Formation of Bright Matter-Wave Solitons during the Collapse of Attractive Bose-Einstein Condensates

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We observe bright matter-wave solitons form during the collapse of ^{85}Rb condensates in a three-dimensional (3D) magnetic trap. The collapse is induced by using a Feshbach resonance to suddenly switch the atomic interactions from repulsive to attractive. Remnant condensates containing several times the critical number of atoms for the onset of instability are observed to survive the collapse. Under these conditions a highly robust configuration of 3D solitons forms such that each soliton satisfies the condition for stability and neighboring solitons exhibit repulsive interactions.

PACS numbers: 03.75.Kk, 03.75.Lm, 05.45.Yv

Bose-Einstein condensates (BECs) with tunable interatomic interactions have been the subject of intense theoretical and experimental interest in recent years. Many aspects of these systems are now well understood. However, several questions remain pertaining to the response of the BEC to a sudden change in the nature of the interactions from repulsive to attractive. Resolving such questions is critical if a complete understanding of these basic atomic systems is to be developed and if related, more complex phenomena, such as the formation of molecular BECs and superfluidity in degenerate Fermi systems, are to be thoroughly explained.

The macroscopic wave function of an atomic BEC obeys a nonlinear Schrödinger equation, known as the Gross-Pitaevskii equation (GPE). The nonlinearity results from the atomic interactions whose nature and magnitude are described by the s-wave scattering length (a). Homogeneous condensates in free space are inherently unstable when the interactions are attractive ($a < 0$). However, in a harmonic trapping potential, the zero-point kinetic energy can stabilize the condensate provided the number of atoms in the BEC (N_0) is less than a critical value (N_{critical})

$$N_0 < N_{\mathrm{critical}} = k \frac{a_{\mathrm{ho}}}{|a|}, \qquad (1)$$

where a_{ho} is the mean harmonic oscillator length which characterizes the kinetic energy in the trap, and k is a dimensionless constant known as the stability coefficient. The exact value of k depends on the ratio of the trap frequencies [1–3] and for our trap we have measured it to be $k = 0.46(6)$ [4].

In the vicinity of a Feshbach resonance, a depends sensitively on the magnitude of an externally applied magnetic field [5–7], allowing the magnitude and sign of the atomic interactions to be tuned to any value. We have previously exploited such a resonance in ^{85}Rb to investigate the collapse of the condensate following a sudden

change in the nature of the interactions from repulsive to attractive [8]. The resulting dynamics of the imploding "Bosenova" were both fascinating and unexpected. Most notable was our observation of an anisotropic burst of atoms that explodes from the condensate during the early stages of collapse, leaving behind a highly excited remnant condensate that survives for many seconds. Curiously, we frequently observed that the number of atoms in the remnant condensate, N_{remnant}, was significantly greater than N_{critical}. It was not understood why this remnant BEC did not undergo further collapse until the number of atoms remaining was less than the value of N_{critical} predicted by the GPE. It was particularly puzzling that the stability condition in Eq. (1) accurately predicted the *onset* of the collapse, but did not seem consistent with number of atoms in the remnant. In the work presented here, we explain this result by showing that the remnant is composed of multiple solitons.

Solitons are localized waves that propagate over long distances without change in shape or attenuation. The existence of solitonic solutions is a common feature of nonlinear wave equations, so that solitons appear in many diverse physical systems. BEC systems described by the GPE can support both dark (local minima in the condensate wave function) and bright (local maxima) solitons [9] depending on whether the interactions are repulsive ($a > 0$) or attractive ($a < 0$), respectively. Dark solitons have been observed in repulsive ^{87}Rb condensates [10,11], while bright solitons have been created using one-dimensional (1D) ^{7}Li condensates with attractive interactions [12,13]. Bright matter-wave solitons form when the attractive atomic interactions exactly balance the wave packet dispersion. Gap bright solitons have also been observed using ^{87}Rb condensates confined in an optical lattice [14]. In this Letter we show that during the collapse of the condensate described in [8], a robust configuration of multiple solitons is created in the three-dimensional (3D)

trap. Comparison with simulations [15] shows that, remarkably, the solitons form with relative phases such that they repel each other and oscillate in the trapping potential for many seconds without degradation. Since the number of atoms in each soliton never exceeds $N_{critical}$ and the repulsive solitons never overlap, the condensate stability condition is never violated.

The experimental apparatus and the magnetic field ramps for probing the condensate collapse have been described in detail elsewhere [8,16]. Stable ^{85}Rb condensates were produced within the positive scattering length region of a broad Feshbach resonance by radio-frequency evaporation in a magnetic trap. The radial (axial) frequency of the cylindrically symmetric trap was $\nu_r = 17.3$ Hz ($\nu_z = 6.8$ Hz). Condensates of up to 15 000 atoms were formed with a shot-to-shot standard deviation of less than 600 atoms. The noncondensate thermal fraction was less than 500 atoms. After the evaporation at 162.2 Gauss, the magnetic field was adiabatically increased to 165.5 G where $a = +9a_0$ ($a_0 = 0.0529$ nm is the Bohr radius). The collapse of the condensate was then induced by quickly switching the scattering length through zero to a variable negative value $a_{collapse}$ by increasing the magnetic field in 0.1 ms. After a variable hold time t_{evolve} at the negative scattering length, one of two imaging procedures was used to interrogate the remnant condensate. In the first, the scattering length was increased to a large positive value in order to use the large repulsive atomic interaction to expand the condensate prior to the standard rapid trap turnoff followed by absorption imaging sequence [8]. This procedure ensured that the size of the condensate was well above the resolution limit of the imaging system [7 μm full width at half maximum (FWHM)], allowing the number of atoms to be measured reliably. In the second, the magnetic trap was turned off immediately and the condensate imaged with minimum delay ($\sim$2 ms), effectively probing the remnant condensate without any expansion.

The number of condensate atoms surviving the collapse was investigated as a function of both $a_{collapse}$ and the initial population of the condensate. The hold time at $a_{collapse}$ was sufficiently long ($t_{evolve} = 50$ ms) to ensure that all collapse-related atom loss had ceased [8]. The results are plotted in Fig. 1 as the ratio of the remnant number to the critical number [calculated using Eq. (1)]. Depending upon the conditions of the collapse, $N_{remnant}$ varied from below the critical number to much greater than the critical number (for $a_{collapse} = -100a_0$, the remnant condensate contained approximately $10\times$ the critical number of atoms). Interestingly, the *fraction* of atoms surviving the collapse depends only on $a_{collapse}$ and is independent of the initial number of condensate atoms (in the range investigated). This fraction varies smoothly from $\sim$60% for $a_{collapse} \simeq -5a_0$ to $\sim$30% for $a_{collapse} \simeq -50a_0$. The lifetime of the remnant condensate at $a_{collapse}$ was several seconds and comparable to the lifetime of a condensate for small positive scattering lengths, even for $N_{remnant} > N_{critical}$.
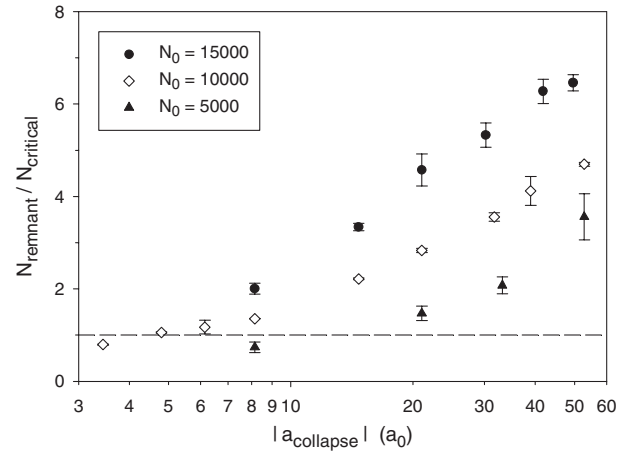

FIG. 1. Dependence of the number of condensate atoms surviving the collapse on the magnitude of the negative scattering length, $a_{collapse}$ (where $a_0 = 0.0529$ nm is the Bohr radius). The data are expressed as the ratio of the number of remnant condensate atoms to the critical number calculated according to Eq. (1). Results are shown for three different initial numbers of condensate atoms; $N_0 = 15\,000$ (●), 10 000 (◇), and 5000 (▲). The dashed line shows the critical number. The error bars represent the statistical spread in the data only.

In a separate set of experiments, the evolution of the size and shape of the unexpanded remnant condensate was investigated. From the images, the axial and radial column density distributions were calculated and fitted with Gaussian profiles to obtain both the axial and radial FWHM. The result of this analysis is shown for the (weaker) axial direction in Fig. 2(a). Any variation in the width of the remnant condensate in the radial direction was below the resolution limit of our imaging system. The remnant condensate is highly excited, with the variation in the axial width being well fitted by sinusoidal oscillations at the two lowest collective mode frequencies ($\nu_1 = 2\nu_z$ and $\nu_2 = 2\nu_r$). The measured frequencies were $\nu_1 = 13.67(1)$ Hz and $\nu_2 = 33.44(3)$ Hz. The amplitude of the ν_2 oscillation was 29(7)% of that of the ν_1 oscillation and contained a significant contribution from the radial focus of the atom burst associated with the collapse [8]. There was no discernible damping during our 3 s observation time.

Close inspection of the images, however, reveals that the variation in the width is not due to the excitation of collective modes. The remnant condensate is observed to separate into two (or more) distinct clouds which we associate with solitons [Figs. 2(b)–2(d)]. The observed oscillation in the axial width of the remnant then results from the oscillation of two solitons along the axial direction of the trap. The solitons persist for over three seconds [Fig. 2(d)] with only a small degradation in the quality of the images due to the presence of more thermal atoms as the condensate melts. The solitons collide over 40 times during this period. The remarkable stability of the soliton dynamics in the 3D trap, leads us to conclude that neigh-
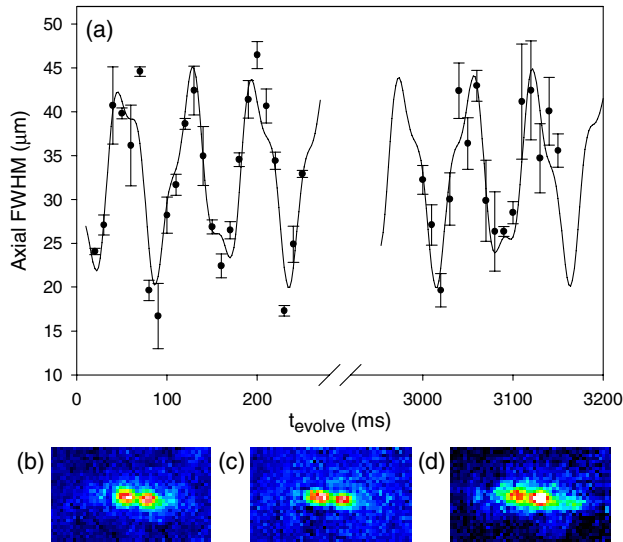
FIG. 2 (color online). Observation of solitons oscillating in the magnetic trap following the collapse at $a_{collapse} = -11.4a_0$ of condensates initially containing approximately 8000 atoms. (a) The evolution of the axial (horizontal) FWHM of the remnant condensate obtained from a single Gaussian fit to the images. Above the resolution limit of the imaging system, the remnant condensate is observed to separate into two solitons as shown in the images taken at (b) 210 ms, (c) 1140 ms, and (d) 3110 ms. Each image is 77×129 μm. The error bars represent the statistical spread in the data only.

boring solitons form with a relative phase, ϕ, that ensures that they interact repulsively ($\pi/2 < \phi < 3\pi/2$) even though the atomic interactions are attractive [17,18]. In this case, the solitons do not pass through each other at the center of the trap, but rather rebound off one another. Consequently, as the solitons never fully overlap, the critical density for collapse is never reached and the solitons remain stable. This conclusion is supported by simulations of our experiment based on a numerical integration of the 3D GPE, which show that multiple solitons are only stable provided they interact repulsively [15]. Similar results were observed for the soliton trains created in the ^{7}Li experiments [12].

The number of solitons that forms depends both on the initial number of condensate atoms and the magnitude of $a_{collapse}$. In Figs. 3(b) and 3(c) we present images and cross sections showing two and three solitons, respectively. These images were acquired with $t_{evolve} = 200$ ms, when the axial width was a maximum and the solitons were maximally separated. For comparison, Fig. 3(a) shows a remnant condensate for which $N_{remnant} < N_{critical}$. Under these conditions the remnant was never observed to separate into two or more solitons. We have observed up to six solitons form [Fig. 3(d)]. However, the quality of the images was significantly reduced when more than three solitons formed because the spacing between solitons became comparable to the imaging resolution limit. In Fig. 3(c) the three solitons contain approximately
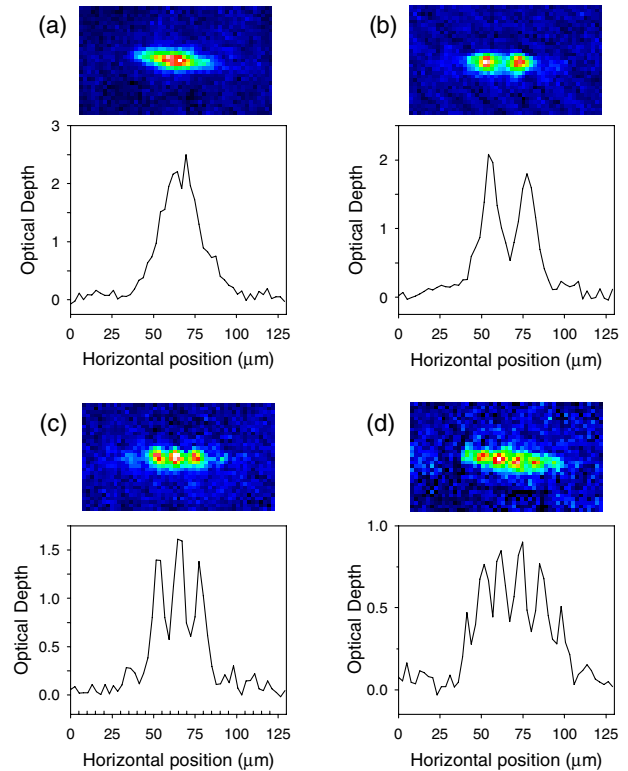

FIG. 3 (color online). Images and cross sections of remnant condensates. (a) When the magnitude of $a_{collapse}$ is sufficiently small a single remnant condensate containing less than the critical number is observed to survive the collapse. When the magnitude of $a_{collapse}$ is larger and/or larger initial condensates are used, the remnant condensate is observed to split into a number of solitons determined by the conditions of the collapse (b)–(d). Each image is 77×129 μm.

same number of atoms. This was not always the case, however, and we often observed an unequal yet (usually) symmetric population distribution across the solitons.

In Fig. 4 we examine in more detail the conditions under which multiple solitons form. For a fixed initial number of atoms in the condensate ($N_0 \approx 8000$), the number of solitons created was investigated as a function of $a_{collapse}$. The images were again acquired with $t_{evolve} = 200$ ms. In Fig. 4(a) the modal number of solitons is plotted. The data show a gradual increase in the number of solitons as the magnitude of $a_{collapse}$ increases. The ratio $N_{remnant}/N_{critical}$ for the same N_0 is also plotted in Fig. 4(a). This ratio exhibits the same trend as a function of $a_{collapse}$ as the observed number of solitons. Most importantly, it never exceeds the number of solitons. From this observation we deduce that the number of atoms in each soliton is always less than or equal to $N_{critical}$. Each soliton is therefore stable against collapse, explaining the unexpected stability of remnant condensates for which $N_{remnant} > N_{critical}$. This point is emphasized in Fig. 4(b) where the ratio $N_{remnant}/N_{critical}$ is plotted against the observed mean number of solitons. All the points lie on or
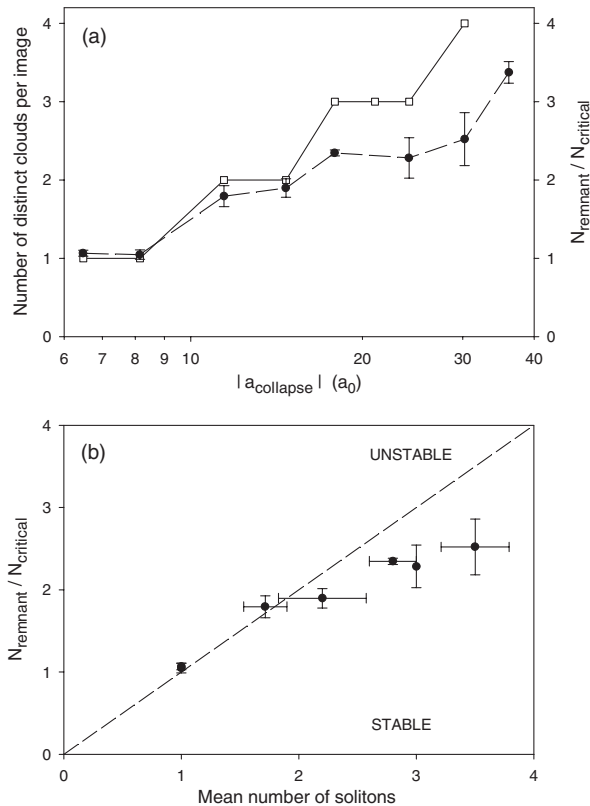
FIG. 4. (a) The modal number of solitons (□) observed after 200 ms of evolution at the collapse field as a function of the magnitude of the negative scattering ($a_{collapse}$). The ratio of the number of remnant condensate atoms to the critical number (●) is also plotted on the right axis (note the two y axes have the same scale). The initial condensate contained approximately 8000 atoms. (b) The ratio of the number of remnant condensate atoms to the critical number is plotted against the mean number of observed solitons to show that each soliton contains less than the critical number of atoms defined by Eq. (1) and is therefore expected to be stable. The error bars represent the statistical spread in the data only.

below the critical condition defined by Eq. (1) (dashed line).

In contrast to the previous observations of bright solitons in 1D confining potentials [12,13], these experiments were performed in a 3D trap with an aspect ratio of $\nu_r/\nu_z \approx 2.5$. Under such conditions, 3D bright solitons are predicted to be stable only if the attractive atomic interaction is smaller than a critical value [18], consistent with our observations. Moreover, the remarkable stability of multiple solitons in our 3D trap indicates that neighboring solitons form with a relative phase that ensures they interact repulsively. Attractive interactions between the solitons would lead to energetic excitations of the soliton wave function during each "collision" (in this case the solitons pass through one another), and ultimately a loss of particles from the BEC [15]. To our knowledge, there is no definitive physical picture of the origin of the relative phase that ensures the

stability of the multiple soliton states. Several authors propose that bright matter-wave solitons form due to a modulation instability of the condensate wave function [19–21], whereby the attractive nonlinearity leads to the growth of perturbations on a scale of the order of the condensate healing length. A sequence of localized collapses follows, such that the density and phase of the condensate evolves to "find" a stable arrangement of solitons. We note that our previous observations of local "spikes" in the condensate density [8] are consistent with this mechanism.

The sudden change in the nature of the atomic interactions in a Bose-Einstein condensate from repulsive to attractive leads to a violent collapse process, in which a significant proportion of the initial condensate is ejected in a highly energetic burst. Yet, surprisingly, out of this violent explosion emerges a robust configuration of solitons that preserves a large fraction of the initial condensate population. It is particularly striking that this behavior holds over a large range of attractive interaction strengths and that the solitons may undergo many collisions, even in a 3D potential, while remaining stable.

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