_3 \simeq \sqrt{\Delta m_{\text{atm}}^2}$, where $\Delta m_{\odot}^2 > 0$ and $\Delta m_{\text{atm}}^2 > 0$ drive the solar and atmospheric neutrino oscillations. In the numerical estimates which follow we use $\Delta m_{\odot}^2 = 7.3 \times 10^{-5} \text{ eV}^2$ [5] and $\Delta m_{\text{atm}}^2 = 2.7 \times 10^{-3} \text{ eV}^2$ [6].

The expression for m_{ν}^{th} and the analysis of interest simplify considerably if the hierarchy between the masses M_1 and M_2 is relatively mild:

$$M_2 \simeq 10 \, M_1 \, . \tag{23}$$

It is not difficult to convince oneself that if Eq. (23) holds and if in addition — in agreement with the hierarchy requirement in m_D — the relation

$$m_{D3} s_{2R(L)} \simeq 10^{-2} m_{D3} \simeq m_{D2}$$
 (24)

holds, the form of m_{ν}^{th} leads to $\sin^2 2\theta_2 \equiv \sin^2 2\theta_{\text{atm}} \simeq 1$ without any further fine-tuning as long as the terms $\propto 1/M_3$ in Eq. (26) give a sub-leading contribution to m_{ν}^{th} . This is ensured provided the inequality

$$M_3 \gg s_{2R}^{-2} M_2 \sim 10^4 M_2 , \qquad (25)$$

implying a strong hierarchy between M_2 and M_3 , holds. The existence of a mild hierarchy between M_1 and M_2 , Eq. (23), and of a strong hierarchy between M_2 and M_3 , Eq. (25) is, as we will see, also compatible with the requirement of effective leptogenesis. Under the conditions (23) — (25) the matrix m_{ν}^{th} takes a relatively simple form:

$$(m_{\nu}^{\text{th}})_{ee} \simeq \frac{(m_{D1} - s_{1L}s_{1R}m_{D2}e^{i\alpha_W})^2}{M_1} + \frac{s_{1L}^2m_{D2}^2e^{2i(\alpha_W + \alpha_R)}}{M_2}$$

$$(m_{\nu}^{\text{th}})_{e\mu} \simeq -\frac{(m_{D1} - s_{1L}s_{1R}m_{D2}e^{i\alpha_W})s_{1R}m_{D2}e^{i\alpha_W}}{M_1} + \frac{m_{D2}^2s_{1L}e^{2i(\alpha_W + \alpha_R)}}{M_2}$$

$$(m_{\nu}^{\text{th}})_{e\tau} \simeq \frac{(m_{D1} - s_{1L}s_{1R}m_{D2}e^{i\alpha_W})(s_{1L}s_{2R} - s_{3R}e^{i\delta_R})m_{D3}^2e^{2i(\beta_W + \delta_L)}}{M_1} - \frac{M_1}{M_2}$$

$$(m_{\nu}^{\text{th}})_{\mu\mu} \simeq -\frac{m_{D2}^2}{M_2}e^{2i(\alpha_R + \beta_W + \delta_L + 2\alpha_R)}$$

$$(m_{\nu}^{\text{th}})_{\mu\tau} \simeq \frac{m_{D2}m_{D3}}{M_2}s_{2R}e^{i(2\alpha_R + \alpha_W + \beta_W + \delta_L)},$$

$$(m_{\nu}^{\text{th}})_{\tau\tau} \simeq -\frac{m_{D3}^2}{M_2}s_{2R}^2e^{2i(\alpha_R + \beta_W + \delta_L)},$$

where we have not written explicitly the sub-leading terms $\propto 1/M_1$ and $\propto 1/M_3$. Further analysis shows that for the generic values of the parameters m_{Di} and $s_{jL(R)}$ we use, the matrix m_{ν}^{th} thus obtained leads to a value of $\theta_{\odot} \equiv \theta_1$ compatible with the observations for, e.g., $M_2 \sim 10^{10}$ GeV. The value of θ_{\odot} is particularly sensitive to the specific value of m_{D1} chosen: a very good agreement with the best fit value $\tan^2 \theta_{\odot} \approx (0.42 - 0.46)$ determined in the analyses of the solar neutrino data is obtained for $m_{D1} \approx (70 - 90)$ MeV.

In what regards the angle θ_3 , for $m_{D1} \sim 100$ MeV and $s_{1L} \simeq (0.10-0.25)$ and the standard values of the other parameters we use, one generically has: $\sin \theta_3 \sim s_{1L}/\sqrt{2}$. Consequently, $\sin^2 \theta_3$ is relatively large: typically one has $\sin^2 \theta_3 \gtrsim 0.005$. However, it is also possible to fine-tune the values of the parameters involved in the analysis to get $\sin^2 \theta_3 < 0.005$ and this latter possibility cannot be ruled out.

The above analysis shows that the assumptions we made about the magnitude of the angles in U_L and U_R and of the values of m_{Di} are very well in agreement with the existing data. We find, in particular, that the angle θ_3 can be relatively large ($\sin^2 \theta_3 \gtrsim 0.005-0.010$), which has important implications for the searches for CP violation in neutrino oscillations. In fact, we shall see that under the assumptions made, the CP violating observables in neutrino oscillations can be sizable.

4.1 Leptogenesis in the bi–Unitary Parametrization

In the context of leptogenesis the lepton asymmetry depends on the parameter $m_D^{\dagger}m_D$, as discussed in Section 2. Using Eq. (18) we find for $(m_D^{\dagger}m_D)_{11}$ to leading order:

$$(m_D^{\dagger} m_D)_{11} \simeq m_{D2}^2 \, s_{1R}^2 + m_{D3}^2 \left(s_{1R}^2 \, s_{2R}^2 - 2 \, \cos \delta_R \, s_{1R} \, s_{2R} \, s_{3R} + s_{3R}^2 \right) \\ \simeq \left(m_{D2}^2 + m_{D3}^2 \, s_{2R}^2 \right) \, s_{1R}^2 \,.$$

$$(27)$$

The decay asymmetry ε_1 receives contribution from

$$\operatorname{Im}\left\{ (m_D^{\dagger} m_D)_{12}^2 \right\} \simeq 2 \left(\cos \alpha_R s_{1R} \left(m_{D2}^2 + m_{D3}^2 s_{2R}^2 \right) - m_{D3}^2 \cos(\alpha_R - \delta_R) s_{2R} s_{3R} \right) \\ \left(\sin \alpha_R s_{1R} \left(m_{D2}^2 + m_{D3}^2 s_{2R}^2 \right) - m_{D3}^2 \sin(\alpha_R - \delta_R) s_{2R} s_{3R} \right) \\ \simeq \left(m_{D2}^2 + m_{D3}^2 s_{2R}^2 \right)^2 s_{1R}^2 \sin 2\alpha_R ,$$

$$(28)$$

as well as from

$$\operatorname{Im}\left\{ (m_{D}^{\dagger}m_{D})_{13}^{2}\right\} \simeq 2 m_{D3}^{4} \left(\cos(\beta_{R}+\delta_{R}) s_{1R} s_{2R} - \cos\beta_{R} s_{3R}\right) \left(\sin(\beta_{R}+\delta_{R}) s_{1R} s_{2R} - \sin\beta_{R} s_{3R}\right) \qquad (29)$$
$$\simeq m_{D3}^{4} s_{1R}^{2} s_{2R}^{2} \sin 2(\beta_{R}+\delta_{R}) ,$$

where s_{3R} was neglected in the last expression. Assuming a hierarchical mass spectrum of the heavy Majorana neutrinos one finds for the decay asymmetry

$$\varepsilon_1 \simeq -\frac{3}{2} \frac{1}{8 \pi v^2} \left(\left(m_{D2}^2 + m_{D3}^2 s_{2R}^2 \right) \sin 2\alpha_R \frac{M_1}{M_2} + \frac{m_{D3}^4 s_{2R}^2}{m_{D2}^2 + m_{D3}^2 s_{2R}^2} \sin 2(\beta_R + \delta_R) \frac{M_1}{M_3} \right). \tag{30}$$

Therefore, we can identify α_R and $(\beta_R + \delta_R)$ as the leptogenesis phases. Note that, as it should, $\varepsilon_1 = 0$ for $\alpha_R, \beta_R, \delta_R = 0, \pi/2, \pi$. Taking $m_{D2} \simeq 10^{-2} m_{D3}$ and $s_{2R} \simeq 10^{-2}$, we can estimate the decay asymmetry as ⁵

$$\varepsilon_1 \sim -10^{-9} \left(\frac{m_{D3}}{\text{GeV}}\right)^2 \left(\sin 2\alpha_R \frac{M_1}{M_2} + 10^3 \sin 2(\beta_R + \delta_R) \frac{M_1}{M_3}\right) . \tag{31}$$

As discussed earlier, the heavy Majorana neutrino masses M_i are also expected to posses a hierarchy. Under the condition (25) of a strong hierarchy between M_2 and M_3 , the second term in the last equation is negligible, and only the phase α_R is relevant for leptogenesis. If further one has $m_{D3} \sim 10^2$ GeV, as is suggested by up–quark mass values, it is possible to have $\varepsilon_1 \sim 10^{-6} - 10^{-7}$ if, e.g., $M_1/M_2 \sim 0.1$, which is compatible with the requirement of a mild hierarchy between M_1 and M_2 , Eq. (23) which in the previous Section was shown to be also compatible with the low energy neutrino phenomenology.

Identifying m_D with the down-quark or charged lepton mass matrix, i.e., $m_{D3} \sim 1$ GeV, leads to a rather small value of ε_1 for $M_1 \ll M_2 \ll M_3$. In this case the lepton asymmetry can only be amplified to the requisite value by the resonance mechanism [33] which is operative for quasi-degenerate heavy Majorana neutrinos. Thus, leptogenesis with hierarchical neutrino Majorana and Dirac masses prefers up-quark type masses for the latter, a fact first noticed in [17].

The "effective mass" \tilde{m}_1 , that represents a sensitive parameter for the Boltzmann equations, is given by

$$\tilde{m}_1 = \frac{(m_D^{\dagger} m_D)_{11}}{M_1} \sim \left(\frac{m_{D3}}{\text{GeV}}\right)^2 \left(\frac{10^9 \text{GeV}}{M_1}\right) 10^{-5} \,\text{eV} \ . \tag{32}$$

⁵Note that the form of this equation resembles Eq. (17) in the second paper in Ref. [8]. A type II see–saw mechanism was used in that work.

For $m_{D3} \sim 10^2$ GeV and M_1 of 10^9 to 10^{10} GeV, acceptable values of 10^{-1} to 10^{-3} eV for \tilde{m}_1 are obtained. This leads to values of $\kappa \sim 10^{-1} - 10^{-3}$. Assuming that the sine of the phases in Eq. (30) is not small, a value of $Y_B \sim 10^{-10}$ is therefore "naturally" obtained for, e.g., $m_{D3} \sim 10^2$ GeV, $M_1 \sim 10^9$ GeV, and $M_2 \sim 10^{10}$ GeV $\ll M_3 \sim 10^{15}$ GeV. The indicated values of M_1 , M_2 and M_3 are also in very good agreement with the constraints Eqs. (23) and (25).

4.2 LFV Charged Lepton Decays

According to Eq. (9), the branching ratios of the LFV charged lepton decays, $BR(\ell_i \to \ell_j + \gamma)$, depend on $(m_D L m_D^{\dagger})$. We have calculated this matrix with the usual approximations for the masses and angles and found the leading terms to be

$$(m_D L m_D^{\dagger})_{21} \simeq m_{D2}^2 s_{1L} L_2,$$

$$(m_D L m_D^{\dagger})_{31} \simeq m_{D3}^2 (s_{1L} s_{2L} - s_{3L} e^{i\delta_L}) L_3 + m_{D3} m_{D2} s_{1L} s_{2R} (L_2 - L_3). \qquad (33)$$

$$(m_D L m_D^{\dagger})_{32} \simeq m_{D3}^2 s_{2L} L_3.$$

Barring accidental cancellations in the expression for $(m_D L m_D^{\dagger})_{31}$, we obtain the following general relation between the branching ratios of interest ⁶:

$$BR(\tau \to \mu + \gamma) \gg BR(\tau \to e + \gamma) \gg BR(\mu \to e + \gamma) .$$
(34)

If we use the generic values of the relevant parameters we work with, we find

$$BR(\tau \to \mu + \gamma) \sim 10^2 BR(\tau \to e + \gamma) \sim 10^5 BR(\mu \to e + \gamma) .$$
(35)

The results do not differ significantly from the results obtained by using a common mass scale M and taking the log-terms out of the matrix, i.e., for $(m_D m_D^{\dagger}) \log M_X/M$ instead of $m_D L m_D^{\dagger}$. Note that though m_D contains U_L and U_R , only the angles and phases of U_L appear in $BR(\ell_i \to \ell_j + \gamma)$, which confirms that Eq. (20) represents a rather good approximation for estimating $BR(\ell_i \to \ell_j + \gamma)$.

5 Leptogenesis, $(\beta\beta)_{0\nu}$ -decay and Low Energy Leptonic *CP* Violation

We shall investigate next whether there exists a relation between the high energy CP violation leading to the decay asymmetry Eq. (30) and low energy observables. We will consider the effective Majorana mass in $(\beta\beta)_{0\nu}$ -decay and the CP violating asymmetry in neutrino oscillations.

⁶Obviously, the relation we obtain does not hold if, e.g., the accidental cancellation $(s_{1L}s_{2L} - s_{3L}e^{i\delta_L}) = 0$ takes place. The latter requires, however, a fine-tuning of the values of 4 parameters.

5.1 The Effective Majorana Mass in $(\beta\beta)_{0\nu}$ -Decay

The observation of $(\beta\beta)_{0\nu}$ -decay would provide direct evidence for the non-conservation of the total lepton charge, which is a necessary ingredient for the leptogenesis mechanism. In $(\beta\beta)_{0\nu}$ -decay one measures the absolute value of the *ee* element of m_{ν} . Given the form of m_D in Eq. (22), it is not difficult to find this element:

$$\langle m \rangle \simeq \frac{(m_{D1} - s_{1L} s_{1R} m_{D2} e^{i\alpha_W})^2}{M_1} + \frac{m_{D2}^2 e^{2i(\alpha_W + \alpha_R)}}{M_2} + \frac{m_{D3}^2 s_{3L}^2 e^{2i(\beta_R + \beta_W + \delta_R)}}{M_3} \,. \tag{36}$$

The term of order m_{D3}^2 contains, in particular, the parameters α_W and β_W , which do not influence the baryon asymmetry. The condition that this term does not contribute significantly to $|\langle m \rangle|$ is

$$\frac{m_{D3}^2}{M_3} s_{3L}^2 \ll \begin{cases} \frac{m_{D2}^2}{M_1} s_{1L}^2 s_{1R}^2 \\ \frac{m_{D2}^2}{M_2} s_{1L}^2 \end{cases} \Rightarrow M_3 \gg \begin{cases} M_1 \left(\frac{m_{D3}}{m_{D2}}\right)^2 \frac{s_{3L}^2}{s_{1L}^2 s_{1R}^2} \sim 10^2 M_1 \\ M_2 \left(\frac{m_{D3}}{m_{D2}}\right)^2 \frac{s_{3L}^2}{s_{1L}^2} \sim M_2 \end{cases}$$
(37)

which is compatible with Eqs. (24) and (26) and the hierarchy between M_3 and $M_{1,2}$, required for generating a sufficient baryon asymmetry (see the discussion after Eq. (31)). The first two terms in (36) can contribute comparably to $|\langle m \rangle|$ unless $M_2 s_{1R}^2 \gg M_1$ or $M_2 s_{1R}^2 \ll M_1$. From the two main possibilities for m_D , the choice of the up–quark type matrix, $m_D \sim m_{\rm up}$, favored by leptogenesis in the case of hierarchical M_i and m_D , leads to $|\langle m \rangle|$ which is larger by a factor of $(m_c/m_{s,\mu})^2 \sim 10^2$ then in the case of $m_D \sim m_{\rm down}$, $m_c (m_s)$ being the charm (strange) quark mass. If $m_{D2} \sim 1$ GeV and $s_{1L(R)}^2 \sim (10^{-2} - 10^{-1})$ (see Eq. (25)), one finds for $M_1 \sim 10^9$ GeV and $M_2 \sim 10^{10}$ GeV that $|\langle m \rangle| \sim (0.001 - 0.01)$ eV⁷, which may be within the reach of the 10t version of the GENIUS experiment [45].

If condition (37) holds, only the leptogenesis parameter α_R appears in the effective Majorana mass $|\langle m \rangle|$. This condition corresponds to a rather strong hierarchy between the masses of the N_i and thus to a decoupling of the heaviest Majorana neutrino N_3 . In this case, one sees from Eq. (30) that only the term proportional to $\sin 2\alpha_R$ contributes to Y_B and there is a direct correlation between the rate of $(\beta\beta)_{0\nu}$ -decay and the baryon asymmetry of the Universe. For a mild hierarchy between the masses M_i , the unknown phases α_W and β_W spoil any simple connection between $|\langle m \rangle|$ and Y_B .

5.2 *CP* Violation in Leptogenesis and in Neutrino Oscillations

Manifest CP violation can be probed in neutrino oscillation experiments (see, e.g., [24]). The corresponding CP violating observables depend on the rephasing invariant quantity [16] J_{CP} — the leptonic analog of the Jarlskog invariant. The following form [15] of J_{CP} is particularly suited for our analysis:

$$J_{CP} = -\frac{\text{Im}(h_{12} h_{23} h_{31})}{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2} , \text{ where } h = m_{\nu} m_{\nu}^{\dagger} .$$
(38)

⁷We would like to emphasize that the numerical values for the observables we give should be taken not too literally: a spread within one order of magnitude, as included in the prediction for $|\langle m \rangle|$ we give, should be allowed for.

The invariant J_{CP} can be calculated using Eqs. (4) and (22). The resulting expression is rather long and which is the leading term depends on the degree of hierarchy of the heavy Majorana neutrino masses. In general, however, we find that J_{CP} vanishes for $m_{D2} = 0$ and/or $1/M_3 = 0$. These approximations correspond effectively to a 2 flavour neutrino mixing case, and consequently to an absence of Dirac–like CP violation. In general, all six phases contribute to J_{CP} , as is suggested by Eq. (19).

As an example, let us consider the hierarchy $M_1 \simeq 10^{-1} M_2 \simeq 10^{-4} M_3$. Then there are four leading terms in J_{CP} , which are proportional to $(M_1 M_2^4 M_3)^{-1}$, $(M_1 M_3)^{-3}$, $(M_1 M_2 M_3^4)^{-1}$ and $(M_1 M_2 M_3)^{-2}$, respectively. They read:

$$\operatorname{Im}(h_{12} h_{23} h_{31}) \simeq \frac{-2 m_{D2}^5 m_{D3}^7 s_{1L}^2 s_{1R}^2 s_{2L} s_{2R}^5}{M_1 M_2^4 M_3} \cos 2\alpha_R \sin \left(\alpha_W - (\beta_W + \delta_L) + 2\alpha_R - 2(\beta_R + \delta_R)\right) \\ - \frac{m_{D2}^5 m_{D3}^7 s_{1L}^2 s_{1R}^4 s_{2L} s_{3R}}{M_1^3 M_3^3} \sin \left(\alpha_W - 2 \beta_R - \beta_W - \delta_L - \delta_R\right) \\ + \frac{2 m_{D2}^4 m_{D3}^8 s_{1L}^2 s_{1R}^2 s_{2L} s_{3L}}{M_1 M_2 M_3^4} \cos 2\alpha_R \sin \delta_L \\ + \frac{m_{D2}^7 m_{D3}^5 s_{1L}^2 s_{1R}^4 s_{2L} s_{2R}}{M_1^2 M_2^2 M_3^2} \left(2 \sin \left(\alpha_W - \beta_W - \delta_L\right) + \sin \left(\alpha_W - 4 \alpha_R - \beta_W - \delta_L\right)\right) \\ + \frac{m_{D2}^5 m_{D3}^5 s_{1L}^2 s_{1R}^4 s_{2L} s_{2R}}{M_1^2 M_2^2 M_3^2} \left(m_{D2}^2 \left(2 \sin \left(\alpha_W - \beta_W - \delta_L\right) + \sin \left(\alpha_W - 4 \alpha_R - \beta_W - \delta_L\right)\right) \\ + m_{D3}^2 s_{2R}^2 \sin \left(\alpha_W - \beta_W - \delta_L\right) + \sin \left(\alpha_W - 4 \alpha_R - \beta_W - \delta_L\right)\right) \\ + (39)$$

which illustrates our general conclusions mentioned above. Choosing a different hierarchy of the heavy Majorana neutrino masses will lead to the presence of different terms with different combinations of the parameters, especially of the phases.

Of the 6 independent physical CP violating phases of the see-saw model, 5 phases are present in Eq. (39), namely α_R , $\alpha_W - \beta_W - \delta_L$, β_R , δ_R and δ_L . This can be understood as being due to the fact that in the approximations we use m_{D1} is neglected. Setting in addition $s_{3L(R)}$ to zero should remove two more phases from the parameter space. Indeed, one finds from Eq. (39) that only three independent phases (α_R , $\alpha_W - \beta_W - \delta_L$ and $\beta_R + \delta_R$) enter into the expression for J_{CP} in this case. Order-of-magnitude-wise we can predict the magnitude of the CP violation to be

$$J_{CP} \sim 10^{-2} \left(\frac{10^9 \,\text{GeV}}{M_1}\right) \left(\frac{10^{10} \,\text{GeV}}{M_2}\right)^4 \left(\frac{10^{14} \,\text{GeV}}{M_3}\right) \sin\left(\alpha_W - (\beta_W + \delta_L) + 2\alpha_R - 2(\beta_R + \delta_R)\right).$$
(40)

One sees that within the approximation discussed in this paper there is no connection, in general, between leptogenesis in Eq. (31) and the low energy CP violation in neutrino oscillations. Successful leptogenesis without low energy CP violation in neutrino oscillations is, in principle, possible. This interesting case has recently been discussed in [11]. However, given the results of the present Subsection, fine-tuning between the values of the different phases has to take place in order to have leptogenesis and $J_{CP} \approx 0$. Even if there is no direct connection between the leptogenesis and the value of J_{CP} , a measurement of J_{CP} can shed light on the "non-leptogenesis" parameters. We note finally that the decoupling of leptogenesis from the low energy Dirac-phase has been noticed to take place in a number of

models as well, e.g. in [10].

We comment finally on an interesting special case of Eq. (39). Suppose that the phases "conspire" to fulfill the relations $\alpha_W - \beta_W - \delta_L = \beta_R = \delta_R = \delta_L = 0$ or, for $s_{3L(R)} = 0$, $\alpha_W - \beta_W - \delta_L = \beta_R + \delta_R = 0$. Then, only one phase contributes to the J_{CP} asymmetry and one finds from Eq. (39) that $J_{CP} \propto \sin 4\alpha_R$. The baryon asymmetry, on the other hand, is proportional to $\sin 2\alpha_R$ as can be seen from Eqs. (7) and (30). This situation allows for a correlation between the relative sign of the baryon asymmetry of the Universe and the CP asymmetry in neutrino oscillations. The two quantities posses opposite signs if $2\alpha_R$ lies between $\pi/2$ and π and the same signs for values of $2\alpha_R$ outside this range. The parameter space allowing for this correlation corresponds to the 3×2 see–saw model with 2 texture zeros in the Dirac mass matrix discussed in Ref. [12].

6 Conclusions

Assuming only a hierarchical structure of the heavy Majorana neutrino masses $M_{1,2,3}, M_1 \ll$ $M_2 \ll M_3$, and of the Dirac mass matrix m_D of the see-saw mechanism (see Eq. (21)), and working in the bi–unitary parametrization of m_D , we find that in order to produce a sufficient amount of baryon asymmetry via the leptogenesis mechanism, the scale of m_D should be given by the up-quark masses. In this class of "hierarchical" see-saw models, the branching ratios of LFV charged lepton decays, whose dependence on m_D is introduced by RGE effects within the SUSY GUT version of the model, are predicted to fulfill the relations $BR(\tau \to \mu + \gamma) \gg BR(\tau \to e + \gamma) \gg BR(\mu \to e + \gamma)$: typically one has $BR(\tau \to e + \gamma)$ $(\mu + \gamma) \sim 10^2 BR(\tau \to e + \gamma) \sim 10^5 BR(\mu \to e + \gamma)$. We find that the effective Majorana mass in $(\beta\beta)_{0\nu}$ -decay depends on the *CP* violating phase controlling the leptogenesis if one of the heavy Majorana neutrinos is much heavier than the other two, $M_3 \gg 10^4 M_2$. A rather mild hierarchy between the masses of the lighter two heavy Majorana neutrinos, $M_2 \sim 10 M_1$, is required for successful leptogenesis. The hierarchical relations $M_3 \gg 10^4 M_2$ and $M_2 \sim 10 M_1$ with, e.g., $M_1 \sim 10^9$ GeV, are also compatible with the low-energy neutrino mixing phenomenology. The CP violation effects in neutrino oscillations can be observable. In general, there is no direct connection between the latter and the CP violation in leptogenesis. We find, however, that if the CP violating phases of the see-saw model "conspire" to satisfy certain relations, the baryon asymmetry of the Universe and the leptonic CP violation rephasing invariant J_{CP} , which determines the magnitude of the CP violation effects in neutrino oscillations, depend on the same CP violating phase and their signs are correlated.

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Figure 1: Connection between low energy lepton number and CP violation with the baryon asymmetry Y_B via the leptogenesis mechanism. Without the left vertical arrow there is none.