



Testing the new CP phase in a supersymmetric model with Q_6 family symmetry by B_s mixing

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ABSTRACT

The new contribution to the non-diagonal matrix element M_{12} of the neutral B_s meson system is investigated in a supersymmetric extension of the standard model based on the discrete Q_6 family symmetry. We assume that CP is explicitly, but softly broken only by the b terms in the soft supersymmetry breaking sector. We find that the new contributions to M_{12} are real, and that nevertheless there exists an observable difference in the CP phase compared with the standard model. We focus our attention on the flavor-specific CP asymmetry a_{fs}^s , and find that a_{fs}^s of the model is mostly negative and its size can be one order of magnitude larger the standard model value. This prediction is consistent with the current experimental value, and might be experimentally tested in future.

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1. Introduction

Recent experimental data from TeVatron, see e.g. [1], give some hints for possible deviations from the Standard Model (SM) in the B_s mixing system. In the standard model the mixing of the neutral mesons is described by the famous box-diagrams. The dispersive part of these diagrams is denoted by M_{12} , it is due to heavy internal particles and therefore sensitive to possible new physics contributions. The absorptive part of the box-diagrams – denoted by Γ_{12} – is due to light internal particles and it cannot be affected by large new physics contributions, see e.g. [2] for more details.

The phases of M_{12} and Γ_{12} alone are unphysical, but the phase difference can be measured. We use the definition $\phi_s = \arg(-M_{12}/\Gamma_{12})$. $|M_{12}|$, $|\Gamma_{12}|$ and ϕ_s can be related to the following observables: the mass difference $\Delta M_s = 2|M_{12}|$ was measured at CDF [3] and DØ [4]. HFAG [5] combines the numbers to $\Delta M_s = 17.78 \pm 0.12 \text{ ps}^{-1}$. From the angular analysis in the decay $B_s \rightarrow J/\psi\phi$ one can extract the decay rate difference $\Delta\Gamma_s = 2|\Gamma_{12}|\cos(\phi_s)$ and the mixing phase $\beta_s = -\arg(-V_{ts}^*V_{tb}/V_{cs}^*V_{cb})$, cf. [6]. The standard model predicts very small numerical values for the mixing phases, $\beta_s \approx (2.2 \pm 0.6)^\circ$ and $\phi_s \approx (0.24 \pm 0.04)^\circ$ [2]. If new physics contributes to M_{12} , then ϕ_s and $-2\beta_s$ are shifted by the same value, which we denote by ϕ_s^Δ , see the note added in [7] for more details. Currently both CDF [8] and DØ [9] did tagged analyses of the decay $B_s \rightarrow J/\psi\phi$ and they obtain values for the mixing phase which differ about 2.2σ [5] from the SM. Similar deviations are obtained by CKMfitter [10], while UTfit [11] sees a slightly bigger effect.

Finally we can relate the box diagrams to flavor-specific CP asymmetries, which are also called semileptonic CP-asymmetries: $a_{fs} = \text{Im}(\Gamma_{12}/M_{12}) = (\Delta\Gamma/\Delta M)\tan\phi_s$. These asymmetries can be extracted directly from experiment [12] or they can be derived from the di-muon asymmetry [13]. The standard model expectation for the semileptonic CP asymmetry in the B_s system is again very small, $a_{fs}^s \approx (2.06 \pm 0.57) \cdot 10^{-5}$ [2]. Currently the experimental uncertainties in a_{fs}^s are still much larger than the standard model value. If the particular strong suppression pattern of the standard model for ϕ_s and a_{fs}^s is not present in a new physics extension, then these quantities might be enhanced considerably (up to a factor of 250, see [2]). In order to distinguish new physics effects from hadronic uncertainties, precise standard model predictions are mandatory. We take the numerical values for the standard model expectations from [2], which uses results of [14–18].

In this Letter we consider a supersymmetric extension of the SM based on the discrete Q_6 family symmetry [19,20], and investigate the extra contribution to M_{12} . In [21] we have stressed a minimal content of the Higgs multiplets, i.e. no extra Higgs multiplet that is

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Table 1

The Q_6 assignment of the chiral matter supermultiplets. The group theory notation is given in Ref. [19]. For completeness we show the Q_6 assignment of the leptons, too.

	Q	Q_3	U^c, D^c	U_3^c, D_3^c	L	L_3	E^c, N^c	E_3^c	N_3^c	H^u, H^d	H_3^u, H_3^d
Q_6	2_1	$1_{+,2}$	2_2	$1_{-,1}$	2_2	$1_{+,0}$	2_2	$1_{+,0}$	$1_{-,3}$	2_2	$1_{-,1}$

Table 2

The symmetry of the different sectors. \mathbf{Y} , \mathbf{h} and \mathbf{m} stand for the Yukawa, tri-linear and soft scalar mass sector, respectively. O_2 in the soft scalar mass sector is accidental. Z_2 is a subgroup of O_2 . CP is explicitly, but softly broken only by the b terms. All the symmetries are compatible with each other, and consequently, the model is renormalizable.

	\mathbf{Y}, \mathbf{h}	\mathbf{m}	μ sector	b terms
Q_6	\bigcirc	\bigcirc	\times	\times
O_2	\times	\bigcirc	\bigcirc	\times
Z_2	\times	\bigcirc	\bigcirc	\bigcirc
CP	\bigcirc	\bigcirc	\bigcirc	\times
R	\bigcirc	\bigcirc	\bigcirc	\bigcirc

$SU(2)_L \times U(1)_Y$ singlet. We have then found that it is possible, without contradicting renormalizability, to have the one + two structure for each family. By the one + two structure for a family we mean a family (including the Higgs sector) with three family members; one member in the Q_6 singlet representation and the other two in the Q_6 doublet representation. As in [21] we assume that CP is explicitly, but softly broken in the soft supersymmetry breaking sector by the b terms only, which consist of dimension-two operators. Therefore, all other parameters of the model are real. We take into account the contribution to M_{12} coming from the supersymmetry breaking sector as well as from the exchange of the flavor-changing neutral Higgs bosons. It turns out that both contributions are real, and that nevertheless there exists an observable difference in the CP phase in the mixing of the neutral mesons. Specifically, we focus our attention on the extra phase ϕ_s^Δ and the flavor-specific CP asymmetry a_{fs}^s , because they are accidentally very small in the SM. We find that a_{fs}^s of the model is mostly negative and can be one order of magnitude larger the SM value in size.

2. The model

The model is briefly described below (the details of the model can be found in [20,21]). The $SU(2)_L$ doublets of the quark and Higgs supermultiplets are denoted by Q and H^u, H^d , respectively. Similarly, $SU(2)_L$ singlets of the quark supermultiplets are denoted by U^c and D^c . (Here we restrict ourselves to the quark sector. The prediction in the lepton sector, which is given in [20], is the same as in the S_3 model of [22,23].) The Q_6 assignment is shown in Table 1, where we assume R parity. In what follows we discuss successively the Yukawa sector, the supersymmetry breaking sector and the Higgs sector. The crucial observation of [21] in achieving the minimality of the Higgs sector is that softly-broken supersymmetry allows for each sector of the model to have certain own symmetries without loosing renormalizability. Table 2 shows the symmetry structure used in [21], where the symbols are explained in the caption.

2.1. The Yukawa sector and the CKM parameters

The superpotential for the Yukawa interactions in the quark sector is given by

$$W_q = Y_a^u Q_3 H_3^u U_3^c + Y_b^u (Q_1 H_2^u + Q_2 H_1^u) U_3^c + Y_b^u Q_3 (H_1^u U_2^c - H_2^u U_1^c) + Y_c^u (Q_1 U_2^c + Q_2 U_1^c) H_3^u \\ + Y_a^d Q_3 H_3^d D_3^c + Y_b^d (Q_1 H_2^d + Q_2 H_1^d) D_3^c + Y_b^d Q_3 (H_1^d D_2^c - H_2^d D_1^c) + Y_c^d (Q_1 D_2^c + Q_2 D_1^c) H_3^d. \quad (1)$$

All the Yukawa couplings are real. So, the VEVs of the Higgs fields have to be complex to obtain the CP phase of the CKM matrix. Thanks to the Z_2 invariance of the scalar potential (see (14)) under

$$H_+^{u,d} = \frac{1}{\sqrt{2}}(H_1^{u,d} + H_2^{u,d}) \rightarrow H_+^{u,d}, \quad H_-^{u,d} = \frac{1}{\sqrt{2}}(H_1^{u,d} - H_2^{u,d}) \rightarrow -H_-^{u,d}, \quad (2)$$

the VEVs¹

$$\langle \hat{H}_-^{0u,d} \rangle = 0, \quad \langle \hat{H}_+^{0u,d} \rangle = \frac{v_+^{u,d}}{\sqrt{2}} \exp i\theta_+^{u,d}, \quad \langle \hat{H}_3^{0u,d} \rangle = \frac{v_3^{u,d}}{\sqrt{2}} \exp i\theta_3^{u,d} \quad (3)$$

can become a local minimum, where we assume that $v_+^{u,d}$ and $v_3^{u,d}$ are real and positive. (The Yukawa interactions do not respect the Z_2 symmetry, but due to the Q_6 family symmetry they cannot induce Z_2 -violating scalar potential terms of dimension less than or equal to four in higher orders in perturbation theory.) From the Yukawa interactions (1) along with the form of the VEVs (3) we obtain the fermion mass matrices. In diagonalizing these mass matrices we found [19] that the CKM mixing matrix can be written as

$$V_{\text{CKM}} = (U_L^u)^\dagger U_L^d = O_L^{uT} P_u^\dagger P_d O_L^d, \quad (4)$$

where O_L^u and O_L^d are orthogonal matrices, and

¹ Fields with a hat are the scalar components of the corresponding superfields.

$$P_{u,d} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \exp(i2\Delta\theta^{u,d}) & 0 \\ -1 & \exp(i2\Delta\theta^{u,d}) & 0 \\ 0 & 0 & \sqrt{2}\exp(i\Delta\theta^{u,d}) \end{pmatrix}, \quad \Delta\theta^{u,d} = \theta_3^{u,d} - \theta_+^{u,d}, \quad (5)$$

$$P_q = P_u^\dagger P_d = \text{diag}(1, \exp(i2\theta_q), \exp(i\theta_q)), \quad \theta_q = \theta_+^u - \theta_+^d - \theta_3^u + \theta_3^d. \quad (6)$$

There are nine independent theory parameters, i.e., $Y_a^{u,d} v_3^{u,d}$, $Y_c^{u,d} v_3^{u,d}$, $Y_b^{u,d} v_+^{u,d}$, $Y_{b'}^{u,d} v_+^{u,d}$ and θ_q , to describe the CKM parameters. So, there is one prediction which can be displayed in different planes. They are presented in [24].

Since the purpose of the present Letter is to calculate the observable CP phases in the B^0 mixing, it is sufficient to consider a single point in the space of the theory parameters.² So, throughout this Letter we use the following theoretical values [21]:

$$m_u/m_t = 0.766 \times 10^{-5}, \quad m_c/m_t = 4.23 \times 10^{-3}, \quad m_d/m_b = 0.895 \times 10^{-3}, \quad m_s/m_b = 1.60 \times 10^{-2},$$

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.9740 & 0.2266 & 0.00362 \\ 0.2265 & 0.9731 & 0.0417 \\ 0.00849 & 0.0410 & 0.9991 \end{pmatrix}, \quad |V_{td}/V_{ts}| = 0.207, \quad (7)$$

$$\sin 2\beta(\phi_1) = 0.690, \quad \gamma(\phi_3) = 63.4^\circ. \quad (8)$$

2.2. Soft-supersymmetry-breaking sector and the phase alignment

As we can see from Table 2 the tri-linear couplings \mathbf{h} and soft scalar mass terms \mathbf{m} have the same family symmetry as the Yukawa sector. Consequently, the tri-linear couplings and the soft scalar mass matrices have the following form:

$$\tilde{\mathbf{m}}_{aLL}^2 = m_{aL}^2 \text{diag}(a_L^a, a_L^a, b_L^a) \quad (a = q, l), \quad (9)$$

$$\tilde{\mathbf{m}}_{aRR}^2 = m_{aR}^2 \text{diag}(a_R^a, a_R^a, b_R^a) \quad (a = u, d, e), \quad (10)$$

$$(\tilde{\mathbf{m}}_{aLR}^2)_{ij} = A_{ij}^a (\mathbf{m}^a)_{ij} = \tilde{A}_{ij}^a \sqrt{m_{aL} m_{aR}} (\mathbf{m}^a)_{ij} \quad (a = u, d, e), \quad (11)$$

where $m_{aL,R}$ denote the average of the squark and slepton masses, respectively, $(a_{L(R)}^a, b_{L(R)}^a)$ are dimensionless free real parameters of $O(1)$, A_{ij}^a are free parameters of dimension one, and \mathbf{m}^a are the fermion mass matrices. Note that $a_{L,R}^a$ and A_{ij}^a are all real, because we impose CP invariance in the tri-linear sector as well as in the soft-scalar mass sector.

The quantities [25,26]

$$\delta_{LL(RR)}^a = U_{aL(R)}^\dagger \tilde{\mathbf{m}}_{aLL(RR)}^2 U_{aL(R)} / m_a^2 \quad \text{and} \quad \delta_{LR}^a = U_{aL}^\dagger \tilde{\mathbf{m}}_{aLR}^2 U_{aR} / m_a^2 \quad (12)$$

in the super CKM basis are used widely to parameterize FCNCs and CP violations coming from the soft supersymmetry breaking sector, where the unitary matrices U 's to rotate the fermions to the mass eigenstates are given in [21]. Note that $\delta_{LL(RR)}^a$ is a function of $\Delta a_{L(R)}^a$, where

$$\Delta a_{L(R)}^a = a_{L(R)}^a - b_{L(R)}^a. \quad (13)$$

The imaginary parts of δ 's contribute to CP violating processes induced in the soft supersymmetry breaking sector. Recall that the phases of \mathbf{m}_{aLR}^2 can come only from the complex VEVs (3). As we can see from (6) the unitary matrices have the form $U_L^{u,d} = P_{u,d} O_L^{u,d}$, where only $P_{u,d}$ are complex. Since $P_{u,d}$ commute with $\mathbf{m}_{aLL,RR}^2$ (because their first 2×2 block is proportional to the identity matrix), $\delta_{LL,RR}^a$ have no imaginary part. Further, \mathbf{m}_{aLR}^2 has the same phase structure as the corresponding fermion mass matrix \mathbf{m}^a , and it turns out that δ_{LR}^a , too, are real. So, the imaginary part of $(\delta_{12,21,13,31,23,32}^d)_{LL,RR,LR,RL}$ which would contribute to $\text{Im} M_{12}^{\text{new}}$ is absent. Therefore, as far as the soft scalar masses and the left-right soft masses in the soft-supersymmetry-breaking sector are concerned, there is no extra CP violating phase.

In Table 3 we show the actual values of the δ 's which should be compared with the experimental bounds.³ These constraints come from the mass differences of the neutral mesons, i.e., ΔM_K , ΔM_d and ΔM_s . We see that no fine tuning of the soft-supersymmetry breaking parameters (except for Δa_R^d for which a fine tuning of about 10% is required) is needed to satisfy the experimental constraints. These contributions from the supersymmetry breaking sector should be added to the contribution coming from the exchange of the flavor-changing neutral Higgs bosons. In the best situation one can have they cancel each other. As we will see, even in this situation, that is, even if we assume that the contributions from the supersymmetry breaking sector can be freely chosen, we are able to make predictions on the CP violating quantities such as the flavor-specific CP asymmetry.

2.3. The neutral Higgs bosons and their mixing

The scalar potential V of the model consists of the μ terms, the scalar soft masses, the b terms and the D terms, and can be written as

² The uncertainties of the observables such as those of the quark masses are reflected into the Yukawa couplings given (36) and (37). We have scanned the allowed parameter space, and found the largest uncertainty is about 6% in the Yukawa couplings. Since the matrix elements (42),(44),(46) depend on the square of the Yukawa couplings, they will suffer from an additional uncertainty of at most 12%.

³ See [20] and references therein.

Table 3

Experimental bounds on δ 's and their theoretical values in the Q_6 model, where $\tilde{m}_{\tilde{q}}$ denotes $m_{\tilde{q}}/500$ GeV, \tilde{A} and $\Delta a_{L(R)}$ are given in (11) and (13).

	Exp. bound	Q_6 model
$\sqrt{ \text{Re}(\delta_{12}^d)_{LL,RR}^2 }$	$4.0 \times 10^{-2} \tilde{m}_{\tilde{q}}$	$(LL) 1.2 \times 10^{-4} \Delta a_L^q, (RR) 1.7 \times 10^{-1} \Delta a_R^d$
$\sqrt{ \text{Re}(\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR} }$	$2.8 \times 10^{-3} \tilde{m}_{\tilde{q}}$	$4.5 \times 10^{-3} \sqrt{\Delta a_L^q \Delta a_R^d}$
$\sqrt{ \text{Re}(\delta_{12}^d)_{LR}^2 }$	$4.4 \times 10^{-3} \tilde{m}_{\tilde{q}}$	$\sim 2 \times 10^{-5} (\tilde{A}_a^d - \tilde{A}_b^d - \tilde{A}_{b'}^d + \tilde{A}_c^d) \tilde{m}_{\tilde{q}}^{-1}$
$\sqrt{ \text{Re}(\delta_{13}^d)_{LL,RR}^2 }$	$9.8 \times 10^{-2} \tilde{m}_{\tilde{q}}$	$(LL) 7.8 \times 10^{-3} \Delta a_L^q, (RR) 1.4 \times 10^{-1} \Delta a_R^d$
$\sqrt{ \text{Re}(\delta_{13}^d)_{LL}(\delta_{13}^d)_{RR} }$	$1.8 \times 10^{-2} \tilde{m}_{\tilde{q}}$	$3.4 \times 10^{-2} \sqrt{\Delta a_L^q \Delta a_R^d}$
$\sqrt{ \text{Re}(\delta_{13}^d)_{LR}^2 }$	$3.3 \times 10^{-2} \tilde{m}_{\tilde{q}}$	$\sim 2 \times 10^{-5} (\tilde{A}_a^d - \tilde{A}_b^d + \tilde{A}_{b'}^d - \tilde{A}_c^d) \tilde{m}_{\tilde{q}}^{-1}$
$ \delta_{23}^d _{LL,RR} $	$8.2 \tilde{m}_{\tilde{q}}^2$	$(LL) 1.5 \times 10^{-2} \Delta a_L^q, (RR) 4.7 \times 10^{-1} \Delta a_R^d$
$ \delta_{23}^d _{LR} $	$1.6 \times 10^{-2} \tilde{m}_{\tilde{q}}^2$	$\sim 5 \times 10^{-5} (\tilde{A}_a^d - \tilde{A}_b^d + \tilde{A}_{b'}^d + 0.1 \tilde{A}_c^d) \tilde{m}_{\tilde{q}}^{-1}$

$$\begin{aligned}
V = & m_{H_+^u}^2 (|\hat{H}_+^{0u}|^2 + |\hat{H}_-^{0u}|^2) + m_{H_+^d}^2 (|\hat{H}_+^{0d}|^2 + |\hat{H}_-^{0d}|^2) + m_{H_3^u}^2 |\hat{H}_3^{0u}|^2 + m_{H_3^d}^2 |\hat{H}_3^{0d}|^2 \\
& + \frac{1}{8} (g_Y^2 + g_2^2) (|\hat{H}_+^{0u}|^2 + |\hat{H}_-^{0u}|^2 + |\hat{H}_3^{0u}|^2 - |\hat{H}_+^{0d}|^2 - |\hat{H}_-^{0d}|^2 - |\hat{H}_3^{0d}|^2)^2 \\
& + [b_{++} \hat{H}_+^{0u} \hat{H}_+^{0d} + b_{--} \hat{H}_-^{0u} \hat{H}_-^{0d} + b_{+3} \hat{H}_+^{0u} \hat{H}_3^{0d} + b_{3+} \hat{H}_3^{0u} \hat{H}_+^{0d} + b_{33} \hat{H}_3^{0u} \hat{H}_3^{0d} + \text{h.c.}],
\end{aligned} \quad (14)$$

where $g_{Y,2}$ are the gauge coupling constants for the $U(1)_Y$ and $SU(2)_L$ gauge groups, and H_{\pm} 's are defined in (2). As announced the scalar potential (14) has the Z_2 symmetry, where H_+ 's and H_3 's are Z_2 even, and H_- 's are Z_2 odd. First we redefine the Higgs fields as $\tilde{H}_+^{0u,0d} = \hat{H}_+^{0u,0d} \exp(-i\theta_+^{u,d})$, $\tilde{H}_3^{0u,0d} = \hat{H}_3^{0u,0d} \exp(-i\theta_3^{u,d})$, and then define

$$\phi_L^u = \cos \gamma^u \tilde{H}_3^{0u} + \sin \gamma^u \tilde{H}_+^{0u}, \quad \phi_H^u = -\sin \gamma^u \tilde{H}_3^{0u} + \cos \gamma^u \tilde{H}_+^{0u}, \quad (15)$$

where

$$\cos \gamma^u = \frac{v_3^u}{\sqrt{(v_3^u)^2 + (v_+^u)^2}}, \quad \sin \gamma^u = \frac{v_+^u}{\sqrt{(v_3^u)^2 + (v_+^u)^2}}, \quad (16)$$

and similarly for the down sector. As we see from (16), only ϕ_L^u and ϕ_L^d have a nonvanishing VEV, which we denote by $\sqrt{2} \langle \phi_L^{u,d} \rangle = \sqrt{(v_3^{u,d})^2 + (v_+^{u,d})^2} = v_{u,d}$. The neutral light and heavy Higgs scalars of the MSSM are then given by

$$\frac{1}{\sqrt{2}} (v + h - iX) = (\phi_L^{d*}) \cos \beta + (\phi_L^u) \sin \beta, \quad (17)$$

$$\frac{1}{\sqrt{2}} (H + iA) = -(\phi_L^{d*}) \sin \beta + (\phi_L^u) \cos \beta, \quad (18)$$

where as in the MSSM $v = \sqrt{v_u^2 + v_d^2}$ and $\tan \beta = v_u/v_d$.

As in the case of the MSSM, the couplings of $\phi_L^{u,d}$ are flavor-diagonal, while the extra heavy fields

$$\hat{H}_-^{0u,0d} = \phi_-^{u,d} = (\varphi_-^{u,d} + i\chi_-^{u,d})/\sqrt{2}, \quad \phi_H^{u,d} = (\varphi_H^{u,d} + i\chi_H^{u,d})/\sqrt{2} \quad (19)$$

can have flavor-changing couplings. The mass matrix for the Z_2 -odd $\phi_-^{u,d}$ can be written as

$$\mathbf{M}_-^2 = \begin{pmatrix} m_{\phi_-^u}^2 & 0 & b_- & -c_- \\ 0 & m_{\phi_-^d}^2 & -c_- & -b_- \\ b_- & -c_- & m_{\phi_-^d}^2 & 0 \\ -c_- & -b_- & 0 & m_{\phi_-^u}^2 \end{pmatrix} \quad (20)$$

in the $(\varphi_-^u, \chi_-^u, \varphi_-^d, \chi_-^d)$ basis, where $m_{\phi_-^{u,d}}^2 = m_{H_-^{u,d}}^2$, $b_- = \text{Re}(b_{--})$, $c_- = \text{Im}(b_{--})$. The mass matrix for the Z_2 -even fields is given by

$$\mathbf{M}_H^2 = \begin{pmatrix} m_{\phi_H^u}^2 & 0 & b_{HH} & -c_{HH} & m_{\phi_{HL}^u}^2/c_\beta & 0 & 0 \\ 0 & m_{\phi_H^d}^2 & -c_{HH} & -b_{HH} & 0 & m_{\phi_{HL}^d}^2/c_\beta & 0 \\ b_{HH} & -c_{HH} & m_{\phi_H^d}^2 & 0 & -m_{\phi_{HL}^d}^2/s_\beta & 0 & 0 \\ -c_{HH} & -b_{HH} & 0 & m_{\phi_H^u}^2 & 0 & m_{\phi_{HL}^u}^2/s_\beta & 0 \\ m_{\phi_{HL}^u}^2/c_\beta & 0 & -m_{\phi_{HL}^d}^2/s_\beta & 0 & m_{\phi_L^u}^2 + m_{\phi_L^d}^2 + s_{2\beta}^2 M_Z^2 & 0 & -c_{2\beta} s_{2\beta} M_Z^2 \\ 0 & m_{\phi_{HL}^d}^2/c_\beta & 0 & m_{\phi_{HL}^u}^2/s_\beta & 0 & m_{\phi_L^u}^2 + m_{\phi_L^d}^2 & 0 \\ 0 & 0 & 0 & 0 & -c_{2\beta} s_{2\beta} M_Z^2 & 0 & c_{2\beta}^2 M_Z^2 \end{pmatrix} \quad (21)$$

in the $(\varphi_H^u, \chi_H^u, \varphi_H^d, \chi_H^d, H, A, h)$ basis, where $m_{\phi_H^u}^2 = \hat{m}_{\phi_H^u}^2 - c_\beta M_Z^2/2$, $m_{\phi_H^d}^2 = \hat{m}_{\phi_H^d}^2 + c_\beta M_Z^2/2$, $c_{a\beta} = \cos a\beta$, $s_{a\beta} = \sin a\beta$,

$$\begin{aligned} \hat{m}_{\phi_H^{u,d}}^2 &= m_{H_{u,d}}^2 \cos^2 \gamma^{u,d} + m_{H_3^{u,d}}^2 \sin^2 \gamma^{u,d}, & m_{\phi_L^{u,d}}^2 &= m_{H_{u,d}}^2 \sin^2 \gamma^{u,d} + m_{H_3^{u,d}}^2 \cos^2 \gamma^{u,d}, \\ m_{\phi_{HL}^{u,d}}^2 &= \frac{1}{2} \sin 2\gamma^{u,d} (m_{H_{u,d}}^2 - m_{H_3^{u,d}}^2), \\ b_{HH} + ic_{HH} &= b_{++} e^{-i(\theta_+^u + \theta_+^d)} \cos \gamma^u \cos \gamma^d - b_{+3} \cos \gamma^u \sin \gamma^d e^{-i(\theta_+^u + \theta_3^d)} \\ &\quad - b_{3+} \sin \gamma^u \cos \gamma^d e^{-i(\theta_3^u + \theta_+^d)} + b_{33} \sin \gamma^u \sin \gamma^d e^{-i(\theta_3^u + \theta_3^d)}. \end{aligned} \quad (22)$$

All the parameters in the mass matrices (20) and (21) are real, and the mass parameters and $\gamma^{u,d}$ are defined in (16). In [21] it was assumed that $m_{\phi_{HL}^{u,d}}^2$ (which express the mixing among the MSSM and extra heavy Higgs fields) are small compared with other mass parameters such as $m_{\phi_H^{u,d}}^2$. Under this assumption the mass matrix squared (21) goes over to the one given in [21].

3. $B^0 - \bar{B}^0$ mixing via heavy neutral Higgs bosons

As a last task we investigate signatures of new physics contributions to the non-diagonal matrix element of the effective Hamiltonians of the neutral meson systems M_{12} . We will see that not only the contributions from the supersymmetry breaking sector (as we have found in Section 2.2), but also those from the flavor-changing neutral Higgs exchanges are real, and that despite being real the new contributions can create a new mixing phase.

The total matrix element M_{12} can be written as

$$M_{12} = M_{12}^{\text{SM}} + M_{12}^{\text{new}} = M_{12}^{\text{SM}} \Delta, \quad (23)$$

and we follow [2] to parameterize new physics effects in the observables ΔM_s , $\Delta \Gamma_s$ and the flavor specific CP asymmetry a_{fs}^s in terms of the complex number $\Delta_s = |\Delta_s| e^{i\phi_s^\Delta}$:

$$\Delta M_s = 2|M_{12}^{\text{SM}} \Delta_s| = \Delta M_s^{\text{SM}} |\Delta_s|, \quad \Delta \Gamma_s = 2|\Gamma_{12}^s| \cos(\phi_s^{\text{SM}} + \phi_s^\Delta), \quad (24)$$

$$\frac{\Delta \Gamma_s}{\Delta M_s} = \frac{|\Gamma_{12}^s|}{|M_{12}^{\text{SM},s}|} \frac{\cos(\phi_s^{\text{SM}} + \phi_s^\Delta)}{|\Delta_s|}, \quad a_{fs}^s = \frac{|\Gamma_{12}^s|}{|M_{12}^{\text{SM},s}|} \frac{\sin(\phi_s^{\text{SM}} + \phi_s^\Delta)}{|\Delta_s|}. \quad (25)$$

Note that Δ given in (23) defines an extra phase ϕ_s^Δ , which is present even if the new contribution M_{12}^{new} is real, because the standard model contribution M_{12}^{SM} is complex. The reason why one nevertheless can obtain a large CP violation compared with the SM is that the SM phase ϕ_s^{SM} is accidentally small (see (28)).

The SM values are given e.g. in [2], in which the results of [14–18] are used. For the present model with the CKM parameters given in (8) we have [21]⁴:

$$2M_{12}^{\text{SM},s} = 20.1(1 \pm 0.40) \exp(-i0.035) \text{ ps}^{-1}, \quad (26)$$

$$2M_{12}^{\text{SM},d} = 0.56(1 \pm 0.45) \exp(i0.77) \text{ ps}^{-1}, \quad (27)$$

$$\phi_s^{\text{SM}} = (4.2 \pm 1.4) \times 10^{-3} \text{ rad}, \quad \Delta \Gamma_s^{\text{SM}} = 0.096 \pm 0.039 \text{ ps}^{-1}, \quad (28)$$

$$a_{fs}^{\text{SM},s} = (2.06 \pm 0.57) \times 10^{-5}, \quad (29)$$

where the errors are dominated by the uncertainty in the decay constants f_B . The corresponding experimental values are given by [5]

$$\Delta M_s^{\text{exp}} = 17.78 \pm 0.12 \text{ ps}^{-1}, \quad (30)$$

$$\Delta M_d^{\text{exp}} = 0.507 \pm 0.005 \text{ ps}^{-1}, \quad (31)$$

$$a_{fs}^{\text{exp},s} (= a_{sl}^{\text{exp},s}) = -0.0037 \pm 0.0094, \quad (32)$$

⁴ The phase for $M_{12}^{\text{SM},s}$ given in [21] $-i0.0035$ should be replaced by $-i0.035$.

and for $\Delta\Gamma_s$ and $\phi_s = \phi_s^{\text{SM}} + \phi_s^\Delta$ there are two regions [5]:

$$\phi_s^{\text{exp}} = -2.36_{-0.29}^{+0.37} \text{ rad}, \quad \Delta\Gamma_s^{\text{exp}} = -0.154_{-0.054}^{+0.070} \text{ ps}^{-1}, \quad (33)$$

$$\phi_s^{\text{exp}} = -0.77_{-0.37}^{+0.29} \text{ rad}, \quad \Delta\Gamma_s^{\text{exp}} = 0.154_{-0.070}^{+0.054} \text{ ps}^{-1}. \quad (34)$$

The above experimental values are 2.2σ away from the SM prediction (28) [2], which may indicate a possible existence of new physics [2,5,10,11]. With this in mind, we proceed with our investigation on possible new effects.

The Lagrangian that describes the mixing of B^0 and \bar{B}^0 (also that of K^0 and \bar{K}^0) is given by

$$\mathcal{L}_{\text{FCNC}} = -[Y_{ij}^{dH} \phi_H^d + Y_{ij}^{d-} \phi_-^d] \bar{d}'_{iL} d'_{jR} + \text{h.c.}, \quad (35)$$

where d' 's are mass eigenstates, the Higgs fields are defined in (15) and (19), and [21]

$$\mathbf{Y}^{dH} \simeq \frac{1}{\tan \gamma^d \cos \beta} \begin{pmatrix} 6.63 \times 10^{-5} & 8.26 \times 10^{-5} & 2.80 \times 10^{-4} \\ -6.224 \times 10^{-5} & 3.74 \times 10^{-4} & 3.37 \times 10^{-4} \\ 4.10 \times 10^{-3} & -6.01 \times 10^{-3} & 2.52 \times 10^{-3} \end{pmatrix} \\ - \frac{\tan \gamma^d}{\cos \beta} \begin{pmatrix} 1.37 \times 10^{-5} & 1.13 \times 10^{-4} & 7.56 \times 10^{-5} \\ 1.98 \times 10^{-5} & -1.88 \times 10^{-4} & -3.72 \times 10^{-4} \\ 1.67 \times 10^{-3} & 6.61 \times 10^{-3} & 0.0131 \end{pmatrix}, \quad (36)$$

$$\mathbf{Y}^{d-} \simeq \frac{\exp i(2\theta_3^d - \theta_+^d)}{\sin \gamma^d \cos \beta} \begin{pmatrix} 0 & -2.53 \times 10^{-4} & -4.72 \times 10^{-4} \\ -2.22 \times 10^{-4} & 0 & -1.04 \times 10^{-4} \\ 7.46 \times 10^{-3} & -1.89 \times 10^{-3} & 0 \end{pmatrix}. \quad (37)$$

The phases appearing in the Yukawa matrices above are given in (3).⁵ Given the FCNC interactions (35) we are now able to compute the extra contribution M_{12}^{new} . To this end we need to compute the inverse of the mass matrices squared (20) and (21), which we denote by Δ^- and Δ^H , respectively. The elements of Δ 's relevant to our purpose are:

$$\Delta_{\varphi_-^d - \varphi_-^d}^- = \Delta_{\chi_-^d - \chi_-^d}^- = \frac{m_{\phi_-^d}^2}{(\bar{M}_-^2)^2} \equiv \frac{1}{(M_-^d)^2}, \quad \Delta_{\varphi_-^d - \chi_-^d}^- = 0, \quad (38)$$

$$\Delta_{\varphi_H^d - \varphi_H^d} = \Delta_{\chi_H^d - \chi_H^d} = \frac{(m_{\phi_{HL}^d}^2)^2 / \cos^2 \beta - m_{\phi_H^d}^2 m_H^2}{(\bar{M}_H^2)^3} \equiv \frac{1}{(M_H^d)^2}, \quad (39)$$

$$\Delta_{\varphi_H^d - \chi_H^d} = 0, \quad (40)$$

where $(\bar{M}_-^2)^4 = \det \mathbf{M}_-^2$ and $(\bar{M}_H^2)^6 = \det \mathbf{M}_H^2 / (\cos^2 2\beta M_Z^2)$ and $m_H^2 = m_{\phi_L^d}^2 + m_{\phi_H^d}^2$. The mass parameters appearing in (38)–(40) are defined in (20) and (21). The fact that $\Delta_{\varphi^d - \varphi^d} = \Delta_{\chi^d - \chi^d}$ and $\Delta_{\varphi^d - \chi^d} = 0$ has an important consequence that although CP is explicitly broken by the b terms in the supersymmetry breaking sector, the new contribution to M_{12} is real, as in the case of the contribution from the supersymmetry breaking sector. Therefore, the new contributions M_{12}^{new} from the φ and χ exchanges take the form [21]

$$M_{12}^{\text{new},K} = 2 \langle \bar{K}^0 | C_K \bar{s}_R^\alpha d_L^\alpha \bar{s}_L^\beta d_R^\beta | K^0 \rangle \simeq 0.56 C_K \text{ GeV}^3, \quad (41)$$

$$C_K = [(Y_{sd}^{dH})^* Y_{ds}^{dH} / (\cos \beta M_H^d)^2 + (Y_{sd}^{d-})^* Y_{ds}^{d-} / (\cos \beta M_-^d)^2], \quad (42)$$

$$M_{12}^{\text{new},d} = 2 \langle \bar{B}_d^0 | C_d(m_b) \bar{b}_R^\alpha d_L^\alpha \bar{b}_L^\beta d_R^\beta | B_d^0 \rangle \simeq 0.36 C_d(m_b) \text{ GeV}^3, \quad (43)$$

$$C_d(m_b) = \eta(m_b) [(Y_{bd}^{dH})^* Y_{db}^{dH} / (\cos \beta M_H^d)^2 + (Y_{bd}^{d-})^* Y_{db}^{d-} / (\cos \beta M_-^d)^2], \quad (44)$$

$$M_{12}^{\text{new},s} = 2 \langle \bar{B}_s^0 | C_s(m_b) \bar{b}_R^\alpha s_L^\alpha \bar{b}_L^\beta s_R^\beta | B_s^0 \rangle \simeq 0.58 C_s(m_b) \text{ GeV}^3, \quad (45)$$

$$C_s(m_b) = \eta(m_b) [(Y_{bs}^{dH})^* Y_{sb}^{dH} / (\cos \beta M_H^d)^2 + (Y_{bs}^{d-})^* Y_{sb}^{d-} / (\cos \beta M_-^d)^2], \quad (46)$$

where $\eta(m_b) \simeq 2.0$ is the one-loop QCD correction, Y 's are elements of the Yukawa matrices (36) and (37). The matrix elements (42), (44), (46) basically suffer from the same size of the uncertainties as (26) and (27). In the following calculations we impose the constraints

$$0.6 < \Delta M_{d,s} / \Delta M_{d,s}^{\text{exp}} < 1.4, \quad 2 |(M_K^{\text{new}})_{12}| < \Delta M_K^{\text{exp}} \simeq 3.49 \times 10^{-15} \text{ GeV}, \quad |\phi_d^\Delta| < 0.17 \text{ rad}. \quad (47)$$

(ϕ_d^Δ is an analog of ϕ_s^Δ for B_d .)

- (i) ϕ_s^Δ . We first compute ϕ_s^Δ . To this end we include all the contributions; the contributions from the φ and χ exchanges and those from the soft supersymmetry breaking terms, where we assume that the later contributions can be freely chosen by varying the a_L , a_R and A_{ij} defined in (10) and (11). We find:

⁵ See the comment in footnote 2.

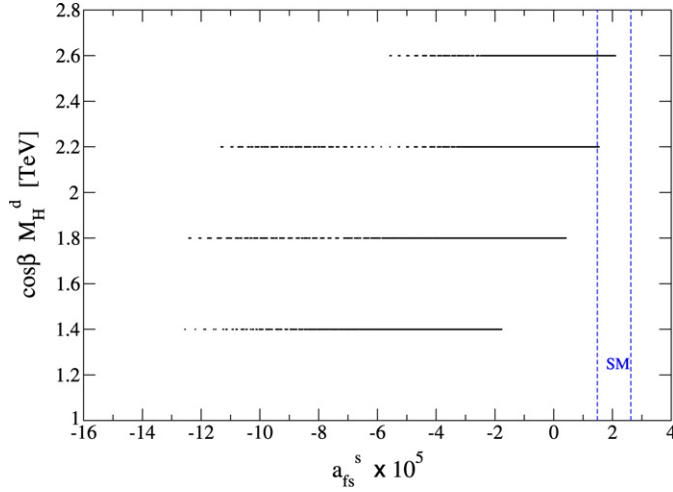


Fig. 1. The prediction of the CP asymmetry $a_{fs}^s \times 10^5$ for different values of the Higgs mass $\cos \beta M_H^d$ [TeV]. The SM value is between two blue lines. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

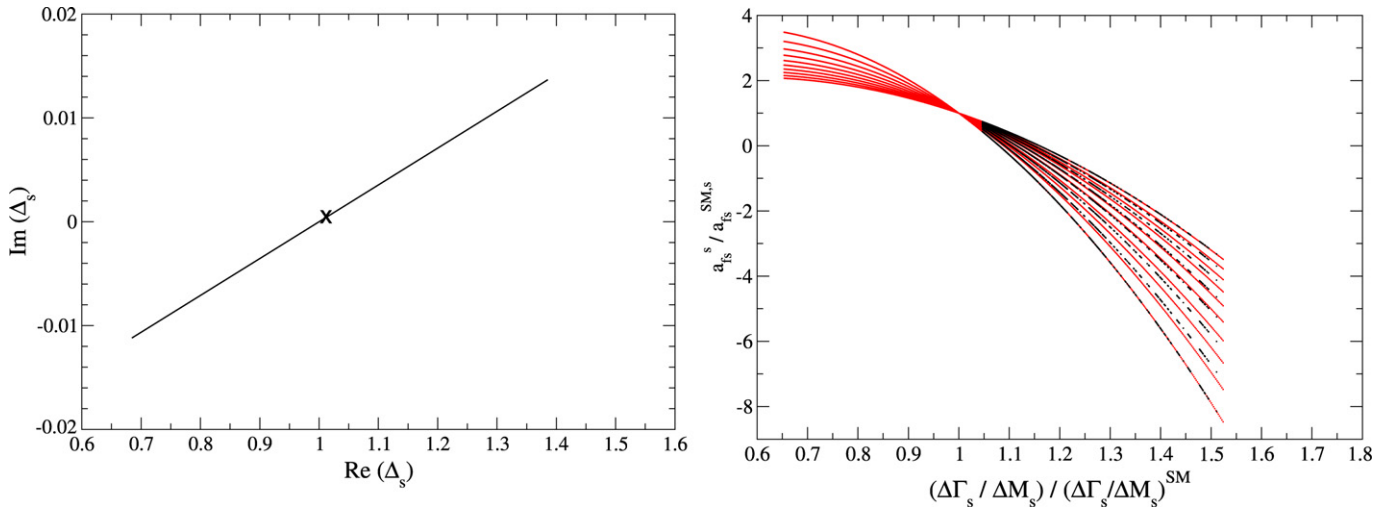


Fig. 2. Right: The prediction in the $(\Delta \Gamma_s / \Delta M_s) / (\Delta \Gamma_s / \Delta M_s)^{SM} - (a_{fs}^s / a_{fs}^{SM})$ plane. The black points are those without the contribution from the soft-supersymmetry breaking terms. The red points are obtained by including the contributions coming from both the Higgs exchanges and soft-supersymmetry breaking terms. Left: The prediction in the $\text{Re}(\Delta_s) - \text{Im}(\Delta_s)$ plane, where the contributions coming from both the Higgs exchanges and soft-supersymmetry breaking terms are included. The cross denotes the SM point. The Higgs mass $\cos \beta M_H^d$ is varied from 1.2 to 3.0 GeV for both panels. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

$$-0.018 \lesssim \phi_s^{SM} + \phi_s^\Delta \lesssim 0.012 \quad \text{and} \quad -0.023 \lesssim \phi_s^\Delta \lesssim 0.009. \quad (48)$$

(If only the Higgs exchanges are taken into account, we find $-0.015 \lesssim \phi_s^{SM} + \phi_s^\Delta \lesssim 0.007$.) So, if the evidence for a new phase (33) or (34) were confirmed, not only the SM, but also the present supersymmetric model might run into a serious problem.⁶

- (ii) a_{fs}^s . Using (25) we next compute $a_{fs}^s / a_{fs}^{SM,s} = \sin(\phi_s^{SM} + \phi_s^\Delta) / (\sin \phi_s^{SM} |\Delta_s|)$. First we consider only the contributions from the Higgs exchanges, where for a given $\cos \beta M_H^d$ we vary the Higgs mixing angle γ^d (16) and $r = M_-^d / M_H^d$ so as to satisfy the constraints (47). The result is plotted in Fig. 1, where we varied $\cos \beta M_H^d$ from 1.2 (the smallest allowed value) to 2.6 TeV. The SM value (29) is between two blue vertical lines. If all three contributions are included, we find

$$-13 \lesssim a_{fs}^s \times 10^5 \lesssim 7. \quad (49)$$

(The experimental value is given in (32).)

- (iii) $(\Delta \Gamma_s / \Delta M_s) - a_{fs}^s$. The prediction of (a_{fs}^s / a_{fs}^{SM}) against $(\Delta \Gamma_s / \Delta M_s) / (\Delta \Gamma_s / \Delta M_s)^{SM}$ is plotted in Fig. 2 (right). The contribution only from the Higgs exchanges is indicated by black. In this area a_{fs}^s is mostly negative and its size may become one order of magnitude larger than the SM value.
- (iv) Δ_s . The prediction in the $\text{Re}(\Delta_s) - \text{Im}(\Delta_s)$ is plotted in Fig. 2 (left), where the cross denotes the SM point. All the contribution are taken into account.

⁶ A similar conclusion has been reached in [27] for the MSSM with large $\tan \beta$ and the Minimal Flavor Violation assumption.

4. Conclusion

We considered a supersymmetric extension of the SM based on the discrete Q_6 family symmetry, which has been recently proposed in [19,20], and investigated the extra contribution to M_{12} , which we denoted by M_{12}^{new} . We assumed that CP is explicitly, but softly broken only by the b terms in the soft supersymmetry breaking sector. Therefore, all other parameters of the model are real, which is consistent with renormalizability [21]. There are two origins for the contribution to M_{12}^{new} ; from the supersymmetry breaking sector and from the exchange of the flavor-changing neutral Higgs bosons. We found that both contributions are real, and that nevertheless we obtain an observable difference in the CP violation. We focus our attention on the extra B_s -mixing phase ϕ_s^Δ and the flavor-specific CP asymmetry a_{fs}^s , because they are accidentally small $\sim O(10^{-3})$ and $\sim O(10^{-5})$, respectively, in the SM. We found that a_{fs}^s in our model is mostly negative and can be indeed one order of magnitude larger the SM value in size. Our results Figs. 1 and 2, which are consistent with the current experimental value [34].

The expected accuracy of the ϕ_s measurement at LHCb with 2 fb^{-1} in the first period is about 0.03 [28], which is in the same order of (48). One can use (25) together with the SM values (26), (28) and (29) to obtain the experimental value of a_{fs}^s for a given ϕ_s . Since $|\Delta_s| = 1.0 \pm 0.3$ [2], with an accuracy of 0.03 for ϕ_s , one arrives at an accuracy of $\sim O(10^{-4})$ for a_{fs}^s , which is again in the same order of (49). Therefore, we may expect that the predicted values of the present model become testable in the later periods of LHCb.

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