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Nonanticommutativity in the presence of a boundary

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In this paper we consider nonanticommutative field theories in $\mathcal{N} = 2$ superspace formalism on three-dimensional manifolds with a boundary. We modify the original Lagrangian in such a way that it preserves half the supersymmetry even in the presence of a boundary. We also analyze the partial breaking of supersymmetry caused by nonanticommutativity between fermionic coordinates. Unlike in four dimensions, in three dimensions a theory with $\mathcal{N} = 1/2$ supersymmetry cannot be obtained by a nonanticommutative deformation of an $\mathcal{N} = 1$ theory. However, in this paper we construct a three-dimensional theory with $\mathcal{N} = 1/2$ supersymmetry by studying a combination of nonanticommutativity and boundary effects, starting from $\mathcal{N} = 2$ supersymmetry.

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I. INTRODUCTION

There are many interesting deformations of field theories which can be realized on the world volume of D-branes in various string theory backgrounds. The presence of a constant $NS - NS$ B-field background gives rise to noncommutativity [1–6]. The concept of noncommutative coordinates can be extended to superspace [7–10]. This concept of spacetime noncommutativity can be extended to include more general deformations of the (super-)Poincaré algebra [11–13], and for Grassmann coordinates this leads to nonanticommutativity [7,8,14–21]. Such deformations are realized on the world volume of D-branes in RR backgrounds [22–26], and the gravity dual of such a field theory has been constructed in Ref. [27]. Also, a graviphoton background gives rise to a noncommutativity between spacetime and superspace coordinates [23,28–31]. Noncommutative deformations generated by the $NS - NS$ and graviphoton backgrounds do not break any supersymmetry. However, the nonanticommutative deformation breaks the supersymmetry corresponding to the deformed superspace coordinate. In four dimensions it is possible to break the supersymmetry generated by one Weyl supercharge while leaving the supersymmetry generated by the other intact. Thus, if we start from a theory with $\mathcal{N} = 1$ supersymmetry in four dimensions and perform a nonanticommutative deformation, we can arrive at a theory with $\mathcal{N} = 1/2$ supersymmetry. Now, a theory with $\mathcal{N} = 1$ supersymmetry in four dimensions has the same amount of supersymmetry as a theory with $\mathcal{N} = 2$ supersymmetry in three dimensions. Thus, from a three-dimensional perspective this corresponds to breaking the supersymmetry from $\mathcal{N} = (1, 1)$ to $\mathcal{N} = (0, 1)$ or $\mathcal{N} = (1, 0)$ supersymmetry [32]. Furthermore, a theory with $\mathcal{N} = 1$ supersymmetry in three dimensions has the same amount of supersymmetry as a theory with $\mathcal{N} = 2$ supersymmetry in two dimensions. However, we cannot carry this argument further, as there are not enough degrees of freedom to perform this nonanticommutative deformation in two dimensions without breaking all supersymmetry. So, we cannot partially break supersymmetry to $\mathcal{N} = 1/2$ supersymmetry in three dimensions by nonanticommutative deformations alone. However, we will show in this paper that we can obtain a theory with $\mathcal{N} = 1/2$ supersymmetry in three dimensions by combining the nonanticommutative deformations with boundary effects.

In determining the Euler-Lagrange equations of a Lagrangian field theory one encounters terms which can be written as a surface integral. In theories that are at most quadratic in derivatives this is the only contribution that remains when an action is varied and its Euler- Lagrange equations are used. Thus, in the presence of a boundary one must specify boundary conditions that ensure the above surface term vanishes. The boundary breaks translation invariance and so it also breaks supersymmetry. In fact, the supersymmetric transformation of most theories transforms into a surface term and this generates a boundary term in the presence of a boundary. This problem can be eliminated by imposing boundary conditions under which this boundary term vanishes. However, the bulk theory can also be modified by introducing a boundary action such that its supersymmetry transformations exactly cancel the boundary term generated by the supersymmetry transformations of the original bulk action. This way half of the original supersymmetry is preserved. This has been done for three-dimensional theories in $\mathcal{N} = 1$ superspace [33,34]. Such boundary effects for $M2$-branes have also been analyzed in $\mathcal{N} = 1$ superspace [35–37]. In this paper we will first generalize these results to a three-dimensional theory in $\mathcal{N} = 2$ superspace and then analyze the nonanticommutative deformation of this theory. We will thus be able to arrive at a theory with $\mathcal{N} = 1/2$ supersymmetry in three dimensions. As nonanticommutativity occurs due to the coupling of D-branes to RR fields, it would be interesting to study a non-Abelian Born-Infeld Lagrangian in this nonanticommutative superspace. In this context the boundary effects analyzed in this paper could be used to study a system of $D2$-branes ending on $D4$-branes in the presence of RR fields. With this motivation we consider the example of a flat-space Born-Infeld Lagrangian [38–41] coupled to scalar matter in three-dimensional $\mathcal{N} = 2$ superspace. See Ref. [42] for a useful review of three-dimensional superspace.

II. BOUNDARY SUPERSYMMETRY

In this section we review the method of introducing a boundary action in order to preserve half the supersymmetry without explicit boundary conditions. We also define our notation. This was originally carried out for $\mathcal{N} = 1$ in Ref. [33] and extended to $\mathcal{N} = 2$ in Ref. [35].

We start from an $\mathcal{N} = 1$ superfield $\phi(\theta)$, where θ is a two component Grassmann parameter. It transforms under supersymmetric transformations as

$$\delta\phi(\theta) = \epsilon^a Q_a \phi(\theta), \quad \text{where } Q_a = \partial_a - (\gamma^\mu\theta)_a\partial_\mu,$$

is the generator of $\mathcal{N} = 1$ supersymmetry. If in component form the field has the following form

$$\phi(\theta) = p + q\theta + r\theta^2,$$

then the supersymmetric transformation can be written as

$$\begin{aligned} \delta p &= \epsilon^a q_a, \\ \delta q_a &= -\epsilon_a r + (\gamma^\mu\epsilon)_a \partial_a p, \\ \delta r &= \epsilon^a(\gamma^\mu\partial_\mu)^b_a q_b. \end{aligned} \tag{1}$$

Now the Lagrangian for an $\mathcal{N} = 1$ theory can be written in terms of such a superfield as

$$\mathcal{L} = D^2[\phi(\theta)]_{\theta=0}, \tag{2}$$

where $D^2 = D^a D_a/2$ and $D_a = \partial_a + (\gamma^\mu\theta)_a\partial_\mu$. This Lagrangian is invariant under these supersymmetric transformations on a manifold without boundaries. However, if there is a boundary, say at $x_3 = 0$, then the supersymmetric transformations of the Lagrangian are given by $\delta\mathcal{L} = -\partial_3(\epsilon\gamma^3 q)$. This breaks the supersymmetry of the resultant theory. However, if we add or subtract the following term $\mathcal{L}_b = \partial_3[\phi(\theta)]_{\theta=0}$, to the original Lagrangian, then the supersymmetric transformation of the total Lagrangian is given by

$$\delta[\mathcal{L} \pm \mathcal{L}_b] = \pm 2\partial_3\epsilon^\pm q_\mp, \tag{3}$$

where $q_\pm = P^\pm q \equiv (1 \pm \gamma^3)q/2$. Hence, we can preserve the supersymmetry generated by either $\epsilon^- Q_+$ or $\epsilon^+ Q_-$ by adding or subtracting $\mathcal{L}_b$ to $\mathcal{L}$. However, we cannot preserve all the supersymmetry. Thus, the Lagrangian which preserves the supersymmetry corresponding to $\epsilon^- Q_+$ is $\mathcal{L}^+$, and to $\epsilon^+ Q_-$ is $\mathcal{L}^-$ where

$$\mathcal{L}^\pm = \mathcal{L} \pm \mathcal{L}_b = (D^2 \mp \partial_3)[\phi]_{\theta=0}. \tag{4}$$

After reviewing boundary supersymmetric theories in $\mathcal{N} = 1$ superspace formalism, we present the straightforward generalization of these results to theories with $\mathcal{N} = 2$ supersymmetry. Thus, we will analyze a Lagrangian with $\mathcal{N} = 2$ supersymmetry,

$$\mathcal{L} = D_1^2 D_2^2[\Phi(\theta_1, \theta_2)]_{\theta_1=\theta_2=0}, \tag{5}$$

where $D_{1a} = \partial_{1a} + (\gamma^\mu\theta_1)_a\partial_\mu$, and $D_{2a} = \partial_{2a} + (\gamma^\mu\theta_2)_a\partial_\mu$, are the standard covariant derivatives which commute with Q_{1a} and Q_{2a}, and Φ is an $\mathcal{N} = 2$ scalar superfield. We can decompose a superfield with $\mathcal{N} = 2$ supersymmetry, into two copies of $\mathcal{N} = 1$ superfields. So, we can write $\Phi(\theta_1, \theta_2)$ as

$$\begin{aligned} \Phi(\theta_1, \theta_2) &= p_1(\theta_1) + q_1(\theta_1)\theta_2 + r_1(\theta_1)\theta_2^2 \\ &= p_2(\theta_2) + q_2(\theta_2)\theta_1 + r_2(\theta_2)\theta_1^2, \end{aligned} \tag{6}$$

where $p_1(\theta_1)$, $p_2(\theta_2)$, $q_1(\theta_1)$, $q_2(\theta_2)$, $r_1(\theta_1)$, $r_2(\theta_2)$ are $\mathcal{N} = 1$ superfields in there own right. So, we can write the Lagrangian as

$$\mathcal{L} = D_1^2[r_1(\theta_1)]_{\theta_1=0} = D_2^2[r_2(\theta_2)]_{\theta_2=0}. \tag{7}$$

The supersymmetry of this theory will be generated by the super-charges $Q_{1a} = \partial_{1a} - (\gamma^\mu\theta_1)_a\partial_\mu$, and $Q_{2a} = \partial_{2a} - (\gamma^\mu\theta_2)_a\partial_\mu$. In absence of a boundary this theory is invariant under the supersymmetry generated by both Q_{1a} and Q_{2a}. However, in the presence of a boundary the supersymmetric transformations generated by both Q_{1a} and Q_{2a} generate boundary terms. So, on the boundary we can again preserve only half of the total supersymmetry. Thus, with a boundary we can only preserve the supersymmetry generated by either $\epsilon^{1+}Q_-$ or $\epsilon^{1-}Q_+$, and by either $\epsilon^{2+}Q_-$ or $\epsilon^{2-}Q_+$. Now, after adding suitable boundary terms that preserve half of the supersymmetry, we get the following four possible Lagrangians:

$$\mathcal{L}^{1\pm 2\pm} = d^{1\pm}d^{2\pm}[\Phi]_{\theta_1=\theta_2=0}, \tag{8}$$

where

$$d^{1\pm} = (D_1^2 \pm \partial_3), \qquad d^{2\pm} = (D_2^2 \pm \partial_3). \tag{9}$$

Now the Lagrangian corresponding to $d^{1\pm}d^{2\pm}$ preserves the supersymmetry generated by $\epsilon^{1\pm}Q_{1\mp}$ and $\epsilon^{2\pm}Q_{2\mp}$. It may be noted this Lagrangian preserves only half the supersymmetry because the supersymmetry generated by $\epsilon^{1\mp}Q_{1\pm}$ and $\epsilon^{2\mp}Q_{2\pm}$ is broken by it.

III. MATTER-BORN-INFELD ACTION

In this paper we will consider the specific example of a matter-Born-Infeld Lagrangian in $\mathcal{N} = 2$ superspace, motivated by the potential application to $D2$-branes ending on $D4$-branes in the presence of RR fields. This Lagrangian will be used for analyzing the partial breaking of supersymmetry due to a combination of nonanticommutative deformations and boundary effects. We first define two spinor superfields Γ_{1a} and Γ_{2a} which we use to construct covariant derivatives for matter fields Φ and $\bar{\Phi}$,

$$\begin{aligned} \nabla_{1a}\Phi &= D_{1a}\Phi - i\Gamma_{1a}\Phi, & \nabla_{2a}\Phi &= D_{2a}\Phi - i\Gamma_{2a}\Phi, \\ \nabla_{1a}\bar{\Phi} &= D_{1a}\bar{\Phi} + i\bar{\Phi}\Gamma_{1a}, & \nabla_{2a}\bar{\Phi} &= D_{2a}\bar{\Phi} + i\bar{\Phi}\Gamma_{2a}. \end{aligned} \tag{10}$$

We can also construct the following field strengths from these spinor superfields:

$$\begin{aligned} \omega_{1a} &= \frac{1}{2}D_1^b D_{1a}\Gamma_{1b} - \frac{i}{2}\{\Gamma_1^b, D_{1b}\Gamma_{1a}\} - \frac{1}{6}[\Gamma_1^b, \{\Gamma_{1b}, \Gamma_{1a}\}], \\ \omega_{2a} &= \frac{1}{2}D_2^b D_{2a}\Gamma_{2b} - \frac{i}{2}\{\Gamma_2^b, D_{2b}\Gamma_{2a}\} - \frac{1}{6}[\Gamma_1^2, \{\Gamma_{2b}, \Gamma_{2a}\}]. \end{aligned} \tag{11}$$

The Born-Infeld Lagrangian can now be written as [38–41]

$$\begin{aligned} \mathcal{L}_{bi} &= D_1^2[\omega_1^a \omega_{1a}]_{\theta_1=0} + D_2^2[\omega_2^a \omega_{2a}]_{\theta_2=0} \\ &\quad + D_1^2 D_2^2[\omega_1^a \omega_{1a}\omega_2^b \omega_{2b} B(K_1, K_2)]_{\theta_1=\theta_2=0}, \end{aligned} \tag{12}$$

where $K_1 = D_1^2[\omega_1^a \omega_{1a}]$, $K_2 = D_2^2[\omega_2^a \omega_{2a}]$ and B must satisfy a constraint equation. For the Abelian Born-Infeld Lagrangian the constraint can be solved and $B(K_1, K_2)$ can be written as

$$\begin{aligned} B(K_1, K_2) = \frac{1}{2}\Big[& 1 - (K_1 + K_2) \\ & + \sqrt{4(1 - (K_1 + K_2) + (K_1 - K_2)^2}\Big]^{-1}. \end{aligned} \tag{13}$$

Now to write the matter Lagrangian we define $\theta_a = (\theta_{1a} - i\theta_{2a})$, $\bar{\theta}_a = (\theta_{1a} + i\theta_{2a})$, and $\partial_a = (\partial_{1a} + i\partial_{2a})/2$, $\bar{\partial}_a = (\partial_{1a} - i\partial_{2a})/2$. We similarly define $D_a = \frac{1}{2}(D_{1a} + iD_{2a})$, $\bar{D}_a = \frac{1}{2}(D_{1a} - iD_{2a})$, and the covariant derivatives $\nabla_a = (\nabla_{1a} + i\nabla_{2a})/2$, $\bar{\nabla}_a = (\nabla_{1a} - i\nabla_{2a})/2$. So, we can write the matter-Born-Infeld Lagrangian on a manifold without boundaries as

$$\begin{aligned} \mathcal{L} &= D_1^2 D_2^2[\nabla^a \Phi \bar{\nabla}_a \bar{\Phi} + \mathcal{V}[\Phi, \bar{\Phi}]]_{\theta_1=\theta_2=0} \\ &\quad + D_1^2[\omega_1^a \omega_{1a}]_{\theta_1=0} + D_2^2[\omega_2^a \omega_{2a}]_{\theta_2=0} \\ &\quad + D_1^2 D_2^2[\omega_1^a \omega_{1a}\omega_2^b \omega_{2b} B(K_1, K_2)]_{\theta_1=\theta_2=0}, \end{aligned} \tag{14}$$

where $\mathcal{V}[\Phi, \bar{\Phi}]$ is a potential term. This Lagrangian is invariant under the following gauge transformation,

$$\Gamma_{1a} \to u\nabla_{1a}u^{-1}, \qquad \Gamma_{2a} \to u\nabla_{2a}u^{-1}. \tag{15}$$

Now, in the presence of a boundary, we can preserve $\mathcal{N} = 1$ supersymmetry by modifying this Lagrangian to

$$\begin{aligned} \mathcal{L}^{1\pm 2\pm} &= d^{1\pm}d^{2\pm}[\nabla^a \Phi \bar{\nabla}_a \bar{\Phi} + \mathcal{V}[\Phi, \bar{\Phi}]]_{\theta_1=\theta_2=0} \\ &\quad + d^{1\pm}[\omega_1^a \omega_{1a}]_{\theta_1=0} + d^{2\pm}[\omega_2^a \omega_{2a}]_{\theta_2=0} \\ &\quad + d^{1\pm}d^{2\pm}[\omega_1^a \omega_{1a}\omega_2^b \omega_{2b} B(K_1, K_2)]_{\theta_1=\theta_2=0}, \end{aligned} \tag{16}$$

where $d^{1\pm}d^{2\pm}$ are given by Eq. (9). These Lagrangians are still invariant under the gauge transformation given by Eq. (15). It may be noted that if the gauge part included Chern-Simons terms then this theory would not be gauge invariant, but gauge invariance could be restored by the addition of further boundary terms which would cancel the boundary piece generated by the gauge transformation. This has been considered in the context of the Aharony-Bergman-Jafferis-Maldacena model in Refs. [35,36,43] in component form, in $\mathcal{N} = 1$ superspace, and for the Abelian case in $\mathcal{N} = 2$ superspace. However, the full boundary action for the non-Abelian $\mathcal{N} = 2$ case has not yet been constructed.

IV. BOUNDARY SUPERCHARGES AND BOUNDARY SUPERFIELDS

In this section we describe the relation between the bulk and boundary supersymmetry [33,35]. For $\mathcal{N} = 1$ supersymmetry, the bulk supercharge Q_a can also be decomposed as $\epsilon^a Q_a = \epsilon(P^+ + P^-)Q = \epsilon^+ Q_- + \epsilon^- Q_+$. These bulk supercharges can be written as $Q_- = Q'_- + \theta_- \partial_3$, and $Q_+ = Q'_+ - \theta_+ \partial_3$. where $Q'_\pm$ are the boundary supercharges given by $Q'_\pm = \partial_\pm - \gamma^s \theta_\mp \partial_s$. Here s is the index for the coordinates along the boundary, i.e., compared to μ the case $\mu = 3$ is excluded for a boundary at fixed x^3. Now by definition $Q_\pm$ are the generators of the half supersymmetry of the bulk fields and $Q'_\pm$ are the standard supersymmetry generators for the boundary fields. We also define $M_+ = \exp(+\theta_- \theta_+ \partial_3)$ and $M_- = \exp(-\theta_+ \theta_- \partial_3)$ and let M_+^{-1} and M_-^{-1} be their inverses. Now we have

$$Q'_- = M_-^{-1} Q_- M_-, \qquad Q'_+ = M_+^{-1} Q_+ M_+. \tag{17}$$

If we write

$$\phi = M_+ \phi'_+ \quad \text{or} \quad \phi = \mathrm{M}_- \phi'_- \tag{18}$$

where $\phi'_\pm$ are given in terms of boundary superfields a' and b' by $\phi'_+ = [a'(\theta_-) + \theta_+ b'(\theta_-)]$ or $\phi'_- = [a'(\theta_+) + \theta_- b'(\theta_+)]$, then

$$\begin{aligned} \epsilon^+ Q_- \phi &= M_- \epsilon^{+\prime} Q'_- \phi'_- \quad \text{or} \\ \epsilon^- Q_+ \phi &= M_+ \epsilon^{-\prime} Q'_+ \phi'_+, \end{aligned} \tag{19}$$

where $Q'_- \phi'_- = Q'_- a'(\theta_+) - \theta_- Q'_- b'(\theta_+)$ and $Q'_+ \phi'_+ = Q'_+ a'(\theta_-) - \theta_+ Q'_+ b'(\theta_-)$. This gives the decomposition of ϕ into boundary superfields depending on which supersymmetry is preserved.

Now, for $\mathcal{N} = 2$ supersymmetry, the bulk supercharges Q_{na} (where $n = 1, 2$) can also be decomposed as $\epsilon^{na} Q_{na} = \epsilon^{n+} Q_{n-} + \epsilon^{n-} Q_{n+}$, and written as $Q_{n-} = Q'_{n-} + \theta_{n-} \partial_3$ and $Q_{n+} = Q'_{n+} - \theta_{n+} \partial_3$, where $Q'_{n\pm}$ are the boundary supercharges given by $Q'_{n\pm} = \partial_{n\pm} - \gamma^s \theta_{n\mp} \partial_s$.

We again define $M_{n+} = \exp(+\theta_{n-}\theta_{n+}\partial_3)$ and $M_{n-} = \exp(-\theta_+\theta_-\partial_3)$ and let M_{n+}^{-1} and M_{n-}^{-1} be there inverses. Then

$$Q'_{n-} = M_{n-}^{-1} Q_{n-} M_{n-}, \qquad Q'_{n+} = M_{n+}^{-1} Q_{n+} M_{n+}. \tag{20}$$

As for $\mathcal{N} = 1$, we write, depending on the supersymmetry preserved,

$$\Phi = M_{2\pm} M_{1\pm} \Phi'_{2\pm 1\pm}, \tag{21}$$

where $\Phi'_{2\pm 1\pm}$ decompose into boundary superfields. Now we have one of the following:

$$\begin{aligned}
\epsilon^{1-} Q_{1+}\Phi &= M_{2\pm} M_{1+} \epsilon^{1-\prime} Q'_{1+} \Phi'_{2\pm 1+},\\
\epsilon^{1+} Q_{1-}\Phi &= M_{2\pm} M_{1-} \epsilon^{1+\prime} Q'_{1-} \Phi'_{2\pm 1-},\\
\epsilon^{2-} Q_{2+}\Phi &= M_{2+} M_{1\pm} \epsilon^{2-\prime} Q'_{2+} \Phi'_{2+ 1\pm},\\
\epsilon^{2+} Q_{2-}\Phi &= M_{2-} M_{1\pm} \epsilon^{2+\prime} Q'_{2-} \Phi'_{2- 1\pm},
\end{aligned} \tag{22}$$

describing the (preserved) supersymmetry transformation of the boundary superfields.

We will now analyze the superalgebra for a bulk $\mathcal{N} = 2$ supersymmetric theory in the presence of a boundary. In the absence of a boundary

$$\{Q_{na}, Q_{mb}\} = 2\gamma^\mu_{ab}\partial_\mu \delta_{nm}, \qquad \{D_{na}, D_{mb}\} = -2\gamma^\mu_{ab}\partial_\mu\delta_{nm}, \qquad \{Q_{na}, D_{mb}\} = 0. \tag{23}$$

Now, defining $D_{n\pm a} = (P_\pm)^b_a D_{nb}$, and similarly for $Q_{n\pm a}$, we can write the full superalgebra in a form adapted to the presence of a boundary as

$$\begin{aligned}
\{Q_{n+a}, Q_{m+b}\} &= 2(\gamma^s_{ab}P_+)\partial_s\delta_{nm}, & \{D_{n+a}, D_{m+b}\} &= -2(\gamma^s_{ab}P_+)\partial_s\delta_{nm},\\
\{Q_{n-a}, Q_{m-b}\} &= 2(\gamma^s_{ab}P_-)\partial_s\delta_{nm}, & \{D_{n-a}, D_{m-b}\} &= -2(\gamma^s_{ab}P_-)\partial_s\delta_{nm},\\
\{Q_{n+a}, Q_{m-b}\} &= -2(P_-)_{ab}\partial_3\delta_{nm}, & \{D_{n+a}, D_{m-b}\} &= 2(P_-)_{ab}\partial_3\delta_{nm},\\
&\{Q_{n\pm a}, D_{m\pm b}\} = 0.
\end{aligned} \tag{24}$$

Contracting $D_{n-a}D_{n+b} = (P_-)_{ab}(\partial_3 - D^2)$ and $D_{n+a}D_{n-b} = -(P_-)_{ab}(\partial_3 + D^2)$ with C^{ab} and using $(P_-)^a_a = 1$, we can also write Eq. (9) as

$$d^{1+} = D_{1+}D_{1-}, \qquad d^{2+} = D_{2+}D_{2-}, \tag{25}$$

$$d^{1-} = D_{1-}D_{1+}, \qquad d^{2-} = D_{2-}D_{2+}. \tag{26}$$

Thus, we can see how the Lagrangian with the measure $d^{1\pm}d^{2\pm}$ preserves the right amount of supersymmetry on the boundary. This is because the Lagrangian corresponding to Eq. (16) can be written as

$$\begin{aligned}
\mathcal{L}^{1\pm 2\pm} = {}& D_{2\pm}D_{2\mp}D_{1\pm}D_{1\mp}[\nabla^a\Phi\bar{\nabla}_a\bar{\Phi} + \mathcal{V}[\Phi,\bar{\Phi}]]_{\theta_1=\theta_2=0}\\
&+ D_{2\pm}D_{2\mp}D_{1\pm}D_{1\mp}\\
&\times[\omega^a_1\omega_{1a}\omega^b_2\omega_{2b}B(K_1,K_2)]_{\theta_{1\mp}=\theta_{2\mp}=0}\\
&+ D_{2\pm}D_{2\mp}[\omega^a_1\omega_{1a}]_{\theta_{2\mp}=0}\\
&+ D_{1\pm}D_{1\mp}[\omega^b_2\omega_{2b}]_{\theta_{1\mp}=0}.
\end{aligned} \tag{27}$$

This Lagrangian is again invariant under the gauge transformation given by Eq. (15). We can write it in terms of boundary superfields as

$$\begin{aligned}
\mathcal{L}^{1\pm 2\pm} = {}& -D'_{2\pm}D'_{1\pm}[\Psi'_{1\mp 2\mp}]_{\theta_{1\mp}=\theta_{2\mp}=0}\\
&+ D'_{2\pm}[\Psi'_{2\mp}]_{\theta_{2\mp}=0} + D'_{1\pm}[\Psi'_{1\mp}]_{\theta_{1\mp}=0},
\end{aligned} \tag{28}$$

where

$$\begin{aligned}
\Psi'_{1\mp 2\mp} = {}& D'_{2\mp}D'_{1\mp}[\nabla^{a\prime}\Phi'\bar{\nabla}'_a\bar{\Phi}' + \mathcal{V}[\Phi',\bar{\Phi}']]_{\theta_{1\mp}=\theta_{2\mp}=0}\\
&+ D'_{2\mp}D'_{1\mp}[\omega^{a\prime}_1\omega'_{1a}\omega^{b\prime}_2\omega'_{2b}B'(K'_1,K'_2)]_{\theta_{1\mp}=\theta_{2\mp}=0},\\
\Psi'_{2\mp} = {}& D'_{2\mp}[\omega^{a\prime}_2\omega'_{2a}]_{\theta_{2\mp}=0},\\
\Psi'_{1\mp} = {}& D'_{1\mp}[\omega^{a\prime}_1\omega'_{1a}]_{\theta_{1\mp}=0}.
\end{aligned} \tag{29}$$

The boundary measure only contains $D'_{2\pm}D'_{1\pm}$. Thus, on the boundary only the supersymmetry generated by $\epsilon^{1\pm\prime}Q'_{1\mp}$ and $\epsilon^{2\pm\prime}Q'_{2\mp}$ is preserved. Furthermore, on the boundary $\epsilon^{1\pm\prime}Q'_{1\mp}$ and $\epsilon^{2\pm\prime}Q'_{2\mp}$ act as independent supercharges. So, we obtain a boundary theory with either (1, 1) supersymmetry or (2, 0) supersymmetry.

V. NONANTICOMMUTATIVITY

In this section we will consider the effect of imposing nonanticommutativity between the Grassmann coordinates [7,8,12–18]. While more general deformations of the super-Poincaré algebra are possible, we only consider nonanticommutativity, i.e., we do not consider $[x^\mu, x^\nu] \neq 0$ or $[x^\mu, \theta] \neq 0$.

We first promote $\theta^{n\pm a}$ to operators $\hat{\theta}^{n\pm a}$ and impose the most general form of nonanticommutativity for an $\mathcal{N} = 2$ supersymmetric theory in three dimensions,

$$\{\hat{\theta}^{n\pm a}, \hat{\theta}^{m\pm b}\} = C^{na\pm mb\pm}, \tag{30}$$

where $C^{na+nb+} = C^{na-nb-} = 0$. It may be noted that if we had started from a theory with $\mathcal{N} = 1$ supersymmetric in three dimensions, it would not be possible to partially break the supersymmetry. This is because in that case the only nonanticommutative deformation that could take place would be $\{\hat{\theta}^{+a}, \hat{\theta}^{-b}\} = C^{a+b-}$ which would break all the supersymmetry of the theory. Hence, it is not possible to obtain in this way a theory with $\mathcal{N} = 1/2$ supersymmetry in three dimensions. However, we will show in the next section we can obtain a theory with $\mathcal{N} = 1/2$ supersymmetry in three dimensions by combining nonanticommutativity with boundary effects.

We can write the Fourier transformation of a scalar superfield on the undeformed superspace as

$$\Phi(\theta^{n\pm}) = \int d^4\pi \exp(-\pi^{n\pm a}\theta_{n\pm a})\Phi(\pi_{n\pm}), \tag{31}$$

where $\exp(-\pi^{n\pm a}\theta_{n\pm a}) = \exp -(\pi^{1+a}\theta_{1+a} + \pi^{1-a}\theta_{1-a} + \pi^{2+a}\theta_{2+a} + \pi^{2-a}\theta_{2-a})$. Now, we can use Weyl ordering and also express the Fourier transformation of a scalar superfield on the deformed superspace as

$$\hat{\Phi}(\hat{\theta}^{n\pm}) = \int d^4\pi \exp(-\pi^{n\pm a}\hat{\theta}_{n\pm a})\Phi(\pi_{n\pm}). \tag{32}$$

Here we have considered the most general form for nonanticommutativity and it breaks all the supersymmetry. Nonanticommutative deformations which partially break the supersymmetry can be obtained from this general case by setting some of the $C^{na\pm mb\pm}$ to zero. We can express the product of two fields on this deformed superspace as

$$\begin{aligned}\hat{\Phi}_1(\hat{\theta}^{n\pm})\hat{\Phi}_2(\hat{\theta}^{n\pm}) = \int d^4\pi d^4\tilde{\pi} \exp(-(\pi + \tilde{\pi})^{n\pm a}\hat{\theta}_{n\pm a})\\ \times \exp(\Delta)\hat{\Phi}_1(\pi^{n\pm})\hat{\Phi}_2(\tilde{\pi}^{n\pm}),\end{aligned} \tag{33}$$

where $\Delta = -C^{na\pm mb\pm}\pi_{n\pm a}\tilde{\pi}_{n\pm b}/2$. This motivates the definition of the star product between ordinary functions. Thus, the nonanticommutativity replaces all the product of fields by star products as follows [7,8,14]:

$$\begin{aligned}\Phi_1(\theta^{n\pm}) \star \Phi_2(\theta^{n\pm}) = \Phi_1(\theta^{n\pm}) \exp\left(\frac{1}{2}C^{na\pm mb\pm}\overleftarrow{\partial}_{n\pm a}\overrightarrow{\tilde{\partial}}_{n\pm b}\right)\\ \times \Phi_2(\tilde{\theta}^{n\pm})_{\tilde{\theta}^{n\pm}=\theta^{n\pm}}\end{aligned} \tag{34}$$

with $\overrightarrow{\partial}_a\theta^b = \delta_a^b$ while $\theta^b\overleftarrow{\partial}_a = -\delta_a^b$, etc.

Now the nonanticommutative bulk supercharges Q_{na} can again be written as $\epsilon^{na}Q_{na} = \epsilon^{n+}Q_{n-} + \epsilon^{n-}Q_{n+}$. We now define $M_{n+\star} = \exp(+\theta_{n-}\theta_{n+}\partial_3)_\star$ and $M_{n-\star} = \exp(-\theta_+\theta_-\partial_3)_\star$ and let $M_{n+\star}^{-1}$ and $M_{n-\star}^{-1}$ be their inverses. Here all the products are understood as star products. Now we can write the relation between the bulk and boundary supercharges in this deformed superspace as

$$\begin{aligned}Q'_{n-} &= M_{n-\star}^{-1} \star Q_{n-}M_{n-\star},\\ Q'_{n+} &= M_{n+\star}^{-1} \star Q_{n+}M_{n+\star}.\end{aligned} \tag{35}$$

Thus, we can also write a relation between the bulk and boundary superfields in this deformed superspace as

$$\Phi = M_{2\pm\star} \star M_{1\pm\star} \star \Phi'_{2\pm1\pm}, \tag{36}$$

where $\Phi'_{2\pm1\pm}$ are boundary superfields.

We have obtained boundary projections of the nonanticommutative superfields. Now we define nonanticommutative field strengths as

$$\begin{aligned}\omega_{1a\star} &= \frac{1}{2}D_1^bD_{1a}\Gamma_{1b} - \frac{i}{2}\{\Gamma_1^b, D_{1b}\Gamma_{1a}\}_\star\\ &\quad - \frac{1}{6}[\Gamma_1^b, \{\Gamma_{1b}, \Gamma_{1a}\}_\star]_\star,\\ \omega_{2a\star} &= \frac{1}{2}D_2^bD_{2a}\Gamma_{2b} - \frac{i}{2}\{\Gamma_2^b, D_{2b}\Gamma_{2a}\}_\star\\ &\quad - \frac{1}{6}[\Gamma_1^2, \{\Gamma_{2b}, \Gamma_{2a}\}_\star]_\star.\end{aligned} \tag{37}$$

The Born-Infeld Lagrangian can now be written as

$$\begin{aligned}\mathcal{L}_{bi} &= D_1^2[\omega_{1\star}^a \star \omega_{1a\star}]_{\theta_1=0} + D_2^2[\omega_{2\star}^a \star \omega_{2a\star}]_{\theta_2=0}\\ &\quad + D_1^2D_2^2[\omega_{1\star}^a \star \omega_{1a\star} \star \omega_{2\star}^b \star \omega_{2b\star}\\ &\quad \star B_\star(K_{1\star}, K_{2\star})]_{\theta_1=\theta_2=0},\end{aligned} \tag{38}$$

where $K_{1\star} = D_1^2[\omega_{1\star}^a \star \omega_{1a\star}]$ and $K_2 = D_2^2[\omega_{2\star}^a \star \omega_{2a\star}]$. Now the Lagrangian for this nonanticommutative theory will be obtained by replacing all products of fields in the Lagrangian given by Eq. (16) with star products,

$$\begin{aligned}\mathcal{L} &= d^{1\pm}d^{2\pm}[\nabla^a \star \Phi \star \bar{\nabla}_a \star \bar{\Phi} + \mathcal{V}[\Phi, \bar{\Phi}]_\star]_{\theta_1=\theta_2=0}\\ &\quad + d^{1\pm}d^{2\pm}[\omega_{1\star}^a \star \omega_{1a\star} \star \omega_{2\star}^b \star \omega_{2b\star}\\ &\quad \star B_\star(K_{1\star}, K_{2\star})]_{\theta_1=\theta_2=0}d^{1\pm}[\omega_{1\star}^a \star \omega_{1a\star}]_{\theta_1=0}\\ &\quad + d^{2\pm}[\omega^{2a\star} \star \omega_{2a\star}]_{\theta_2=0}.\end{aligned} \tag{39}$$

Here the nonanticommutative potential term $\mathcal{V}[\Phi, \bar{\Phi}]_\star$ is again obtained by replacing the product of superfields in the original potential term by star products. We can again write it in terms of boundary superfields as

$$\begin{aligned}\mathcal{L}^{1\pm2\pm} &= -D'_{2\pm}D'_{1\pm}[\Psi'_{1\mp2\mp\star}]_{\theta_{1\mp}=\theta_{2\mp}=0}\\ &\quad + D'_{2\pm}[\Psi'_{2\mp\star}]_{\theta_{2\mp}=0} + D'_{1\pm}[\Psi'_{1\mp\star}]_{\theta_{1\mp}=0},\end{aligned} \tag{40}$$

where

$$\begin{aligned}\Psi'_{1\mp2\mp\star} &= D'_{2\mp}D'_{1\mp}[\nabla^{a\prime} \star \Phi' \star \bar{\nabla}'_a \star \bar{\Phi}' + \mathcal{V}[\Phi', \bar{\Phi}']_\star\\ &\quad + \omega_{1\star}^{a\prime} \star \omega'_{1a\star} \star \omega_{2\star}^{b\prime} \star \omega'_{2b\star}\\ &\quad \star B'_\star(K'_{1\star}, K'_{2\star})]_{\theta_{1\mp}=\theta_{2\mp}=0},\\ \Psi'_{2\mp\star} &= D'_{2\mp}[\omega_{2\star}^{a\prime} \star \omega'_{2a\star}]_{\theta_{2\mp}=0},\\ \Psi'_{1\mp\star} &= D'_{1\mp}[\omega_{1\star}^{a\prime} \star \omega'_{1a\star}]_{\theta_{1\mp}=0}.\end{aligned} \tag{41}$$

This Lagrangian is invariant under the nonanticommutative gauge transformation given by

$$\Gamma_{1a} \to u \star \nabla_{1a} \star u^{-1}, \qquad \Gamma_{2a} \to u \star \nabla_{2a} \star u^{-1}. \quad (42)$$

The boundary measure corresponding to $d^{1\pm} d^{2\pm}$ contains only $D'_{2\pm} D'_{1\pm}$ and so the boundary effects again break half the supersymmetry. However, now the nonanticommutativity also partially breaks supersymmetry. By combining the boundary effects with nonanticommutativity, it is possible to obtain theories with $\mathcal{N} = 1$ supersymmetry or $\mathcal{N} = 1/2$ supersymmetry in the bulk. In the next session we will analyze various combinations of these boundary effects with nonanticommutativity.

VI. PARTIALLY BREAKING SUPERSYMMETRY

Various amount of supersymmetry can be broken by a combination of boundary effects with nonanticommutativity. The projection of the generators of bulk supersymmetry again reproduces the correct generators of the boundary supersymmetry. Thus, we have one of

$$\begin{aligned}
\epsilon^{1-} Q_{1+} \Phi &= M_{2\pm\star} \star M_{1+\star} \star \epsilon^{1-\prime} Q'_{1+} \Phi'_{2\pm 1+}, \\
\epsilon^{1+} Q_{1-} \Phi &= M_{2\pm\star} \star M_{1-\star} \star \epsilon^{1+\prime} Q'_{1-} \Phi'_{2\pm 1-}, \\
\epsilon^{2-} Q_{2+} \Phi &= M_{2+\star} \star M_{1\pm\star} \star \epsilon^{2-\prime} Q'_{2+} \Phi'_{2+1\pm}, \\
\epsilon^{2+} Q_{2-} \Phi &= M_{2-\star} \star M_{1\pm\star} \star \epsilon^{2+\prime} Q'_{2-} \Phi'_{2-1\pm},
\end{aligned} \quad (43)$$

with the combination of supersymmetry generators which are left unbroken depending both on the choice of the boundary projection and the nonanticommutative deformation.

If all the remaining components of $C^{na\pm mb\pm}$ are nonzero then all supersymmetry is broken. In fact, all supersymmetry will also be broken if the nonzero components of $C^{na\pm mb\pm}$ break the supersymmetry that is preserved by the boundary action. For example, after introducing the boundary, if the measure is changed to $d^{1+} d^{2+}$ then the supersymmetry corresponding to Q_{1-} and Q_{2-} is left unbroken, but if the nonanticommutativity is then imposed in such a way that C^{1a-2b-} is nonzero then all the supersymmetry will be broken.

It is also possible to impose nonanticommutativity in such a way that it breaks the same supersymmetry that would be broken by the boundary. In this case half the supersymmetry of the original theory survives. Thus, if the measure changes to $d^{1+} d^{2+}$ and C^{1a+2b+} is nonzero, then the supersymmetry corresponding to Q_{1-} and Q_{2-} remains unbroken. So, we get an $\mathcal{N} = 1$ theory in the bulk which corresponds to $\mathcal{N} = (0, 2)$ supersymmetry on the boundary, with the star product defined with only C^{1a+2b+} being nonzero. Similarly, if the measure changes to $d^{1-} d^{2-}$ and C^{1a-2b-} is nonzero, we preserve $\mathcal{N} = (2, 0)$ supersymmetry on the boundary. However, if the measure changes to $d^{1+} d^{2-}$ and $C^{1a1+2b-}$ is nonzero, or if the measure changes to $d^{1-} d^{2+}$ and C^{1a-2b+} is nonzero, then in both these cases we get $\mathcal{N} = (1, 1)$ supersymmetry on the boundary.

The most interesting case is when half the supersymmetry left over after introducing the boundary is broken. For example, if the measure is changed to $d^{1+} d^{2+}$ and C^{a1+b2-} is nonzero, only the supersymmetry corresponding to Q_{1-} is left unbroken. This corresponds to $\mathcal{N} = (0, 1)$ on the boundary and thus $\mathcal{N} = 1/2$ in the bulk. Now for the same measure, if instead C^{1a-2b+} is nonzero, then the supersymmetry corresponding to Q_{2-} is left unbroken which again corresponds to $\mathcal{N} = (0, 1)$ on the boundary and $\mathcal{N} = 1/2$ in the bulk. Similarly, if we change the measure to $d^{1-} d^{2-}$ and let either C^{1a+2b-} or C^{1a-2b+} be nonzero, then $\mathcal{N} = (1, 0)$ supersymmetry is preserved on the boundary. Further possibilities correspond to $d^{1-} d^{2+}$ or $d^{1+} d^{2-}$ with C^{1a+2b+} or C^{1a-2b-} nonzero. Again the bulk theory preserves $\mathcal{N} = 1/2$ supersymmetry corresponding to $\mathcal{N} = (0, 1)$ or $\mathcal{N} = (1, 0)$ on the boundary.

VII. CONCLUSION

In this paper we have shown how a three-dimensional $\mathcal{N} = 1/2$ theory can be realized by starting with an $\mathcal{N} = 2$ theory and breaking supersymmetry through a nonanticommutative deformation together with the inclusion of a boundary. Most of the analysis is general but we also discussed a Born-Infeld Lagrangian coupled to a matter field using $\mathcal{N} = 2$ superspace, motivated by the potential application to $D2$-branes.

In summary, the supersymmetry of the $\mathcal{N} = 2$ theory was broken by the presence of a boundary. However, we modified the original theory by adding a boundary action to it such that the supersymmetric transformation of this boundary piece exactly cancels the boundary term generated from the supersymmetric transformation of the bulk theory. This way we were able to preserve half the supersymmetry of the original theory, i.e., $\mathcal{N} = 1$ in three dimensions. Depending on the choice of boundary action, this corresponds to a two-dimensional theory with $\mathcal{N} = (1, 1)$ or $\mathcal{N} = (2, 0)$ supersymmetry. We then analyzed the breaking of supersymmetry due to nonanticommutative deformations, including the correct boundary projections of the bulk superfields in this general nonanticommutative superspace. We showed that, depending on the precise anticommutative deformation, it is possible to construct theories with $\mathcal{N} = 1$ or $\mathcal{N} = 1/2$ supersymmetry in three dimensions. This was done by combining the breaking of supersymmetry by the boundary with the breaking of supersymmetry by the nonanticommutativity. The reason both effects are required is that, unlike what happens in four dimensions, it is not possible to obtain a theory with $\mathcal{N} = 1/2$ supersymmetry in three dimensions by only imposing nonanticommutativity.

One interesting application of the methods in this paper would be to a nonanticommutative deformation of a system of $M2$-branes with boundary on an $M5$-brane. The low energy Lagrangian for multiple $M2$-branes is thought to be described by the Aharony-Bergman-Jafferis-Maldacena Chern-Simons matter theory [44–47] which can be studied in $\mathcal{N} = 2$ superspace. The formalism developed in the present paper could be directly applied to this system. However, there is an additional complication that the gauge transformation of the Chern-Simons Lagrangian generates a surface term. Thus, we would need to add another boundary term (or choose some suitable boundary conditions) such that the combined gauge variation of the original theory along with this boundary piece will be gauge invariant. This has been done in the bosonic case [43] and in $\mathcal{N} = 1$ superspace [35–37] where it was shown that, not unexpectedly, the Lagrangian on the boundary is (a gauged version of) a Wess-Zumino-Witten model. It is expected that a similar result will hold in $\mathcal{N} = 2$ superspace but the non-Abelian Chern-Simons action is more complicated in this case, and the detailed construction of the boundary action has not yet been performed.

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