

Formation mechanisms for ice stream lateral shear margin moraines

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Abstract

The lateral shear margins of palaeo-ice stream beds are occasionally marked by large curvi-linear moraines but little work has addressed how they might form. In this paper we review the characteristics of 'lateral shear margin moraines' and present two quantitative models for their formation; (i) differential erosion rates related to lateral variations in the stream velocity; and (ii), lateral incursion from inter-stream ridges after the thinner stream retreats, where the shear margin moraine is consequently just a terminal moraine of the inter-stream ridge. Other possible mechanisms of formation are addressed qualitatively. Both of our quantitative theories are, in principle, feasible, but the incursion model is less plausible and inconsistent with observations. The differential erosion model can predict moraines tens of metres thick and several kilometres wide which can be formed in a thousand years. The width of the moraine depends on the competition between a term which varies across the ice-stream because it is dependent on ice velocity and which always causes erosion, and a term related to the local accumulation rate which causes erosion if is positive and deposition if negative. These terms combine to create the possibility of deposition at the shear margins of ice streams in ablation areas. The width of the moraine has a tendency to increase with the width of the stream. The conditions which permit shear margin moraines to form are quite constrained; too much ablation and deposition occurs everywhere rather than just near the lateral margins of the ice streams; too little ablation and there is erosion everywhere. This is consistent with the infrequent observations of lateral shear margin moraines.

1 Introduction

The discharge of continental ice sheets is focussed into relatively narrow corridors of ice, known as ice streams, which play an important role in their mass balance and their links to the ocean-climate system (Bamber et al., 2000; Joughin and Tulaczyk, 2002). Ice streams are commonly fed by tributaries (Joughin *et al*, 1999) and as the

flow velocity increases, the lateral transition to the slow-moving ice outside of the ice stream generates intense shear zones (Raymond *et al.*, 2001). These shear zones are only a few hundred metres wide and are characterised by prominent bands of crevasses, which are readily identifiable both on the ground and from remote sensing and provide an easy way to identify the onset of ice stream activity (Hodge and Doppelhammer, 1996).

The importance of ice stream shear margins lies in the fact that they can support very high shear stresses (>100 kPa), which often exceed that of the bed (order of 10k Pa) (Jackson and Kamb, 1997; Raymond *et al.*, 2001). Consequently, they play a crucial role in the force balance of an ice stream and side drag has been calculated to support between 50-100% of the driving force, becoming increasingly important in the lower reaches of marine-terminating ice streams (Echelmeyer *et al.*, 1994; Raymond *et al.*, 2001). It is for this reason that ice stream flow, and hence ice flux, is acutely sensitive to the width of the ice stream (Raymond *et al.*, 2001). Given the recent observations from West Antarctica that show changes in ice stream width and lateral migration of lateral margins (Jacobel *et al.*, 1996; Conway *et al.*, 2002), there is an increased urgency associated with understanding the processes that occur at ice stream lateral margins and the factors that control their position (Raymond *et al.*, 2001).

Most studies to date have focused on measuring and modelling the strain rates and mechanics within ice stream shear margins and the associated influence on meltwater production and ice dynamics (Echelmeyer *et al.*, 1994; Jackson and Kamb, 1997; Whillans and Van der Veen, 1997; 2002; Harrison *et al.*, 1998; Schoof, 2004; Van der Veen *et al.*, 2007). Due to their inaccessibility, fewer studies have focussed on the subglacial processes beneath ice stream shear zones, although several papers have attempted to link ice dynamics with basal conditions (e.g. Raymond, 1996; Jacobsen and Raymond, 1998; Schoof, 2004). Radar data appear to show higher basal reflectivity (indicative of meltwater) under the shear zone compared to the ice outside of the lateral margin of the stream (e.g. Catania *et al.*, 2003) but geophysical techniques are hindered by the noise imparted by crevasse structures. More recent deployment of low frequency (2 MHz) ice-penetrating radar across the lateral margin of Whillans Ice Stream in West Antarctica indicated that lubrication within the shear zone is different from that under the central parts of the ice stream, indicating that the 'jump' in basal lubrication lies towards the inner portion of the shear zone (Raymond *et al.*, 2006). Raymond *et al.* (2006) also noted that roughness elements may possibly exist under the shear margin of Whillans Ice Stream but the bed reflection data is not able to confirm or refute this possibility.

In contrast to the severe logistical constraints of studying the bed beneath active (or recently stagnated) ice stream shear margins, the shear margins of palaeo-ice streams are typically very obvious on the now-exposed beds of palaeo-ice sheets (e.g. Stokes and Clark, 1999). Interestingly, observations of palaeo-ice stream shear margins reveal that, in places, the lateral margin of the ice stream is marked by large subglacial ridges, 10s of metres high, 10s metres wide, and several kilometers long (cf. Dyke and Morris, 1988; Stokes and Clark, 2002). There are very few observations of such features and

their mechanism of formation is unknown. This paper is motivated by the idea that if we can understand more about the genesis of these ridges, we learn more about the processes that occur at and control the position of ice stream shear margins.

The paper relies on the till-continuity/till deformation model of glacial sediment transport (Boulton et al., 1985). Till is assumed to deform in a layer beneath an ice stream, with velocity strongly correlated with the overlying ice velocity. If the ice flow is extending, the till flow is also extending. In many situations, the ice thickness is maintained by surface accumulation, but the till thickness will decrease on account of the extension - i.e. erosion. If ablation is occurring, the opposite effects occur; the ice flow compresses and till will build up. Modelling of a variety of different circumstances based on these principles has been carried out by e.g. Alley et al. (1987), Alley (1991), Jenson and others (1996), Boulton (1996), Hindmarsh (1997), Clark and Pollard (1998) and Pollard and DeConto (2003). These studies apply to larger length scales than are considered here, and there may be a lower length scale to which viscous theories apply (Hindmarsh, 1997); if the viscous theory can be shown to work for landforms such as ribbed moraine and drumlins, then this indicates that the modelling approach used here remains valid. A study of ribbed moraines suggests that the viscous theory remains a useable description down to length-scales of 100m (Dunlop, 2004), comparable with the narrowest shear moraines.

Firstly, we review the relevant data from palaeo-ice stream beds. Following this, a simplified theory based on lateral variations of velocity and differential erosion is presented, and shown to fulfil some basic criteria in terms of moraine thickness and breadth. The assumptions going into the simplification are reviewed, and a more sophisticated version of the theory is then presented, which makes some quantitative difference but does not affect the broad outline of the picture. The increased complexity of the theory arises from a more complete discussion of lateral flow processes, and of deformation styles within till. An alternative theory is also presented based on flow incursion from inter-stream ridges following climatically induced retreat of the ice-stream, and shown to fit the observations less well. Both theories are essentially time-dependent, and argue that lateral shear margin moraines are associated with retreat processes. This is in contrast to other possible mechanisms of formation which we briefly discuss and evaluate.

2 Observations of Lateral Shear Margin Moraines

Descriptions of lateral shear margin moraines are rare compared to the ubiquity of other subglacial landforms on former ice sheet beds (e.g. flutes, drumlins, ribbed moraines, etc.). The first detailed description of a lateral shear margin moraine was reported by Dyke and Morris (1988) who identified a single narrow ridge of till, 68 km long and lying perfectly parallel to the edge of a drumlin field on Prince of Wales Island, Canadian Arctic Archipelago, see Figure 1. They noted that the ridge was much lower and narrower than the adjacent drumlins but that it had an ice moulded appear-

ance and that individual segments of the ridge partially resembled heel-to-toe drumlins. Dyke and Morris (1988) stated that “it is difficult to envisage what might cause a single body of drift to become excessively attenuated at the edge of a drumlin field” but provided two possible explanations.

The first explanation was that it might represent a lateral moraine formed at the lateral margin of a brief but rapid ice flow (surge?) but this was rejected because there is strong evidence that the area outside of the drumlin field was not ice-free (see Dyke et al., 1992 for comprehensive reconstruction of glacial history). The second explanation, therefore, postulated that the ridge marks the boundary between rapidly flowing warm-based ice to the north and east and slow-flowing cold-based ice to the south and west (Figure 1). Dyke and Morris (1988) and later, Dyke et al. (1992) preferred this explanation, coining the term ‘lateral shear moraine’ and suggesting that the feature marks a curvilinear vertical shear zone at the lateral margin of an ice stream.

Later work by Kleman and Borgström (1994) identified a similar ridge formed subglacially at the boundary between warm and cold-based ice at a site in northern Sweden. They described a 600 m ridge of till marking the boundary of a small area of relict periglacial landscape, inferred to have been preserved under cold-based ice. Because the adjacent flutes in this location are orientated at a very small angle towards the cold-based patch, Kleman and Borgström (1994) suggested that ice flow, and hence subglacial sediment transport, led to the accumulation of debris at the boundary of the cold-based patch. They defined a lateral shear moraine as ‘a ridge parallel or sub-parallel to a flow line between glacially lineated and a relict [formerly cold-based] surface’ and suggested they were a diagnostic landform of a thermal boundary.

The only other well-documented location of lateral shear moraines is on Storkerson Peninsula, north-eastern Victoria Island, which, incidentally, lies across M’Clintock Channel from Prince of Wales Island at the north-western edge of the Laurentide Ice Sheet. Here, Hodgson (1994) was the first to describe massive, extended ridges that border a drumlin field thought to have been produced by an ice stream, see Figure 2. He noted the similarity of these ridges to the one reported by Dyke and Morris (1988) and Dyke et al. (1992) but also pointed out that the area outside of the ridges (and the ice stream) was probably not cold-based. This is because eskers are found outside and spanning the ice stream margin (see Figure 2). Alternatively, the area outside the ice stream may have been cold-based during ice stream activity but reverted to warm-based during esker formation. The ridges identified by Hodgson (1994) were the focus of further research by Stokes and Clark (2002). They identified four distinct ridges at the lateral margin of the M’Clintock Channel Ice Stream (cf. Hodgson, 1994; Clark and Stokes, 2001). The southernmost moraine is shown in Figure 2 and represents a 22 km long sinuous ridge of till. Taken together, the ridges are generally 10-20 km in length, 250-1000 m wide and 10-60 m above the surrounding terrain. They generally appear more ruggedly streamlined than the adjacent ice stream bedforms and the elevation along their crestline fluctuates by 10s of metres, possibly due to subsequent modification. Stokes and Clark (2002) also noted that two of the ridges appear further away (and slightly offset) from the last inferred position of the ice stream

lateral margin and speculated that they may have formed when the ice stream was in a slightly different location (implying lateral margin migration prior to ice stream shutdown).

More recent observations of shear margin moraines are reported by Ottesen et al. (2005) who use their presence to define the width of a palaeo-ice stream in the Vestfjorden-Trænadjupet drainage system on the mid-Norwegian continental shelf. These ridges are several 10s of kilometres long, a few kilometres wide, and 10-20 m high but the mechanisms of formation are not discussed. The available data on the dimensions and characteristics of lateral shear margin moraines are summarised in Table 1. It is likely that other shear margin moraines remain to be detected. More recent mapping of the whole of Victoria Island by Storrar and Stokes (2006), for example, identified another shear margin moraine further south from those on Storkerson Peninsula. Notwithstanding this, almost 20 years of searching palaeo-ice sheet beds has yielded only a small (<30) population of such landforms. It would appear, therefore, that they are either rarely formed or that they are rarely preserved.

Have shear margin moraines been identified under active ice streams? There is no convincing evidence of their occurrence but Clarke et al. (2000) identified several features described as entrained morainal debris at the base of a recently abandoned shear margin of Ice Stream B in West Antarctica. Further offshore in West Antarctica, there are also descriptions of 'laterally accreting ridges' marking the position of the lateral margins of palaeo-ice stream tracks (Shipp et al., 1999).

The observations and data on shear margin moraines are largely descriptive and no work has specifically addressed the formation of these features. Stokes and Clark (2002) did, however, speculate on five possible mechanisms which included: 1. Melt-water processes depositing sediment in englacial and subglacial streams; 2. Melt-out and deposition of entrained englacial debris; 3. Downstream sediment recycling ('bank' erosion-deposition, analogous to fluvial deposition of bars); 4. Differential erosion; 5. Lateral advection of sediment towards the lateral margin. Based on a large (up to 40 m) 'step' in the topography coinciding with the ice stream margin, Stokes and Clark (2002) tentatively favoured the sediment recycling hypothesis, envisaging a situation whereby sediment is 'mined' from the ice stream margin (analogous to a river bank) and is then deposited and streamlined in the down-stream direction. The association between topographic steps and ice stream shear margin moraines has not, however, been reported elsewhere.

In this paper, we explore aspects of the latter two mechanisms; (i) that they are the result of lateral variations in the erosional power of ice-streams, and (ii) that they are the results of lateral advection of sediment towards the ice stream shear margin; specifically, that the retreat of the ice-stream allows the inter-stream ridge to advance into the area formerly occupied by the ice-stream, and that the lateral moraines represent a type of terminal moraine. Following this analysis, we evaluate their plausibility and discuss other possible theories of formation.

It is important to make a distinction between subglacial shear margin moraines and 'interlobate complexes' that are inferred to form between neighbouring ice lobes/streams

(Punkari, 1995, 1997). Firstly, lateral shear margin moraines form at the boundary between fast and slow-flowing ice (which may also be a thermal boundary), whereas interlobate complexes form between regions of similar ice velocities (Punkari, 1997). Secondly, lateral shear margin moraines are, according to limited observations (although no exposures have been recorded), built from subglacial till (see Table 1), whereas interlobate complexes comprise glaciofluvial material transported by supraglacial and englacial meltwater channels (Punkari, 1997). Thirdly, lateral shear margin moraines occur as narrow linear to curvilinear ice-moulded ridges, whereas interlobate complexes generally occur as composite ridges and hummocks and are often found in association with eskers (Punkari, 1997).

3 Variation in lateral erosion from lateral variations in ice-stream dynamic fields

Lateral variations in ice-stream downstream velocity create lateral variations in the downstream till flux gradient, and consequently differential erosion. However, the requirement of the ice-stream flow to balance a locally uniform accumulation rate requires a transverse velocity component to be set up in the ice-stream in order to ensure mass conservation in the ice-sheet (Hindmarsh, 2006). In the accumulation area, ice moves very slowly from near the lateral margin to the centre of the ice-stream. In the ablation zone the movement is the other way. If till and ice flow are reasonably strongly correlated, this creates an additional stretching or shortening in *till* strain rates across the ice stream (i.e. a lateral gradient in the lateral velocity) which turns out to acts against lateral variation in the erosive power. If stretching terms dominated till erosion, erosion would be as uniform as the accumulation rate pattern, which is not expected to vary significantly across an ice-stream. However, our analysis shows there is also an erosional/depositional term which depends upon the ice thickness gradient (normalised by ice thickness) and till thickness gradient (normalised by till thickness) being different, which in general is true. This term is proportional to the ice velocity.

Consequently, there is (i) an erosion or deposition term independent of lateral position which increases with thinning and accumulation, (i.e. where the ice is extending), and decreases with ablation and thickening, (i.e. where the ice is compressing); and (ii) a erosive term which varies across the ice-stream in proportion to the velocity. The two terms are additive, creating variations in erosion or deposition across the ice-stream. The argument can be put very simply, which we do now, and then refined, to show its robustness.

3.1 Basic Principles

Consider a coordinate system (x, y, z) , with x aligned with the flow of the ice-stream, y transverse to the flow and z pointing vertically upwards. We recall that in an ice-

stream, velocities are low at the lateral margin, increase rapidly through the shear zone, and reach a plateau in the middle. This can be justified theoretically (Raymond, 1996). The statement of continuity in the ice is

$$\frac{\partial H}{\partial t} + H \frac{\partial u}{\partial x} + u \frac{\partial H}{\partial x} + v \frac{\partial H}{\partial y} + H \frac{\partial v}{\partial y} = a, \quad (1)$$

where H is the ice thickness, (u, v, w) are the velocities and a is the accumulation rate. Assume $v \frac{\partial H}{\partial y}$ to be small, and noting that in general $\frac{\partial H}{\partial x} < 0$, (1) can be rewritten

$$\frac{\partial v}{\partial y} = \frac{1}{H} \left(a - \frac{\partial H}{\partial t} \right) - \frac{u}{H} \frac{\partial H}{\partial x} - \frac{\partial u}{\partial x}. \quad (2)$$

This term can be integrated

$$v(y) = \int_0^y \left(\frac{1}{H} \left(a - \frac{\partial H}{\partial t} \right) - \frac{u}{H} \frac{\partial H}{\partial x} \right) dy' - \int_0^y \frac{\partial u}{\partial x} dy' = -v_L, \quad (3)$$

where v_L is the prescribed velocity at the shear margin arising from flow into the stream from flanking ice-rises. Initially it is assumed that till is deforming in a plug with velocity equal to the ice velocity. In this case, till continuity is

$$\frac{\partial D}{\partial t} + D \frac{\partial u}{\partial x} + u \frac{\partial D}{\partial x} + v \frac{\partial D}{\partial y} + D \frac{\partial v}{\partial y} = 0. \quad (4)$$

We first consider the simple case where D is uniform across the stream, so that $\frac{\partial D}{\partial y}$ is zero. Then, till continuity simplifies to

$$\frac{\partial D}{\partial t} = -D \frac{\partial v}{\partial y} - D \frac{\partial u}{\partial x} - u \frac{\partial D}{\partial x}, \quad (5)$$

so by using (2) the erosion rate can be computed.

$$\frac{\partial D}{\partial t} = -\frac{D}{H} \left(\left(a - \frac{\partial H}{\partial t} \right) - u \left(\frac{\partial H}{\partial x} - \frac{H}{D} \frac{\partial D}{\partial x} \right) \right). \quad (6)$$

Note that $\frac{\partial u}{\partial x}$ has cancelled in this expression. It is a good approximation to take H , $\frac{\partial H}{\partial t}$ and $\frac{\partial H}{\partial x}$ to be constant across an ice-stream, meaning that the lateral erosion rate variation is dominated by the variation in u . To simplify matters further, let us take $u \frac{\partial D}{\partial x}$ to be zero, and examine what happens to a uniform till layer. The term $-u \frac{\partial H}{\partial x} > 0$ nearly always, and the term $(a - \frac{\partial H}{\partial t}) > 0$ under many circumstances e.g. a steady Antarctic ice stream, or a steady extending Antarctic ice-stream. Then there will be erosion across the width of the ice stream. If however the term $(a - \frac{\partial H}{\partial t}) < 0$ (ablation or compressional flow) then one can have till deposition everywhere, or till deposition at the shear margins but erosion in the middle, or erosion throughout.

It is important to realise that the source of the till is upstream, even though there are transverse velocities. Something that is slightly non-intuitive is that since the velocity

fields are identical in ice and till (in this first analysis of the problem) and the ice flux divergence has to balance a uniform accumulation/ablation rate, one wonders why should the till erosion/deposition rate not be uniform. The difference arises from the advective ice flux divergence contribution $u \frac{\partial H}{\partial x}$ which is in general different from the advective till flux divergence $u \frac{\partial D}{\partial x}$. In this first analysis we take this term to be zero, but more generally there is no reason why $\frac{\partial H}{\partial x}$ should be the same as $\frac{H}{D} \frac{\partial D}{\partial x}$, and it is this which permits lateral variation in the erosion rate.

Some examples in Figure 3 show the conditions which create differential erosion/deposition near the flanks of ice-streams. The derivation of the downstream velocity field is discussed below, and is shown in Figure 4; parameters used are listed at the end of the next section. Under particular conditions, notably when the shear margin is experiencing ablation or when the ice is thickening, one can have erosion in the middle of the ice-stream but deposition at the sides. Under this view lateral shear margin moraines are an unusual variant of terminal moraine, which occur under special combinations of ablation, ice velocity and surface slope. Their lack of streamlining and irregular thickness along profile can at least in principle be explained by this as follows. If lateral deposition only occurs in the ablation zone, then the amount of deposition (thickness of till) will reflect amongst other things the time interval that the ablation zone occupied that particular area. Variations in till thickness can be explained by variability in the ice sheet margin retreat rate. Note in Figure 3 that the predominant direction of the lateral velocity depends upon the surface mass-balance regime in the area.

3.2 Calculation of the lateral variation in downstream velocity

Across a very long flat and weak-bedded ice stream, the appropriate equation for the along-flow mechanical balance is (Raymond, 1996)

$$\frac{\partial (H \bar{\tau}_{xy})}{\partial y} = \rho g H \frac{\partial H}{\partial x} + \tau_{bx}, \quad (7a)$$

$$\tau_{bx} = \begin{cases} \tau_b^*, & \text{Plastic Bed} \\ C |u|^{1/\ell}, & \text{Viscous bed} \end{cases}, \quad (7b)$$

$$\bar{\tau}_{xy} = \bar{B} \left| \frac{1}{2} \frac{\partial u}{\partial y} \right|^{\frac{1}{n}-1} \frac{\partial u}{\partial y} \quad (7c)$$

where $\bar{\tau}_{xy}$ is the mean value of the shear stress acting in the vertical plane, ρ_i is the density of ice, g is the acceleration due to gravity, τ_{bx} is the basal shear stress (strictly, tangential traction) acting in the downstream direction, τ_b^* is the plastic strength of till, u is the downstream velocity component, C and ℓ are material constants for the till, and \bar{B}, n are material properties for the ice; \bar{B} is the mean value over the vertical column. Boundary conditions are $\bar{\tau}_{xy}(y=0) = 0$ at the stream centre and $u(y=L) = 0$ at the stream shear margin. It is assumed that the bed is sufficiently weak that

vertical variations in the velocity are small. If ℓ is large, the viscous law tends towards plasticity. Both C and τ_b^* include the effects of the unknown effective pressure; as is usual, we assume it to be set by the properties of the sub-glacial drainage system.

The mechanical equations (7) can be readily solved numerically to obtain the downstream velocity u as a function of lateral position y . Our aim now is to examine how the downstream velocity varies with position when affected by variations in the basal shear stress τ_{bx} . In particular we will be interested in how the width of the shear zone depends upon τ_{bx} and through this on the very poorly-known bed parameters τ_b^* , C and ℓ .

Figure 4 shows some velocity plots for different bed conditions. The different cases are indicated in the caption. As the bed becomes more viscous, the shear zone broadens, and the difference between the greatest deposition rate and the most negative erosion rate decreases. A large contrast is helpful to the creation of lateral shear margin moraines, as it increases the range of circumstances where one can get the necessary simultaneous erosion and deposition to create shear moraines. Thus, the apparent association of shear moraines with streamlined features, inferred high velocities and implicit weak beds is at least consistent with the theory.

Parameters used were $\frac{\partial H}{\partial x} = 0.002$, $H = 1000\text{m}$, $\rho_i = 917\text{kg.m}^{-3}$, $g = 9.81\text{m.s}^{-2}$, $n = 3$, $L = 20\text{km}$, $\bar{B} = 2.7 \times 10^8\text{Pa.yr}^{1/3}$, $\ell = 3$. The resistant cases 2 and 3 had mean values $C = (2.15 \times 10^3, 10^4)\text{Pa.}(\text{yr.m}^{-1})^{1/3}$. The effective pressure had a mean value of 10kPa and where it varied in space (Case 4), it was linearly from 5kPa in the centre to 15kPa at the lateral margin.

3.3 Strong lateral variations and time evolution.

So far we have computed deposition/erosion rates on the basis that the till was of uniform thickness across the stream, and used these to calculate resultant till thicknesses. In fact, the erosion/deposition rates are influenced by the lateral variations in till thickness, which will develop through time. Here, we consider the effect of lateral variations in thickness upon the flow. Now, the till continuity equation is

$$\frac{\partial D}{\partial t} + v \frac{\partial D}{\partial y} + \frac{D}{H} \left(\left(a - \frac{\partial H}{\partial t} \right) - u \frac{\partial H}{\partial x} \right) = 0. \quad (8)$$

where $v(y)$ is given by (3). We are still taking $\frac{\partial D}{\partial x}$ to be zero. In the regions of interest, where lateral moraine is being built, we rather expect $\frac{\partial D}{\partial x}$ to be of the opposite sign to $\frac{\partial H}{\partial x}$ so this assumption, motivated by tractability, does not seriously bias our results by producing deposition where none exists. Clearly, as the till thins non-uniformly the bed resistance will become non-uniform and the downstream velocity will change, but the results in the preceding section indicate that the basic pattern remains relatively constant. We therefore take the velocity distribution from the perfectly slippery bed, and use this to compute v .

The results from four time-dependent evolutions are shown in Figure 5, the different cases corresponding to the accumulation/ablation cases in Figure 3. Cases are

shown which include the effect of transverse velocity as well as ignoring it. In the accumulation area, transverse advection ($v\partial D/\partial y$) has the effect of broadening the area of reduced erosion associated with the shear zone; in the ablation area, transverse advection sharpens up the zone of deposition, in other words causes the lateral shear margin moraine to be thinner and higher.

4 Velocity/Flux relationships for shearing flow

So far a somewhat simple approach to till deformation has been followed, which assumes that till flows as a plug with thickness D . We consider qualitatively what will happen if there is internal deformation within the till in a quasi-viscous way.

The velocities and fluxes are computed following Alley (1989) and Hindmarsh (1998). Till flux can arise either from internal deformation within the till or from till sliding over the base. We assume that when considering internal deformation, the strain-rate in the till is given by a double power law rheology (Boulton and Hindmarsh, 1987)

$$\frac{d\mathbf{u}}{dz} = \frac{A_d |\mathbf{T}_t|^{\ell-1} \mathbf{T}_t}{p_e(z)^b} = \frac{A_d |\mathbf{T}_t|^{\ell-1} \mathbf{T}_t}{(\hat{p}_c + \beta(D-z))^m}, \quad (9)$$

where A_d is a rate factor and a and b are till material parameters, and, following Hindmarsh (1997), view this as a parameterisation of many complex failure processes. After integrating with respect to z over the interval $0 \leq z \leq D$ we find

$$\mathbf{u}_b = \frac{A_d |\mathbf{T}_t|^{\ell-1} \mathbf{T}_t}{\beta(m-1)} (\hat{p}_c^{1-m} - (\hat{p}_c + \beta D)^{1-m}), \quad (10)$$

and the flux is given by

$$\mathbf{q} = \frac{A_d |\mathbf{T}_t|^{\ell-1} \mathbf{T}_t}{(m-1)(m-2)\beta^2} (\hat{p}_c^{2-m} - (\hat{p}_c + \beta D)^{1-m} (\hat{p}_c + \Omega D)), \quad (11)$$

where

$$\Omega = \beta(m-1), \beta = (1-\phi)(\rho_s - \rho_w)g \quad (12)$$

where g is the acceleration due to gravity, ρ_w , ρ_i and ρ_s are the densities of ice, water and sediment grains, and ϕ is the porosity of the sediment. We use the values 9.81m.s^{-2} , 1000kg.m^{-3} , 917kg.m^{-3} , 2700kg.m^{-3} and 0.2 respectively in this paper.

An effective thickness D_e (see also Schoof, 2007, for a discussion on effective till thickness) may be defined by

$$D_e = \frac{|\mathbf{q}|}{|\mathbf{u}_b|} = \frac{1}{(m-2)\beta} \frac{\hat{p}_c^{2-m} - (\hat{p}_c + \beta D)^{1-m} (\hat{p}_c + \Omega D)}{\hat{p}_c^{1-m} - (\hat{p}_c + \beta D)^{1-m}}.$$

Note that this is independent of the shear stress, but depends on the interfacial effective pressure \hat{p}_c . For deep till ($\beta D \gg \hat{p}_c$), this becomes

$$D_e = \frac{1}{(m-2)\beta} \frac{\hat{p}_c^{2-m} - \Omega\beta^{1-m}D^{2-m}}{\hat{p}_c^{1-m} - (\beta D)^{1-m}}$$

with different further simplifications depending on the magnitude of m

$$D_e = \begin{cases} \frac{\hat{p}_c}{(m-2)\beta} & m > 2 \\ \frac{(m-1)\beta^{1-m}}{(2-m)} \frac{D^{2-b}}{\hat{p}_c^{1-b}} & 1 < m < 2 \\ \frac{m-1}{m-2} D & m < 1 \end{cases} .$$

The lower values of m have the effective thickness increasing with till thickness, so in qualitative terms the results will be similar for the case of plug flow. However, for the case $m > 2$, the most plastic-like behaviour, the effective thickness is independent of thickness (when the sediment thickness is large), and increases with effective pressure. Figure 6 shows graphs of effective till thickness for different effective pressure indices, with $\beta = 1.3 \times 10^4 \text{Pa}\cdot\text{m}^{-1}$ and $\hat{p}_c = 10^4 \text{Pa}$. It seems that in general $D_e \leq D$, at least in situations where there is no extrusion flow.

When we are no longer in plug flow, the till continuity equation is

$$\frac{\partial D}{\partial t} + v \frac{\partial D_e}{\partial y} + \frac{D_e}{H} \left(\left(a - \frac{\partial H}{\partial t} \right) - u \frac{\partial H}{\partial x} \right) = 0.$$

The third term shows D_e affects the rate of lowering, presumably reducing it as $D_e \leq D$. The second term shows that the advection term depends upon $\frac{\partial D_e}{\partial y}$ rather than directly on D .

For small thicknesses, the effective thickness increases with the thickness and the evolutions are like those shown with the solid line in Figure 5. For $m > 2$ and large till thicknesses (e.g. for the case shown, $> 10\text{m}$), the effective thickness reaches a constant value. In this case, the term $v \frac{\partial D_e}{\partial y}$ will be negligible, and the results are qualitatively similar to those with the dashed line in the same figure. Since the cases $D_e \propto D$ and D_e is constant bound the theoretical cases, we expect evolutions to lie somewhere between the solid and dashed lines in Figure 5. As both produce reasonable simulations of lateral shear moraine formation the somewhat involved matter of flux/velocity relationships does not appear to require further investigation in this context.

5 An alternative theory: incursion from inter-stream ridges

An alternative till-continuity-based hypothesis can be constructed on the notion that when an ice stream thins and retreats, it no longer prevents incursion of ice from the inter-stream ridges, which can advance on the recently deglaciated ice stream bed.

There is no substantial geomorphic evidence from underneath ice-stream ridges in the form of cross-cutting lineations from the zone that was initially ice-stream and subsequently overridden by the ridge. However, we arguably do not know enough about the conditions under which palimpsest landforms are preserved for this to be a conclusive objection, so we proceed with a modelling study to investigate the feasibility of the mechanism.

For this, we need a time-dependent ice-sheet model; unlike the previous case the incorporation of membrane stresses is not necessary, so the shallow ice approximation (Hutter, 1983) is used to describe ice mechanics. Thus, for the ice the continuity equation is

$$\frac{\partial H}{\partial t} + \nabla \cdot \mathbf{Q} = a, \quad (13)$$

$$\mathbf{Q} = -\frac{2A(\rho g)^n}{n+2} H^{n+2} \left| \frac{\partial s}{\partial x} \right|^{n-1} \frac{\partial s}{\partial x} + H \mathbf{u}_b \quad (14)$$

and for the till

$$\frac{\partial D}{\partial t} + \nabla \cdot \mathbf{q} = 0. \quad (15)$$

In broad terms, an ice-sheet is allowed to grow. Lubricating till is provided in a strip, so as to form an ice-stream. Accumulation and ablation areas are specified so as to limit the size of the ice-sheet. The ablation is increased, and the stream and inter-stream ridge both retreat - the stream more so, because of its lower slope. During the initial period, while the till lubricates, its surface profile is not allowed to change. This is contrived, but it allows us to look at the changes in till thickness after the change in ablation.

The results of one simulation are presented in Figures 7 and 8. Ice flow always occurs in the downslope direction. Parameters are as above; also $A = \bar{B}^{-1/3} = 10^{-16} \text{Pa}^{-3} \cdot \text{yr}^{-1}$. The accumulation rate is $1 \text{m} \cdot \text{yr}^{-1}$; the ablation rate is increased from $5 \text{m} \cdot \text{year}^{-1}$ to $10 \text{m} \cdot \text{year}^{-1}$ to cause retreat. The till parameters are $m = \ell = 2.5$, $A_d = 10 \text{yr}^{-1}$.

As can be seen, terminal moraines for the ice-stream as well as a lateral shear moraine are created. The lateral shear moraine is really better described as a corner moraine. This example took a fair amount of experimentation adjusting the forcing parameters to create, and such clear moraines were not in general created.

6 Summary of Conclusions from Modelling

1. If till velocity is reasonably strongly correlated with ice velocity, this creates differential till erosion/deposition patterns across an ice stream on account of the strong lateral variations in downstream velocity. This erosion stems from till being thinned by stretching.

2. The erosion/deposition consists of (i) a quantity uniform across the breadth of the ice stream, which can be either erosional or depositional. Erosion increases with accumulation or extending flow, and decreases with ablation or compressing flow; and (ii) a spatially varying term which is always erosional, with the greatest erosion occurring in the fastest parts of the ice stream.
3. Deposition can only occur when there is ablation or compressing flow
4. Typical erosion/deposition rates are of the order of millimetres to centimetres per year.
5. We associate lateral shear margin moraines with the retreat of the ablation zone through an area. Changes in ablation rate or in occupancy time will affect the thickness of such moraine, offering an explanation of their somewhat irregular along-flow profiles.
6. Low ablation or thinning by glaciodynamic processes including surging reasonably explain the absence of lateral moraines.
7. These conclusions appear to be independent of detailed consideration of how the deformation of till varies with depth, provided that there is some flux of till correlated with ice velocity.
8. Our modelling of transverse variation in erosion effectively parameterised the along-flow direction. Modelling could be improved by considering map view modelling with longitudinal stresses.
9. Time-dependent modelling using a simpler mechanical model (the shallow ice approximation) shows that ice-stream retreat and lateral incursion from ice-stream ridges could produce lateral moraines, but the predicted features appear to be associated with ice-stream terminal moraines, and the lateral moraines are really corner moraines.
10. We suggest that lateral shear margin moraines are relatively rarely formed, rather than rarely recognised (compared to the ubiquity of other subglacial bedforms) and that this is due to the unique glaciological setting. Their formation requires a metres-thick layer of unconsolidated sediment; an abrupt lateral transition in ice velocity; and relatively high ablation rates.
11. Till continuity/deforming bed mechanisms offer a feasible explanation for lateral shear margin moraines but other possible mechanisms require testing and there is an urgent need to examine the internal structure of shear margin moraines.

7 Discussion

7.1 Evaluation of model plausibility

A final satisfying discussion would include a statement on how well the theories match the data in Table 1 and in Figures 1 and 2, in terms of the moraine thickness and width. In both theories these observables are dependent on unknown parameters, and we have to cast the discussion in the more restrictive form of whether the theories are plausible explanations of the observations. Since the lateral incursion mechanism seems to require specific associations between lateral and terminal moraines that are not generally observed, we conclude it to be unlikely to be an adequate explanation of the observations we have reported.

Figure 5 shows that moraines tens of metres thick and several kilometres wide can be formed in a thousand years. The width of the moraine depends on the competition between (i) the term $u\partial H/\partial x$ (downstream velocity times slope where the bed is flat) which always causes erosion, and (ii) $a - \partial H/\partial t$ (accumulation rate - rate of change of thickness) which causes erosion if it is positive and deposition if negative. The first term becomes very small near the lateral margin of an ice-stream, meaning that lower erosion or deposition can occur in this area. Exactly where across the ice-stream erosion changes to deposition depends upon the competition between the two terms. Since how the velocity pattern varies across the ice-stream depends on the ice-stream width, the width of the moraine is also dependent on the ice-stream width; if the ice-stream were narrower, the lateral moraine would be narrower.

From the above discussion we can see that the rate of erosion and deposition depend upon the ice-stream velocity and the accumulation/ablation rate. It seems that the ratio of the two which permits lateral moraines to form is quite constrained; too much ablation and deposition occurs everywhere rather than just near the lateral margins of the ice streams. Moreover, the ablation zone has to be in the same region for some considerable time to create the substantial moraines.

7.2 Alternative mechanisms of formation

Our analysis suggests that differential erosion rates related to lateral variations in ice stream velocity offer a plausible explanation for the formation of lateral shear margin moraines. Numerical modelling is also able to generate such moraines from lateral incursion of interstream ridges after the (thinner) ice stream has retreated. Since this mechanism suggests that shear margin moraines may actually be a form of terminal moraine and should be found in association with terminal moraines we reject it: they have never been observed in such situations. Moreover, after the ice stream has retreated, the advance from the inter-stream ridge might be expected to flow across the trough. In this case, we would expect to observe cross-cutting lineations in the trough (and maybe flutings/lineations pointing obliquely towards the moraines from outside the trough). Again, this situation has never been observed.

There are several other possible explanations for the formation of lateral shear margin moraines, which were briefly outlined in Stokes and Clark (2002) and which we now discuss. These are: (1), meltwater processes depositing sediment in englacial and subglacial streams; (2), melt-out and deposition of entrained englacial debris; (3), downstream sediment recycling from a step at the lateral margin; and (4), lateral advection of sediment towards the ice stream margin.

The shear created between the slow-moving interstream ridges and the ice stream results in strain-heating and high melting rates at ice stream margins. The generation of crevasses in this area may also allow any surface meltwater to penetrate to the bed. For these reasons, Punkari (1997) suggested that the margins of ice streams, specifically interlobate areas of the Scandinavian Ice Sheet, were prime locations for the net accumulation of glacial and glaciofluvial deposits. This led Stokes and Clark (2002) to consider whether meltwater processes play a role in forming lateral shear margin moraines. They discarded the hypothesis on the grounds that, based on their morphology and surficial deposits, all of the examples mentioned in the literature are described as being composed of till (e.g. Hodgson, 1994). This would appear to undermine theories of formation which appeal directly to meltwater erosion and deposition and liken shear margin moraines to eskers. Moreover, unlike eskers, which are commonly sinuous, sharp-crested, and may have tributaries; shear margin moraines are curvi-linear, have streamlined crests, and have never been observed to have tributaries. This, too, would appear to indicate that shear margin moraines *sensu stricto* are not glaciofluvial features. Although we think it unlikely that shear margin moraines are related to meltwater processes, we add a note of caution because there are no detailed observations of their internal structure. Large and well-defined moraine ridges can be formed almost entirely of glacio-fluvial material (e.g. Salpausselkä moraines in southern Finland: Fyfe, 1990) and glaciofluvial deposits, including eskers, have been observed at ice stream shear margins and in interlobate areas (Punkari, 1997). In order to ultimately resolve the potential role of meltwater processes, there is a clear need to examine the internal structure of ice stream shear margin moraines.

Related to meltwater processes is the idea that entrained englacial debris may melt out and be preferentially deposited at ice stream shear margins. If sediment is eroded and entrained in the onset zones of ice streams, or through the process of basal freeze-on under the main trunk of the ice stream (cf. Christofferson et al., 2006; Rempel, in press), the elevated levels of strain heating (cf. Jacobsen and Raymond, 1998; Schoof, 2004) could produce sufficient melting and deposition of englacial debris at ice stream shear margins. Therefore, given sufficiently long time scales, melt-out could be a feasible mechanism for producing ice stream shear margin moraines. This process could be seen as being related to our ideas which link both differential erosion and ablation, although not subglacial melting.

The process of downstream sediment recycling was favoured by Stokes and Clark (2002) who reported a series of shear margin moraines in association with a topographic step at the ice stream margin. They suggested that as the ice stream eroded down through a thick layer of soft sediments it produced a step at the ice stream mar-

gin and that the moraines are composed of sediment mined from lateral erosion into the step and subsequently smeared downstream. They draw analogy with fluvial erosion on the outside of a meander bend and subsequent deposition in bars. The appeal of this mechanism, as stated by Stokes and Clark (2002) is that sediment supply from upstream is the primary control on their development and this would explain the large variability in their size and elevation above surrounding terrain. It would also explain why ice stream shear margin moraines are not found at the inferred margins of all palaeo-ice streams. It should be noted, however, that the secondary transverse flows encountered in the fluvial setting are not plausible in the glacial setting. Moreover, not all shear margin moraines are found in association with topographic steps. The central idea of differential erosion is appealing, however, and our new analysis in this paper would appear to support its important role, without necessarily invoking steps at the ice stream margin or downstream recycling and streamlining.

A further mechanism discussed by Stokes and Clark (2002) was lateral advection of sediment towards the lateral margin from outside or from within the ice stream. Our analysis in this paper suggests that sediment from outside the margin might be transported towards and deposited at the margin after the thinner ice stream has retreated but that this mechanism is inconsistent with observations. It is also plausible that the interstream ridges deliver sediment towards the shear margin whilst the neighbouring ice stream remains active. We would argue that this is unlikely because most adjacent interstream ridges in both palaeo and contemporary settings were/are characterised by inactive cold-based ice, which is not capable of eroding and transporting significant amounts of subglacial sediment.

A further possibility is that sediment is transported from within the ice stream towards the margin. This is a second-order prediction of our theory, but is difficult to detect and in general the flow vectors of ice streams do not point obliquely towards the ice stream margin. This mechanism was, however, favoured by Kleman and Borgström (1994) who observed flutings pointing at a small angle towards a lateral shear moraine in northern Sweden. They suggested that the lateral shear moraine may have formed because subglacial debris was transported towards the patch of cold-based ice and accumulated at the frozen boundary. There is no evidence that this feature was formed at the margin of an ice stream and so this mechanism may not be valid for the specific case of ice stream shear margin moraines. Kleman and Borgström (1994) do, however, highlight the importance of frozen-thawed boundaries, which are known to exist at the lateral margins of many active ice streams and which may or may not play a role in shear margin moraine formation. Stokes and Clark (2002), for example, also raised the possibility that vigorous flow within the ice stream may squeeze and stack subglacial sediment up against the thermal boundary and that substantial basal melting in the low pressure marginal zone may also allow material to be squeezed upwards, drawing water-rich till from beneath the ice stream.

Our theory does predict movement towards the margins in the ablation zone, which is where we predict shear margin moraines to form, so the observations of Kleman and Borgström (1994) are entirely consistent with our theory. However, the source of the

till is from upstream rather than from the sides, so it is not really correct to view our theory as postulating lateral accretion; transverse motion is a second-order effect associated with our model, and the oblique fluting angles seen by Kleman and Borgström (1994) are consistent with this. If one wants to make a distinction between lateral shearing and squeezing (both are driven by thinning of ice towards the lateral margin) squeezing happens when the ice motion is decoupled from the sediment motion. This implies that the same is true for motion along the ice-stream. The geomorphological implications of this remain unclear at the moment as most theories assume the flows of ice and sediment to be strongly coupled.

An appealing aspect of our 'differential erosion' model is that it can explain the apparent scarcity of ice stream shear margin moraines. The conditions which permit them to form are quite constrained because with too much ablation, deposition occurs everywhere (rather than just at the lateral margins) and with too little ablation there is erosion everywhere (including near the ice stream margins). A possible complicating factor with some of the other theories discussed above (e.g. meltwater processes) is that they do not necessarily explain why lateral shear moraines are apparently so rarely created and have never been formally identified under active ice streams. The alternative is that lateral shear moraines are often created but are rarely preserved. Given that ice stream shear margins are highly dynamic features susceptible to migration (e.g. Jacobel et al., 1996; Jacobsen and Raymond, 1998) this possibility should certainly be entertained in future work. Technological advances in geophysical exploration of active sub-ice stream environments will also go a long way towards solving this particular conundrum.

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Location	Length	Height	Width	Composition	Reference
Prince of Wales Island, northern Canada (Laurentide IS)	68km, possibly longer	< 30m above surrounding terrain	< 1000m	Till	Dyke and Morris (1988); Dyke et al. (1992)
Storkerson Peninsula, Victoria Island, northern Canada (Laurentide IS)	10-20km	10-60m	250-1000m	Till	Hodgson (1994); Stokes and Clark (2002)
Vestfjorden-Trænadjupet, mid-Norwegian continental shelf, (Fennoscandian IS)	35-70km	20-50m	2000-6000m	Till (inferred)	Ottesen et al. (2005)
Arvestuottar, northern Sweden, (Fennoscandian IS)	0.6km	not reported	Not reported; probably <100m	Till	Kleman and Borgström (1994)
Firth of Tay, off-shore eastern Scotland, (British IS)	not reported	not reported	6000m	Till	Golledge and Stoker (2006)

Table 1: Examples and dimensions of lateral shear margin moraines

List of Figures

- 1 Landsat ETM+ satellite image (R,G,B: 7,4,2) of part of a lateral shear margin moraine (red arrows) formed in the shear zone of the former Crooked Lake Ice Stream on Prince of Wales Island, Canadian Arctic Archipelago. The ice stream lineations indicate flow to the north-west in the top right of the image, whereas the area south and west of the moraine is inferred to have been preserved under cold based ice (see Dyke et al., 1992 and De Angelis and Kleman, 2005 for discussion of the reconstructed glacial history). 23

2	Landsat ETM+ satellite image (R,G,B: 7,4,2) of a lateral shear margin moraine (red arrows) formed in a part of the shear zone of the M'Clintock Channel Ice Stream on Storkerson Peninsula, Victoria Island, Canadian Arctic Archipelago. The ice stream lineations indicate flow to the north on the right of the image, whereas the area to the west of the moraine is outside of the margin and show no evidence of stream-lining (see Hodgson, 1994 and Clark and Stokes, 2001 for discussion of the reconstructed glacial history).	24
3	Erosion/deposition rates and transverse velocities for different accumulation/ablation rates in the marginal zones of ice-streams. Corresponding downstream velocity is shown in Case 1, Figure 4. Other parameters specified at end of section 3.2	25
4	Erosion/deposition rates and downstream velocities for different basal properties. Case 1 is a perfectly slippery bed (also used in Figure 3; Cases 2 and 3 have increasing bed resistance, while Case 4 is the same as Case 2, but with effective pressure increasing from centre of stream to flank. Accumulation rate is -1m/year, the corresponding transverse velocity may be see in Figure 3. Other parameters specified at end of section 3.2	26
5	Time dependent evolutions of till thickness for indicated accumulation/ablation cases; solid line includes the effect of transverse advection ($v\partial D/\partial t$), dotted lines exclude it. Corresponding erosion deposition rates and transverse velocity may be see in Figure 3. Time interval between lines is 200 years. Other parameters specified at end of section 3.2	27
6	Effective till thickness (flux/upper surface velocity) as a function of thickness for different effective pressure indices m	28
7	Incursion simulations (a): Ice and bed geometry at start of simulation, immediately prior to the increase in ablation.	29
8	Incursion simulations (b): Ice and bed geometry approximately 600 years after warming begins. Ice has retreated and terminal and lateral moraines established.	30

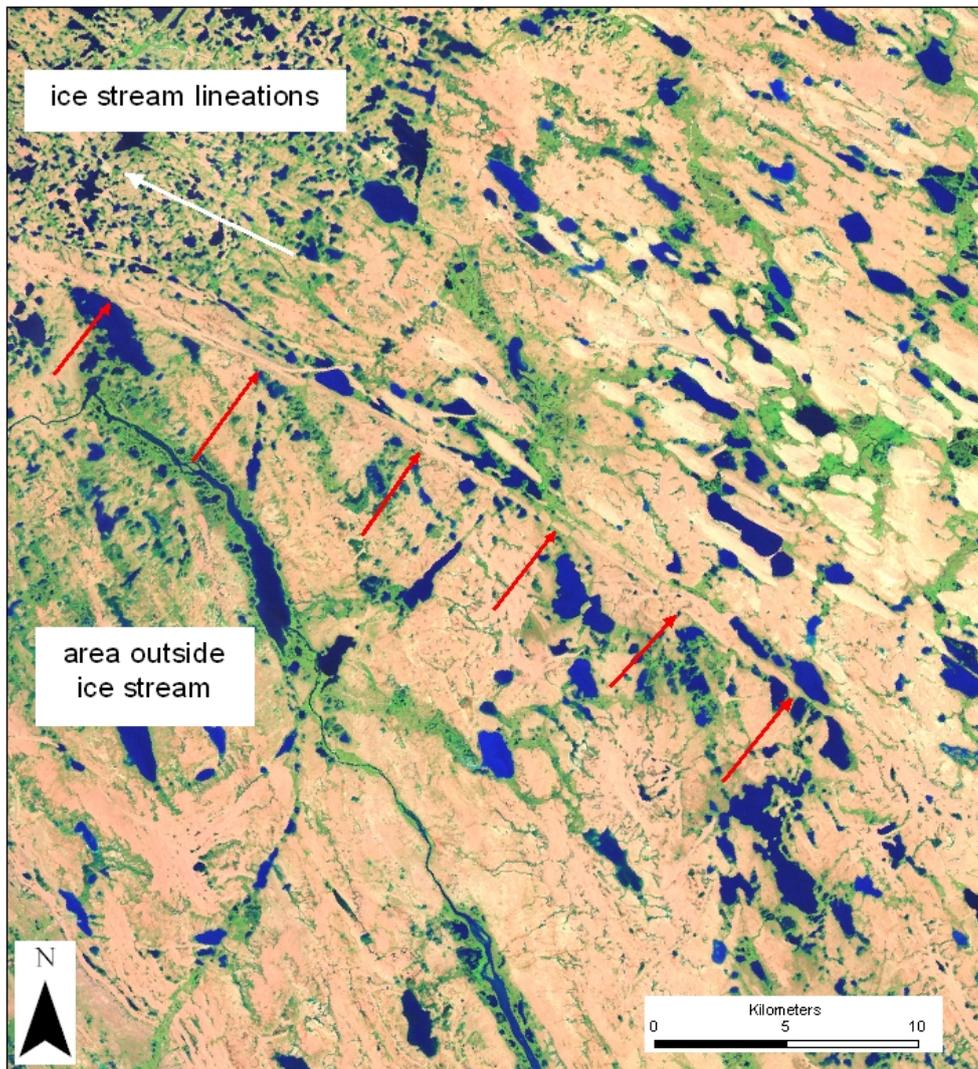


Figure 1: Landsat ETM+ satellite image (R,G,B: 7,4,2) of part of a lateral shear margin moraine (red arrows) formed in the shear zone of the former Crooked Lake Ice Stream on Prince of Wales Island, Canadian Arctic Archipelago. The ice stream lineations indicate flow to the north-west in the top right of the image, whereas the area south and west of the moraine is inferred to have been preserved under cold based ice (see Dyke et al., 1992 and De Angelis and Kleman, 2005 for discussion of the reconstructed glacial history).

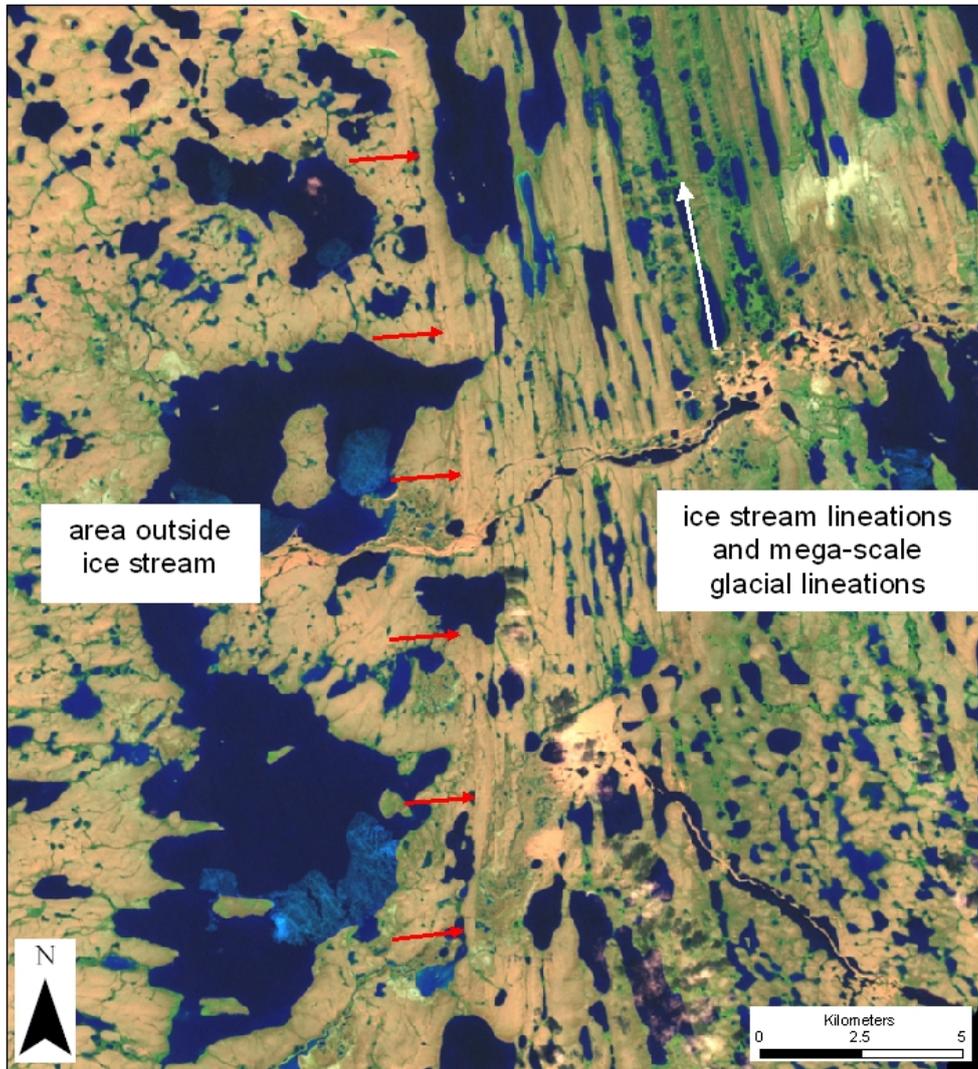


Figure 2: Landsat ETM+ satellite image (R,G,B: 7,4,2) of a lateral shear margin moraine (red arrows) formed in a part of the shear zone of the M'Clintock Channel Ice Stream on Storkerson Peninsula, Victoria Island, Canadian Arctic Archipelago. The ice stream lineations indicate flow to the north on the right of the image, whereas the area to the west of the moraine is outside of the margin and show no evidence of stream-lining (see Hodgson, 1994 and Clark and Stokes, 2001 for discussion of the reconstructed glacial history).

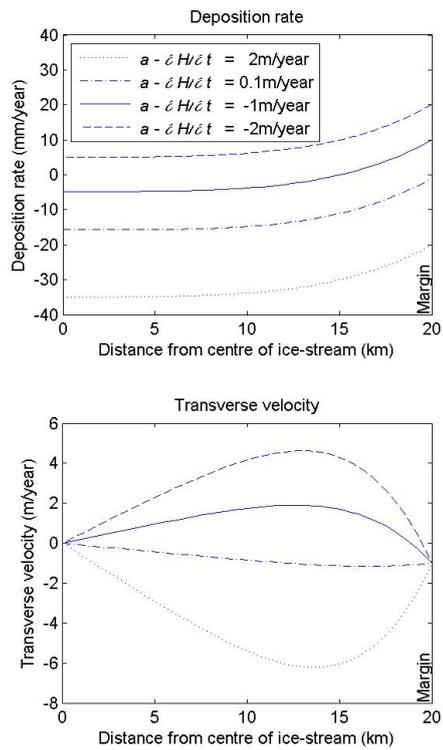


Figure 3: Erosion/deposition rates and transverse velocities for different accumulation/ablation rates in the marginal zones of ice-streams. Corresponding downstream velocity is shown in Case 1, Figure 4. Other parameters specified at end of section 3.2

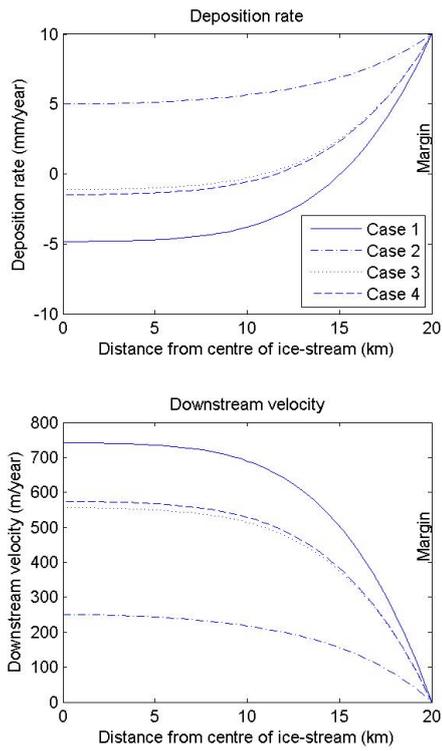


Figure 4: Erosion/deposition rates and downstream velocities for different basal properties. Case 1 is a perfectly slippery bed (also used in Figure 3; Cases 2 and 3 have increasing bed resistance, while Case 4 is the same as Case 2, but with effective pressure increasing from centre of stream to flank. Accumulation rate is -1m/year, the corresponding transverse velocity may be see in Figure 3. Other parameters specified at end of section 3.2

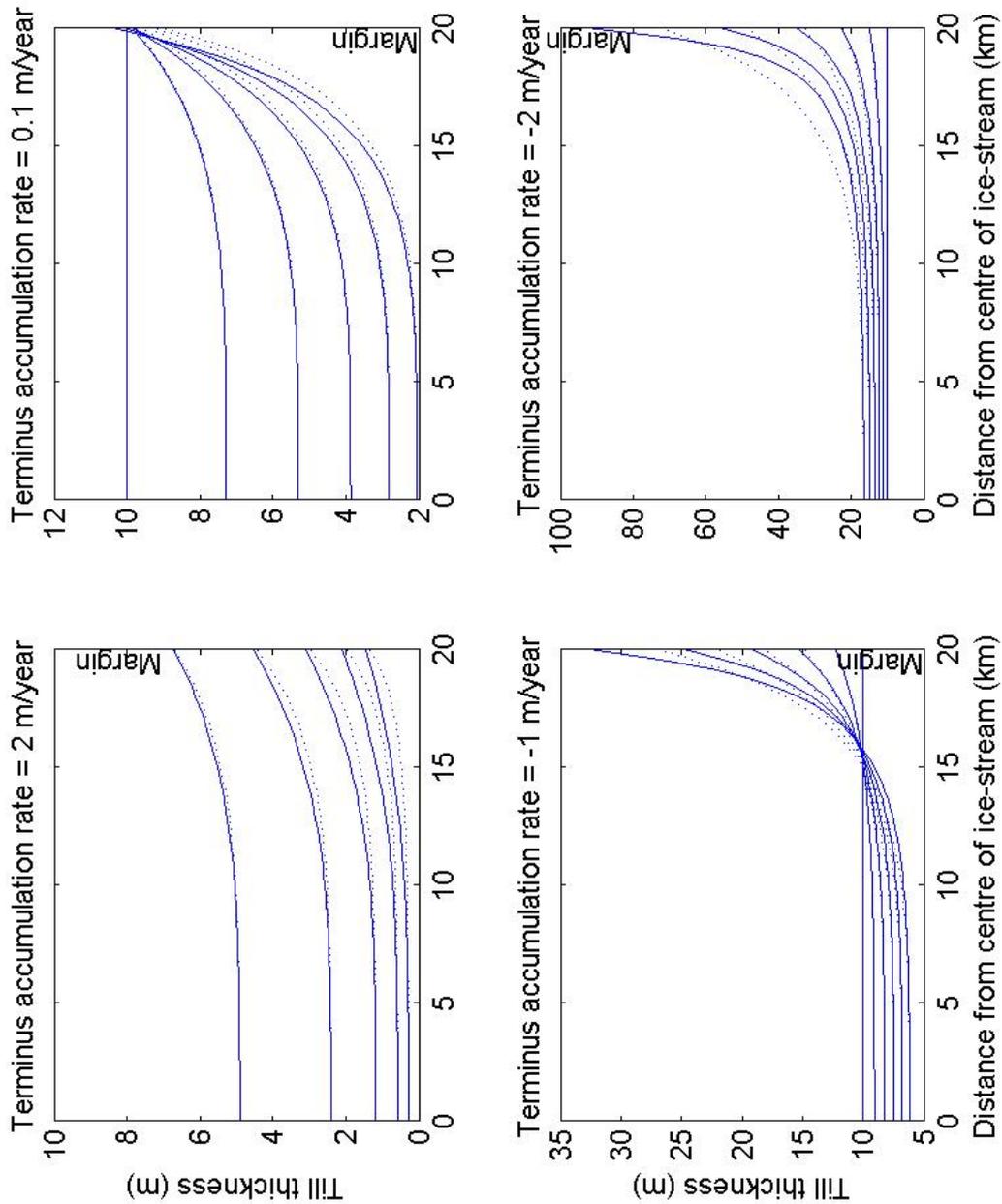


Figure 5: Time dependent evolutions of till thickness for indicated accumulation/ablation cases; solid line includes the effect of transverse advection ($v\partial D/\partial t$), dotted lines exclude it. Corresponding erosion deposition rates and transverse velocity may be see in Figure 3. Time interval between lines is 200 years. Other parameters specified at end of section 3.2

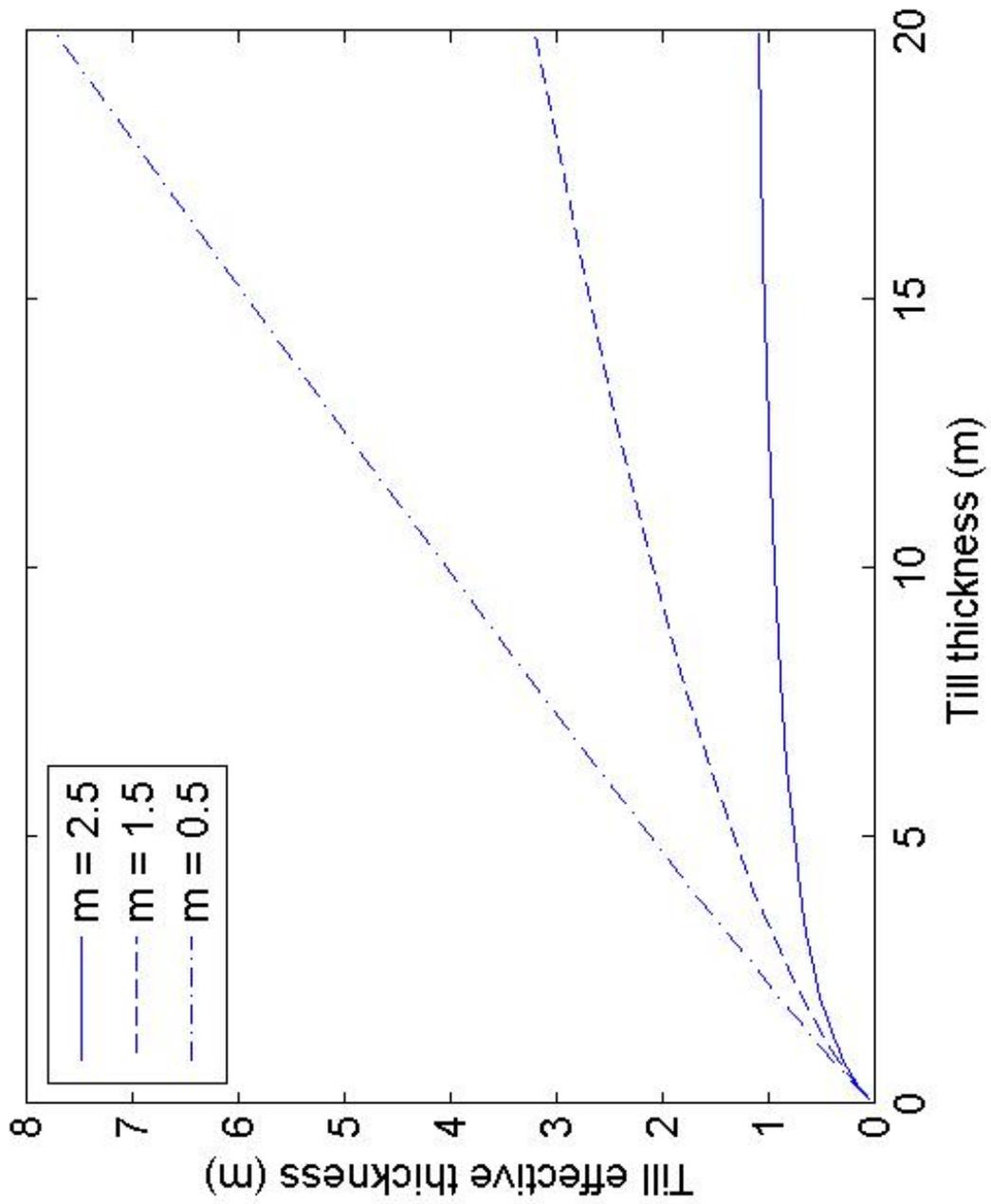


Figure 6: Effective till thickness (flux/upper surface velocity) as a function of thickness for different effective pressure indices m .

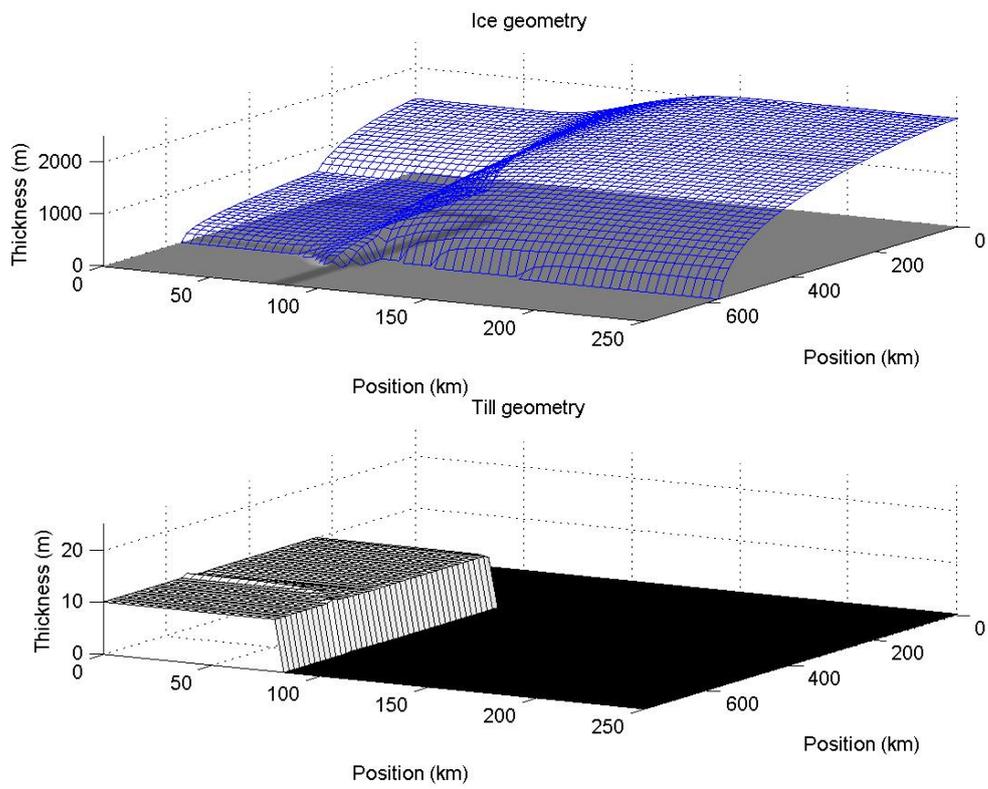


Figure 7: IncurSION simulations (a): Ice and bed geometry at start of simulation, immediately prior to the increase in ablation.

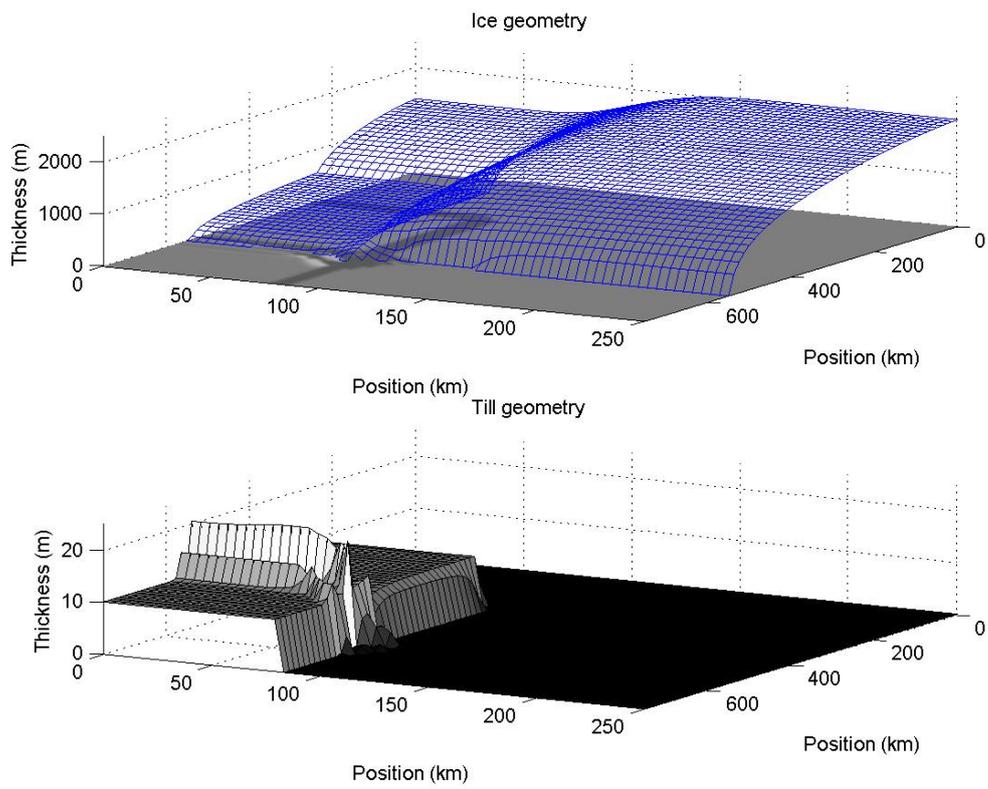


Figure 8: Ingression simulations (b): Ice and bed geometry approximately 600 years after warming begins. Ice has retreated and terminal and lateral moraines established.