

**SOCIAL SECURITY INCENTIVES
AND HUMAN CAPITAL INVESTMENT***

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While the effect of social security systems on retirement decisions has received much attention, there are no analytical results on the impact of these systems on individual incentives to invest in human capital. We integrate human capital investment and retirement decisions in an analytical life-cycle model with full certainty and investigate how different social security schemes may affect human capital investment and labor supply. We analyze and compare three different social security systems, differing on whether benefits are conditional on withdrawal from the labor market and on previous income. (JEL: H55, I21, J26)

1. Introduction

In this paper, we extend the previous literature by integrating human capital investment and retirement decisions in an analytical life-cycle

model. While the effect of social security systems on retirement decisions has received much attention, the impact of these systems on individual incentives to invest in human capital has not been analyzed analytically. The effect of social security system on the incentives of the previous generation to invest in the public education provided to the future generation has been analyzed already by Pogue and Sgontz (1977). We study the effects of social security rules on private human capital investment and retirement.

Our analysis highlights two important features of social security systems: (i) actuarial adjustment, and (ii) the link between individual

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social security contributions and benefits. We expect that actuarial adjustment encourages later retirement because the present value of social security benefits is unaffected by the retirement age, and we expect that the link between social security contributions and benefits has a positive effect on human capital investment because the return on human capital investment increases. Finally, we analyze the interaction between these two links. How does actuarial adjustment affect human capital investment, and how does the link between social security contributions and benefits affect retirement behavior?

Since uncertainty and ability differences are absent, social security cannot offer efficiency gains in our framework. However, Browning (1975) and Cooley and Soares (1999) have demonstrated that social security systems may be maintained as a voting equilibrium, even when young generations are hurt. As past contributions are sunk costs, the old and middle-aged generations may vote for redistributive reasons in favor of a pay-as-you-go (PAYG) system. We do not replicate the analysis of a voting game, but take social security rules as given. Instead of asking what the optimal social security system would be, we focus on the consequences for steady state generations of adopting alternative social security rules.

We analyze and compare the effects of three different social security components on private retirement and education decisions. Social security benefits are financed by a proportional tax rate on labor income in each system, and the three components include: (i) flat-rate old-age benefits paid to individuals who are older than a given entitlement age, (ii) flat-rate retirement subsidies paid to retired individuals, and (iii) earnings-related retirement subsidies paid to retired individuals as a proportion of wage income during a given period before retirement. By naming the first component “old-age benefits”, we highlight that these benefits are not conditional on withdrawal from the labor market. Retirement subsidies, on the other hand, are conditional on withdrawal from the labor market, thus effectively subsidizing retirement. The first component is actuarially fair, because the present value of social security benefits is independent of the retirement age. The last two

components do not include actuarial adjustment. The third component introduces a link between social security contributions and benefits, whereas social security benefits do not depend on past contributions in the first two components. We do not focus on earnings-related old-age benefits, because this system would replicate the market outcome in our framework.

We include both private human capital investment and retirement decisions. The duration of education is kept constant, and the level of human capital depends on effort and resources devoted by the individual to education. Effort could also include the monetarized value of time used to study. As we do not want to take stance on how time, effort, and other expenses are combined in the production of human capital, we do not include explicitly a choice of time devoted to studies.

Our results suggest that actuarial social security schemes encourage later retirement and thus increase the incentive to invest in human capital compared to non-actuarial schemes. We also find that a stronger correspondence between earnings history and social security benefits increases the incentive to invest in human capital and postpones retirement.

Most related to our paper, Jensen, Lau and Poutvaara (2004) study the effects of alternative social security rules on human capital formation, retirement and welfare using a computational general equilibrium model. Our paper complements theirs in four respects. First, this paper derives the effects of social security rules analytically, rather than numerically. Second, Jensen et al. (2004) analyze the productivity process in which human capital formation continues over the whole life-time, and there is no period of formal schooling. This paper studies formal education which takes place before entry into the labor market. Third, this paper assumes that education has effort costs, while Jensen et al. assume that it only has time costs. Fourth, Jensen et al. (2004) compare only systems in which the link between previous earnings and benefit rate is either linear or non-existent, while the analytical framework of this paper allows deriving results for arbitrary convex combinations of different systems.

The paper is organized as follows. The model is first introduced without social security in Section 2, and three alternative components of social security are then introduced and compared in Section 3. Using general specifications of production and utility functions, we derive and compare the economic effects on human capital investment and retirement of marginal changes in the weights of the three social security components. In Section 4, we apply a Cobb-Douglas specification of the utility function, which allows us to rank all three systems with respect to the distortions they cause in human capital investment and retirement behavior. Section 5 presents a model where investment in human capital is held constant. Section 6 concludes.

2. Human capital and retirement without social security

To illustrate how different social security systems may affect the supply and quality of labor, we construct a simple life-cycle model with endogenous human capital formation and retirement. After completing education, an individual decides how long he or she will be active in the labor market and when to retire. In other words, the retirement age is endogenous in the model. All agents are identical, and we analyze the optimal behavior of a representative agent. Public expenditures are not included in this section.

Human capital investment includes education obtained at universities and other institutions of higher learning, as well as any courses and training obtained elsewhere. Some inputs have to be purchased, like tuition and course material. In addition to purchased inputs, studying requires effort, the utility cost of which we measure as equivalent units of lifetime consumption. Before us, this simplification has been used by Cremer and Pestieau (2003) to present the utility cost of working as a quadratic loss which is deducted from resources available for consumption. We keep time spent on education constant, while allowing effort to vary. We denote individual investment in education by H , measured as equivalent units of the consumption good, and refer to it simply as the cost of education. The time horizon after education is completed

is normalized at unity for each agent, and there is no uncertainty about life expectancy or the return on education. Even though modelling human capital formation as a pure consumption expenditure is a strong assumption, the model could be solved numerically also with an alternative assumption that human capital formation requires also time.¹

Perfect competition prevails in each market, which implies that output and factor prices are given to all individuals in the model. The price of the homogeneous consumption good is normalized at unity. We assume that lifetime utility depends on consumption over the lifetime, and on time spent on retirement:

$$\tilde{U} = \int_{t=0}^1 e^{-\rho t} u(c(t)) dt + \int_{t=0}^1 e^{-\rho t} v(t, \psi_t) dt,$$

in which $c(t)$ is the flow of consumption at time t , and $u(c(t))$ denotes utility from consumption at time t , and ρ is a subjective discount factor. The dummy variable ψ_t obtains value one if the individual is retired at time t , and zero otherwise. Retirement is an irreversible action, implying that with a retirement age $1 - R$, $\psi_t = 0$ when $0 \leq t < 1 - R$ and $\psi_t = 1$ when $1 - R \leq t \leq 1$. The utility flow from retirement, $v(t, \psi_t)$, may depend on individual age t . This allows us to capture, for example, the possibility that the utility cost of working could increase with age. We assume that $v(t, 1) > v(t, 0) \forall t \in [0; 1]$. Our assumption that utility is separable between consumption and time spent on retirement considerably simplifies the analysis. However, this assumes away any retirement-consumption-drop puzzle.

The consumption good can be borrowed or lent internationally at a zero interest rate. This simplification allows us to present lifetime utility \tilde{U} in a more concise form, as a function of lifetime consumption C and retirement age R :

$$(1) \quad \tilde{U} = U(C) + V(R).$$

¹ See Jensen et al. (2004) for an analysis in which education has only time cost. The similarities and differences between Jensen et al. (2004) and this paper are discussed in section 4.

We assume that $u(c(t))$ is a strictly increasing and concave function of $c(t)$, which suffices to guarantee that U also is a concave function of consumption C . The assumption that $\frac{d}{dt}[v(t,1) - v(t,0)] \geq 0$ is sufficient to guarantee that $V(R)$ is a concave function of R . We assume that $\lim_{C \rightarrow 0^+} U'(C) = \infty$, $\lim_{R \rightarrow 0^+} V'(R) = \infty$ and $\lim_{H \rightarrow 0^+} w'(H) = \infty$ in order to guarantee interior solutions. The wage rate, w , is a concave function of human capital investment H , $w = w(H)$.

The lifetime budget constraint states that the value of lifetime expenditures on consumption and human capital investment cannot exceed lifetime income from the supply of labor services:

$$(2) \quad (1 - R) \cdot w(H) = C + H,$$

where $(1 - R)$ is the duration of working life, as well as the point in time at which the individual retires from the labor market.

The representative agent maximizes lifetime utility (1) with regard to H and R subject to the human capital production function and the lifetime budget constraint (2). The first-order condition with respect to human capital is:

$$(1 - R) \cdot w'(H) = 1,$$

where the left-hand side is the return on human capital investment, and the right-hand side is the opportunity cost in terms of foregone consumption. The first-order condition with respect to retirement is:

$$V'(R) = w(H) \cdot U'(C),$$

where the left-hand side is the marginal utility of retirement, and the marginal cost on the right-hand side is equal to foregone labor income times the marginal utility of consumption goods. These two equations determine optimal choices of human capital investment and the duration of retirement.

3. Human capital investment and retirement with social security

We use the life-cycle model to analyze steady state effects of a social security system with

three different components. Social security benefits are financed by a proportional tax rate on labor income, and the three components include (i) flat-rate old-age benefits paid to individuals who are older than a given entitlement age, (ii) flat-rate retirement subsidies paid to retired individuals, and (iii) earnings-related retirement subsidies paid to retired individuals as a proportion of wage income during a given period before retirement. The first component is actuarially fair, since the present value of social security benefits is independent of the retirement age. The last two components do not include actuarial adjustment, and both systems effectively subsidize retirement since they drive the private cost of retirement below the net wage. The third component introduces a link between social security contributions and benefits, whereas social security benefits do not depend on past contributions in the first two components.

3.1 Population and production

We analyze an economy with a continuum of agents across and within generations. New agents are born to the economy at a constant rate that is equal to the mortality rate, and the population growth rate is therefore equal to zero. Without loss of generality, we choose the indexation of generations such that $g \in [0, 1]$ corresponds to the age of the given generation.² In other words, the oldest generation has index one and the youngest generation has index zero. Agents are identical within and across generations, and we analyze the behavior of a representative agent. Each generation is associated with a density function that is equal to one, which implies that the total population is equal to one:

$$\int_{g=0}^1 dg = 1.$$

Although the model represents a continuum of overlapping generations, we simplify the anal-

² We could include two generational indices: one representing the point of time, and another representing the age of each generation. Since we analyze the steady state equilibrium, we do not need this distinction and thus exclude the time index.

ysis by excluding generation-specific indices whenever convenient.

The production function is linear in human capital, and there are no other factor inputs in the model. Prices and quantities are constant in steady state because the interest rate and the growth rate are normalized at zero. Variables do therefore not carry time indices.

3.2 Introducing different benefit schemes

In the first social security component, each person is entitled to old-age benefits at age $1 - \hat{R}$. The old-age benefits are constant and equal to z per unit of time, which implies that each individual receives a lump sum lifetime social security payment of $B \equiv \hat{R}z$ from the government. The benefits are financed by a proportional tax rate on labor income, τ_1 , and the public budget constraint with respect to this component is:

$$\tau_1 \int_{g=0}^{1-R} w(H_g) dg = \int_{g=1-\hat{R}}^1 z dg.$$

where H_g denotes human capital investment by generation g . The left-hand side is equal to tax payments from current generations who work, and the right-hand side reflects social security payments to current old generations. Note that R and \hat{R} may differ. Since $H_g = H, \forall g$ in steady state, we can simplify the public budget constraint to:

$$\tau_1(1 - R) \cdot w(H) = \hat{R}z = B.$$

The second social security component includes a uniform benefit flow, say a given monthly benefit to retired persons. The flat-rate retirement subsidy rate is denoted by b , and the payments are financed by a proportional tax rate on labor income, τ_2 . In this case, the public budget constraint is:

$$\tau_2 \int_{g=0}^{1-R} w(H_g) dg = \int_{g=1-R}^1 b dg.$$

Again, the left-hand side is equal to tax payments from current generations who work, and the right-hand side is equal to social security

payments to current old generations. Simplification yields

$$\tau_2(1 - R) \cdot w(H) = Rb.$$

Finally, we introduce a social security component in which benefits depend on wage income during a given period before retirement. In particular, social security benefits are determined as a proportion, p , of wage income during a period, n , before retirement.³ Defining $x \equiv np$, social security benefits for individual i are determined by:

$$b_i = x \cdot w(H_i),$$

where x is, from the individual's perspective, an exogenous fraction of the wage rate. From the government's perspective, x is endogenous to satisfy the budget constraint. The earnings-related retirement subsidies are financed by a proportional tax rate, τ_3 , on labor income, and the public budget constraint is given by:

$$\tau_3 \int_{g=0}^{1-R} w(H_g) dg = \int_{g=1-R}^1 x \cdot w(H_g) dg,$$

where the left-hand side is equal to tax payments from individuals who work. The right-hand side is equal to aggregate social security payments to retired generations. Since all individuals are identical in the model,

$$\tau_3(1 - R) \cdot w(H) = Rx \cdot w(H).$$

Combining the three social security systems, the budget constraint for the representative agent is:

$$(4) \quad (1 - \tau_1 - \tau_2 - \tau_3)(1 - R) \cdot w(H) + B + Rb + Rx \cdot w(H) = C + H.$$

The three social security components introduce distortions in the economic decision making. A system with perfect correspondence between an

³ Since the wage rate is constant in the model, the length of the period is not important to the results, unless the period is sufficiently long to postpone retirement.

individual's own social security tax payments and benefits received would in our framework replicate the solution without social security.

3.3 Incentive effects of social security

The representative agent maximizes lifetime utility (1) subject to the lifetime budget constraint (4). The first-order condition with respect to human capital investment is:

$$[(1 - \tau_1 - \tau_2 - \tau_3)(1 - R) \cdot w' + Rx \cdot w' - 1]U' = 0,$$

and the first-order condition with respect to retirement is:

$$(5) \quad [-(1 - \tau_1 - \tau_2 - \tau_3) \cdot w + b + x \cdot w]U' + V' = 0.$$

The first-order condition with respect to human capital investment simplifies to:

$$(1 - \tau_1 - \tau_2 - \tau_3)(1 - R) \cdot w' + Rx \cdot w' = 1,$$

where the left-hand side is the return on human capital investment and the right-hand side is the opportunity cost in terms of foregone consumption. The second term on the left-hand side measures the return on human capital investment through its effects on social security benefits. Social security taxes decrease the return on human capital investment, whereas earnings-related retirement subsidies partially offset this decrease.⁴ The first-order condition with respect to retirement can be written as:

$$V' = [(1 - \tau_1 - \tau_2 - \tau_3) \cdot w - b - x \cdot w]U',$$

where the left-hand side is the marginal utility of retirement, and the marginal cost on the right-hand side is equal to the net income loss due to retirement times the marginal utility of consumption goods. Note that both social secu-

urity taxes and retirement subsidies reduce the marginal cost of retirement.

Using Cramer's rule, we analyze and compare the three different social security components with respect to private retirement and education decisions. The results (proofs available on the web page of this journal) can be summarized as:

Proposition 1 An increase in the tax rate to finance any component of the social security system discourages human capital investment and encourages early retirement.

Proposition 2 Increasing the share of flat-rate retirement subsidies at the expense of either flat-rate old-age benefits or earnings-related retirement subsidies discourages human capital investment and encourages early retirement.

The social security system affects human capital investment in two ways. First, it may change the return on human capital investment at any given retirement age. The system in which social security benefits depend on wage income before retirement encourages human capital investment compared to systems without the link. Second, the social security system may indirectly affect human capital investment through the impact on retirement age, which affects the amortization period of human capital investment. We find that actuarial adjustment has a positive effect on human capital investment, because it postpones retirement. Hence, replacing earnings-related retirement subsidies or flat-rate old-age benefits with flat-rate retirement subsidies discourages human capital investment.

Retirement decisions are also affected in two ways by the social security system. First, social security benefits lower the private opportunity cost of retirement. The private opportunity cost of retirement is reduced by the retirement benefit in the two non-actuarial social security systems compared to the actuarial system. Replacing flat-rate old-age benefits with flat-rate retirement subsidies thus encourages early retirement. Second, the social security system indirectly affects the retirement age through human capital investment, since the level of human capital affects individual productivity. Increasing the

⁴ It is useful to contrast our results with Heckman (1976). Heckman assumes that the demand for leisure is constant and the opportunity cost of human capital investment is equal to foregone labor income. With those assumptions, labor income taxes are non-distortionary.

share of flat-rate retirement subsidies compared to earnings-related retirement subsidies therefore encourages early retirement.

It is not possible to say anything decisive about education and retirement decisions across the flat-rate old-age benefit and earnings-related retirement subsidy components. We relegate this issue to the next section, where we restrict the lifetime utility function to be of the Cobb-Douglas variety.

4. Cobb-Douglas specification

We next apply a Cobb-Douglas specification of the utility function and assume that each agent maximizes:

$$(6) \quad \tilde{U} = \ln(C) + \beta \ln(R),$$

where $\beta > 0$ is the relative weight of utility from retirement. The individual stock of human capital is determined by H^α , where $0 < \alpha < 1$. The marginal productivity of human capital investment is thus diminishing, which implies that human capital investment is strictly positive and bounded. The representative individual maximizes lifetime utility, (6) subject to the lifetime budget constraint:

$$(7) \quad (1 - \tau_1 - \tau_2 - \tau_3)(1 - R) \cdot H^\alpha + B + Rb + Rx \cdot H^\alpha = C + H.$$

The first term on the left-hand side is lifetime wage income after tax, the second term is the sum of flat-rate old-age benefits, the third term is the sum of flat-rate retirement subsidies, and the fourth term is the sum of earnings-related retirement subsidies. Solving the individual maximization problem, we find that the first-order conditions with respect to human capital investment and retirement simplify to

$$(8) \quad H = \left[\frac{\alpha(1 - \tau_1 - \tau_2 - \tau_3)(1 - \tau_1 - \tau_2)}{1 - \tau_1 + \beta(1 - \alpha(1 - \tau_1 - \tau_2))} \right]^{\frac{1}{1-\alpha}},$$

$$(9) \quad R = \frac{\tau_2 + \tau_3 + \beta(1 - \alpha(1 - \tau_1 - \tau_2))}{1 - \tau_1 + \beta(1 - \alpha(1 - \tau_1 - \tau_2))}.$$

These two equations allow us to compare incentive effects across flat-rate old-age benefit

and earnings-related retirement subsidy components. The results are derived in Appendix A, and they show:

Proposition 3 Increasing the share of earnings-related retirement subsidies at the expense of flat-rate old-age benefits increases human capital investment and leads to earlier retirement.

This proposition suggests that the link between social security contributions and benefits is more important than the actuarial link with respect to human capital investment, whereas actuarial adjustment is more important with respect to retirement decisions. The intuitive explanation is that first-order effects (the effect of linking benefits to earnings on human capital investment, and the effect of actuarial adjustment on retirement age) are more important than second-order effects (the effect of retirement decision on human capital investment and the effect of human capital investment on retirement decision).

Using a Cobb-Douglas representation of the lifetime utility function, the results of propositions 2 and 3 can be summarized as:

Proposition 4 Measured by human capital investment, the descending order of the three social security systems is: earnings-related retirement subsidies, flat-rate old-age benefits and flat-rate retirement subsidies. Measured by the retirement age, the descending order is: flat-rate old-age benefits, earnings-related retirement subsidies and flat-rate retirement subsidies.

The results we have derived with Cobb-Douglas specification are in line with the results that Jensen et al. (2004) derive using a computational general equilibrium model with a different utility function. They assume a life-cycle with 60 periods with the periodic utility function from consumption being of constant relative risk aversion, and utility from leisure being quadratic. Also they find that whether there are old-age benefits or retirement subsidies is more important for retirement behavior than whether benefits are earnings-related. However, they assume that human capital formation continues over the life time. Education has only time cost, and is thus effective

tively tax deductible. This paper complements theirs by deriving the effects of social security rules in the case when education takes place before entry into the labor market, and has non-deductible costs, whether monetary or effort. This paper complements Jensen et al. (2004) also by deriving the effects analytically. Their numerical results, in turn, serve as sensitivity analysis for the analytical results derived here.

5. Results with exogenous human capital

We next demonstrate the importance of allowing for endogenous human capital formation by comparing the results with predictions from the same model with exogenous human capital formation instead. In the terminology of our model, exogenous human capital formation amounts to assuming that H is fixed at \hat{H} . This leaves the individual maximization problem, apart from intertemporal allocation of consumption, with only one decision variable, R , and subsequently one first-order condition, (5). We prove in Appendix B:

Proposition 5 When human capital investment is exogenous, earnings-related retirement subsidies and flat-rate retirement subsidies are equivalent in their effects on the timing of retirement. With a given social security tax rate, an increase in either component at the expense of flat-rate old-age benefits leads to earlier retirement.

Proposition 5 suggests that flat-rate retirement subsidies are equally distorting as earnings-related retirement subsidies, and both are more distorting than flat-rate old-age benefits. This is in marked contrast with the previous results where human capital formation is endogenous. With endogenous human capital formation, flat-rate retirement subsidies always result in larger distortions than the two other systems.

6. Conclusions and implications

We have analyzed the interaction between social security rules, human capital investment, and

the timing of retirement. Our results highlight two important links in social security systems: (i) actuarial adjustment and (ii) the link between contributions made and benefits received. We find that actuarially adjusted systems lead to later retirement than systems with a weaker actuarial adjustment. This result corresponds to the empirical finding by Börsch-Supan (2000), who suggests that retirement before age 60 would be reduced by more than a third if the German social security system were reformed and made actuarially fair. We also find that the link between benefits and contributions encourages human capital investment. The results stress the importance of incentives embedded in social security rules, since distortions arise even when agents are identical and redistribution is absent in equilibrium.

Since uncertainty and ability differences are not present, social security can not offer efficiency gains in our framework. However, the public finance literature identifies several ways in which redistribution may improve welfare and efficiency when uncertainty is present. For example, Sinn (1997) suggests that adverse selection problems may exclude a private insurance market for career risk. When agents have private knowledge of their productivity, any provider of voluntary income redistribution contracts would suffer from adverse selection problems. Diamond and Mirrlees (1986) analyze the optimal structure of social security benefits with exogenous productivity and disability risk. In both of these contributions, redistributive taxation can be interpreted as a substitute for the missing private insurance market. An optimal social security system should balance these benefits of redistribution against the costs outlined in our study. In any case, our results call for devoting proper attention to incentives concerning human capital formation when considering reforms in social security policy. An important challenge for future work would be to evaluate empirically the relationship between social security systems and human capital investments. In addition to the incentive effects highlighted here, such an empirical study should take into account also differences in general wage taxation.

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Appendix A

Retirement and human capital across benefit schemes

Maximizing the Cobb-Douglas specification of lifetime utility (6) subject to the lifetime budget constraint (7) yields the following first-order conditions:

$$(A1) \quad \frac{(1 - \tau_1 - \tau_2 - \tau_3)(1 - R)\alpha H^{\alpha-1} + R\alpha H^{\alpha-1}x - 1}{C} = 0$$

$$(A2) \quad \frac{-(1 - \tau_1 - \tau_2 - \tau_3)H^\alpha + b + H^\alpha x}{C} + \frac{\beta}{R} = 0$$

The public budget constraint for the flat-rate old-age benefit component is $B = \tau_1 (1 - R)H^\alpha$, that for flat-rate retirement subsidies is $b = \tau_2 \frac{1-R}{R} H^\alpha$, and that for earnings-related retirement subsidies is $x = \tau_3 \frac{1-R}{R}$. When we substitute these expressions into (A1) and (A2), we obtain:

$$(A3) \quad (1 - \tau_1 - \tau_2)(1 - R)\alpha H^{\alpha-1} - 1 = 0,$$

$$(A4) \quad -(1 - \tau_1)RH^\alpha + \tau_2 H^\alpha + \tau_3 H^\alpha + \beta((1 - R)H^\alpha - H) = 0.$$

(A3) and (A4) yield (8) and (9). We differentiate (8) and (9) with respect to τ_1 and τ_3 such that $d\tau_3 = -d\tau_1$. These derivations reveal that $\frac{dH}{d\tau_1} \Big|_{d\tau_3 = -d\tau_1} < 0$ and $\frac{dR}{d\tau_1} \Big|_{d\tau_3 = -d\tau_1} < 0$.

Appendix B

Proof of Proposition 5

We totally differentiate the first-order condition (5) with respect to the individual decision variable R and social security tax rates τ_1 , τ_2 , and τ_3 . We obtain:

$$A_{11}dR = [X_{11} \ X_{12} \ X_{13}] \begin{bmatrix} d\tau_1 \\ d\tau_2 \\ d\tau_3 \end{bmatrix}, \text{ where}$$

$$A_{11} = [-(1 - \tau_1 - \tau_2 - \tau_3) \cdot w + b + x \cdot w]^2 U'' + V''$$

$$X_{11} = -wU'$$

$$X_{12} = -\frac{1}{R} \cdot wU'$$

$$X_{13} = -\frac{1}{R} \cdot wU'.$$

this yields

$$\begin{aligned} \frac{dR}{d\tau_1} &= \frac{-wU'}{A_{11}} > 0; \quad \frac{dR}{d\tau_2} = \frac{dR}{d\tau_3} = \frac{-wU'}{RA_{11}} > 0, \\ \frac{dR}{d\tau_1} + \frac{dR}{d\tau_2} \Big|_{\tau_2 = -\tau_1} &= \frac{dR}{d\tau_1} + \frac{dR}{d\tau_3} \Big|_{\tau_3 = -\tau_1} = \left(1 - \frac{1}{R}\right) \frac{-wU'}{A_{11}} > 0; \\ \frac{dR}{d\tau_2} + \frac{dR}{d\tau_3} \Big|_{\tau_3 = -\tau_2} &= 0. \end{aligned}$$