

Effect of Extra Dimensions on Gravitational Waves from Cosmic Strings

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We show how the motion of cosmic superstrings in extra dimensions can modify the gravitational wave signal from cusps. Additional dimensions both round off cusps, as well as reducing the probability of their formation, and thus give a significant dimension dependent damping of the gravitational waves. We look at the implication of this effect for LIGO and LISA, as well as commenting on more general frequency bands.

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The notion that nature might have extra dimensions has been with us for some time, but only recently have we revisited it with a view to obtaining direct observational or experimental consequences. The idea of large extra dimensions (LED’s) [1] has given new possibilities both for compactifying nature’s extra dimensions, as well as allowing a much richer gravitational phenomenology. In particular, brane inflation [2] uses ideas from string theory, with inflation driven by the motion of a brane on some stabilized internal manifold [3]. A key side effect of brane inflation is the formation of cosmic strings, [4,5] as a by product of brane annihilation (for reviews see [6]), which can have a wide range of physical parameters and properties. The observation of such cosmic strings would therefore provide direct evidence for string theory, as well as giving us valuable information on inflation and the early Universe.

Cosmic strings [7] were originally popular as an alternative to inflation, but were soon found to be inconsistent with the emerging measurements of fluctuations in the microwave background [8], although their existence is not entirely ruled out [9]. From the cosmological point of view, the internal structure of the cosmic string is irrelevant, and the string is taken to have zero width with a Nambu action: $S = -\mu \int d^2\sigma \sqrt{\gamma}$, where μ is the mass per unit length of the string. Together with rules for intercommutation [10], or how crossing strings interact, this gives the basic physics of how a network of cosmic strings will evolve. Incorporating gravitational effects via a linearized approximation indicates how fast energy is lost from the network [11] and putting all these pieces together gives the scaling picture of the original cosmic string scenario [12].

For cosmic superstrings the picture is similar, but there are crucial differences. One is that the strings will now not necessarily intercommute when they intersect [13], a simple way of understanding this is to imagine that the strings “miss” in the internal LED’s. This clearly has a significant impact on one of the drivers of network evolution, and leads to a denser network [5,14].

Currently, gravitational wave experiments are most likely to detect cosmic strings, with constraints on parameter space [15] being derived using the Damour-Vilenkin (DV) results [16]. This calculation was performed in 4 spacetime dimensions; however, while the reduced intercommutation probability was taken into account, to our knowledge there has been no systematic investigation of the impact of motion in the extra dimensions on the gravitational waves from cosmic strings. In this Letter we include these extra dynamical degrees of freedom, and find a potentially significant moderation of the DV result, even when a phenomenologically motivated cutoff is imposed. The basic physics behind the effect is the extra degrees of freedom associated with the extra dimensions which not only reduce the probability of cusp formation, but also round off the cusp producing a narrowing of the gravitational wave beam and hence a loss of power. The combination of these effects drops the gravitational wave event rate, power, and hence detectable signal, thus altering current bounds [15] from gravitational wave experiments.

To understand how this comes about, recall that a string obeying the Nambu action sweeps out a world sheet in spacetime: $X^\mu = (t, \frac{1}{2}[\mathbf{a}(\sigma_-) + \mathbf{b}(\sigma_+)])$, where $\sigma_\pm = t \pm \sigma$ are the light cone variables on the world sheet ($\sigma \in [0, L]$ for a closed loop of length L). With the conventional gauge choice, \mathbf{a}' and \mathbf{b}' are constrained to lie on a unit “Kibble-Turok” sphere [17]. At a cusp, $\mathbf{a}' = \mathbf{b}'$, and the left and right moving velocities coincide, $\dot{X}_+^\mu = \dot{X}_-^\mu = \dot{\ell}^\mu = (1, \mathbf{n}')$; the string instantaneously reaches the speed of light, and thus there is a strong gravitational interaction. Cusps are therefore transient but powerful events, and beam out a strong pulse of gravitational radiation in a cone centered on the cusp. As they are generic on string trajectories in 3 space dimensions (3d) this can lead to a potentially measurable gravitational signal.

In the seminal papers [16], DV examined gravitational wave bursts (GWBs) from cosmic strings, assessing for what range of mass per unit length the string could poten-

tially be visible to the next generation of gravitational wave detectors. They first computed the amplitude of an individual cusp GWB as a function of the mass per unit length of the string, obtaining the logarithmic cusp waveform

$$h^{\text{cusp}}(f, \theta) \sim \frac{G\mu L^{2/3}}{r|f|^{1/3}} H[\theta_m - \theta], \quad (1)$$

where $f = \omega_m/2\pi = 2m/L$ is the frequency, H is the Heaviside step function, with θ the angle between the wave vector \mathbf{k} and the cusp vector $\mathbf{n}' = \mathbf{a}' = \mathbf{b}'$, and $\theta_m \simeq (2/Lf)^{1/3}$ a cutoff giving the opening angle of the cone in which the GWB beams out from the cusp.

In an expanding universe, the waveform frequency is redshifted in the obvious way, $f \rightarrow (1+z)f$, and r in the asymptotic waveform must be replaced by the physical distance, $a_0 r = (1+z)D_A(z)$, where $D_A(z)$ is the angular diameter distance at redshift z . To find the background for a cosmological network of strings, DV used the one scale model, $L \sim \alpha t$, $n_L(t) \sim 1/(\alpha t^3)$, to write the loop length and network density in terms of cosmological time. ($\alpha \sim 50G\mu$ is a constant representing the rate of energy loss from string loops [11].) The expected number of cusp events per unit spacetime volume is then given by $\nu(z) \sim C n_L / PT_L \sim 2C/P \alpha^2 t^4$, where C is the average number of cusps per loop period $T_L = L/2 \sim \alpha t/2$, and P is the intercommutation probability [13], which DV take in the range 10^{-1} – 10^{-3} . From this they obtain an estimate of the rate of GWB's per unit spacetime volume at redshift z as

$$d\dot{N} \sim \frac{\nu(z)}{(1+z)} \frac{\pi \theta_m^2(z) D_A(z)^2}{(1+z)H(z)} dz. \quad (2)$$

The final step of the DV argument is to integrate out until a desired event rate at an experimentally motivated fiducial frequency is obtained, then invert to find the redshift which dominates the signal. Evaluating the gravitational wave at this redshift and frequency then gives the amplitude. In practice, DV use interpolating functions for the angular diameter and cosmological time, which allows them to approximate these expressions analytically, and obtain a direct form of the amplitude (the black lines in Figs. 1 and 2).

With extra dimensions, the motion of the string in the internal dimensions causes it to appear to slow down in our noncompact space dimensions, which allows the left and right moving modes to misalign in momentum space, thus avoiding an exact cusp, which becomes a highly special feature in higher space dimensions. We need to generalize the notion of a ‘‘cusp’’ and estimate its probability. A near cusp event (NCE) is a local minimum of $|\mathbf{a}' - \mathbf{b}'| = 2\Delta \ll 1$, and is parametrized by Δ , which measures how close to an exact cusp (EC) this event is. Assuming a uniform distribution of solutions in parameter space, and modeling simple higher dimensional loop solutions (see [18] for full details and toy loop solutions), we find that the number of NCE's with $|\mathbf{a}' - \mathbf{b}'|_{\min} \leq 2\Delta$ in a generic loop

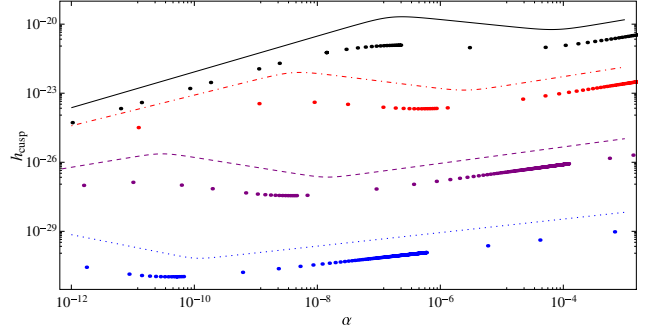


FIG. 1 (color online). A direct comparison with the DV plot [16], showing the GWB amplitude at $f = 150$ Hz as a function of α . Solid lines show the interpolating function result, the dots correspond to exact numerical results. From top to bottom the plots are 3d DV in black, in red (dot-dashed) $n = 1$, purple (dashed) $n = 3$, and blue (dotted) $n = 6$.

is $\mathcal{N}(\Delta) \simeq \Delta^n$ (since all loops have $|\mathbf{a}' - \mathbf{b}'| \leq 2$ at all points on their trajectory).

We now compute the waveform for a NCE. Since these strings are formed in brane inflation scenarios, the flux stabilization procedure that prevents dangerous cosmological moduli evolution [3] should also prevent the excitation of internal Kaluza-Klein degrees of freedom. Thus, we can use the standard Einstein propagator in calculating the gravitational radiation from a cusp. The main difference between the EC and the NCE is that the 4-velocity $\dot{X}^\mu = (1, (\mathbf{a}' + \mathbf{b}')/2)$ need not be null, and that the individual left and right moving velocities need not be aligned. The effect of this misalignment is similar to the misalignment between the cusp direction vector and the gravitational wave vector, and performing the computation in detail [18] shows that the waveform of the NCE is the same as (1), with the proviso that the cone opening angle is decreased to $\theta_\Delta = \theta_m - \Delta$.

Cosmologically, a general network will have a range of NCE's with different Δ values, up to and including the cutoff value when the GWB beaming cone closes off. We must therefore calculate the GWB event rate \dot{N} as a function of Δ , replacing the solid angle $\theta_m^2(z)$ by $(\theta_m(z) - \Delta)^2$, and $\nu(z) \rightarrow \nu(z, \Delta) = C(\Delta) n_L / PT_L$, where $C(\Delta)$ is the

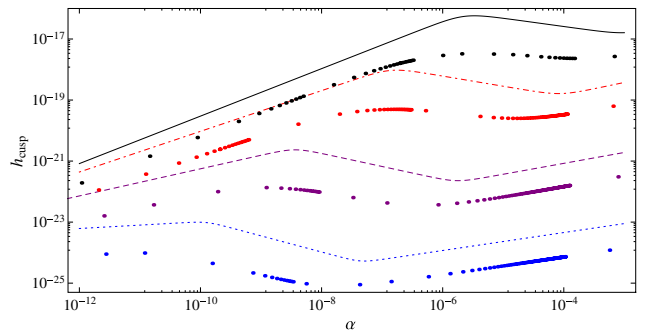


FIG. 2 (color online). As Fig. 1, but with $f = 3.9$ mHz appropriate to the LISA detector.

local probability density of NCE's for the network. Assuming that the loops are spread evenly in the parameter space of solutions, $\mathcal{C}(\Delta) = \mathcal{N}'(\Delta) = n\Delta^{n-1}$, and we integrate over Δ to obtain the nett effect of all possible NCE's:

$$\frac{d\dot{N}_{\text{NCE}}}{dz} = \frac{2\theta_m(z)^{n+2}}{(n+1)(n+2)} \frac{n_L(z)}{PT_L(z)} \frac{\pi D_A(z)^2}{(1+z)^2 H(z)}. \quad (3)$$

Figures 1 and 2 show the gravitational wave amplitude for the Laser Interferometer Gravitational Wave Observatory (LIGO) and Laser Interferometer Space Antenna (LISA) detectors, respectively. In each case, we used the same fiducial frequencies as DV: the waveform has the same profile; hence, the signal to noise analysis remains the same. For direct comparison we used the interpolating function method, but also include an exact numerical computation for the concordance cosmology ($\Omega_r = 4.6 \times 10^{-5}$, $\Omega_m = 0.28$, $\Omega_\Lambda = 1 - \Omega_m - \Omega_r$).

Clearly, the motion in the extra dimensions has a significant effect on the GWB amplitude; however, to what extent is this result a feature of our assumptions? The basic reason for the suppression of the signal is the distribution over the near cusp parameter Δ . This was derived assuming a uniform distribution in solution space, and a zero width string. Let us deal with each in turn.

One objection to having a uniform distribution in solution space is the notion that compact extra dimensions must somehow constrain the allowed parameter space of the string. Since cosmic strings form from the collision of a brane and antibrane, it seems likely that they have significant initial momentum in the extra dimensions; thus, it seems reasonable not to curtail solution space in this way. However, one might worry that if the loop wraps back and forth across the extra dimension(s) the string has more opportunity to self intersect, and that this will result in a restriction on parameter space. We modeled this [18] by exploring the self intersection of a 4d family of loops with a 3d limit. In 3d, about 30% of the parameter space had self intersections, but in 4d, once again the measure of solution space with self intersections became zero by a similar parametric argument as for the cusp.

The clear outcome of testing exact loop trajectories is that for a zero width string, there is no restriction on parameter space from compact extra dimensions. However, cosmic strings have finite width w , and while this is smaller than the internal LED size R , we would expect the ratio w/R to enter into the parametric computation. We model this by restricting $\Delta \in [0, \Delta_0]$ with Δ_0 related to w/R , and normalize \mathcal{C} so that $\mathcal{N}(\Delta_0) = 1$, i.e., $\mathcal{C}(\Delta) = n\Delta^{n-1}/\Delta_0^n$. This modifies the dependence of $d\dot{N}_{\text{NCE}}/dz$ on θ_m to

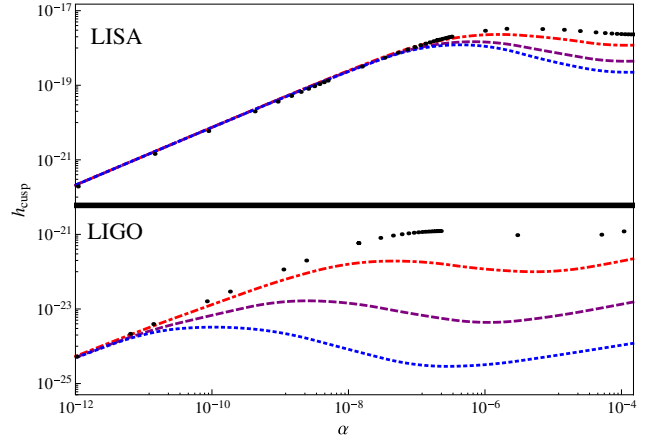


FIG. 3 (color online). A plot of the amplitude for LISA and LIGO with the modified measure (4) and $P = 10^{-3}$, fixing $\Delta_0 = 10^{-3}$, and varying n . From top to bottom: the 3d result, and the extra dimension plots with $n = 1, 3$, and 6 , respectively.

$$\begin{aligned} & \int_0^{\min\{\Delta_0, \theta_m\}} \mathcal{C}(\Delta)(\theta_m(z) - \Delta)^2 \\ &= \frac{2\theta_m(z)^{n+2} H[\Delta_0 - \theta_m]}{\Delta_0^n (n+1)(n+2)} + \left(\theta_m^2(z) - \frac{2n\Delta_0\theta_m(z)}{n+1} \right. \\ & \quad \left. + \frac{n\Delta_0^2}{n+2} \right) H[\theta_m - \Delta_0]. \end{aligned} \quad (4)$$

To test this alternate expression, we took values of $\Delta_0 = 0.1 - 10^{-4}$ and $n = 1, 3, 6$ (see Figs. 3 and 4). From (4), we see that the effect of Δ_0 is to shift the behavior from (3) for $\theta_m < \Delta_0$, towards a $\theta_m(z)^2$ form as θ_m grows. For $\theta_m > 50\Delta_0$, the 3d result is recovered. Since $\theta_m(z) \propto (G\mu)^{-1/3}$, the results converge to the 3d value at larger Δ_0 for smaller $G\mu$.

In summary, we have studied the impact of motion in extra dimensions on the GWB signal from cusp events on

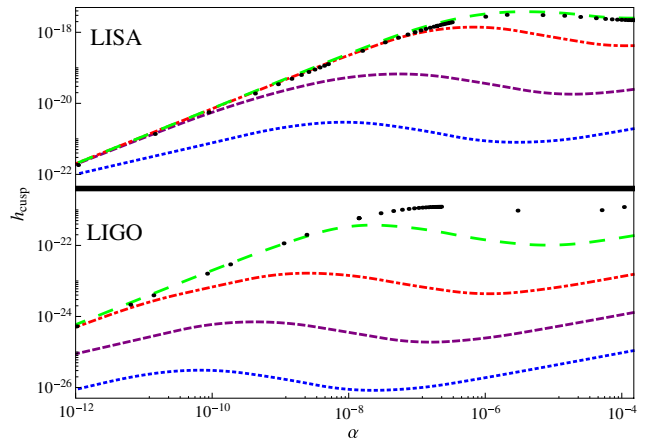


FIG. 4 (color online). A plot of the amplitude for LISA and LIGO with the modified measure (4) and $P = 10^{-3}$, varying Δ_0 with fixed $n = 3$. From top to bottom: the 3d DV result, then $\Delta_0 = 10^{-4}, 10^{-3}, 10^{-2}$, and 0.1 , respectively.

cosmic string loops. We find a potentially significant modulation of the signal, even after taking into account finite width effects and the size of the extra dimension. Clearly further work is required to get better control of the approximations being used, in particular, to take into account more complex compactification geometries, however, it does seem that motion in internal dimensions is important. Although we have focused on LIGO and LISA, we should also comment on alternative gravitational wave detectors. It is not difficult to see from the interpolating function approximation that the higher the frequency, the greater the damping on the signal due to the extra dimensions. Coupled with the inherent damping of the signal at higher dimensions, we conclude that high frequency GW detectors, such as the electromagnetic detector of Cruise *et al.* [19], have little hope of seeing this signal. On the other hand, at low frequencies, such as those probed by the pulsar timing array [20], the damping due to extra dimensions is diminished. For the pulsar limits, the confusion noise, as defined by DV, will be similar for both cosmic superstrings and strings, since at such low frequencies we are at the limit of the approximation used in the calculation of the waveform, and the GWB cone has completely opened out. This would suggest that we can no longer trust the waveform (1) or its extra dimensional analog that we have used throughout. Finally, from the dependence of the signal on n , the possibility arises that a positive detection of gravitational radiation would not only confirm the general brane inflation scenario, but could also provide a means of determining the number of (effective) extra dimensions.

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