Negative refraction can make non-diffracting beams

M. A. Kaliteevski[1,2], S. Brand[1]*, R. A. Abram[1], A. J. Gallant[3] and J. M. Chamberlain[1]

[1]*Department of Physics, Durham University, South Road, Durham DH1 3LE, United Kingdom*
[2]*Ioffe Institue, St-Petersburg, Russia*
[3]*School of Engineering, Durham University, South Road, Durham DH1 3LE, United Kingdom*

**Corresponding author: stuart.brand@dur.ac.uk*

Abstract: We report the results of simulations relating to the illumination of a structure consisting of a slab constructed from a 2-D hexagonal array of metal rods with a terahertz frequency source. As a consequence of negative refraction an essentially non-divergent beam pattern is observed. Although the results presented relate to the terahertz regime they should also be applicable at other frequencies.

OCIS codes: Photonic crystals(160.5298); Electromagnetic optics (260.2110).

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13. Dispersion relations have been calculated using a complex photonic bandstructure method described in [11], and finite difference time domain software OMNISIM© has been employed to simulate the beam propagation. Due to minor convergence problems the frequency scales for the two methods are slightly different. In order to provide matching of the two scales, the bandstructure has been renormalized to provide matching of the frequencies at the J point which can be determined for both calculation techniques.
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The diffraction of waves is a ubiquitous feature of electromagnetism but at the same time is a process which can severely restrict the performance of electromagnetic devices [1]. In essence diffraction causes a beam of radiation with wavelength λ and width w to diverge by an angle δ

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which satisfies the relation sin $\delta \approx \lambda/w$. In an attempt to eliminate, or at least to reduce the divergence of beams, Bessel beams [2], and more recently Airy packets [3], have been employed, where the transverse profile of the beam intensity is a non-monotonic function of position. However, it is still necessary for the characteristic width of the beam to be a substantial number of wavelengths to provide something approaching non-divergent behaviour over large distances. Here we demonstrate that an essentially non-divergent beam of terahertz radiation with a width of a few wavelengths can be produced by the use of a flat lens made from a metallic photonic crystal in the form of a 2-D hexagonal array of metal rods operating in the negative refraction regime.

In 1968 Veselago [4] pointed out that for materials in which both the dielectric constant ε and the magnetic permeability μ have negative values, $\mathbf{E}\times\mathbf{H}$ is antiparallel to the wave vector $\mathbf{k}$. Such materials have been called left-handed materials (LHMs) in contrast to conventional materials, which can be called right-handed materials (RHMs). LHMs are also called negative index materials or negative refraction materials because the negative angle of refraction that occurs when radiation is incident from an RHM can be formally described by Snell's law using a negative refractive index. Later, it was recognized that negative refraction can occur near an optical resonance in conventional material with positive magnetic permeability[5], in quasi-homogeneous metamaterials [6] and in photonic crystals [7-9]. Various lensing–related applications of LHMs have been discussed, and in particular it has been shown that a flat slab of LHM having a refractive index of the same absolute value as the surrounding material, but opposite sign, acts as a superlens, providing imaging with subwavelength resolution [5,6,10]. However, in this paper we consider the case illustrated in Fig. 1 where the slab of LHM has a negative refractive index n with a magnitude different from that of the surrounding RHM. The slab is illuminated by a point source in the 2-D plane (a line source in 3-D, parallel to the axis of the metallic rods in the slab).

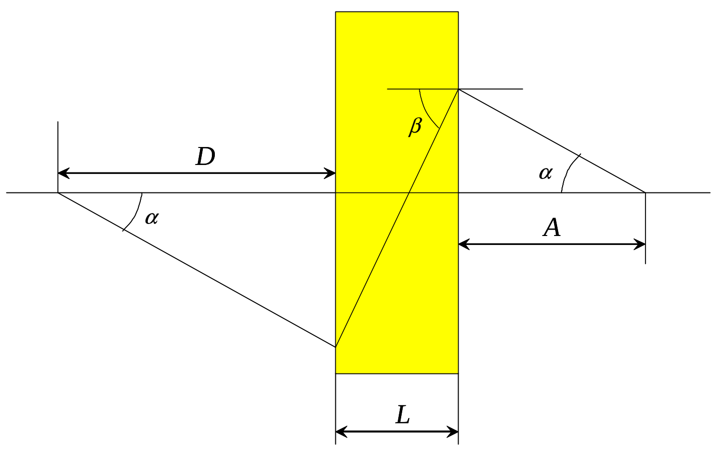

Fig. 1. Propagation of rays from a 2-D point source through a slab of negatively refracting material.

In Fig. 1, a ray emerging from a source located at distance A from the right side of the slab crosses the optic axis at a distance D from the other side of the slab, where D depends on the angle α between the ray and the optical axis. To be specific, we take the medium surrounding the slab to be vacuum when Snell's law gives the relationship

$$\sin\alpha = |n|\sin\beta \tag{1}$$

and it follows that

$$D + A = L \tan \beta / \tan \alpha = L \sqrt{1 - \sin^2 \alpha} \Big/ \sqrt{|n|^2 - \sin^2 \alpha} \ . \quad (2)$$

It is interesting to note that if $|n| < 1$, the radiation incident on the slab can experience total internal reflection, and only those rays for which the angle of incidence α is smaller than a critical angle α_{tot}, given by $\sin \alpha_{tot} = |n|$, can enter the slab. When $\alpha \rightarrow 0$ rays cross the optic axis near $D_0 = L / |n| - A$, but when the incident angle corresponds to α_{tot}, $D_0 \rightarrow \infty$. Hence, the image of the source will not be a single point (in the 2-D plane), as would be the case if $|n| = 1$, but will be stretched from the position D_0 to infinity. Since D_0 can be made arbitrarily large by reducing $|n|$, the result suggests that it should be possible to use a slab with a negative effective refractive index of small magnitude to produce an essentially non-divergent beam of radiation from a point source in the 2-D plane. In this paper we illustrate how a non-divergent beam with a width of a few wavelengths can be produced at terahertz frequencies using a slab in the form of a metallic photonic crystal having the required properties.

Figure 2 shows the bandstructure, $f(k)$, for the two-dimensional photonic crystal considered, which is in the form of gold (plasma frequency = 8.9 eV) circular rods of diameter 80 μm arranged in a hexagonal lattice with period 200 μm. For radiation in the E polarization, with electric field parallel to the rods, such a structure exhibits an effective plasma frequency [11,12] of about 1 THz. The dispersion in the upper band is such that negative refraction occurs when radiation is incident on the crystal from vacuum [9]. In particular near the top of

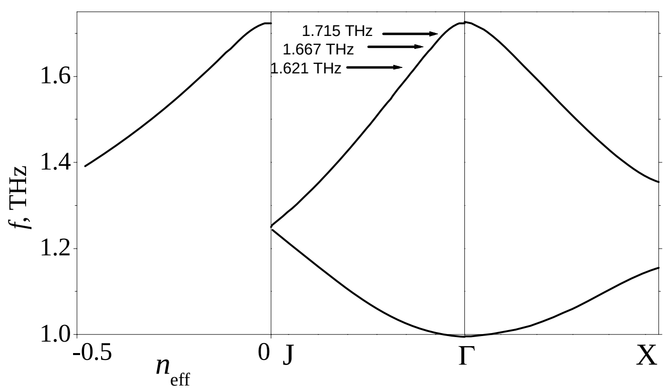

Fig. 2. Band structure [13] of the hexagonal photonic crystal with lattice constant of 200 μm formed by metallic rods of diameter 80 μm. Horizontal arrows indicate the frequencies for which the modelling of the field pattern is shown in Fig. 3.

the upper band the equifrequency surfaces are circular and the group velocity and the Poynting vector averaged over a unit cell are antiparallel to the Bloch wavevector. It follows that when radiation is incident on the photonic crystal, the conditions of continuity of transverse wavevector and flow of energy away from the interface and into the crystal require a negative angle of refraction. Further, straightforward trigonometry shows that the negative refraction can be described by a scalar effective refractive index $n = -k(f)/k_0$ where $k(f)$ and k_0 are the magnitudes of the wavevectors at frequency f in the photonic crystal and the vacuum respectively. Also, $k(f)$ decreases as the frequency is increased, and near the top of the band the effective refractive index can be very small in magnitude, making large values of D_0 possible [13].

Figures 3(a-c) show the 2-D field patterns formed by a flat slab of photonic crystal of thickness L = 1200 μm irradiated with line sources (perpendicular to the plane shown) of frequencies 1.621 THz, 1.667 THz and 1.715 THz placed at a distance A = 1.8 mm to the right

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of the structure. For a frequency of 1.621 THz (wavelength λ = 185 µm) the beam has a width of about 3λ and it propagates for about 10 mm (50λ) before there is significant divergence. The divergence of the beam is illustrated in Fig. 4a by the black solid line, which shows the intensity profile at a distance of 7.5 mm to the left of the slab (approximately in the middle of Fig. 3), and by the black dashed line which shows the profile at 16 mm (at the left edge of Fig. 3). With increasing frequency, $|n|$ reduces, as shown in Fig. 2, and D_0 increases, as is implicit in Figs. 3(b) and 3(c).

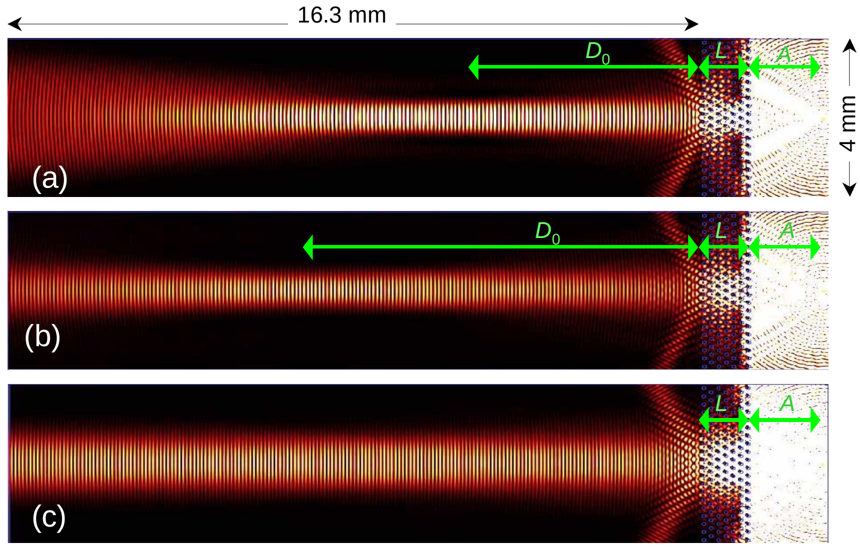

Fig. 3. The field patterns formed when a line source irradiates the photonic crystal slab at frequencies of (a) 1.621 THz, (b) 1.667 THz and (c) 1.715 THz. The distance between the source and the edge of the slab is A =1.8 mm.

For a frequency of 1.667 THz the divergence can hardly be seen in Fig. 3b, but the spreading of the Gaussian-like profile is clearly seen in Fig. 4b. For a frequency of 1.715 THz, which is close to the top of the band, the beam has a waist of about 5λ and shows no sign of divergence at a propagation distance corresponding to 100λ.

For all three beam patterns shown in Figs. 3 and 4 the beam profile is Gaussian-like in the region close to the slab (at distance 7.5 mm). The wavelengths, effective refractive indexes, angles of total internal reflection, distances D_0 and waist widths w are shown in Table 1.

Table 1. Parameters relating to the results shown in Figs. 3 and 4.

f (THz)	λ(µm)	n	α_{tot} (degrees)	D_0 (µm)	w (µm)
1.621	185	-0.15	8.6	5250	400
1.667	180	-0.095	5.5	9300	550
1.715	175	-0.04	2.3	24500	810

Figure 3 shows that there are two side-lobes in the region where the beams emerge from the left side of the slab in Fig. 3. These are evanescent in nature and there is little energy associated with them, but they are an inherent feature of the intensity pattern. Conventional diffracting behaviour occurs if a screen is placed to block the side-lobes. We should point out that the collimation of the beam is not due to the same self-collimating/self-guiding effect

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described by Kosaka et al [14] and Chigrin et al. [15] for the dielectric structures they considered. The collimation there is a result of the existence of a straight-line section of the equifrequency contours, meaning that the group velocity is in the same direction for a range of different wavevectors. In the present case, with essentially circular equifrequency contours, the collimation of the beam and the form of the side lobes are a consequence of the detailed interference effects at the particular frequency.

The red and green lines in Fig. 4 show the beam profiles when the source is separated from the slab by the distances A = 4.5 mm and A = 5.3 mm respectively. It is apparent that the width of the beam is not increased in proportion to A, as would be expected on purely geometrical grounds, and in fact for all three source distances considered, the parameters of the non-diffracting beam differ only slightly (although when the distance is smaller than 1 mm, results not shown here demonstrate that the divergence of the beam becomes significantly more pronounced). However, the data in Table 1 do show a clear increase in the waist of the beam with decreasing angle of total reflection, which is the opposite to what would be expected on geometrical grounds. These results can be explained, at least intuitively, by a Fourier argument where the width of the beam is determined by the spread of its transverse wavevectors within the slab, which in turn is determined principally by sin α_{tot} . Then, as α_{tot} is decreased, the wavevector spread is decreased and, through the usual Fourier relationship, there is an increase in the beam width in real space.

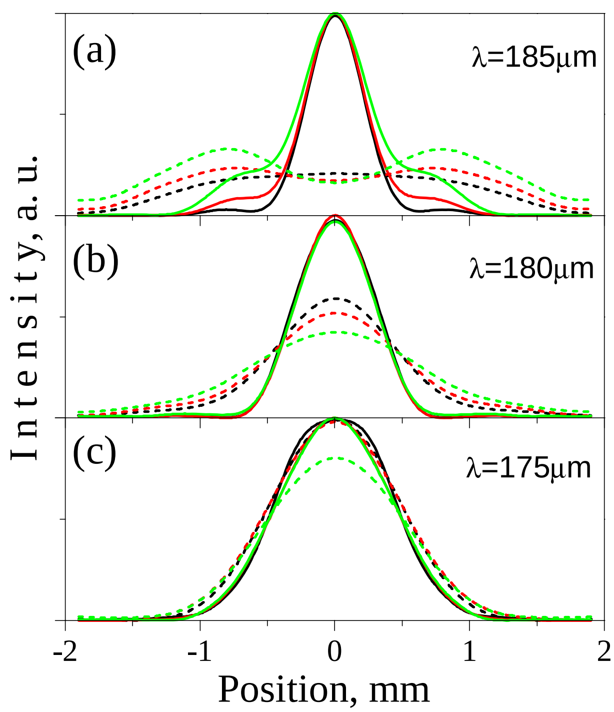

Fig. 4. Intensity profile of the beam for frequencies (a) 1.621 THz, (b) 1.667 THz and (c) 1.715 THz at distances from the left-hand edge of the slab of 7.5 mm (solid line), 16 mm (dashed line), and for distances of the source from the right-hand edge of the slab of A = 1.8 mm (black), A = 4.5 mm (red), A =5.3 mm (green).

Since there is omnidirectional emission from the line source, the fraction of total power impinging on the slab with an angle of incidence less than α_{tot} is only α_{tot} /180, and of this, somewhat less than 10% is transferred into the collimated beam. However, more efficient illumination schemes could be employed to increase the fraction of emitted power that is transmitted.

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In conclusion, we have shown that left-handed materials can be employed to produce an essentially non-diffracting beam. Although this has been demonstrated for a system operating in the THz frequency regime, a region in which we have a particular interest, it should also be applicable at other frequencies.

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