Optimal taxation with monopolistic competition

Leslie J. Reinhorn

Economics Department

University of Durham

23-26 Old Elvet

Durham DH1 3HY

United Kingdom

phone +44 191 334 6365

fax +44 191 334 6341

reinhorn@hotmail.com

27 June 2011

Abstract

This paper studies optimal taxation in a Dixit–Stiglitz model of monopolistic competition. In this setting, taxes may be used as an instrument to offset distortions caused by producer markups. Since markups tend to be higher in industries where firms face less elastic demand, tax rates will be pushed lower in these industries. This tends to work against the familiar inverse elasticities intuition associated with the Ramsey tax rule. However, a key feature of the model is that the Ramsey rule responds to the industry demand curve (Chamberlin's DD) while the monopolistic markup is a response to the demand curve faced by firms (Chamberlin's dd). Hence the elasticities of both these curves influence the optimal tax rate, but in opposite directions.

1 Introduction

This paper addresses optimal taxation in a multi-sector version of the Dixit–Stiglitz (1977) model of monopolistic competition. Since firms with market power create economic distortions, taxes will be used in part to offset the adverse welfare consequences of producer markups. Thus, the focus of study is taxation that not only raises revenue efficiently, it must also have an optimal corrective component to combat the monopolistic distortions. This is potentially a rather complex problem with multiple policy objectives but only a limited number of policy instruments. Nonetheless, the problem is an important one since varying degrees of imperfect competition are present in many markets yet the theory of optimal taxation has focused primarily on the perfectly competitive case.[1] Fortunately, despite the complexity of the problem some valuable insights are available.

Since markups respond inversely to the elasticity of demand, corrective policy will work in the opposite direction: taxes will respond positively to the elasticity of demand. This is in contrast to the familiar Ramsey rule for efficient taxation which tends to favor an inverse elasticities tax rule. So the two policy objectives respond in opposite ways to the elasticity of demand — a pro-elasticities rule for the corrective component and an inverse elasticities rule for the efficient component. The optimal policy is a combination of the two. But note that the Ramsey rule responds to elasticities of industry demand curves (Chamberlin's DD). By contrast, the markups, and hence the tax corrections, respond to elasticities of firm demand curves (Chamberlin's dd). Thus the optimal balance between inverse elasticities and pro-elasticities tax rules depends on careful measurement of the different elasticities.

The optimal tax problem for the Dixit–Stiglitz economy — an imperfectly competitive economy with zero profits and heterogeneous goods — has the same form as the optimal tax problem for an imperfectly competitive economy with positive profits and homogeneous goods. The latter problem has been studied by Myles (1989). We exploit the equivalence between the two problems and use the methods of Myles to show the compensated effect of the optimal tax system on the number of firms in the free entry equilibrium. The results are consistent with the corrective role for taxation. If an industry faces a large monopolistic distortion, the tax system's direct response is to cause only a small reduction in entry.

The rest of the paper is organized as follows. Sections 2 and 3 present the model and its equilibrium respectively. Section 4 studies the optimal tax problem when quantities are the control variables. Section 5 considers the case where prices are the control variables. Section 6 is a brief conclusion.

---

[1] Section 6 of Auerbach and Hines (2002) and chapter 11 of Myles (1995) discuss optimal taxation under imperfect competition.

2 Model

There are I monopolistically competitive industries, labeled $i = 1, \ldots, I$. The representative consumer has utility function

$$U(\ell, Y_1, \ldots, Y_I)$$

where ℓ is leisure and Y_i is an aggregator for industry i:

$$Y_i := \int_0^{n_i} u_i(q_i(j)) dj.$$

Each firm in industry i produces a distinct variety, so n_i is both the number of firms and the extent of variety in the industry. The function u_i gives the utility contribution for each variety of industry i consumption. The representative consumer is endowed with L units of time, so $L - \ell$ is labor supply.

Labor is the only factor of production. Thus firms' production functions can be inverted to yield cost functions. Specifically, each of the n_i firms in industry i has the same cost function

$$C_i(q)$$

where q is the firm's output and the costs are measured in units of labor. There are fixed costs: $C_i(0^+) > 0$ for $i \geq 1$, where $C_i(0^+) := \lim_{q \downarrow 0} C_i(q)$.

Government purchases consist of g units of labor with $g < L$. Since g is exogenous and fixed, its effect on the consumer's utility is omitted. The labor income tax rate is t_0. The sales tax rate in industry i is t_i.

With labor as numeraire, let $p_i(j)$ (or simply p_{ij}) denote the price charged by firm $j \in [0, n_i]$ in industry $i \geq 1$. The corresponding consumer price is $P_i(j) := (1 + t_i)p_i(j)$ (or simply P_{ij}). In a symmetric equilibrium, $p_{ij} = p_i$ for all $j \in [0, n_i]$ and $P_i := (1 + t_i)p_i$. The consumer price for labor is denoted $P_0 := 1 - t_0$.

The model's technical assumptions are in appendix A.

3 Equilibrium

This section describes agent behavior and the economy's equilibrium. The representative consumer chooses $(\ell, q_1(\cdot), \ldots, q_I(\cdot)) \geq 0$ to

$$\text{maximize} \quad U(\ell, Y_1, \ldots, Y_I)$$

$$\text{subject to} \quad Y_i = \int_0^{n_i} u_i(q_i(j)) dj$$

$$\sum_{i=1}^{I} \int_0^{n_i} (1 + t_i) p_i(j) q_i(j) dj \leq (1 - t_0)(L - \ell).$$

There is no income from dividends since free entry drives profits to zero. Let U_0, U_1, ..., U_I denote the partial derivatives of U. The first order conditions for an interior solution are

$$U_0 \tau_i p_{ij} = U_i u_i'(q_{ij}) \qquad i \geq 1, \quad j \in [0, n_i]$$

where $\tau_i := (1 + t_i)/(1 - t_0)$. Thus the inverse demand curve for variety j in industry i is proportional to $u_i'(q_{ij})$ and hence the elasticity of the dd demand curve is $\epsilon_{ij} = u_i'(q_{ij})/[q_{ij} u_i''(q_{ij})]$.

At this point it is worth emphasizing the way the model captures the dd/DD distinction. We just saw that the dd elasticity for firms in industry i depends only on the properties of u_i. As for DD, this is demand for aggregate industry output. Here, since Y_i plays the role of industry aggregate, DD demand is based on utility derived from Y_i which is governed by U. So demand properties derived from U are the model's DD, while those derived from u_i are the model's dd.

In a symmetric equilibrium the consumer's first order conditions and the budget constraint are

$$U_0 \tau_i p_i = U_i u_i' \qquad i \geq 1$$

$$\ell + \sum_{i=1}^{I} \tau_i p_i q_i n_i = L.$$

Corner solutions can be ignored since, under assumptions 1(b) and 5 in appendix A, the government will never choose tax rates that lead to a corner. Similarly, the definition of τ_i requires $1 - t_0 \neq 0$. The government would never choose $t_0 = 1$ since that would leave the consumer with no income.

Firms are monopolists relative to their respective dd demand curves. Hence markups satisfy

$$\frac{p_{ij}}{C_i'(q_{ij})} = \frac{1}{1 + 1/\epsilon_{ij}} = \frac{u_i'(q_{ij})}{u_i'(q_{ij}) + q_{ij} u_i''(q_{ij})}.$$

In a symmetric equilibrium with free entry this yields

$$\frac{p_i}{C_i'(q_i)} = \frac{u_i'(q_i)}{u_i'(q_i) + q_i u_i''(q_i)} \qquad \text{and} \qquad p_i q_i = C_i(q_i)$$

where the latter is the zero profit condition. Eliminate p_i from these equations to get

$$\frac{q_i C_i'(q_i)}{C_i(q_i)} = 1 + \frac{q_i u_i''(q_i)}{u_i'(q_i)} \tag{1}$$

(Spence 1976, equation (71)). Under the model's assumptions, for each industry $i \geq 1$ this equation has a unique solution $q_i > 0$. Hence output per firm is determined independently of the government's policy, and so is the producer price $p_i = C_i(q_i)/q_i$. Policy does, however, affect the number of firms in each industry.[2]

---

[2] This result — that policy has no effect on output per firm, but does affect entry — is a consequence of the form of the dd demand curves. From the consumer's first order condition ($U_0 \tau_i p_{ij} = U_i u_i'(q_{ij})$) we find that firm j in industry i faces inverse demand $p_{ij} = \psi_i u_i'(q_{ij})$ where $\psi_i := \tau_i^{-1} U_i/U_0$ is exogenous to the firm. If the government were to reduce τ_i, dd would shift outward. This alone would raise profits, so entry would occur and this would shift demand inward. But the only mechanism for shifting demand inward is a reduction in ψ_i. Ultimately, the original reduction in τ_i must be exactly offset by a reduction in U_i/U_0 in equilibrium. Otherwise, profits would not return to zero.

With these results, the equilibrium conditions are

$$U_0 \tau_i C_i = U_i q_i u_i' \qquad i \geq 1 \tag{2}$$

$$\ell + \sum_{i=1}^{I} \tau_i n_i C_i = L \tag{3}$$

$$g + \ell + \sum_{i=1}^{I} n_i C_i \leq L \tag{4}$$

where the zero profit condition, $p_i q_i = C_i$, has been used to eliminate p_i in (2) and (3). Inequality (4) is the resource constraint. Recall that government consumption consists of g units of labor/leisure and that q_i, u_i, u_i', and C_i are evaluated at the level of q_i that is consistent with profit maximization and zero profits. The government's budget constraint is automatically satisfied when the consumer's budget and the resource constraint are satisfied (Walras' law).

An equilibrium is an intersection between the offer curve and the resource constraint. The offer curve is the set of vectors $(\ell, n_1, \ldots, n_I)$ that satisfy the consumer's optimality conditions (2) and (3). The curve is traced out as the vector of policy variables $(\tau_1, \ldots, \tau_I)$ is allowed to take on all admissible values. Eliminate $(\tau_1, \ldots, \tau_I)$ from (2) and (3) to get a single equation in $(\ell, n_1, \ldots, n_I)$ which generates the entire offer curve:

$$U_0 \ell + \sum_{i=1}^{I} U_i q_i u_i' n_i = U_0 L. \tag{5}$$

To find the policy variables τ_i that correspond to a particular solution to (5), invert (2):

$$\tau_i = \frac{U_i}{U_0} \frac{q_i u_i'}{C_i}. \tag{6}$$

Thus an equilibrium is a vector $(\ell, n_1, \ldots, n_I)$ that solves (4) and (5) simultaneously.

The model has a free normalization since $1 - t_0, 1 + t_1, \ldots, 1 + t_I$ can all be multiplied by any positive scale factor without affecting the values of $\tau_1, \ldots, \tau_I$, and hence without affecting the corresponding allocation.

4   Optimal tax problem: quantities as controls

This section characterizes the optimal tax rates in terms of demand properties, with a DD inverse elasticity Ramsey effect, a dd pro-elasticity corrective effect, and interactions between them.

The optimal tax problem is to choose $(\ell, n_1, \ldots, n_I) \geq \mathbf{0}$ to

$$\text{maximize} \quad U(\ell, n_1 u_1, \ldots, n_I u_I)$$
$$\text{subject to} \quad \text{(4) and (5)}$$

with $u_i = u_i(q_i)$ and q_i determined as above. The optimal tax rates are found from (6). Appendix B shows that this problem has a solution.

4.1 First order conditions

The Lagrangian is

$$\mathcal{L} = U - \lambda \left[g + \ell + \sum_{j=1}^{I} n_j C_j - L \right] + \eta \left[\ell + \sum_{j=1}^{I} \frac{U_j}{U_0} q_j u_j' n_j - L \right]$$

where λ and η are Lagrange multipliers associated with (4) and (5) respectively. Let U^* be the optimal value function. By the envelope theorem, $\lambda = -\partial U^*/\partial g$ (since g is not an argument of U). Due to the sign of the weak inequality (4), $\lambda \geq 0$. It is natural to expect η to be positive. To see why, consider how the Lagrangian would change if an exogenous lump sum tax T were added to the model. The consumer's lump sum income on the right hand side of (3) would become $L - T$ (after a suitable normalization of values), while the resource constraint would be unaffected. Therefore, at the optimum, $\eta = \partial U^*/\partial T \mid_{T=0}$. Intuitively, $\partial U^*/\partial T \mid_{T=0}$ should be positive since it represents a marginal shift in policy away from distortionary taxes and toward a lump sum tax.

To facilitate comparison with the perfectly competitive case it is convenient to employ the change of variables $Y_i := n_i u_i$ and to maximize with respect to Y_i rather than n_i. Clearly, this does not change the problem. Then the Lagrangian is

$$\mathcal{L} = U - \lambda \left[g + \ell + \sum_{j=1}^{I} \frac{C_j}{u_j} Y_j - L \right] + \eta \left[\ell + \sum_{j=1}^{I} \frac{U_j}{U_0} \frac{q_j u_j'}{u_j} Y_j - L \right].$$

The first order conditions are

$$0 = U_0 - \lambda + \eta \left[1 + \sum_{j=1}^{I} \frac{q_j u_j'}{u_j} Y_j \frac{\partial}{\partial \ell} \left(\frac{U_j}{U_0} \right) \right] \tag{7}$$

$$0 = U_i - \lambda \frac{C_i}{u_i} + \eta \left[\frac{U_i}{U_0} \frac{q_i u_i'}{u_i} + \sum_{j=1}^{I} \frac{q_j u_j'}{u_j} Y_j \frac{\partial}{\partial Y_i} \left(\frac{U_j}{U_0} \right) \right] \qquad i \geq 1. \tag{8}$$

As a benchmark, consider the optimal tax policy when a lump sum tax T is available. As indicated above, $\eta = \partial U^*/\partial T \mid_{T=0}$. So if T is chosen optimally rather than set equal to zero, $\eta = 0$. In that case, (7) and (8) yield $U_i/U_0 = C_i/u_i$. Then from (6), $\tau_i = q_i u_i'/u_i$ which gives the following result.

4.1.1 Lemma   *When lump sum taxation is available, the optimal policy is a subsidy for each industry:* $\tau_i < 1$ *for all* $i \geq 1$.[3,4]

---

[3] Recall τ_i is defined as $(1+t_i)/(1-t_0)$ so $\tau_i < 1$ yields a subsidy ($t_i < 0$) under any reasonable normalization with $t_0 \geq 0$.

[4] This result is related to the consumer's love of variety. Under the latter, the consumer would prefer q units of output from each of 10 different varieties rather than $10q$ units from a single variety: $10u_i(q) > u_i(10q)$, or more generally, $nu_i(q) > u_i(nq)$ for all $n > 1$. This yields $u_i(q)/q > u_i(nq)/(nq)$ for all $n > 1$, hence $q \mapsto u_i(q)/q$ is a decreasing function of q. From basic microeconomics, if an "average" function is declining, the marginal curve must lie below it. Hence, a consumer with a love of variety has $u_i'(q) < u_i(q)/q$ for all $q > 0$, and this implies $\tau_i < 1$.

Proof Since the optimum is $\tau_i = q_i u_i' / u_i$, the goal is to show that $q_i u_i'(q_i) < u_i(q_i)$. By assumption 2 in appendix A, u_i is strictly concave so it lies below its tangent lines: $u_i(q) < u_i(\bar{q}) + (q - \bar{q}) u_i'(\bar{q})$ for all $q \neq \bar{q}$. If we take $q = 0$ and note again from assumption 2 that $u_i(0) = 0$, we get $0 < u_i(\bar{q}) - \bar{q} u_i'(\bar{q})$ for all $\bar{q} > 0$. In particular, if we take $\bar{q} = q_i$, the equilibrium quantity, we get the desired result. ∎

These subsidies are purely corrective. They undo some of the welfare damage from imperfect competition, and they are financed with the lump sum tax.

The benchmark with lump sum taxation, $\tau_i = q_i u_i' / u_i$, states that optimal taxes respond positively to the elasticity of the *variety utility function*. (I.e., the subsidy is smaller if this elasticity is larger.) As noted on page 303 of Dixit and Stiglitz (1977), the numerator of this expression, $q_i u_i'$, is proportional to firm revenue, the factor that motivates private sector activity. The denominator is u_i which is the full social value of the firm's output. To the extent that this ratio is small, the monopolistically competitive market fails to provide adequate incentives to firms. The optimal response is a large subsidy (small τ_i).

Under reasonable conditions this benchmark for τ_i yields the pro-elasticities tax rule: optimal corrective taxes respond positively to the elasticity of the *dd* demand curve.

4.1.2 Lemma *Let q_i and q_j be the equilibrium levels of output per firm in industries i and j respectively. Suppose the dd demand curve is more elastic in industry i than industry j at the equilibrium output and at all equiproportionate quantity reductions from equilibrium.[5] Then if lump sum taxation is available, the optimal policy has $\tau_i > \tau_j$.*

Proof Recall from section 3 that the magnitude of the *dd* elasticity is $-u'(q)/[q u''(q)]$. So the hypothesis is

$$-\frac{q_i u_i''(\xi q_i)}{u_i'(\xi q_i)} < -\frac{q_j u_j''(\xi q_j)}{u_j'(\xi q_j)}$$

for all $\xi \in (0, 1]$. Integrate this relationship with respect to ξ over the interval $[\bar{\xi}, 1]$, and make use of the identity $-q u''(\xi q)/u'(\xi q) = -\frac{d}{d\xi}[\log(u'(\xi q))]$:

$$-\log(u_i'(q_i)) + \log(u_i'(\bar{\xi} q_i)) < -\log(u_j'(q_j)) + \log(u_j'(\bar{\xi} q_j))$$

for all $\bar{\xi} \in (0, 1)$. Take the exponential of both sides of this inequality to get

$$u_i'(\bar{\xi} q_i)/u_i'(q_i) < u_j'(\bar{\xi} q_j)/u_j'(q_j)$$

for all $\bar{\xi} \in (0, 1)$. Integrate again, this time with respect to $\bar{\xi}$ over the interval $(0, 1)$, and make use of $u(0) = 0$ from assumption 2 in appendix A:

$$\frac{u_i(q_i)}{q_i u_i'(q_i)} < \frac{u_j(q_j)}{q_j u_j'(q_j)}.$$

_____

[5]An "equiproportionate quantity reduction from equilibrium" means that quantity per firm is ξq_i in industry i and ξq_j in industry j for a scale factor $\xi < 1$.

This proves the lemma since, with lump sum taxation, the optimal policy has $\tau = qu'/u$. ∎

Return now to the case where no lump sum transfer is available. We begin with two specific examples that illustrate some of the general features of the optimal tax policy.

4.2 Two examples

The examples below illustrate how optimal tax policy changes as we move from perfect competition to monopolistic competition. In the first, if competition were perfect, uniform commodity taxation would be optimal. When, instead, firms are monopolistically competitive the example shows how optimal taxes are adjusted away from uniform rates, and in particular, it shows the pro-elasticities corrective tax rule at work. In the second example, perfect competition would deliver a pure inverse elasticities Ramsey optimal tax rule. So when we have monopolistic competition instead, we see the interaction between the pro-elasticities corrective role for taxation and the inverse elasticities efficiency role.

In the first example,

$$U(\ell, Y_1, \ldots, Y_I) := F(\ell) + \sum_{i=1}^{I} \delta_i \log(Y_i)$$

for some concave F. The utility contribution functions u_i, and the cost functions C_i, are not specified — they need only satisfy the model's technical assumptions. Direct substitution into (8) yields $U_i = \lambda C_i / u_i$ which, together with (6), gives $\tau_i \propto q_i u_i' / u_i$, a pro-elasticities corrective tax rule. (Again, this uses lemma 4.1.2's positive relationship between the elasticity of utility and the elasticity of dd.) The constant of proportionality is $\lambda / U_0 = \lambda / F'(\ell)$ which is a measure of the marginal burden of taxation. So the optimal policy has a two part interpretation. First, take the purely corrective subsidies from the case where lump sum transfers were available (lemma 4.1.1). But since lump sum transfers are not actually available, the second part of the policy raises the required revenue by scaling up all the commodity tax rates in equal proportion. Since the scaling up leaves *relative* tax rates unaffected, in the end we still have a pro-elasticities dd tax rule. By comparison, as noted above, if competition were perfect this U would result in an optimal policy of uniform commodity tax rates (Besley and Jewitt, 1995).[6] When competition is imperfect, the optimal taxes are a natural combination of the pro-elasticities corrective policy and the perfectly competitive uniform policy.

---

[6] With U as given here, e^U, which represents the same preferences, has the following two properties which are sufficient to ensure that under perfect competition uniform taxation is optimal: first, e^U is separable between leisure and the vector $(Y_1, \ldots, Y_I)$ and second, e^U is homogeneous of positive degree in $(Y_1, \ldots, Y_I)$. As a consequence, the parameters δ_i have no bearing on the optimal taxes. A large δ_i might seem to encourage a low tax rate on commodity i, but this is not the case. In order for the government to maximize the Cobb–Douglas part of the utility function, resources devoted to industry i should be proportional to δ_i. Consumer sovereignty ensures that consumer expenditure is proportional to δ_i, so optimal policy entails proportionality between consumer expenditure and resource cost. This is precisely what we get from uniform taxation since consumer prices are proportional to producer prices.

For the second example, the functional forms are

$$U(\ell, Y_1, \ldots, Y_I) \;=\; \ell + \sum_{i=1}^{I} \delta_i Y_i^{1-1/E_i} / (1 - 1/E_i)$$

$$u_i(q) \;=\; q^{\rho_i}$$

$$C_i(q) \;=\; F_i + c_i q.$$

All parameters are positive and ρ_i is restricted to be less than one. If all of the $E_i \to 1$, we get the log case of the previous example. The elasticity of the firms' dd demand curve in industry i is $\epsilon_i = u_i'/(q u_i'') = -1/(1 - \rho_i)$. The magnitude of ϵ_i is positively related to ρ_i as required for lemma 4.1.2. Assume the labor endowment L is sufficiently large that the consumer chooses $\ell > 0$. Although U does not satisfy assumption 1 in appendix A, it is nonetheless sufficiently well behaved to ensure the existence of an interior optimal tax equilibrium. Also, since U delivers an inverse elasticities tax rule under perfect competition (Myles, 1995, page 107), it is a natural choice to underscore the effects of imperfect competition.

Output per firm in industry i is determined by (1) which yields

$$q_i = \frac{\rho_i F_i}{(1 - \rho_i) c_i} \quad \text{and, by zero profits,} \quad p_i = \frac{c_i}{\rho_i}.$$

In equilibrium, the industry i aggregator is $Y_i = n_i q_i^{\rho_i}$ with q_i as above. Then the equilibrium conditions (4) and (5) are

$$g + \ell + \sum_{i=1}^{I} Y_i q_i^{1-\rho_i} c_i / \rho_i \;\le\; L$$

$$\ell + \sum_{i=1}^{I} Y_i^{1-1/E_i} \delta_i \rho_i \;=\; L.$$

The government chooses ℓ and the Y_is to maximize U subject to these two conditions. Substitute for ℓ from the latter. The Lagrangian is

$$\mathcal{L} \;=\; L + \sum_{i=1}^{I} Y_i^{1-1/E_i} \delta_i \left(\frac{1}{1 - 1/E_i} - \rho_i \right) + \lambda \left[-g + \sum_{i=1}^{I} \left(Y_i^{1-1/E_i} \delta_i \rho_i - Y_i q_i^{1-\rho_i} c_i / \rho_i \right) \right].$$

The first order condition for Y_i yields

$$Y_i^{-1/E_i} \delta_i \left[1 + (\lambda - 1) \rho_i (1 - 1/E_i) \right] \;=\; \lambda q_i^{1-\rho_i} c_i / \rho_i. \tag{9}$$

Then from (6),

$$\tau_i \;=\; \frac{\lambda \rho_i}{1 + (\lambda - 1) \rho_i (1 - 1/E_i)}, \tag{10}$$

or equivalently,

$$1 - \rho_i / \tau_i \;=\; (1 - 1/\lambda)(1 - \rho_i + \rho_i / E_i). \tag{11}$$

10

Since $\lambda = -\partial U^*/\partial g$, if optimal lump sum taxation were available, λ would equal the consumer's marginal utility from the labor endowment, and with quasi-linear preferences that marginal utility is one. In that case, (10) is the benchmark $\tau_i = \rho_i$, the pro-elasticities purely corrective tax policy which depends only on the *dd* elasticity. In the absence of lump sum taxation, λ exceeds one.[7] The other reference point for comparison is perfect competition, $\rho_i = 1$. Then (11) is the Ramsey inverse elasticities rule $1 - 1/\tau_i = (1 - 1/\lambda)E_i^{-1}$, which depends only on the *DD* elasticity.

In the general case, (11) shows that the ingredients for the optimal policy consist of two pure effects: the corrective benchmark ρ_i and the Ramsey rule $1/E_i$. These ingredients are blended in a way that tends to dilute the full impact of each pure effect. Finally, (11) shows that major policy errors could occur if one uses the familiar tax rule for the perfectly competitive case when the economy is actually imperfectly competitive.

4.3   Distance function

The first order conditions (7) and (8) are somewhat messy for analysis. A cleaner approach uses the distance function (Deaton and Muellbauer 1980). This provides a representation for preferences which has certain homogeneity and concavity properties that Deaton (1979) exploits to provide a very neat and simple optimal tax formula for the perfectly competitive case. Deaton's result can be extended for monopolistic competition.

Let $d(u, \ell, Y_1, \ldots, Y_I)$ be the distance function for the utility function U.[8] Denote first order partial derivatives of d with respect to $\ell, Y_1, \ldots, Y_I$ by $d_0, d_1, \ldots, d_I$ respectively, and similarly, denote second order partial derivatives by d_{ij} for $i, j \geq 0$. Define

$$\rho_0 := 1 \text{ and } \rho_i := q_i u_i'/u_i \text{ for } i \geq 1$$

$$z_0 := 1 \text{ and } z_i := C_i/u_i \text{ for } i \geq 1$$

(12)

and modify notation so that $Y_0 := \ell$. Then from (6), and using $d_i/d_j = U_i/U_j$,

$$\tau_i = \frac{1 + t_i}{1 - t_0} = \frac{d_i \rho_i / z_i}{d_0 \rho_0 / z_0} = \frac{d_i \rho_i}{d_0 z_i}.$$

(13)

Following Deaton (1979), the optimal tax problem is to choose values for u and $(Y_0, Y_1, \ldots, Y_I) \geq \mathbf{0}$ to

---

[7]From (9), $Y_i^{1-1/E_i} \delta_i \rho_i = \lambda Y_i q_i^{1-\rho_i} c_i / [1 + (\lambda - 1)\rho_i(1 - 1/E_i)]$. Take the sum over i and use the two constraints from the optimization problem to get $g + \sum_{i=1}^I Y_i q_i^{1-\rho_i} c_i / \rho_i \leq \lambda \sum_{i=1}^I Y_i q_i^{1-\rho_i} c_i / [1 + (\lambda - 1)\rho_i(1 - 1/E_i)]$. Since $g > 0$ the following inequality must be satisfied: $\sum_{i=1}^I Y_i q_i^{1-\rho_i} c_i \{\lambda/[1 + (\lambda - 1)\rho_i(1 - 1/E_i)] - 1/\rho_i\} > 0$. Since $\lambda/[1 + (\lambda - 1)\rho_i(1 - 1/E_i)] = 1/[\rho_i(1 - 1/E_i) + \lambda^{-1}(1 - \rho_i + \rho_i/E_i)]$, and since $\rho_i < 1$, the left hand side of this inequality is an increasing function of $\lambda \in (0, 1]$. Furthermore, the inequality is violated at $\lambda = 1$. It follows that λ must exceed one.

[8]The distance function is defined implicitly by $U(\ell/d, Y_1/d, \ldots, Y_I/d) = u$. The utility function $U(\cdot)$ is recovered by solving $d(u, \ell, Y_1, \ldots, Y_I) = 1$ for u. For each value of u the distance function is concave, homogeneous of degree 1, and increasing (under the additional assumption that U is monotone) in $(\ell, Y_1, \ldots, Y_I)$. See Deaton and Muellbauer (1980) for further details.

maximize u subject to

$$d(u, Y_0, Y_1, \ldots, Y_I) \quad = \quad 1 \tag{14}$$

$$\sum_{i=0}^{I} d_i \rho_i Y_i \quad = \quad d_0 L \tag{15}$$

$$g + \sum_{i=0}^{I} z_i Y_i \quad \leq \quad L. \tag{16}$$

The Lagrangian is

$$\mathcal{L} = u + \nu(d-1) + \theta \left(\sum_{j=0}^{I} d_j \rho_j Y_j - d_0 L \right) + \phi \left(L - g - \sum_{j=0}^{I} z_j Y_j \right).$$

As in section 4.1, at an optimum $\phi \geq 0$ and for all cases of interest $\theta > 0$. The first order condition for Y_i, $i \geq 0$, is

$$0 = \nu d_i + \theta \left(d_i \rho_i + \sum_{j=0}^{I} d_{ji} \rho_j Y_j - d_{0i} L \right) - \phi z_i.$$

Define

$$H_i := \rho_i - \frac{d_{0i}}{d_i} L + \sum_{j=0}^{I} \frac{d_{ji}}{d_i} \rho_j Y_j \qquad i \geq 0 \tag{17}$$

so the first order condition can be written

$$0 = \nu d_i + \theta d_i H_i - \phi z_i \qquad i \geq 0. \tag{18}$$

Multiply (18) by Y_i and sum from $i = 0$ to I. Simplify using (14), (15), and (16) with equality. Also, use Euler's theorem. (Each d_j is homogeneous of degree 0 in $(Y_0, Y_1, \ldots, Y_I)$.) This yields

$$0 = \nu + \theta d_0 L - \phi(L - g).$$

Use this to substitute for ν in (18): $0 = \theta d_i(H_i - d_0 L) - \phi(z_i - d_i(L - g))$, or equivalently

$$\phi(z_i/d_i - L + g) = \theta(H_i - d_0 L) \qquad i \geq 0. \tag{19}$$

Take (19) at $i = 0$ and subtract from it (19) at $i \geq 1$: $\phi(z_0/d_0 - z_i/d_i) = \theta(H_0 - H_i)$. Multiply by d_0/ϕ; substitute for z_i, $i \geq 1$, from (13); and use $z_0 := 1$ to get[9] $1 - \rho_i/\tau_i = (\theta d_0/\phi)(H_0 - H_i)$. Finally use (19) at $i = 0$ to substitute for θ/ϕ:

$$1 - \rho_i/\tau_i = \frac{1 - d_0(L - g)}{H_0 - d_0 L} (H_0 - H_i) \qquad i \geq 1. \tag{20}$$

---

[9] In the expression that follows, ϕ must not equal zero. From the envelope theorem $\phi = -\partial U^*/\partial g$, so if ϕ were zero there would be no reduction in utility when the government requires a bit more revenue. In this case, the optimal taxation problem would be particularly uninteresting.

When competition is perfect, (20) reduces to the formula in Deaton (1979). The perfectly competitive case is captured with $\rho_i = 1$ for all $i \geq 0$ in which case (15) and (17) collapse to $1 = d_0 L$ and $H_i = 1 - d_{0i} L/d_i$ respectively. Then direct substitution into (20) gives

$$1 - 1/\tau_i = \frac{g}{L} \cdot \left(1 - \frac{d_{0i}/d_i}{d_{00}/d_0} \right).$$

Since $d_{00} < 0$ by concavity, τ_i is larger for larger values of d_{0i}/d_i — more on this below.

Return now to the general case (20). Since $[1 - d_0(L - g)]/(H_0 - d_0 L) = \theta d_0/\phi$ from (19) at $i = 0$, and since $\theta d_0/\phi > 0$, (20) shows that $\tau_i > \rho_i$ if and only if $H_0 - H_i > 0$. The results may be summarized as follows.

4.3.1 Proposition *When lump sum transfers are available the benchmark for optimal taxes is $\tau_i = \rho_i$. In the absence of such transfers, the optimal τ_i is given by (20). Hence τ_i exceeds the benchmark as $H_0 - H_i$ exceeds zero. From the definitions in (17) it follows that τ_i/ρ_i is larger for larger values of*

(a) $1 - \rho_i$

(b) $-d_{00} L/d_0 + d_{0i} L/d_i$

(c) $\sum_{j=0}^{I} \rho_j d_{0j} Y_j/d_0 - \sum_{j=0}^{I} \rho_j d_{ij} Y_j/d_i$

where (c) *uses symmetry $d_{ij} = d_{ji}$.*

Since $1 - \rho_i > 0$, condition (a) pushes up τ_i/ρ_i. But note, the higher the benchmark ρ_i, the smaller the upward adjustment in (a). This smooths out differences in tax rates and moderates the lump sum tax case, which should be expected since here taxes are necessarily distortionary. That is, the corrective subsidies that address the monopoly problems generate a revenue requirement and here this requirement must be met with distortionary taxes, so the subsidies should be exercised in moderation.

Condition (b) is the Deaton (1979) result for the perfectly competitive case. Embedded within this is the familiar Ramsey inverse elasticities tax rule. This is somewhat difficult to see here due to general equilibrium effects, however the second example in section 4.2 highlights this point. To interpret (b) in the general case, terms of the form $d_{ij} Y_j/d_i = \partial \log(d_i)/\partial \log(Y_j)$ are compensated inverse demand elasticities. Roughly, they show how the consumer's marginal utility from i is affected by a change in Y_j along an indifference surface. Thus $d_{ij} > 0$ indicates that i and j are complementary. So (b) suggests relatively higher tax rates for complements of leisure. That is, since the direct effect of taxes is to push up the prices of goods $i \geq 1$ relative to leisure, this distortion can be offset in part by taxing leisure indirectly through taxes on its complements.

Now consider (c) and focus on the second sum since the first is common for all i. Condition (b) addressed cross-price effects between i and leisure but what about other cross-price effects? In (b), the complements of leisure received higher tax rates. The feature of leisure that drives this is simply that it is supplied to

13

the market (as labor). More generally, the complements of any good in net supply should receive upward adjustments to their tax rates; and the complements of any good in net demand should receive downward adjustments (conversely for substitutes). Since goods $j \geq 1$ are in net demand, their complements should receive downward adjustments: $d_{ij} > 0 \Rightarrow \tau_i \downarrow$ for $j \geq 1$, and conversely for substitutes. The second sum in (c) quantifies this.[10] Under perfect competition these adjustments taken together cancel out the $j = 0$ term in (c). That is, since the perfectly competitive case is represented by $\rho_j = 1$ for all j, the sums in (c) are then zero due to homogeneity. But when imperfect competition is present and $\rho_j < 1$ for $j \geq 1$, this no longer applies. In this case, the adjustments transmitted from goods $j \geq 1$ to τ_i are reduced according to the degree of distortion in industry j. E.g., if j is highly distorted so that ρ_j is small and the benchmark is a considerable subsidy to j, the adjustment "$d_{ij} > 0 \Rightarrow \tau_i \downarrow$" is reduced, as shown in (c).

The following special case highlights the role of (c). Suppose $\rho_i = \rho < 1$ for all $i \geq 1$ (and $\rho_0 := 1$). I.e., the extent of the monopolistic distortion is the same in all industries. Then by degree zero homogeneity, (c) simplifies to $(1 - \rho)d_{00}\ell/d_0 - (1 - \rho)d_{0i}\ell/d_i$. This offsets part of (b). So the effect of imperfect competition is to counteract the Ramsey rule in (b). Furthermore, this effect is more pronounced when the monopolistic distortion is larger (when ρ is smaller).

Now we can put the pieces together. The optimal policy has four components. Two of these are "pure:" the corrective benchmark ρ_i, and Ramsey efficient taxation in (b) with the latter extending the inverse elasticities tax rule to general equilibrium. The overall policy is not merely the sum of these two. Rather, they interact with one another and as a result each of the pure components is weakened or moderated to some degree. The benchmark is weakened by (a); Ramsey, by (c). Nonetheless, the broad picture is one of two main influences. Corrective taxation tends to respond positively to the elasticity of the dd demand curve while efficient taxation tends to respond inversely to the elasticity of DD.

5   Optimal tax problem: prices as controls

The analysis thus far has used primal variables $(Y_0, Y_1, \ldots, Y_I)$ as controls. Recall that Y_0 is leisure and $Y_i := n_i u_i(q_i)$ for $i \geq 1$. An alternative approach is to use prices or the tax rates as controls. This approach will show the effect of the optimal tax system on the number of firms in each industry. There is a direct effect in which industries with a greater departure from perfect competition experience a smaller percentage decline in the number of firms. Thus the tax system spares those industries already hit hard by monopolistic competition. There is also an indirect effect that works through the impact of the tax system on an industry's complements and substitutes.

Let $T_0 := 1 - t_0$. Define ρ_i and z_i as in (12) and let $T_i := z_i(1 + t_i)/\rho_i$ for $i \geq 1$, from which the tax

---

[10] The $j = 0$ terms in (c) fit more appropriately within (b), which would then become $-d_{00}(L - \ell)/d_0 + d_{0i}(L - \ell)/d_i$, and would have the same interpretation as previously. These terms are presented in (c) to facilitate comparison with the perfectly competitive case.

rates can be recovered easily. The T variables correspond to consumer prices in the perfectly competitive case while the z variables correspond to producer prices. Then the equilibrium conditions (2), (3), and (4) can be written

$$U_i T_0 = U_0 T_i \qquad i \geq 1 \tag{21}$$

$$\sum_{i=0}^{I} \rho_i T_i Y_i = T_0 L \tag{22}$$

$$g + \sum_{i=0}^{I} z_i Y_i \leq L. \tag{23}$$

Consider an artificial economy in which the representative consumer chooses $(Y_0, Y_1, \ldots, Y_I) \geq \mathbf{0}$ to

$$\text{maximize} \quad U(Y_0, Y_1, \ldots, Y_I) \quad \text{subject to} \quad \sum_{i=0}^{I} T_i Y_i \leq M$$

where $(\mathbf{T}, M) \geq \mathbf{0}$ is taken parametrically. Recall that the consumer in the Dixit–Stiglitz economy actually chooses quantities of leisure and the output of each firm, not industry aggregates like Y_i for $i \geq 1$. If $\mathbf{T} \gg \mathbf{0}$ and $M > 0$, the problem has a unique solution characterized by (21) and

$$\sum_{i=0}^{I} T_i Y_i = M. \tag{24}$$

Let $\mathbf{Y}(\mathbf{T}, M) \gg \mathbf{0}$ be the maximizer and $V(\mathbf{T}, M)$ the indirect utility function. By construction, this maximizer satisfies the equilibrium condition (21) for the Dixit–Stiglitz economy. Conversely, any vector $(Y_0, Y_1, \ldots, Y_I)$ that satisfies (21) solves the artificial consumer's problem for some M.

If $M > 0$ is chosen appropriately, the solution for the artificial economy also satisfies (22). To see this, note that if $M = 0$, $\mathbf{Y}(\mathbf{T}, 0) = \mathbf{0}$ so the left hand side of (22) is less than the right hand side. Under the assumption of normal goods, for larger values of M the LHS increases and eventually exceeds the RHS. In particular, for each $\mathbf{T} \gg \mathbf{0}$ there is a unique $M > 0$, denoted $M(\mathbf{T})$, such that

$$\sum_{i=0}^{I} \rho_i T_i Y_i(\mathbf{T}, M(\mathbf{T})) \equiv T_0 L. \tag{25}$$

Observe that $M(\cdot)$ is homogeneous of degree one. Since $\rho_i < 1$ for $i \geq 1$, (24) and (25) imply that $M(\mathbf{T}) > T_0 L$ for all $\mathbf{T} \gg \mathbf{0}$.

Based on the above results, the optimal tax problem has an equivalent representation in which the government chooses $\mathbf{T} \gg \mathbf{0}$ to

$$\left. \begin{aligned} &\text{maximize} \quad V(\mathbf{T}, M(\mathbf{T})) \\ &\text{subject to} \quad g + \sum_{i=0}^{I} z_i Y_i(\mathbf{T}, M(\mathbf{T})) \leq L \end{aligned} \right\} \quad \text{Problem (P)}$$

where V is the indirect utility function defined above.

Notice the restriction $\mathbf{T} \gg \mathbf{0}$. This is required to ensure that the consumer's problem has a solution, i.e., to ensure that $\mathbf{Y}(\mathbf{T}, M)$ is defined. However, with this restriction the optimal tax problem might not have a solution since its constraint set is not compact. This issue is addressed in appendix C.

Problem (P) above has the form of an optimal tax problem for an economy with homogeneous goods (with quantities measured by Y_i and consumer prices by T_i) and feedback effects $M(\mathbf{T})$ from prices to the consumer's lump sum income. Such feedback effects are found when firms earn positive profits that are not all taxed away. Since $M(\mathbf{T}) > T_0 L$ the feedback effects here are indeed additions to the consumer's income, much like profits. Thus the optimal tax problem for the economy with free entry (zero profits) and heterogeneous goods has the same form as the optimal tax problem for an economy with positive profits and homogeneous goods. This latter economy must be imperfectly competitive since the resource constraint indicates constant returns technology yet profits are positive. Myles (1989) analyzes a problem of the form (P) and his approach motivates what follows.

The Lagrangian for problem (P) is

$$V(\mathbf{T}, M(\mathbf{T})) + \lambda \left(L - g - \sum_{j=0}^{I} z_j Y_j(\mathbf{T}, M(\mathbf{T})) \right).$$

The first order condition for T_i, $i \geq 1$, is

$$\frac{\partial V}{\partial T_i} + \frac{\partial V}{\partial M} \frac{\partial M}{\partial T_i} - \lambda \left(\sum_{j=0}^{I} z_j \left[\frac{\partial Y_j}{\partial T_i} + \frac{\partial Y_j}{\partial M} \frac{\partial M}{\partial T_i} \right] \right) = 0.$$

Use Roy's identity to replace $\partial V/\partial T_i$ with $-Y_i \partial V/\partial M$, and use the Slutsky equation to replace $\partial Y_j/\partial T_i$ with $\partial Y_j^c/\partial T_i - Y_i \partial Y_j/\partial M$ where the superscript c indicates the Hicksian compensated demand from the utility function U. Then, by symmetry of the Slutsky matrix, replace $\partial Y_j^c/\partial T_i$ with $\partial Y_i^c/\partial T_j$:

$$-Y_i \frac{\partial V}{\partial M} + \frac{\partial V}{\partial M} \frac{\partial M}{\partial T_i} - \lambda \left(\sum_{j=0}^{I} z_j \left[\frac{\partial Y_i^c}{\partial T_j} - Y_i \frac{\partial Y_j}{\partial M} + \frac{\partial Y_j}{\partial M} \frac{\partial M}{\partial T_i} \right] \right) = 0.$$

Collect terms:

$$-Y_i \left(\frac{\partial V}{\partial M} - \lambda \sum_{j=0}^{I} z_j \frac{\partial Y_j}{\partial M} \right) + \frac{\partial M}{\partial T_i} \left(\frac{\partial V}{\partial M} - \lambda \sum_{j=0}^{I} z_j \frac{\partial Y_j}{\partial M} \right) - \lambda \sum_{j=0}^{I} z_j \frac{\partial Y_i^c}{\partial T_j} = 0 \qquad i \geq 1.$$

Implicit differentiation of (25), and the same Slutsky steps as above, gives $\partial M/\partial T_i$:

$$\frac{\partial M}{\partial T_i} = Y_i - \frac{\rho_i Y_i + \sum_{j=0}^{I} \rho_j T_j \partial Y_i^c/\partial T_j}{\sum_{j=0}^{I} \rho_j T_j \partial Y_j/\partial M} \qquad i \geq 1$$

which, when substituted above yields

$$-\left(\rho_i Y_i + \sum_{j=0}^{I} \rho_j T_j \frac{\partial Y_i^c}{\partial T_j} \right) \frac{\partial V/\partial M - \lambda \sum_{j=0}^{I} z_j \partial Y_j/\partial M}{\sum_{j=0}^{I} \rho_j T_j \partial Y_j/\partial M} = \lambda \sum_{j=0}^{I} z_j \frac{\partial Y_i^c}{\partial T_j} \qquad i \geq 1.$$

16

Divide through by Y_i and define Θ as follows:[11]

$$\Theta := -\frac{\partial V/\partial M - \lambda \sum_{j=0}^{I} z_j \partial Y_j/\partial M}{\sum_{j=0}^{I} \rho_j T_j \partial Y_j/\partial M}$$

to get

$$\sum_{j=0}^{I} z_j \frac{\partial \log Y_i^c}{\partial T_j} = \frac{\Theta}{\lambda} \left(\rho_i + \sum_{j=0}^{I} \rho_j T_j \frac{\partial \log Y_i^c}{\partial T_j} \right) \qquad i \geq 1. \tag{26}$$

This is a Ramsey rule for consumption quantities, modified for the economy with monopolistic competition. Note that if competition were perfect so that $\rho_j = 1$ for all $j \geq 0$, then by homogeneity (26) would reduce to

$$\sum_{j=0}^{I} z_j \frac{\partial \log Y_i^c}{\partial T_j} = \frac{\Theta}{\lambda} \qquad i \geq 1$$

which is the Ramsey quantity rule under perfect competition (e.g., Samuelson 1986): for each commodity, the optimal tax system reduces compensated demand by an equal percentage, to a first order approximation.[12]

With monopolistic competition, the Ramsey quantity rule is modified by the term in parentheses in (26). Since $Y_i := n_i u_i(q_i)$ for $i \geq 1$, and since q_i is unaffected by the tax system, the percentage change in Y_i^c equals the percentage change in n_i^c. Thus (26) tells us the compensated effect of the optimal tax system on the number of firms in industry i. In the parentheses, ρ_i gives a direct effect while the sum gives an indirect effect via i's Hicksian substitutes ($\partial \log Y_i^c/\partial T_j > 0$) and complements ($\partial \log Y_i^c/\partial T_j < 0$).

5.1 Proposition *Subject to approximation, the optimal tax system has the following compensated effect on the number of firms n_i that enter industry i, $i \geq 1$:*

(a) *a direct effect whereby industries with a higher degree of monopolistic competition (smaller ρ_i) have a smaller percentage reduction in the number of firms; and*

(b) *an indirect effect whereby the percentage reduction in n_i^c is greater if i's Hicksian substitutes have a relatively small degree of monopolistic competition (large ρ_j) and if i's Hicksian complements have a relatively large degree of monopolistic competition (small ρ_j).*

---

[11] Θ is the increase in utility that would arise from a marginal shift toward lump sum taxation. More specifically, let T_{LS} denote an exogenous lump sum tax. This affects the equilibrium conditions only through the consumer's budget equation (3), which has become (25) in this artificial economy. So the right hand side of (25) becomes $T_0 L - T_{LS}$. This affects the definition of $M(\cdot)$ and, through implicit differentiation, gives $\partial M/\partial T_{LS}$. Let V^* denote the maximum value function for problem (P). Then by the envelope theorem $\Theta = \partial V^*/\partial T_{LS}$ which we expect to be positive, and $\lambda = -\partial V^*/\partial g \geq 0$ which is strictly positive in all cases of interest.

[12] In the perfectly competitive case the z_j terms that appear in (26) are equivalent to producer prices. Due to homogeneity, the left hand side of (26) equals $-\sum_{j=0}^{I}(T_j - z_j)\partial \log Y_i^c/\partial T_j$, where $T_j - z_j$ is the amount of the tax in sector j. (Since $T_0 := 1 - t_0$ and $z_0 := 1$, $T_0 - z_0$ is also the amount of the tax on leisure in sector 0.) Hence the jth term in the sum is the approximate percentage change in Y_i^c caused by the jth tax. The whole sum then approximates the total percentage change in Y_i^c caused by the entire tax system, holding utility constant.

When monopolistic competition is present, the artificial consumer price is $T_j := z_j(1 + t_j)/\rho_j$ for $j \geq 1$, and the change in this price caused by taxation is $T_j - z_j/\rho_j$. So, again by homogeneity, the approximate total percentage change in Y_i^c is $-\sum_{j=0}^{I}(z_j/\rho_j)\partial \log Y_i^c/\partial T_j$, which differs in magnitude from the left hand side of (26) by the presence of ρ_j. Nonetheless, the left hand side of (26) still gives an approximation for the total percentage change in Y_i^c when the effect of imperfect competition is not too severe, i.e., when the terms ρ_j are not significantly less than one.

Part (a) of the proposition tells us the optimal tax system avoids overkill: if an industry already bears a heavy burden from imperfect competition, then the tax system responds with a relatively lighter burden. Part (b) is a partial offset to this: if industry j has a small ρ_j and hence is treated lightly under (a), then its complements get an increased burden under (b). The interpretation of this proposition is thus similar to proposition 4.3.1 above. Taxes should be used to undo some of the damage caused by monopolistic distortions, but with moderation due to general equilibrium effects.

6 Conclusion

This paper addresses optimal taxation in a monopolistically competitive economy. In an ideal world the government would have a rich enough set of policy instruments to fully correct market imperfections and to raise sufficient revenue. In a more realistic setting, the available instruments must serve a double duty. Here, commodity taxes were used both (i) as a corrective instrument against the adverse effects of producer markups and also (ii) as a (distortionary) source of government revenue. For objective (i), if the firms in an industry face relatively less elastic demand and hence choose relatively large markups, the optimal policy should try to offset this with a lower tax rate — low elasticity, low tax rate.[13] This works in the opposite direction to the familiar inverse elasticities tax rule that addresses objective (ii). Thus under imperfect competition there are two opposing influences on the optimal tax rates.

If we abstract from distributional concerns and consider a representative consumer economy, the optimal tax formula is given in section 4.3.[14] The formula does not simply add the solutions to (i) and (ii) above. Interactions arise. In large part, these interactions are due to the fact that for (i) it is Chamberlin's dd demand curve that influences the producer markups while for (ii) it is the DD demand curve that influences the Ramsey rule. The elasticities of both these demand curves must be taken into account when designing an optimal tax scheme. More generally, misleading policy prescriptions are likely to arise if imperfect competition is ignored.

The tax system affects the number of firms in a free entry equilibrium. When taxes are set optimally, the compensated effect on entry has both a direct effect and an indirect effect. For the direct effect, those industries hit hardest by monopolistic distortions should be hit softest by tax distortions — they should experience the smallest percentage reduction in entry. The indirect effect takes into account general equilibrium linkages and moderates the impact of the direct effect.

---

[13] This reduction in tax rates is not a reward for monopolists. Rather, it is "compensation" for consumers.

[14] The analysis may be extended to the case of heterogeneous consumers. The optimal tax formula becomes rather unwieldy but it continues to be influenced by many of the same factors as in the representative consumer case.

Appendix A Technical assumptions

The following five assumptions provide a set of conditions under which the consumer's and firms' problems have solutions, and equilibrium exists. Furthermore, under the restriction that identical firms be treated identically, equilibrium is unique in the sense that each choice of tax rates $(t_0, t_1, \ldots, t_I)$ can generate at most one equilibrium.

Assumption 1(a) is standard and it implies that U is locally non-satiated. Assumption 1(b) states that any indifference surface of U that has a non-empty intersection with the interior of the non-negative orthant is in fact contained entirely within the interior of the non-negative orthant. This rules out corner solutions. Assumption 1(c) will be used for comparative statics analysis. The assumption of normal goods (positive income elasticity of demand) is not unreasonable at this level of aggregation (industry aggregates and a representative consumer).

The second assumption places smoothness and concavity restrictions on the functions u_i. For each firm, the third assumption ensures that the absolute value of the *dd* elasticity exceeds one and is non-increasing in quantity. This plays a role in making the firms' profit functions bounded and concave. Assumptions 2 and 3 are satisfied, for example, by the constant elasticity functions $u_i(q) = A_i q^{\rho_i}$ for $A_i > 0 < \rho_i < 1$.

The fourth assumption restricts the cost functions, again with the aim of ensuring that the profit functions are well-behaved and that the equilibrium is unique. These restrictions will be satisfied, for example, if the cost functions are affine (fixed cost followed by constant marginal cost).

The last assumption is that the consumer's offer curve has a non-empty intersection with the resource constraint, i.e., equilibrium exists. Essentially, this requires that government purchases g not be too large a burden on available resources.

Assumption 1:

(a) U is defined on the non-negative orthant where it is a continuous function that does not attain a maximum. On the interior of the non-negative orthant, U is strictly quasi-concave and twice continuously differentiable.

(b) Suppose $(\ell, Y_1, \ldots, Y_I) \gg \mathbf{0}$ and $0 \in \{\hat{\ell}, \hat{Y}_1, \ldots, \hat{Y}_I\}$. Then $U(\ell, Y_1, \ldots, Y_I) > U(\hat{\ell}, \hat{Y}_1, \ldots, \hat{Y}_I)$.

(c) The gradient of U never vanishes. The determinant of the bordered Hessian matrix of U never vanishes. All of the goods $\ell, Y_1, \ldots, Y_I$ are normal under U.

Assumption 2: For all $i \geq 1$, u_i is defined and continuous on $\mathbb{R}_+$ and is three times continuously differentiable on $\mathbb{R}_{++}$; u_i is strictly increasing and strictly concave; $u'_i \uparrow \infty$ as $q_i \downarrow 0$ and $u'_i \downarrow 0$ as $q_i \uparrow \infty$;

$u_i(0) = 0.$[15]

Assumption 3: For all $i \geq 1$,

(a) $-1 < qu_i''(q)/u_i'(q) < 0 \quad \forall q > 0$,

(b) $q \mapsto qu_i''(q)/u_i'(q)$ is continuous at $q = 0$,

(c) $d[-qu_i''(q)/u_i'(q)]/dq \geq 0 \quad \forall q > 0$.

Assumption 4: For all $i \geq 1$, C_i maps $\mathbb{R}_+$ to $\mathbb{R}_+$. On $\mathbb{R}_{++}$ each C_i is twice continuously differentiable, strictly increasing, and convex. $C_i(0) = 0$, $C_i(0^+) > 0$, and $d[qC_i'(q)/C_i(q)]/dq > 0$ for all $q > 0$ — the ratio of marginal costs to average costs is increasing in output.

Assumption 5: There exists $(\ell, n_1, \ldots, n_I) \gg \mathbf{0}$ that simultaneously solves (4) and (5).

Appendix B   Existence of an optimum for section 4

The proof simply amounts to showing that the problem can be written as the maximization of a continuous function on a non-empty compact set.

Define

$$S := \{(\ell, n_1, \ldots, n_I) \gg \mathbf{0} : (5) \text{ is satisfied}\}.$$

By continuity, S is closed relative to the interior of the non-negative orthant. That is, it is the intersection of the interior of the non-negative orthant with some closed set $K \in \mathbb{R}^{I+1}$. Therefore the closure of S in $\mathbb{R}^{I+1}$ is the union of S and (perhaps) some points on the boundary of the non-negative orthant. Assumptions 1(b) and 5 make it impossible for any of these boundary points to solve the optimization problem. Thus the problem can be restated: Choose $(\ell, n_1, \ldots, n_I) \geq \mathbf{0}$ to

$$\text{maximize} \quad U(\ell, n_1 u_1, \ldots, n_I u_I)$$
$$\text{subject to} \quad (4) \text{ and } (\ell, n_1, \ldots, n_I) \in \text{closure}(S).$$

It follows immediately that a maximum exists. Furthermore, any maximum must be in the interior of the non-negative orthant under assumptions 1(b) and 5.

Appendix C   Existence of an optimal T for section 5

Truncate the consumption set $\{(Y_0, Y_1, \ldots, Y_I) \geq \mathbf{0}\}$ by intersecting it with a compact convex set K that contains the feasible set (defined by (23)) in its relative interior. Let $\mathbf{Y}^K(\mathbf{T}, M)$ be the solution to the

---

[15] The condition $u_i(0) = 0$ is not merely a normalization. If it is violated, utility is affected by the *existence* of a variety even if it is not consumed. This paper will not address tax policy in the presence of existence values.

artificial consumer's problem after truncation. This is defined for all $(\mathbf{T}, M) \geq \mathbf{0}$. When $M = 0$ there may be multiple solutions. If so, pick any one of them as $\mathbf{Y}^K(\mathbf{T}, 0)$.

The function $\mathbf{Y}^K(\mathbf{T}, M)$ is not necessarily monotonic in M when it maps into the boundary of K. Therefore, the procedure used in the text to define $M(\mathbf{T})$ will not work here: there may be some $\mathbf{T}$ for which no M will solve (25), and there may be other $\mathbf{T}$ for which there are multiple solutions. Thus, an alternative procedure is required.

Let $\mathbf{Y}^{K*}$ be the unique maximizer of U on the truncated consumption set (with no budget constraint). The consumer in the artificial economy will choose $\mathbf{Y}^K(\mathbf{T}, M) = \mathbf{Y}^{K*}$ whenever $M \geq \sum_{i=0}^{I} T_i Y_i^{K*}$. Since $\mathbf{Y}^{K*}$ fails to satisfy (23), such large values of M are not feasible. So if $\mathbf{T}$ is in the unit simplex Δ, there is no loss of generality in restricting $M \leq \sum_{i=0}^{I} Y_i^{K*}$. Let

$$
\begin{aligned}
X &:= \Delta \times \left[0, \sum_{i=0}^{I} Y_i^{K*}\right] \\
X' &:= \Delta \times \left(0, \sum_{i=0}^{I} Y_i^{K*}\right]
\end{aligned}
$$

so that M can equal 0 in X but not in X'. Now let

$$
S := \{(\mathbf{T}, M) \in X : \sum_{i=0}^{I} \rho_i T_i Y_i^K(\mathbf{T}, M) = T_0 L\},
$$

cf (25). By continuity, S is compact. (For each i, $0 \leq T_i Y_i^K(\mathbf{T}, M) \leq M$ implies continuity at $M = 0$.) The resource constraint (23) must be treated with some care because of the possible discontinuity in $\mathbf{Y}^K(\mathbf{T}, M)$ when $M = 0$. Let

$$
R := \{(\mathbf{T}, M) \in X' : g + \sum_{i=0}^{I} z_i Y_i^K(\mathbf{T}, M) \leq L\}.
$$

This set is closed relative to X'. The closure of R in $\mathbb{R}_+^{I+2}$ consists of R together with (perhaps) some points with $M = 0$. But no solution to the government's optimization problem can possibly have $M = 0$ since $\mathbf{Y}^K(\mathbf{T}, 0)$ must be a boundary point of the non-negative orthant for any $\mathbf{T}$ in the simplex.

The problem can now be stated as

$$
\begin{aligned}
\text{maximize} \quad & V^K(\mathbf{T}, M) \\
\text{subject to} \quad & (\mathbf{T}, M) \in S \cap \text{closure}(R)
\end{aligned}
$$

which must have a solution $(\mathbf{T}^*, M^*)$, even if V^K is only upper semi-continuous when $M = 0$. Since this solution satisfies the feasibility constraint (23), and since by choice of K any feasible allocation is interior to K, it follows that $(\mathbf{T}^*, M^*)$ also solves the original problem (without truncation to K).

References

[1] Auerbach, Alan J. and James R. Hines, Jr. (2002). "Taxation and economic efficiency," chapter 21 in Alan J. Auerbach and Martin Feldstein, editors, *Handbook of Public Economics*, volume 3, Amsterdam: Elsevier.

[2] Besley, Timothy and Ian Jewitt (1995). "Uniform taxation and consumer preferences," *Journal of Public Economics* 58, 73–84.

[3] Deaton, Angus (1979). "The distance function in consumer behaviour with applications to index numbers and optimal taxation," *Review of Economic Studies* 46, 391–405.

[4] Deaton, Angus and John Muellbauer (1980). *Economics and Consumer Behavior*, Cambridge: Cambridge University Press.

[5] Dixit, Avinash K. and Joseph E. Stiglitz (1977). "Monopolistic competition and optimum product diversity," *American Economic Review* 67, 297–308.

[6] Myles, Gareth D. (1989). "Ramsey tax rules for economies with imperfect competition," *Journal of Public Economics* 38, 95–115.

[7] Myles, Gareth D. (1995). *Public Economics*, Cambridge: Cambridge University Press.

[8] Samuelson, Paul A. (1986). "Theory of optimal taxation," *Journal of Public Economics* 30, 137–143.

[9] Spence, A. Michael (1976). "Product selection, fixed costs, and monopolistic competition," *Review of Economic Studies* 43, 217–235.