Finite-size effects for jet quenching

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ABSTRACT: We study corrections to the drag force exerted on a quark moving through a quark-gluon plasma of finite extent, using holographic methods. Interestingly we find that the leading correction is negative, i.e. it reduces the magnitude of the drag force as compared to its value in infinite volume.

Contents

1 Introduction

During the last few years, much work has been devoted to studying strongly coupled processes in the quark-gluon plasma by making use of the gauge/gravity duality. In particular, a lot of attention has focused on the phenomenon of jet quenching: quark-antiquark pairs which are produced near the boundary of the quark-gluon plasma do not lead to two back-to-back jets, but rather give rise to only one observed jet. A simple qualitative explanation of this phenomenon lies in the fact that the quark which needs to move through the plasma before it can escape loses a lot of its initial energy due to interaction with the medium. While this is a qualitatively and intuitively simple explanation, it is hard to obtain a quantitatively correct answer from it. Starting with the work of [1, 2], the ultra-relativistic quenched quark produced inside the strongly coupled quark-gluon plasma was modelled holographically with an open string ending on the boundary of AdS space, hanging deep in the interior of an AdS black hole. This initial work, as well as subsequent generalisations to other dimensions, to presence of chemical potentials or higher derivative corrections, all focused on the study of quark motion in an infinite-volume medium.

A realistic quark-antiquark pair produced in colliders, however, propagates in a plasma of finite extent. The question then arises whether this has any influence on the jet quenching parameter as computed in an infinite volume system. In this paper we initiate the study of finite-size corrections to the quark motion using the gauge/gravity correspondence.

To achieve this we will study, in a holographically dual picture, a string which is moving in the background of a black hole in global AdS space, as opposed to the planar AdS black holes in the Poincaré patch which were studied so far. Black holes in the Poincaré patch have horizons which are non-compact and planar, and as such they are dual to a quark-gluon plasma of infinite extent. In contrast, black holes in

global AdS space have spherical horizons and are dual to a quark-gluon plasma on a three-sphere.

The only scales in the finite-temperature super-Yang-Mills theory on a sphere are the temperature T and the radius of the three sphere L, so that the only dimensionless quantity one can form is the product TL. Therefore, all results, including the dimensionless drag force which we will obtain, will be functions of this quantity. Although global AdS and planar black holes are genuinely different gravitational configurations, there is a limit in which a global black hole reduces to the planar one. In the limit of a large global AdS black hole, when the size of the horizon is much larger than the AdS scale, $\rho_H \gg L$, the global AdS black hole reduces to the planar one. This limit translates to the condition $TL \gg 1$, which can be interpreted either as the large-volume limit at fixed temperature, or as the high-temperature limit at fixed sphere size. Hence, in the large black hole limit $\rho_H \gg L$ we expect to recover the result of the infinite volume system (the planar black hole), amended by a tower of finite-size corrections.

Indeed this is what we will find in this note. Interestingly, the leading correction which we find is negative, i.e. it reduces the magnitude of the drag force. This is in contrast to what one usually encounters in situations with large objects moving in a non-relativistic fashion through fluids in finite-size containers. In these systems one usually finds that the drag force is increased by the presence of walls (see e.g. [3, 4]). The basic intuitive reason for this behaviour is that the walls impose an additional friction force on the fluid, creating a contra-flow which further obstructs propagation of the object.

In our setup the situation is a bit different, since the quark-gluon plasma is placed on a sphere, i.e. in a container without boundaries. However, in our case finiteness of space is imposed through the periodicity condition which any excitation in the fluid has to satisfy. Therefore, not any arbitrary excitation which is present in infinite volume can be excited in this system. This is a possible reason why the force is reduced with respect to the one in infinite volume. It would be very interesting to test finite-size corrections for the quark-gluon plasma in other systems, hopefully those which do have boundaries, and use holography to check which of the features observed here persist. One possible system which one could consider is the numerical solution of [5] which describes a classically stable finite energy black hole localised in the infrared part of the geometry.

Note added: When this paper was completed we became aware of the work [6] which has some overlap with our results in section 2.

2   Dragged string in a global AdS black hole

The system we will study is the Schwarzschild black hole in *global* AdS$_5$ space, in contrast to the Poincaré patch studied before [1, 2]. The metric of the global AdS$_5$

black hole is given by

$$ds^2 = -h(\rho)d\tau^2 + \frac{1}{h(\rho)}d\rho^2 + \rho^2\big(d\theta^2 + \cos^2\theta(d\phi^2 + \cos^2\phi d\chi^2)\big),$$

$$\rho_0^2 = \frac{8GM}{3\pi}, \qquad h(\rho) = 1 - \frac{\rho_0^2}{\rho^2} + \frac{\rho^2}{L^2}.$$

(2.1)

Here G is the Newton constant, M is the mass of the black hole and L is the radius of the AdS space. The boundary of the AdS space is at $\rho \to \infty$, while in the interior there is a black hole horizon at position ρ_H,

$$\rho_H = L\sqrt{\frac{\sqrt{1 + 4\rho_0^2/L^2} - 1}{2}}, \qquad \rho_0 = \rho_H\sqrt{1 + \frac{\rho_H^2}{L^2}}.$$

(2.2)

The temperature of the black hole is given by [7]

$$T = \frac{\rho_H}{\pi L^2}\left(1 + \frac{L^2}{2\rho_H^2}\right).$$

(2.3)

As the temperature is decreased, on encounters a first-order phase transition at the critical temperature $T_{\mathrm{HP}} = 3/(2\pi L)$ below which pure AdS$_5$ space (with periodic time circle) is preferred over AdS$_5$-Schwarzschild [8, 9]. At this temperature the boundary theory undergoes a "deconfinement/confinement" phase transition, in the sense that the free energy of the system is of order N_c^2 above the temperature T_{HP} and of the order one below it. We also note that the only scales in the super-Yang-Mills theory are the AdS radius L (which is the same as the size of the boundary sphere) and the temperature T, so that the only dimensionless and physically relevant parameter is TL. Hence the limit of large TL can equivalently be interpreted either as the high-temperature limit at fixed volume or the large-volume limit at fixed temperature. We will be working at fixed temperature and interpret $TL \to \infty$ as the limit of large volume.

The AdS$_5$ Schwarzschild black hole should be compared with the planar AdS black hole,

$$ds^2 = \frac{r^2}{L^2}\left(-\left(1 - \frac{r_H^4}{r^4}\right)dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2}\frac{dr^2}{1 - \frac{r_H^4}{r^4}}.$$

(2.4)

the boundary of which is $\mathbb{R}^3 \times S^1$ with S^1 being the time circle. We will later use the fact that the global AdS$_5$ Schwarzschild black hole reduces to the planar one in the limit of a large black hole.

In order to describe the dragged string in the global AdS black hole, we use world-sheet coordinates aligned with τ and ρ, and an embedding given by

$$\theta = \omega\tau + f(\rho).$$

(2.5)

With this ansatz the action for the string becomes

$$S = \int \sqrt{1 + \rho^2 f'^2 h(\rho) - \frac{\rho^2 \omega^2}{h(\rho)}} \, . \tag{2.6}$$

Next, we introduce the "conserved momentum" $\pi_f = \partial S / \partial f'$, conjugate to f, in terms of which f' is

$$f'(\rho) = \frac{\pi_f}{\rho \, h(\rho)} \sqrt{\frac{\rho^2 \omega^2 - h}{\pi_f^2 - \rho^2 h}} \, . \tag{2.7}$$

Substituting this back into the action we get

$$S = \rho \sqrt{\frac{\rho^2 \omega^2 - h}{\pi_f^2 - \rho^2 h}} \, . \tag{2.8}$$

As in the planar case, we fix π_f from the requirement that f' and the action are real functions. In other words, we require that when the numerator inside the square root of (2.7) and (2.8) changes sign, the denominator changes sign as well. The position $\rho = \rho_*$ at which this happens is determined by

$$\rho_*^2 \omega^2 - h(\rho_*) = 0 \, , \qquad \pi_f^2 = \rho_*^2 h(\rho_*) = \rho_*^4 \omega^2 \, . \tag{2.9}$$

This can be solved to obtain

$$\pi_f = \omega L^2 \frac{-1 \pm \sqrt{1 + 4\frac{\rho_0^2}{L^2}(1 - \omega^2 L^2)}}{2(1 - \omega^2 L^2)} \, . \tag{2.10}$$

We note that we should take the plus sign in this expression, since in the pure AdS background (i.e. $\rho_0 = 0$) there is no dissipation and the drag force should be zero.

Obtaining the shape of the dragged string in the global AdS space is more complicated than in the planar case, since it requires integration of (2.7) using the expression (2.10). This can be done numerically, but we do not need this information in order to extract the drag force itself.

In order to compute the loss of the energy of the dragged string, we need to evaluate the flow of the momentum down the string, towards the horizon. Following [1] we thus need to compute

$$\frac{\mathrm{d}p_\theta}{\mathrm{d}t} = \sqrt{-g} P^\rho{}_\theta \, , \quad P^\alpha{}_\mu \equiv -\frac{1}{2\pi\alpha'} G_{\mu\nu} g^{\alpha\beta} \partial_\beta X^\alpha \, , \tag{2.11}$$

where P^α_μ is the conserved world-sheet current of the space-time energy momentum, and $g_{\alpha\beta}$ and $G_{\mu\nu}$ are the induced metric on the world-sheet and the target space metric respectively. We find

$$\frac{\mathrm{d}p_\theta}{\mathrm{d}t} = -\frac{1}{2\pi\alpha'} \pi_f \, , \tag{2.12}$$

with π_f given by (2.10).

3 Expansion near the planar AdS black hole

In order to make contact with results for the drag force obtained in the planar case, we now wish to expand the result (2.12) given (2.10). In the case of non-relativistic fluids with small Reynolds number, the drag force acting on a moving object can typically be written as [3, 4] $\vec{F} = f(\eta, d)\,\vec{v}$ where the function $f(\eta, d)$ depends on the viscous properties of the fluid (through the viscosity constant η) and the size of the object d. This expression is valid only if the size of the container in which the particle moves is infinite. In the case of a container with finite size D, one expects that f will also depend on the dimensionless ratio d/D. Typically the function f is then hard to compute and it is extracted only experimentally. We now proceed to analyse these type of corrections to the drag force for our system.

Before continuing, let us make a comparison of (2.10) with the analogue quantity in the planar case. If the string is moving along the x-direction in the planar background (2.4), with embedding given by $x = vt + \xi(r)$, then the momentum conjugate to ξ is [1]

$$\pi_\xi = \frac{r_H^2}{L^2} \frac{v}{\sqrt{1 - v^2}}\,. \tag{3.1}$$

The drag force for this planar case is

$$\frac{\mathrm{d}p_x}{\mathrm{d}t} = -\frac{1}{2\pi\alpha'}\pi_\xi = -\frac{r_H^2/L^2}{2\pi\alpha'} \frac{v}{\sqrt{1 - v^2}} = -\frac{\pi\sqrt{\lambda}}{2}\tilde{T}^2 \frac{v}{\sqrt{1 - v^2}}\,,$$

$$\text{with} \quad L^4 = \lambda\alpha'^2\,, \qquad \tilde{T} = \frac{r_H}{\pi L^2}\,, \tag{3.2}$$

and the 't Hooft coupling $\lambda = g_{\mathrm{YM}}^2 N$.

In order to compare this to the global AdS black hole result, we take the limit of a large black hole

$$\rho_H \gg L\,. \tag{3.3}$$

In this limit the S^3 at the boundary becomes a plane, the coordinate $L\theta$ becomes x and hence the momentum π_f/L becomes π_ξ. The velocities are related by $\omega L \to v$. Keeping v fixed we can expand (2.10) in powers of e.g. L/ρ_0, to get

$$\frac{\pi_f}{L} = \frac{-\omega L}{2(1 - \omega^2 L^2)} + \frac{\omega L}{\sqrt{1 - \omega^2 L^2}} \frac{\rho_0}{L} \left(1 + \frac{1}{8}\frac{L^2}{\rho_0^2}\frac{1}{1 - \omega^2 L^2} + \cdots\right)\,. \tag{3.4}$$

In order to express the drag force in terms of gauge theory quantities, we then need the temperature expressed in terms of the parameter ρ_0 appearing in the metric, using (2.3). It is also convenient to introduce the dimensionless force $\mathcal{F} \equiv FL^2$, for which we finally find

$$\mathcal{F} = -\frac{v\sqrt{\lambda}}{2\pi\sqrt{1 - v^2}} \left[\pi^2(TL)^2 - \frac{1 + \sqrt{1 - v^2}}{2\sqrt{1 - v^2}} + \frac{1}{8\pi^2}\frac{3v^2 - 2}{(1 - v^2)(TL)^2} + \cdots\right]\,. \tag{3.5}$$

The leading term agrees with the flat result (3.2) and there is a whole tower of corrections on top. We discuss these results in the next section.

4 Discussion

There are several comments which we want to make about the result (3.5). We see that all corrections to the leading, infinite-volume results are given in terms of the dimensionless quantity TL. As we have explained in the introduction, in conformal field theory on a sphere and at finite temperature, this is the only dimensionless quantity available. Considering now that the estimate for the mean free path is given by [10] $l_{\mathrm{mfp}} \sim 1/\lambda^2 T$, one can interpret the dimensionless ratio TL as the ratio of the size of the plasma container L and an "effective size" of the quark, set by the mean free path $\sim 1/T$. Hence the form of the corrections is as expected on general grounds. However, while the fact that the leading-order force [1] is proportional to T^2 is fixed by dimensional analysis, the functional dependence on the velocity in the various corrections is not.

We also note that the sign of the first correction to the infinite volume result of [1] is always opposite from the leading term, i.e. the magnitude of the drag force seems to be decreased due to the finite-volume effects. This is a quite unusual behaviour as compared to most non-relativistic Newtonian fluids.

It is conceivable that an explanation of this interesting feature lies in the fact that our system is placed in a container without boundary, namely the three-sphere. This basically requires that all hydrodynamical modes with which the quarks can interact, and which it can dump energy into, are periodic. Therefore, fewer available hydrodynamical modes could lead to a smaller drag force, somewhat similar to the discussion in [11]. It would be very interesting to check in other duals of plasmas in finite size systems if such a behaviour persists.

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