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# How large can the SM contribution to CP violation in $D_0$ - $\bar{D}_0$ mixing be?

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**ABSTRACT:** We reconsider the leading HQE contributions to the absorptive part of the mixing amplitude of neutral  $D$  mesons by taking also  $\alpha_s$  corrections and subleading  $1/m_c$  corrections into account. We show that these contributions to  $\Gamma_{12}$  do not vanish in the exact  $SU(3)_F$  limit and they also can have a large phase. Moreover, we give an example of a new physics model that can enhance these leading HQE terms to  $\Gamma_{12}$ , which are orders of magnitude lower than the current experimental expectation, by one to two orders of magnitude.

**KEYWORDS:** Beyond Standard Model, Heavy Quark Physics, CP violation, Standard Model

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## 1 Introduction

Meson-antimeson oscillations have long since provided a rich area for theoretical studies. In an application of pure quantum mechanics, the concept of neutral kaon mixing [1] led to the understanding of the observed CP violation in the decay  $K_L \rightarrow \pi\pi$  [2]. Furthermore, hints to the mass of the  $c$  quark [3] were obtained from  $K$  mixing before the first evidence of the  $J/\Psi$  [4, 5]. Also  $B_d - \bar{B}_d$  mixing [6, 7] provided information on the mass of the  $t$  quark prior to its discovery [8, 9]. The latter was possible due to the sensitivity of meson-antimeson mixing to heavy virtual particles propagating in an internal loop of the transition. Even today, the absence of a Standard Model (SM) tree-level background turns precision measurements of meson-antimeson mixing into an excellent probe for new physics effects; recently, a possible indication of such effects in the phase  $\Phi_s$  of the  $B_s - \bar{B}_s$  system [10, 11] has stirred vivid discussions [12–14].

Among the four mixing systems ( $K^0$ ,  $D^0$ ,  $B_d$  and  $B_s$ ) the  $D^0$  system is in a sense unique. The mixing mechanism relies on internal  $d$ -type quarks; due to the smaller mass of the  $b$  quark compared to the  $t$  quark, the kinematics of the dispersive part of the mixing amplitude are not completely dominated by the heavy third generation quark. Furthermore, due to the specific structure of the Cabibbo-Kobayashi-Maskawa couplings [15], the absorptive part  $\Gamma_{12}$  will feature an extremely efficient Glashow-Iliopoulos-Maiani (GIM) mechanism [16]. We will discuss this in detail later on and show that it leads to a suppression of the leading contribution by several orders of magnitude.

On the experimental side, the first evidence for charmed meson oscillation was reported by the *Belle* collaboration [17, 18], by *BABAR* [19] and later by CDF [20]. Currently, the relative decay width difference  $y$  and the relative mass difference  $x$  have been measured

to about 20% accuracy. The Heavy Flavor Averaging Group (HFAG) quotes the best-fit values [21]

$$y := \frac{\Delta\Gamma}{2\Gamma_{D^0}} = (7.3 \pm 1.8) \times 10^{-3}, \quad x := \frac{\Delta M}{\Gamma_{D^0}} = 9.1_{-2.6}^{+2.5} \times 10^{-3}. \quad (1.1)$$

The theory status for the  $D^0$  system is, unfortunately, in a slightly worse shape. There are two main approaches to this issue: Heavy Quark Expansion (HQE) [22–24] is an expansion of the bilocal  $\Delta C = 2$  matrix elements as a series of local operators of increasing dimension, which are suppressed by powers of the heavy quark mass. While this technique provides an excellent tool to study  $B$  mixing [25], the  $D^0$  system predictions [26–29] differ from experiment by up to a factor  $10^3$ . There are four main lines of argumentation why such a behavior is observed. First of all, a breakdown of the expansion in powers of the charm mass, which may not qualify it as “heavy” in the sense of the expansion, is possible. Comparing the ratios of the typical hadronic scale  $\Lambda$  and the heavy quark mass in the  $B$  and  $D^0$  system ( $\Lambda/m_b \approx 0.05$  versus  $\Lambda/m_c \approx 0.25$ ), the expansion parameter has increased by almost a factor of five. Secondly, one can invoke a violation of quark-hadron duality due to non-perturbative long distance effects; reliable quark-level predictions for the decay width difference may consequently serve as a probe of quark-hadron duality in the charm system. Furthermore, such a deviation could arise, if the severe GIM cancellations present in the leading terms of the OPE are lifted for higher dimensional operators. Finally, new physics may enhance the SM result for  $D^0 - \bar{D}^0$  mixing.

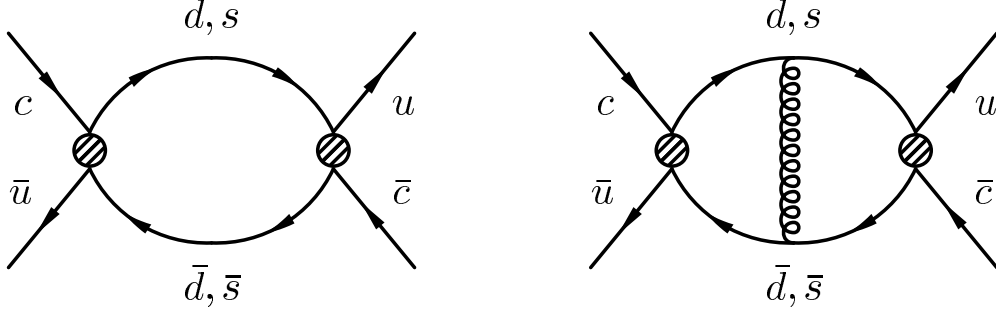
A second way to access the calculation of the  $D^0 - \bar{D}^0$  decay width difference  $\Delta\Gamma$  is based on exclusive techniques [30, 31]. In principle one has therefore to determine all contributions to  $\Gamma_{D^0}$  and  $\Gamma_{\bar{D}^0}$  with high precision, which is clearly beyond our current ability. As a first step to determine the size of  $\Delta\Gamma$  within the exclusive approach the authors of [30,31] take only the difference of the phase space of the corresponding final states into account.

In this work we reexamine the HQE for the relative decay width difference  $y$ . We begin with a short introduction of the formalism for the  $D^0 - \bar{D}^0$  mixing via box diagrams. Section 3 deals with the leading-order (LO) HQE predictions for the mixing matrix element  $\Gamma_{12}$  in next-to-leading order (NLO) in  $\alpha_s$ . Furthermore, we will show that, contrary to expectation, the large cancellations due to GIM mechanism can generate a sizable imaginary part in  $\Gamma_{12}$ . After a brief discussion of possible effects of higher order terms in the HQE, a new physics model is presented, which can substantially enhance the leading HQE term. We finish with a conclusion.

## 2 Mixing formalism

The mixing of neutral mesons is described by box diagrams with the absorptive part  $\Gamma_{12}$  and the dispersive part  $M_{12}$ . The observable mass and decay rate differences are given by ( $\phi := \arg[-M_{12}/\Gamma_{12}]$ )

$$\begin{aligned} (\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 &= 4|M_{12}|^2 - |\Gamma_{12}|^2, \\ \Delta M \Delta\Gamma &= 4|M_{12}||\Gamma_{12}|\cos(\phi). \end{aligned} \quad (2.1)$$



**Figure 1.** Contributions to  $\Gamma_{12}$  from operators of dimension 6 ( $D = 6$ ). The leading order QCD diagram is shown in the left panel, an example for  $\alpha_s$  corrections is shown in the right panel.

If  $|\Gamma_{12}/M_{12}| \ll 1$ , as in the case of the  $B_s$  system ( $\approx 5 \cdot 10^{-3}$ ) or if  $\phi \ll 1$ , one gets the famous approximate formulae

$$\Delta M = 2|M_{12}|, \quad \Delta \Gamma = 2|\Gamma_{12}| \cos \phi. \quad (2.2)$$

The experimental values for  $x$  and  $y$  suggest that in the  $D^0$  system  $|\Gamma_{12}/M_{12}| \approx \mathcal{O}(1)$ , the size of the mixing phase  $\phi$  will be discussed below.

### 3 Leading HQE predictions

The absorptive part of the box diagram with internal  $s$  and  $d$  quarks can be decomposed according to the CKM structure as

$$\Gamma_{12} = -(\lambda_s^2 \Gamma_{ss} + 2\lambda_s \lambda_d \Gamma_{sd} + \lambda_d^2 \Gamma_{dd}), \quad (3.1)$$

with  $\lambda_x = V_{cx}V_{ux}^*$ . The application of the heavy quark expansion (HQE), which turned out to be very successful in the  $B$  system, to the charm system typically meets major doubts. Our strategy in this work is the following: instead of trying to clarify the convergence of the HQE in the charm system in advance, we simply start with the leading term and determine corrections to it. The size of these corrections will give us an estimate for the convergence of the HQE in the  $D^0$  system. To this end we first investigate the contribution of dimension-6 ( $D = 6$ ) operators to  $\Gamma_{12}$ , see figure 1. Next we include NLO-QCD corrections, which were calculated for the  $B_s$  system [13, 32–34] and subleading terms in the HQE (dimension-7 operators), which were obtained in [35, 36]. To investigate the size of  $\alpha_s$  and  $1/m_c$  corrections in more detail we decompose  $\Gamma_{ss, sd, dd}$  into the Wilson coefficients  $G$  and  $G_S$  of the  $\Delta C = 2$  operators  $Q$  and  $Q_S$  (for more details see [13])

$$\Gamma_{xx}^{D=6,7} = \frac{G_F^2 m_c^2}{24\pi M_{D^0}} [G^{xx} \langle D^0 | Q | \bar{D}^0 \rangle + G_S^{xx} \langle D^0 | Q_S | \bar{D}^0 \rangle] + \Gamma_{xx}^{\frac{1}{m_c}}, \quad (3.2)$$

with

$$\begin{aligned} Q &= \bar{u}_\alpha \gamma^\mu (1 - \gamma_5) c_\alpha \cdot \bar{u}_\beta \gamma_\mu (1 - \gamma_5) c_\beta, \\ Q_S &= \bar{u}_\alpha (1 + \gamma_5) c_\alpha \cdot \bar{u}_\beta (1 + \gamma_5) c_\beta. \end{aligned} \quad (3.3)$$

The effect of the QCD corrections has already been discussed in [37]. In our numerics we carefully expand in  $\alpha_s$ : the leading order QCD contribution consists of leading order  $\Delta C = 1$  Wilson coefficients inserted in the left diagram of figure (1), while our NLO result consists of NLO  $\Delta C = 1$  Wilson coefficients inserted in both diagrams of figure (1) and consistently throwing away all terms which are explicitly of  $\mathcal{O}(\alpha_s^2)$ . Following [38] we have also summed terms like  $z \ln z$  to all orders; therefore, we use in our numerics  $\bar{z} = \bar{m}_s(\bar{m}_c)^2 / \bar{m}_c(\bar{m}_c)^2 \approx 0.0092$ . The matrix elements in eq. (3.3) are parameterized as

$$\begin{aligned} \langle D^0 | Q | \bar{D}^0 \rangle &= \frac{8}{3} f_{D^0}^2 M_{D^0}^2 B(\mu), \\ \langle D^0 | Q_S | \bar{D}^0 \rangle &= -\frac{5}{3} f_{D^0}^2 M_{D^0}^2 \left( \frac{M_{D^0}}{m_c(\mu) + m_u(\mu)} \right)^2 B_S(\mu). \end{aligned} \quad (3.4)$$

We take  $f_D = 212(14)$  MeV from [39] and we derive  $B(m_c) = 0.9 \pm 0.1$  and  $B_S(m_c) \simeq 1.3$  from [40, 41]. Finally we use the  $\overline{\text{MS}}$  scheme for the charm mass,  $\bar{m}_c(\bar{m}_c) = 1.27$  GeV. For clarity we show in the following only the results for the central values of our parameters, the error estimates will be presented at the end of the next section. We obtain

	LO	NLO	$\Delta\text{NLO/LO}$
$G^{ss}$	$0.25^{+0.09}_{-0.06}$	$0.37^{+0.18}_{-0.20}$	+48%
$G^{ds}$	$0.26^{+0.09}_{-0.06}$	$0.39^{+0.19}_{-0.21}$	+49%
$G^{dd}$	$0.28^{+0.09}_{-0.06}$	$0.42^{+0.19}_{-0.22}$	+49%
$G_S^{ss}$	$1.97^{+0.15}_{-0.29}$	$1.34^{+0.19}_{-0.23}$	−32%
$G_S^{ds}$	$1.98^{+0.15}_{-0.29}$	$1.34^{+0.19}_{-0.23}$	−32%
$G_S^{dd}$	$1.98^{+0.15}_{-0.29}$	$1.35^{+0.19}_{-0.23}$	−32%

For the error estimate we vary  $\mu_1$  between 1 GeV and  $2m_c$ . Combining  $G$  and  $G_S$  to  $\Gamma_{xx}$  we get (in units of  $\text{ps}^{-1}$ )

	LO	$\Delta\text{NLO-QCD}$	$\Delta 1/m_c$	$\Sigma$
$\Gamma_{ss}$	3.52	−0.94 (−27%)	−0.76 (−22%)	1.82
$\Gamma_{ds}$	3.54	−0.93 (−26%)	−0.76 (−22%)	1.84
$\Gamma_{dd}$	3.55	−0.92 (−26%)	−0.76 (−22%)	1.87

Using instead the operator basis suggested in [13] with

$$\begin{aligned} \tilde{Q}_S &= \bar{u}_\alpha(1 + \gamma_5)c_\beta \cdot \bar{u}_\beta(1 + \gamma_5)c_\alpha, \\ \langle D^0 | \tilde{Q}_S | \bar{D}^0 \rangle &= \frac{1}{3} f_{D^0}^2 M_{D^0}^2 \left( \frac{M_{D^0}}{m_c(\mu) + m_u(\mu)} \right)^2 \tilde{B}_S(\mu), \end{aligned} \quad (3.5)$$

and  $\tilde{B}_S(m_c) \simeq 1.2$  leads to

	LO	$\Delta\text{NLO-QCD}$	$\Delta 1/m_c$	$\Sigma$
$\Gamma_{ss}$	1.77	+0.02 (+1%)	−0.34 (−19%)	1.46
$\Gamma_{ds}$	1.78	+0.03 (+2%)	−0.34 (−19%)	1.48
$\Gamma_{dd}$	1.80	+0.05 (+3%)	−0.34 (−19%)	1.51

All in all we get large QCD (up to 50%) and large  $1/m_c$  corrections (up to 30%) to the leading  $D = 6$  term, which considerably lower the LO values. In the  $(Q, \tilde{Q}_S)$ -basis numerical cancellations can occur which mimic very small QCD corrections for  $\Gamma_{xy}$ . Despite large corrections, the HQE seems not to be completely off. From our above investigations we see no hints for a breakdown of OPE. The same argument can be obtained from the comparison of  $B$  and  $D$  meson lifetimes. In the HQE one obtains

$$\frac{\tau_B}{\tau_{D^0}} = \frac{\Gamma_{0,D^0} + \delta\Gamma_{D^0}}{\Gamma_{0,B} + \delta\Gamma_B} \approx \frac{\Gamma_{0,D^0}}{\Gamma_{0,B}} \left(1 + \frac{\delta\Gamma_{D^0}}{\Gamma_{0,D^0}}\right) \left(1 - \frac{\delta\Gamma_B}{\Gamma_{0,B}}\right), \quad (3.6)$$

where the leading term  $\Gamma_0 \propto m_{b,c}^5 V_{\text{CKM}}^2$  corresponds to the free quark decay and all higher terms in the HQE are comprised in  $\delta\Gamma$ . For the ratio  $\Gamma_{0,D^0}/\Gamma_{0,B}$  one gets a value close to one. Higher order HQE corrections in the  $B$  system are known to be smaller than 10 % [25]. Using the experimental values for the lifetimes we get

$$\frac{\tau_B}{\tau_{D^0}} \approx 1.4 \dots 4 (\text{Exp.}) \approx 1 \cdot \left(1 + \frac{\delta\Gamma_{D^0}}{\Gamma_{0,D^0}}\right). \quad (3.7)$$

From this rough estimate one expects higher order HQE corrections in the  $D$  system of up to 300 %. So clearly no precision determination will be possible within the HQE, but the estimates should still be within the right order of magnitude.

## 4 Cancellations

As is well known huge GIM cancellations [16] arise in the leading HQE terms for  $D^0$  mixing. To make these effects more obvious, we use the unitarity of the CKM matrix ( $\lambda_d + \lambda_s + \lambda_b = 0$ ) to rewrite the expression for the absorptive part in eq. (3.1) as

$$\Gamma_{12} = -\lambda_s^2 (\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd}) + 2\lambda_s\lambda_b (\Gamma_{sd} - \Gamma_{dd}) - \lambda_b^2 \Gamma_{dd}. \quad (4.1)$$

Note that the CKM structures differ enormously in their numerical values:  $\lambda_{d,s} \propto \lambda$  and  $\lambda_b \propto \lambda^5$  in terms of the Wolfenstein parameter  $\lambda \approx 0.2255$ . In the limit of exact  $\text{SU}(3)_F$  symmetry,  $\Gamma_{ss} = \Gamma_{sd} = \Gamma_{dd}$  holds and therefore, contrary to many statements<sup>1</sup> in the literature,  $\Gamma_{12} = -\lambda_b^2 \Gamma_{dd}$  is not zero although strongly CKM suppressed. Next we expand the arising terms in  $\bar{z}$ . Using eq. (3.2) we get in LO

$$\begin{aligned} \Gamma_{ss}^{D=6} &= 3.55477 - 3.22581\bar{z} - 14.877\bar{z}^2 + \dots, \\ \Gamma_{sd}^{D=6} &= 3.55477 - 1.61291\bar{z} - 7.43849\bar{z}^2 + \dots. \end{aligned} \quad (4.2)$$

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<sup>1</sup>These statements are obtained assuming  $V_{ub} = 0$ , which we have shown to be not justified for the HQE approach.

The first term in the above equations obviously corresponds to  $\Gamma_{dd}^{D=6}$ . For the combinations in eq. (4.1) we get

$$\begin{aligned}(\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd})^{D=6} &= -36.91\bar{z}^3 \approx \lambda^{7.0}, \\ (\Gamma_{sd} - \Gamma_{dd})^{D=6} &= -1.613\bar{z} \approx \lambda^{2.8}.\end{aligned}\quad (4.3)$$

To make the comparison with the arising CKM structures more obvious, we have expressed the size of these combinations also in terms of powers of the Wolfenstein parameter  $\lambda$ . As is well known, we find in the first term of eq. (4.1) an extremely effective GIM cancellation, only terms of order  $\bar{z}^3$  survive. In NLO we get

$$\begin{aligned}\Gamma_{ss}^{D=6,7} &= 1.8696 - 5.5231\bar{z} - 13.8143\bar{z}^2 + \dots, \\ \Gamma_{sd}^{D=6,7} &= 1.8696 - 2.7616\bar{z} - 7.4906\bar{z}^2 + \dots.\end{aligned}\quad (4.4)$$

The arising combinations in eq. (4.1) read now

$$\begin{aligned}(\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd})^{D=6,7} &= 1.17\bar{z}^2 - 59.5\bar{z}^3 \approx \lambda^{6.2} - \lambda^{6.7}, \\ (\Gamma_{sd} - \Gamma_{dd})^{D=6,7} &= -2.76\bar{z} \approx \lambda^{2.5}.\end{aligned}\quad (4.5)$$

The fact that now the first term of eq. (4.1) is of order  $\bar{z}^2$  compared to  $\bar{z}^3$  in the case of the LO-QCD value was discussed in detail in [33] and later on confirmed in [37]. These numbers are now combined with CKM structures, whose exact values read

$$\begin{aligned}\lambda_d &= -c_{12}c_{23}c_{13}s_{12} - c_{12}^2c_{13}s_{23}s_{13}e^{i\delta_{13}} = \mathcal{O}(\lambda^1 + i\lambda^5), \\ \lambda_s &= +c_{12}c_{23}c_{13}s_{12} - s_{12}^2c_{13}s_{23}s_{13}e^{i\delta_{13}} = \mathcal{O}(\lambda^1 + i\lambda^7), \\ \lambda_b &= c_{13}s_{23}s_{13}e^{i\delta_{13}} = \mathcal{O}(\lambda^5 + i\lambda^5),\end{aligned}\quad (4.6)$$

with  $c_{ij} = \cos(\theta_{ij})$  and  $s_{ij} = \sin(\theta_{ij})$ . Looking at eq. (4.6), it is of course tempting to throw away the small imaginary parts of  $\lambda_d$  and  $\lambda_s$ , but we will show below that this is not justified. Doing so and keeping only the leading term in the CKM structure ( $c_{12}c_{23}c_{13}s_{12}$ ), which is equivalent to approximate  $\lambda_b = 0$ , one gets a real  $\Gamma_{12}$  which vanishes in the exact  $SU(3)_F$  limit. Keeping the exact expressions, we see that the first term in eq. (4.1) is leading in CKM ( $\mathcal{O}[\lambda^2 + i\lambda^8]$ ) and has a negligible imaginary part, but it is suppressed by  $1.2\bar{z}^2 \approx \lambda^{6.2}$ . The second term in eq. (4.1) is subleading in CKM ( $\mathcal{O}[\lambda^6 + i\lambda^6]$ ) and it can have a sizeable phase. This term is less suppressed by  $SU(3)_F$  breaking ( $\approx 2.7\bar{z} \approx \lambda^{2.5}$ ). The third term in eq. (4.1) is not suppressed at all by  $SU(3)_F$  breaking, but it is strongly CKM suppressed ( $\mathcal{O}[\lambda^{10} + i\lambda^{10}]$ ). For clarity we compare the different contributions of eq. (4.1) in the following table<sup>2</sup>

	1 <sup>st</sup> term	2 <sup>nd</sup> term	3 <sup>rd</sup> term
LO	$37\lambda_s^2\bar{z}^3$ $\approx \lambda^{9.0}$	$3.2\lambda_s\lambda_b\bar{z}$ $\approx \lambda^{8.4}$	$3.6\lambda_b^2$ $\approx \lambda^{9.1}$
NLO	$1.17\lambda_s^2\bar{z}^2$ $\approx \lambda^{8.2}$	$5.5\lambda_s\lambda_b\bar{z}$ $\approx \lambda^{8.0}$	$1.87\lambda_b^2$ $\approx \lambda^{9.6}$

<sup>2</sup>With ‘NLO’ we denote the sum of leading order value,  $\mathcal{O}(\alpha_s)$  QCD corrections and subleading  $\frac{1}{m_c}$  contributions.

From this simple power counting, we see that a priori no contribution to eq. (4.1) can be neglected. Taking into account the hierarchy of the CKM matrix elements<sup>3</sup>, we find that the first two terms of eq. (4.1) are of similar size, while the third term is suppressed. Moreover, the second term can give rise to a large phase in  $\Gamma_{12}$ , while the first term has only a negligible phase. To make our arguments more solid we perform the full numerics using the CKM values from [43] and obtain for the three contributions of eq. (4.1)

$$\begin{aligned}
10^7 \Gamma_{12}^{D=6,7} &= -14.6409 + 0.0009i && (1^{\text{st}} \text{ term}) \\
&\quad - 6.68 - 15.8i && (2^{\text{nd}} \text{ term}) \\
&\quad + 0.27 - 0.28i && (3^{\text{rd}} \text{ term}) \\
&= -21.1 - 16.0i = (11 \dots 39) e^{-i(0.5 \dots 2.6)}. && (4.7)
\end{aligned}$$

Here we show for the first time the errors; they are estimated by varying  $\mu_1$  between 1 GeV and  $2m_c$  and by taking into account the results for both choices of the operator basis. The first term in eq. (4.1) turns out to be very sensitive with respect to the exact values of the bag parameters and its real part is approximately of the same size as the second term, which features a large imaginary part. Furthermore, even the third term can give a non-negligible contribution, in particular to the imaginary part.

To summarize, we have demonstrated that the typical approximation  $\lambda_b \approx 0$ , which is equivalent to neglecting the imaginary parts of  $\lambda_d$  and  $\lambda_s$  is wrong for the case of the leading (D=6,7) HQE prediction for  $y$  and yields the wrong conclusion that  $\Gamma_{12}^{D=6,7 \text{ NLO}}$  cannot have a sizeable phase. We get for the first terms in the OPE a value for  $y$  of

$$y^{D=6,7 \text{ NLO}} \leq |\Gamma_{12}| \cdot \tau_{D^0} = 4.7 \cdot 10^{-7} \dots 1.6 \cdot 10^{-6}. \quad (4.8)$$

The range of values was again estimated on the basis of the renormalization scheme dependence and the choice of the operator basis. These values are still a factor of  $0.5 \dots 1.6 \cdot 10^4$  smaller than the experimental number. This is in contrast to our previous expectations that the HQE should give at least the right order of magnitude. Moreover, we do not confirm the observation made in [37] that the NLO result for  $\Gamma_{12}$  is almost an order of magnitude larger than the LO result.

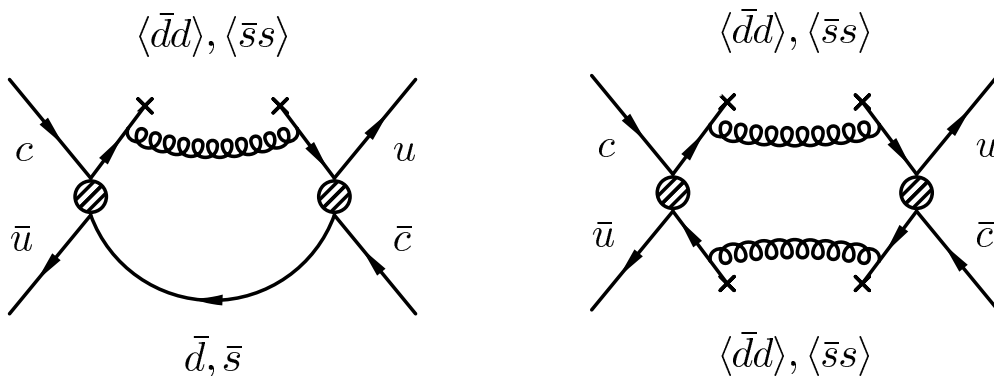
## 5 Higher HQE predictions

In [26–28] higher order terms in the HQE of  $D^0$  mixing were discussed. If the GIM cancellation is not as effective as in the leading HQE term, operators of dimension 9 and dimension 12, see figure (2), might be numerically dominant. In order to obtain an imaginary part of the loop integral, the operators of dimension 9 have to be dressed with at least one gluon and the operators of dimension 12 with at least two gluons. If we normalize the leading term (left figure of figure (1)) to 1, we expect the  $D = 9$  diagram of figure (2) to be of the order  $\mathcal{O}(\alpha_s(4\pi)\langle\bar{q}q\rangle/m_c^3) \approx 0.03$  and the  $D = 12$  diagram of figure (2) to be of the order

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<sup>3</sup> Actually  $|V_{ub}|$  is numerically of order  $\lambda^4$  and therefore  $\lambda_b \propto \lambda^6$  (see [42]), but in the literature  $V_{ub} = A\lambda^3(\rho - i\eta)$  with small values of  $\rho$  and  $\eta$  is commonly used.





**Figure 2.** Contributions to  $\Gamma_{12}$  from operators of dimension 9 ( $D = 9$ , left panel) and dimension 12 ( $D = 12$ , right panel). To obtain an imaginary part the  $D = 9$  diagrams have to be dressed with at least one gluon and the  $D = 12$  diagrams with at least 2 gluons.

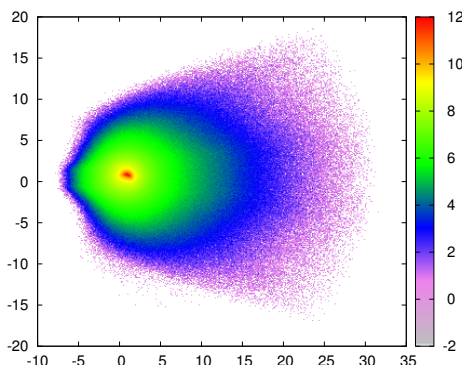
$\mathcal{O}(\alpha_s^2(4\pi)^2\langle\bar{q}q\rangle^2/m_c^6) \approx 10^{-3}$ . As explained above, the formally leading term of  $D = 6$  is strongly GIM suppressed to a value of about  $2 \cdot 10^{-5}$  and the big question is now how severe are the GIM cancellations in the  $D = 9, 12$  contribution. For the contributions to  $y$  we get the naive expectations

$y$	no GIM	with GIM
$D = 6, 7$	$2 \cdot 10^{-2}$	$5 \cdot 10^{-7}$
$D = 9$	$5 \cdot 10^{-4}$	?
$D = 12$	$2 \cdot 10^{-5}$	?

If there would be no GIM cancellations in the higher OPE terms, then the  $D = 9$  or  $D = 12$  contributions could be orders of magnitudes larger than the  $D = 6$  term, but in order to explain the experimental number still an additional numerical enhancement factor of about 15 has to be present. For more substantiated statements  $\Gamma_{12}^{D=9,12}$  has to be determined explicitly, which is beyond the scope of this work [44]. This calculation is also necessary in order to clarify to what extent the large phase in  $\Gamma_{12}$  from the first OPE term will survive. In order to determine the possible SM ranges of the physical phase  $\phi$ , in addition one has to determine  $M_{12}$ .

## 6 New physics

Finally we would like to address the question, whether new physics (NP) can enhance  $\Gamma_{12}$ . In the  $B_s$  system it is argued [45] that  $\Gamma_{12}$  is due to real intermediate states, so one cannot have sizeable NP contributions. Moreover, the mixing phase in the  $B_s$  system is close to zero, so the cosine in eq. (2.2) is close to one and therefore NP can at most modify  $\phi$ , which results in lowering the value of  $\Delta\Gamma$  compared to the SM prediction. In principle there is a loophole in the above argument. To  $\Gamma_{12}$  also  $\Delta B = 1$  penguin operators contribute, whose Wilson coefficients might be modified by NP effects. But these effects would also change all tree-level  $B$  decays. Since this is not observed at a significant scale, it is safe to say that within the hadronic uncertainties  $\Gamma_{12} = \Gamma_{12}^{\text{SM}}$  and therefore the argument of [45] holds.



**Figure 3.** The enhancement factor  $\Gamma_{12}^{\text{SM4}}/\Gamma_{12}^{\text{SM3}}$  using the possible values for  $V_{\text{CKM4}}$  found in [42]. The color encoded scale denotes the logarithm of the number of allowed parameter points of  $V_{\text{CKM4}}$ .

Since in the  $D^0$  system the QCD uncertainties are much larger, also the possible effects might be larger but not dramatic. The peculiarity of the  $D^0$  system — the leading term in the HQE is strongly suppressed due to GIM cancellation — gives us however a possibility to enhance  $\Gamma_{12}$  by a large factor, if we manage to soften the GIM cancellation. This might be accommodated either by weakening the  $\text{SU}(3)_F$  suppression in the first two terms of eq. (4.1), see, e.g., Petrov et al. [29], or by enhancing the CKM factors of the last two terms in eq. (4.1). The latter can be realized in a model with an additional fourth fermion family (SM4). The usual CKM matrix is replaced by a four dimensional one ( $V_{\text{CKM4}}$ ) and the unitary condition now reads  $\lambda_d + \lambda_s + \lambda_b + \lambda_{b'} = 0$ . Eq. (4.1) is replaced by

$$\Gamma_{12} = -\lambda_s^2 (\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd}) + 2\lambda_s (\lambda_b + \lambda_{b'}) (\Gamma_{sd} - \Gamma_{dd}) - (\lambda_b + \lambda_{b'})^2 \Gamma_{dd}.$$

In [42] an exploratory study of the allowed parameter space of  $V_{\text{CKM4}}$  was performed and as expected only very small modifications of  $\lambda_d$  and  $\lambda_s$  are experimentally allowed. In almost all physical applications these modifications are numerically much smaller than the corresponding hadronic uncertainties and therefore invisible. However, in the  $D^0$  mixing system it might happen, that all dominant contributions cancel and only these modifications survive. In the SM the first two terms of eq. (4.1) are numerical equal. In the SM4 the numerical hierarchy depends on the possible size of  $\lambda_{b'}$ , see eq. (6.1). In particular, it was found in [42] that currently a value of  $\lambda_{b'}$  of the order  $\lambda^3$  is not excluded. This means that the second term of eq. (6.1) could be greatly enhanced by the existence of a fourth family and also the third term would now become relevant. Using experimentally allowed data points for  $V_{\text{CKM4}}$  from [42] we have determined the possible values of  $\Gamma_{12}$  in the SM4 from eq. (6.1); enhancement factors of a few tens are possible, see figure (3). It should be noted that this enhancement is very sensitive to the exact values of the bag parameters; e.g. a reduction of  $B_S$  by 30% triples the range of possible enhancement factors.

## 7 Discussion and conclusions

In this work we investigated the leading HQE contribution to the absorptive part of  $D$  mixing and the leading corrections to it. We found that the size of these corrections is

large, but not dramatic ( $\approx 50\%$  QCD,  $\approx 30\%$   $1/m_c$ ). So we see no signal for a breakdown of the OPE and it seems that the HQE might be appropriate to estimate the order of magnitude of  $\Gamma_{12}$ . For a further investigation of the question of the convergence of the HQE for the case of the  $D$  system a systematic study of  $D$  meson lifetimes within that framework might be very helpful.

Above we have explained in detail that  $\Gamma_{12}^{D=6,7}$  gives, due to huge GIM cancellations, a value of  $y$  which is about a factor of about 10000 smaller than the current experimental expectation, but it can have a large phase and it also does not vanish in the exact  $SU(3)_F$  limit. The possibility of a sizeable phase in  $\Gamma_{12}$  is a new result. The important question is now, how big can the physical phase  $\phi$  be?

Due to the peculiarity of the  $D^0$  system — the extreme GIM cancellations — it might also be possible that the HQE result is dominated by  $D = 9$  and  $D = 12$  contributions, if there the GIM cancellations are less pronounced, see e.g. [28]. To quantify that possibility these higher dimensional corrections have to be determined explicitly, i.e. beyond the estimates presented in [26–28]. This calculation will also show what values are possible for the phase of  $\Gamma_{12}$ . To determine finally the physical phase  $\phi$  one has to determine in addition  $M_{12}$ .

Currently, estimates about the possible size of the phase in  $D$  mixing are typically based on the assumption that the  $\lambda_b$ -term can be neglected. The remaining contribution proportional to  $\lambda_s^2(\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd})$  leaves almost no space for a CP-violating phase. In this paper we have shown that this widely used assumption is wrong for the leading ( $D=6,7$ ) HQE term: the  $\lambda_b$ -term is as sizeable as the pure  $\lambda_s$  term and it can have a huge phase. Although we can not proof at the current stage that a sizeable phase can survive after all corrections are included in the HQE calculation, we think one should meet claims that already small values for a  $D$  mixing phase are an unambiguous sign for new physics with some caution. This question has to be studied in more detail both within the inclusive approach — as explained above — and within the exclusive approach.

To become more concrete, let us speculate: if the first term in eq. (4.1) is enhanced by a factor  $\eta^3$  due to higher dimensional terms in the HQE, where the GIM-suppression is much weaker compared to the leading HQE terms, as advocated e.g. in [28], the same effect — but less pronounced — will also be active in the second term of eq. (4.1). For the first term, the authors of [28] see a possibility of enhancing the effect of order  $z^2$  up to  $\sqrt{z}$  (by 3 powers of  $\sqrt{z}$ ). For the second term we expect an enhancement from  $z$  to  $\sqrt{z}$  (by 1 power of  $\sqrt{z}$ ). So the enhancement of the second term of eq. (4.1) is a factor of the order of  $\eta$ . Now we can make some numerical predictions for  $y$  and  $\text{Im } \Gamma_{12} / \text{Re } \Gamma_{12}$  depending on the numerical enhancement factor  $\eta$  - we use only values for  $\eta$  that are within the estimates of [28]:

$\eta$	$\frac{\text{Im}\Gamma_{12}}{\text{Re}\Gamma_{12}}$	$y^{\text{Theory}}$	Deviation from exp. central value of $y$
18.24	0.32%	0.73%	$0\sigma$
14.54	0.50%	0.37%	$2\sigma$
11.64	0.78%	0.19%	$3\sigma$
4.33	5.62%	0.01%	$4\sigma$

(7.1)

We see that the new contribution, we have found in this paper could lead to a relative size of the imaginary part of  $\Gamma_{12}$  of the order of up to 1%, while the absolute value of  $\Gamma_{12}$  is close to the experimental value of  $y$ . A size of the physical phase  $\phi$  of the order of 1% within the standard model is typically excluded in the literature, where the correction we have worked out in this paper is not taken into account. But we would like to warn the reader: for the numbers in eq. (7.1) we have purely speculated about the size of the enhancement factor  $\eta$  — but it is within the estimates of [28]. Its concrete value has to be determined by a calculation of the higher order terms in the HQE. This task, however, is beyond the scope of the current paper; we plan to investigate this question in [44].

Finally we have shown that new physics, in particular a small violation of the unitarity of the  $3 \times 3$  CKM matrix, can enhance the leading HQE prediction for  $\Gamma_{12}$  by a double-digit factor.

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## References

- [1] M. Gell-Mann and A. Pais, *Behavior of neutral particles under charge conjugation*, *Phys. Rev.* **97** (1955) 1387 [SPIRES].
- [2] J.H. Christenson, J.W. Cronin, V.L. Fitch and R. Turlay, *Evidence for the  $2\pi$  decay of the  $K_2^0$  meson*, *Phys. Rev. Lett.* **13** (1964) 138 [SPIRES].
- [3] M.K. Gaillard and B.W. Lee, *Rare decay modes of the  $K$ -mesons in gauge theories*, *Phys. Rev. D* **10** (1974) 897 [SPIRES].
- [4] E598 collaboration, J.J. Aubert et al., *Experimental observation of a heavy particle  $J$* , *Phys. Rev. Lett.* **33** (1974) 1404 [SPIRES].
- [5] SLAC-SP-017 collaboration, J.E. Augustin et al., *Discovery of a narrow resonance in  $e^+e^-$  annihilation*, *Phys. Rev. Lett.* **33** (1974) 1406 [SPIRES].
- [6] ARGUS collaboration, H. Albrecht et al., *Observation of  $B^0$ - $\bar{B}^0$  mixing*, *Phys. Lett. B* **192** (1987) 245 [SPIRES].
- [7] UA1 collaboration, C. Albajar et al., *Search for  $B^0$ - $\bar{B}^0$  oscillations at the CERN proton-anti-proton collider. 2*, *Phys. Lett. B* **186** (1987) 247 [Erratum *ibid.* **B 197** (1987) 565] [SPIRES].
- [8] D0 collaboration, S. Abachi et al., *Observation of the top quark*, *Phys. Rev. Lett.* **74** (1995) 2632 [hep-ex/9503003] [SPIRES].
- [9] CDF collaboration, F. Abe et al., *Observation of top quark production in  $\bar{p}p$  collisions*, *Phys. Rev. Lett.* **74** (1995) 2626 [hep-ex/9503002] [SPIRES].

- [10] CDF collaboration, T. Aaltonen et al., *First flavor-tagged determination of bounds on mixing-induced CP-violation in  $B_s^0 \rightarrow J/\psi\phi$  decays*, *Phys. Rev. Lett.* **100** (2008) 161802 [[arXiv:0712.2397](#)] [[SPIRES](#)].
- [11] D0 collaboration, V.M. Abazov et al., *Measurement of  $B_s^0$  mixing parameters from the flavor-tagged decay  $B_s^0 \rightarrow J/\psi\phi$* , *Phys. Rev. Lett.* **101** (2008) 241801 [[arXiv:0802.2255](#)] [[SPIRES](#)].
- [12] CKMFITTER GROUP collaboration, J. Charles et al., *CP violation and the CKM matrix: assessing the impact of the asymmetric B factories*, *Eur. Phys. J. C* **41** (2005) 1 [[hep-ph/0406184](#)] [[SPIRES](#)].
- [13] A. Lenz and U. Nierste, *Theoretical update of  $B_s$ - $\bar{B}_s$  mixing*, *JHEP* **06** (2007) 072 [[hep-ph/0612167](#)] [[SPIRES](#)].
- [14] M. Bona et al., *New physics from flavour*, [arXiv:0906.0953](#) [[SPIRES](#)].
- [15] M. Kobayashi and T. Maskawa, *CP violation in the renormalizable theory of weak interaction*, *Prog. Theor. Phys.* **49** (1973) 652 [[SPIRES](#)].
- [16] S.L. Glashow, J. Iliopoulos and L. Maiani, *Weak interactions with lepton-hadron symmetry*, *Phys. Rev. D* **2** (1970) 1285 [[SPIRES](#)].
- [17] BELLE collaboration, M. Staric et al., *Evidence for  $D^0$ - $\bar{D}^0$  mixing*, *Phys. Rev. Lett.* **98** (2007) 211803 [[hep-ex/0703036](#)] [[SPIRES](#)].
- [18] BELLE collaboration, K. Abe et al., *Measurement of  $D^0$ - $\bar{D}^0$  mixing in  $D^0 \rightarrow K_S\pi^+\pi^-$  decays*, *Phys. Rev. Lett.* **99** (2007) 131803 [[arXiv:0704.1000](#)] [[SPIRES](#)].
- [19] BABAR collaboration, B. Aubert et al., *Evidence for  $D^0$ - $\bar{D}^0$  mixing*, *Phys. Rev. Lett.* **98** (2007) 211802 [[hep-ex/0703020](#)] [[SPIRES](#)].
- [20] CDF collaboration, T. Aaltonen et al., *Evidence for  $D^0$ - $\bar{D}^0$  mixing using the CDF II detector*, *Phys. Rev. Lett.* **100** (2008) 121802 [[arXiv:0712.1567](#)] [[SPIRES](#)].
- [21] HEAVY FLAVOR AVERAGING GROUP collaboration, E. Barberio et al., *Averages of b-hadron and c-hadron properties at the end of 2007*, [arXiv:0808.1297](#) [[SPIRES](#)].
- [22] M.A. Shifman and M.B. Voloshin, *Preasymptotic effects in inclusive weak decays of charmed particles*, *Sov. J. Nucl. Phys.* **41** (1985) 120 [[SPIRES](#)].
- [23] M.A. Shifman and M.B. Voloshin, *Hierarchy of lifetimes of charmed and beautiful hadrons*, *Sov. Phys. JETP* **64** (1986) 698 [*Zh. Eksp. Teor. Fiz.* **91** (1986) 1180] [[SPIRES](#)].
- [24] I.I.Y. Bigi, N.G. Uraltsev and A.I. Vainshtein, *Nonperturbative corrections to inclusive beauty and charm decays: qCD versus phenomenological models*, *Phys. Lett. B* **293** (1992) 430 [*Erratum ibid.* **B 297** (1993) 477] [[hep-ph/9207214](#)] [[SPIRES](#)].
- [25] A. Lenz, *Mixing and lifetimes of b-hadrons*, *AIP Conf. Proc.* **1026** (2008) 36 [[arXiv:0802.0977](#)] [[SPIRES](#)].
- [26] H. Georgi,  *$D$ - $\bar{D}$  mixing in heavy quark effective field theory*, *Phys. Lett. B* **297** (1992) 353 [[hep-ph/9209291](#)] [[SPIRES](#)].
- [27] T. Ohl, G. Ricciardi and E.H. Simmons,  *$D$ - $\bar{D}$  mixing in heavy quark effective field theory: the sequel*, *Nucl. Phys. B* **403** (1993) 605 [[hep-ph/9301212](#)] [[SPIRES](#)].
- [28] I.I.Y. Bigi and N.G. Uraltsev,  *$D^0$ - $\bar{D}^0$  oscillations as a probe of quark-hadron duality*, *Nucl. Phys. B* **592** (2001) 92 [[hep-ph/0005089](#)] [[SPIRES](#)].

- [29] E. Golowich, S. Pakvasa and A.A. Petrov, *New physics contributions to the lifetime difference in  $D^0$ - $\bar{D}^0$  mixing*, *Phys. Rev. Lett.* **98** (2007) 181801 [[hep-ph/0610039](#)] [[SPIRES](#)].
- [30] A.F. Falk, Y. Grossman, Z. Ligeti, Y. Nir and A.A. Petrov, *The  $D^0$ - $\bar{D}^0$  mass difference from a dispersion relation*, *Phys. Rev. D* **69** (2004) 114021 [[hep-ph/0402204](#)] [[SPIRES](#)].
- [31] A.F. Falk, Y. Grossman, Z. Ligeti and A.A. Petrov, *SU(3) breaking and  $D^0$ - $\bar{D}^0$  mixing*, *Phys. Rev. D* **65** (2002) 054034 [[hep-ph/0110317](#)] [[SPIRES](#)].
- [32] M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, *Next-to-leading order QCD corrections to the lifetime difference of  $B/s$  mesons*, *Phys. Lett. B* **459** (1999) 631 [[hep-ph/9808385](#)] [[SPIRES](#)].
- [33] M. Beneke, G. Buchalla, A. Lenz and U. Nierste, *CP asymmetry in flavour-specific  $B$  decays beyond leading logarithms*, *Phys. Lett. B* **576** (2003) 173 [[hep-ph/0307344](#)] [[SPIRES](#)].
- [34] M. Ciuchini, E. Franco, V. Lubicz, F. Mescia and C. Tarantino, *Lifetime differences and CP-violation parameters of neutral  $B$  mesons at the next-to-leading order in QCD*, *JHEP* **08** (2003) 031 [[hep-ph/0308029](#)] [[SPIRES](#)].
- [35] M. Beneke, G. Buchalla and I. Dunietz, *Width difference in the  $B_s$ - $\bar{B}_s$  system*, *Phys. Rev. D* **54** (1996) 4419 [[hep-ph/9605259](#)] [[SPIRES](#)].
- [36] A.S. Dighe, T. Hurth, C.S. Kim and T. Yoshikawa, *Measurement of the lifetime difference of  $B/d$  mesons: possible and worthwhile?*, *Nucl. Phys. B* **624** (2002) 377 [[hep-ph/0109088](#)] [[SPIRES](#)].
- [37] E. Golowich and A.A. Petrov, *Short distance analysis of  $D^0$ - $\bar{D}^0$  mixing*, *Phys. Lett. B* **625** (2005) 53 [[hep-ph/0506185](#)] [[SPIRES](#)].
- [38] M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, *The  $B^+$ - $B_d^0$  lifetime difference beyond leading logarithms*, *Nucl. Phys. B* **639** (2002) 389 [[hep-ph/0202106](#)] [[SPIRES](#)].
- [39] V. Lubicz and C. Tarantino, *Flavour physics and Lattice QCD: averages of lattice inputs for the Unitarity Triangle Analysis*, *Nuovo Cim.* **123B** (2008) 674 [[arXiv:0807.4605](#)] [[SPIRES](#)].
- [40] D. Becirevic, V. Giménez, G. Martinelli, M. Papinutto and J. Reyes,  *$B$ -parameters of the complete set of matrix elements of  $\Delta B = 2$  operators from the lattice*, *JHEP* **04** (2002) 025 [[hep-lat/0110091](#)] [[SPIRES](#)].
- [41] M. Gorbahn, S. Jager, U. Nierste and S. Trine, *The supersymmetric Higgs sector and  $B^-$   $B$ bar mixing for large  $\tan \beta$* , [arXiv:0901.2065](#) [[SPIRES](#)].
- [42] M. Bobrowski, A. Lenz, J. Riedl and J. Rohrwild, *How much space is left for a new family of fermions?*, *Phys. Rev. D* **79** (2009) 113006 [[arXiv:0902.4883](#)] [[SPIRES](#)].
- [43] CKMFITTER GROUP collaboration, J. Charles et al., *CP violation and the CKM matrix: assessing the impact of the asymmetric  $B$  factories*, *Eur. Phys. J. C* **41** (2005) 1 [[hep-ph/0406184](#)] [[SPIRES](#)].
- [44] M. Bobrowski, V.M. Braun, A. Lenz, U. Nierste and T. Prill in preparation.
- [45] Y. Grossman, *The  $B/s$  width difference beyond the standard model*, *Phys. Lett. B* **380** (1996) 99 [[hep-ph/9603244](#)] [[SPIRES](#)].