

# A dynamical model of supernova feedback: gas outflows from the interstellar medium

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## ABSTRACT

We present a dynamical model of supernova feedback which follows the evolution of pressurized bubbles driven by supernovae in a multiphase interstellar medium (ISM). The bubbles are followed until the point of break-out into the halo, starting from an initial adiabatic phase to a radiative phase. We show that a key property which sets the fate of bubbles in the ISM is the gas surface density, through the work done by the expansion of bubbles and its role in setting the gas scaleheight. The multiphase description of the ISM is essential, and neglecting it leads to order-of-magnitude differences in the predicted outflow rates. We compare our predicted mass loading and outflow velocities to observations of local and high-redshift galaxies and find good agreement over a wide range of stellar masses and velocities. With the aim of analysing the dependence of the mass loading of the outflow,  $\beta$  (i.e. the ratio between the outflow and star formation rates), on galaxy properties, we embed our model in the galaxy formation simulation, GALFORM, set in the  $\Lambda$  cold dark matter framework. We find that a dependence of  $\beta$  solely on the circular velocity, as is widely assumed in the literature, is actually a poor description of the outflow rate, as large variations with redshift and galaxy properties are obtained. Moreover, we find that below a circular velocity of  $\approx 80 \text{ km s}^{-1}$ , the mass loading saturates. A more fundamental relation is that between  $\beta$  and the gas scaleheight of the disc,  $h_g$ , and the gas fraction,  $f_{\text{gas}}$ , as  $\beta \propto h_g^{1.1} f_{\text{gas}}^{0.4}$ , or the gas surface density,  $\Sigma_g$ , and the gas fraction, as  $\beta \propto \Sigma_g^{-0.6} f_{\text{gas}}^{0.8}$ . We find that using the new mass loading model leads to a shallower faint-end slope in the predicted optical and near-IR galaxy luminosity functions.

**Key words:** supernovae: general – ISM: bubbles – ISM: supernova remnants – galaxies: evolution – galaxies: formation – galaxies: ISM.

## 1 INTRODUCTION

An outstanding problem in astrophysics is to understand how galaxies form in dark matter (DM) haloes. The problem is highly non-linear: the stellar mass function of galaxies differs substantially from the DM halo mass function, with the stellar mass function being shallower at the low-mass end and steeper at the high-mass end than the halo mass function (see Baugh 2006). The main physical driver of these differences is thought to be gas cooling and feedback (Larson 1974; Rees & Ostriker 1977; White & Rees 1978; Dekel & Silk 1986; White & Frenk 1991; Cole et al. 2000; Bower et al. 2006; Croton et al. 2006). Feedback from supernovae (SNe) and active galactic nuclei (AGN) is thought to suppress star formation (SF) in low and high stellar mass galaxies, respectively, lowering

the cold baryon fraction in these galaxies (e.g. Fukugita, Hogan & Peebles 1998; Mandelbaum et al. 2006; Liu et al. 2010).

Observations suggest that SN-driven outflows are common in galaxies (e.g. Martin 1999; Heckman et al. 2000; Shapley et al. 2003; Rupke, Veilleux & Sanders 2005; Schwartz et al. 2006; Sato et al. 2009; Weiner et al. 2009; Chen et al. 2010; Rubin et al. 2010; Banerji et al. 2011; see Veilleux, Cecil & Bland-Hawthorn 2005 for a review). In many cases, the inferred outflow rate exceeds the star formation rate (SFR; Martin 1999, 2005; Bouché et al. 2012), suggesting that SN feedback potentially has a large impact on galaxy evolution. The outflow rates inferred from absorption line studies correlate with galaxy properties such as SFRs and near-ultraviolet to optical colours, indicating that the influence of SN feedback might be differential with SFR and stellar mass (e.g. Martin 2005; Kornei et al. 2012). Photometric and kinematic observations of atomic hydrogen shells and holes in the interstellar medium (ISM) of local galaxies, in addition to SN remnants observed in X-rays and

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radio, imply that SNe lead to the formation of bubbles within the ISM and that the mass carried away is large and able to substantially change the gas reservoirs of galaxies (e.g. Heiles 1979; Maciejewski et al. 1996; Pidopryhora, Lockman & Shields 2007). SN feedback is also thought to be responsible for the metal enrichment of the intergalactic medium (IGM; e.g. see Putman, Peek & Joungh 2012 for a recent review).

Although the importance of SN feedback is clear from observations, the wide range of phenomenological models of SN feedback found in the literature reflect the uncertainty in how this process affects the ISM of galaxies and the IGM. The key questions are how does the mass loading of winds driven by SNe,  $\beta = \dot{M}_{\text{out}}/\text{SFR}$  (the ratio between the outflow rate,  $\dot{M}_{\text{out}}$ , and the SFR), depend on galaxy properties and what is the effect of winds on the evolution of galaxies?

A common assumption made in galaxy formation modelling is that the mass loading (sometimes called the ‘mass entrainment’ of the wind) depends exclusively on the energy input by SNe and the circular velocity of the galaxy, which is taken as a proxy for the depth of the gravitational potential well (e.g. White & Rees 1978; White & Frenk 1991). The specific form of the dependence contains adjustable parameters which are set by requiring that the model fits observations, such as the stellar mass function or luminosity function (LF), etc. (e.g. Cole et al. 2000; Springel et al. 2001; Benson et al. 2003; Croton et al. 2006). Simple, physically motivated forms for the explicit dependence of  $\beta$  on  $v_{\text{circ}}$  are based on arguments which invoke momentum-driven or energy-driven winds, corresponding to dependences of  $\beta \propto v_{\text{circ}}^{-1}$  and  $\beta \propto v_{\text{circ}}^{-2}$ , respectively (e.g. Silk 1997, 2003; Hatton et al. 2003; Murray, Quataert & Thompson 2005; Stringer et al. 2012; see Benson 2010 for a review).

Hydrodynamic simulations commonly assume constant wind velocities, adopting a kinetic feedback scheme in which SNe inject momentum to neighbouring particles, which are assumed to become dynamically decoupled from the other particles for a period of time (Springel & Hernquist 2003; Scannapieco et al. 2006; Dalla Vecchia & Schaye 2008; Narayanan et al. 2008; Schaye et al. 2010). Alternatively, simple scaling relations between the outflow velocity and the halo circular velocity may be assumed (e.g. Davé, Oppenheimer & Finlator 2011). These calculations can qualitatively reproduce the properties of disc galaxies (Scannapieco et al. 2012). The wind speed is a free parameter in these simulations with values of  $v_w \approx 300\text{--}1000 \text{ km s}^{-1}$  typically used [see Schaye et al. (2010) for an analysis of the impact of changing  $v_w$  on the predicted evolution of the global density of SFR in a hydrodynamical simulation, and Scannapieco et al. (2012) for a comparison between different simulations].

However, such a scheme where the wind speed,  $v_w$ , is constant fails to reproduce the stellar mass function, suggesting that this parametrization is too effective in intermediate stellar mass galaxies, but not efficient enough in low stellar mass galaxies (Crain et al. 2009; Davé et al. 2011; Bower, Benson & Crain 2012). In addition to these problems, Bower et al. (2012), Guo et al. (2013) and Weinmann et al. (2012) show that simple phenomenological recipes for SN feedback are not able to explain the observed shallow low-mass end of the stellar mass function (Drory et al. 2005; Li & White 2009; Marchesini et al. 2009; Caputi et al. 2011; Bielby et al. 2012). This problem can be alleviated by introducing an ad hoc dependence of the time it takes for the outflowing gas to fall back on to the galaxy on redshift (Henriques et al. 2013). A possible explanation for this is that such parametrizations do not accurately describe the complex process of outflows driven by SNe in the ISM

and their subsequent propagation through the hot halo gas around galaxies.

Creasey, Theuns & Bower (2013) analysed the effect of a single SN in the ISM by simulating a column through the disc of a galaxy with very high mass and spatial resolution. Creasey et al. varied the initial conditions in the disc with the aim of covering different gas surface densities and gas-to-stellar mass ratios, and found that the mass outflow rate depends strongly on the local properties of the ISM, such as the gas surface density. Similar conclusions were reached by Hopkins, Quataert & Murray (2012) in four simulations of individual galaxies including different types of feedback in addition to SN feedback. The SN feedback scheme used in Hopkins et al. was not fully resolved and hence depends on subgrid modelling of the momentum deposition of the different types of feedback. Regardless of the details of each simulation, both studies point to a breakdown of the classical parametrizations used for  $\beta$ . However, since the simulations of both Creasey et al. and Hopkins et al. cover a narrow range of environments, the generality of their results is not clear.

In this paper, we implement a fully numerical treatment of SN feedback due to bubbles inflated by SNe which expand into the ISM. We follow the bubbles during the adiabatic and radiative phases assuming spherical symmetry, starting in the star-forming regions in the ISM and continuing until the bubble breaks out of the galactic disc or is confined. The aims of this paper are (i) to study the effect of different physical processes on the expansion of bubbles, such as the multiphase ISM, the gravity from stars and DM, the temporal changes in the ambient pressure, etc., and (ii) to extend previous theoretical work by using the new dynamical SN feedback model in the cosmological semi-analytic model of galaxy formation, GALFORM. Semi-analytic models have the advantage of being able to simulate large cosmological volumes containing millions of galaxies over cosmic epochs and making multiwavelength predictions (Baugh 2006). This approach makes it possible to study a wide enough range of properties and epochs to reach robust conclusions about the dependence of  $\beta$  on galaxy properties and to characterize the combination of properties that best quantifies the mass outflow rates in galaxies.

Previous dynamical models of SN feedback in the context of cosmological galaxy formation have focused on the evolution of bubbles either in the ISM or in the hot halo. For instance, Larson (1974) (see also Monaco 2004b; Shu, Mo & Shu-DeMao 2005) implemented analytic solutions for the evolution of bubbles until their break-out from the ISM by neglecting gravity, external pressure and temporal changes in the ambient gas. Bertone, Stoehr & White (2005), Bertone, De Lucia & Thomas (2007) and Samui, Subramanian & Srianand (2008) followed the evolution of bubbles in the hot halo assuming an ad hoc mass outflow rate and wind velocity from the disc into the halo. Dekel & Silk (1986) implemented a simpler model which aimed to estimate the mass ejection rate from both the ISM and the halo, using analytic solutions for the evolution of bubbles in the ISM to calculate an average rate of mass injection from the ISM into the halo. Efstathiou (2000) went a step further, implementing bubble evolution in a multiphase ISM with the hot phase dominating the filling factor, using analytic solutions for the evolution of adiabatic bubbles. We improve upon previous calculations by including the effects of gravity, radiative losses, external pressure from the diffuse medium and temporal changes in the ambient gas on the expansion of bubbles, all embedded in a multiphase medium. We use the information about the radial profiles of galaxies to calculate mass outflow rates locally. In addition to the sophistication of our calculation, another key difference in

our work is that bubbles expand into the warm component of the ISM instead of the hot component, as is assumed in some previous work. This is motivated by the results from detailed simulations and observations in our Galaxy which point to a rather small volume filling factor of hot gas,  $\lesssim 20$  per cent, with little mass contained in this gas phase (e.g. Mac Low, McCray & Norman 1989; Ferrière 2001; de Avillez & Breitschwerdt 2004; see Haffner et al. 2009 for a review on the warm phase of the ISM).

In this paper, we focus on the ejection of gas from the disc and do not attempt to model the expansion of bubbles in the hot halo or the rate of gas ejection from the halo into the IGM. In Paper II, we will implement a full model of the expansion of bubbles in the hot halo, following a similar approach to that adopted in this paper, and analyse the rate at which mass and metals escape the halo and go into the IGM, and how this depends on galaxy and halo properties (Lagos, Baugh & Lacey, in preparation).

This paper is organized as follows. Section 2 describes the dynamical model of SN feedback and the evolution of individual bubbles in the ISM. Section 2.2 describes the calculation of the properties of the diffuse medium and how we locate giant molecular clouds (GMCs) in the disc. In Section 3, we describe how we include the full dynamical model of SN feedback in the galaxy formation simulation GALFORM. In Section 4, we analyse the properties of bubbles and the mass and metal outflow rate, and their dependence on galaxy properties. We also present analytic derivations of some of the relations found in this work, giving insight into the physics which sets the outflow rate. We study the physical regimes of SN feedback and compare with observations of mass outflow rates and velocities in galaxies. In Section 5, we present a new parametrization of the outflow rate that accurately describes the full dynamical calculation of SN feedback and compare this to parametrizations that are widely used in the literature. In Section 6, we show how the new SN feedback model affects the galaxy LF and the SFR density evolution. We discuss our results and present our conclusions in Section 7. In Appendix A, we describe how we calculate the recycled fraction and yield from SNe, in Appendix B we explain how we calculate the stellar and DM mass enclosed by bubbles, and in Appendix C we describe how we calculate the overall rates of break-out and confinement of bubbles in the ISM.

## 2 MODELLING SUPERBUBBLE EXPANSION DRIVEN BY SNE

In this section, we describe the physical treatment we apply to bubbles and their expansion in the ISM. We consider that galaxies have an ISM which is initially characterized by two gas phases: the diffuse, atomic phase and the dense, molecular phase. The molecular gas is assumed to be locked up in GMCs and stars are allowed to form only in these regions. We use the empirical relation proposed by Blitz & Rosolowsky (2006) which connects the atomic-to-molecular surface density ratio to the hydrostatic gas pressure (see Section 3.1 for details). We use the observed molecular SFR coefficient,  $\nu_{\text{SF}}$ , to calculate the rate at which stars form from molecular gas (e.g. Bigiel et al. 2008, 2011).

The onset of SF in GMCs results in SNe. SNe inject mechanical energy and momentum into the surrounding medium, which pressurizes the immediate region inflating a cavity of hot gas, called an SN-driven bubble. We follow the evolution of the bubbles from an initial adiabatic phase to a possible radiative phase. The interiors of bubbles correspond to a third phase in the ISM of galaxies: a hot, low-density gas phase. Bubbles start their expansion conserving energy, but soon after the expansion starts (after a cooling time),

the interiors of bubbles become radiative. Bubbles then enter into a pressure-driven phase, in which the interior gas is still hot and highly pressurized. Once this interior gas cools radiatively, bubbles continue their evolution conserving momentum.

The main considerations we take into account when following the evolution of bubbles are as follows.

- (i) The injection of energy by SNe lasts for a finite period of time, which corresponds to the lifetime of a GMC.
- (ii) The gravity of stars and DM is included and can decelerate the expansion of bubbles.
- (iii) Temporal changes are followed in the atomic, molecular, stellar and DM contents, with bubbles evolving in this dynamical environment.
- (iv) We allow bubbles to be offset from the centre of the galaxy but they are centred on the mid-plane of the disc. We therefore consider local properties when calculating the expansion of bubbles.
- (v) Metal enrichment in the ISM due to massive stars takes place through bubbles.
- (vi) We follow the radiative cooling in the interior of bubbles to make an accurate estimate of the transition between the adiabatic and radiative stages of bubble evolution.

We solve the equations describing the evolution of bubbles numerically to prevent having to apply restrictive assumptions to features we would like to test, such as the effect of ambient pressure and gravity on the expansion of bubbles. We make three key assumptions when solving for the evolution of bubbles.

- (i) SF taking place in a single GMC gives rise to a new generation of SNe. We assume that the group of SNe in a single GMC inflate a single bubble. Thus, each bubble is accelerated by a number of SNe, the value of which depends on the SFR in the GMC and the initial mass function (IMF) of stars.
- (ii) We assume that bubbles are spherically symmetric. Observations of SN remnants show that the geometry of bubbles is close to spherical in most cases (e.g. Green 2009). This assumption does not restrict the level of accuracy that can be added into the equations of momentum and energy describing the evolution of bubbles.
- (iii) We assume that bubbles expand only through the diffuse atomic medium and that the gas in GMCs is not affected by these expanding bubbles. This is motivated by the fact that GMCs are characterized by large gas densities which tend to reflect the energy carried out by bubbles rather than absorbing it (e.g. McKee & Cowie 1975; Elmegreen 1999). In addition, Dale, Ercolano & Bonnell (2012) and Walch et al. (2012) show that at the moment of explosion of massive stars, the surrounding gas has already been photoionized by the radiation emitted by those stars. Hopkins et al. (2012) show that this effect is also present in their simulations of individual galaxies. This implies that SNe can efficiently accelerate the surrounding diffuse gas, causing the adiabatic expansion of a bubble to last for longer.

In Section 2.1, we describe the three evolutionary stages for a single bubble outlined above and give the equations we use to determine the mass, radius, velocity and temperature of the expanding bubbles. In Section 2.2, we describe how we estimate the properties of GMCs and the diffuse medium, and how we connect these to the global properties of galaxies.

### 2.1 Expansion of a single bubble

Let us consider a bubble located at a distance  $d$  from the galactic centre and expanding in a diffuse medium characterized by density

$\rho_d$ , velocity dispersion  $\sigma_d$ , pressure  $P_d$ , internal energy density  $u_d$  and metallicity  $Z_g$ .

A single GMC has an SFR of  $\psi_{\text{GMC}}$  and lasts for a time  $\tau_{\text{life, GMC}}$ . Within the cloud, the rate of SNe events is  $\eta_{\text{SN}} \psi_{\text{GMC}}$ , where  $\eta_{\text{SN}}$  is the number of SNe per solar mass of stars formed. The latter depends on the IMF adopted. Individual SNe release  $E_{\text{SN}} = 10^{51}$  erg (Arnett et al. 1989; Woosley & Weaver 1995). With these definitions in mind, we set out the equations we use to follow the expansion of bubbles in the following three subsections.

### 2.1.1 The adiabatic expansion

The pressure generated by SNe can significantly exceed that of the ISM, producing a hot cavity. When radiative losses are negligible, the hot cavity evolves like a stellar wind bubble which cools adiabatically. The interior of the bubble is thermalized and its motion drives a shock into the ISM and starts to sweep up the surrounding gas (Ostriker & McKee 1988). The inner structure of the bubble corresponds to a thick shell of gas swept up from the ambient ISM. The top panel of Fig. 1 shows a schematic of the inner structure of bubbles in this stage, which we refer to with the label ‘ad’. The internal gas density profile is illustrated in the bottom-right corner.

The bubble at this stage is characterized by kinetic and thermal energies  $E_K$  and  $E_{\text{th}}$ , respectively, a radius  $R$  and an expansion speed  $v_s = dR/dt$ , which evolve with time. The total mass of the bubble,  $m_b$ , corresponds to the sum of the mass injected by SNe,  $m_{\text{inj}}$ , and the swept-up from the diffuse ISM,  $m_{\text{sw}}$ . The rate of mass injection depends on  $\psi_{\text{GMC}}$  and the fraction of the total mass that is returned to the medium by massive stars,  $R_{\text{SN}}$ , through  $\dot{m}_{\text{inj}} = R_{\text{SN}} \psi_{\text{GMC}}$ . Explicit expressions for  $\eta_{\text{SN}}$  and  $R_{\text{SN}}$  are given in Appendix A.

The expansion of the inflated bubble is described by the equations of energy and mass conservation,

$$E = E_{\text{th}} + E_K = \kappa_E m_b v_s^2 \quad (1)$$

$$\frac{dE}{dt} = \dot{E}_{\text{inj}} + 4\pi R^2 v_s \cdot \left( u_d - \rho_d \frac{G M_t(R, d)}{R} - \rho_t \frac{G m_b}{R} \right) \quad (2)$$

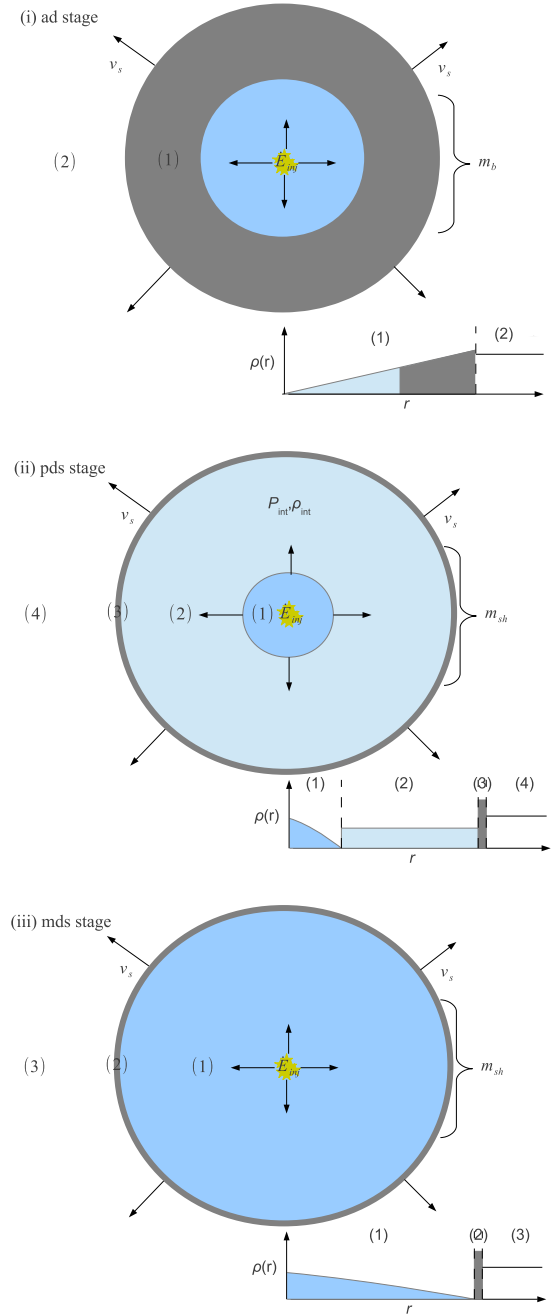
$$\frac{dm_b}{dt} = \dot{m}_{\text{inj}} + 4\pi R^2 \rho_d v_s. \quad (3)$$

Here,  $E$  is the total energy of the bubble in the adiabatic stage and  $\dot{E}_{\text{inj}}$  is the energy injection rate from SNe.

The total stellar plus DM mass enclosed by a bubble is  $M_t(R, d)$  and the average density of stars and DM within the bubble is  $\rho_t$ . Both terms act to decelerate the expansion of the bubble and come from the gravitational term  $\int_0^{V_b} \rho(r) v(r) g(r) dV$  in the energy conservation equation, where  $V_b$  is the volume enclosed by the bubble. The term  $G\rho_t m_b/R$  represents the increase of gravitational energy internal to the bubble due to the expanding shell (see Appendix B for a description of the calculation of the stellar and DM profiles and the mass enclosed in  $R$ ). Note that here we neglect the self-gravity of the bubble, given that  $m_b \ll M_t(R, d)$ .

The ratio  $E/(m_b v_s^2) = \kappa_E$  is calculated using a single power-law dependence of the velocity and density on the radius inside the bubble ( $\rho \propto r$  and  $v \propto r$ ), which gives  $\kappa_E = 3/4$ , for a ratio of specific heats of  $\gamma = 5/3$  (corresponding to a monatomic gas; Ostriker & McKee 1988). The energy injection rate is calculated from the SN rate,  $\eta_{\text{SN}} \psi_{\text{GMC}}$ , and the mechanical energy produced by an individual SN,  $E_{\text{SN}}$ ,

$$\dot{E}_{\text{inj}} = E_{\text{SN}} \eta_{\text{SN}} \psi_{\text{GMC}}. \quad (4)$$



**Figure 1.** Schematic of the inner structure of bubbles in three of the expansion stages considered in our dynamical model of SNe (see Section 2). SNe inject energy at a rate  $\dot{E}_{\text{inj}}$ , at the centre of the bubble and the ambient medium surrounds the bubble. A schematic of the gas densities as a function of radius depicting the inner structure of the bubble is shown in the bottom right of each panel. Top panel: the adiabatic (‘ad’) stage. The overpressurized region initially expands adiabatically, with the density increasing towards the edge of the bubble due to the swept-up gas, producing a thick shell. Middle panel: the pressure-driven snowplough (‘pds’) stage. Once the cooling time becomes shorter than the expansion time, the internal mass collapses to a shell. The interior mass fuelled by the injected mass from SNe remains adiabatic. The interior accelerates the outer shell through pressure. Bottom panel: the momentum-driven snowplough (‘mds’) stage. Once the cooling time in the interior becomes shorter than the expansion time in the ‘pds’ stage, the interior mass collapses to the shell and forms a bubble with a cooled, low-density interior. The mass and energy injected by SNe modify directly the motion of the outer shell through momentum injection.

Note that the pressure of the diffuse medium does not affect the energy of the bubbles, given that the diffuse ISM is static with respect to the bubbles. This means that there is no coherent motion in the ISM, only random motions characterized by a velocity dispersion  $\sigma_d$ .

For the rate of change in the mass internal to the bubble in equation (3), the right-hand side of the equation corresponds to the rate at which mass is incorporated from the diffuse medium into the bubble. We also keep track of the swept-up mass,  $m_{sw}$ , in order to subtract it from the diffuse ISM component when solving the SF equations (see Section 3),

$$\frac{dm_{sw}}{dt} = 4\pi R^2 \rho_d v_s. \quad (5)$$

Metals produced by nucleosynthesis in stars and ejected by SNe are added to the hot cavities. The rate of metal injection by SNe into the hot cavity depends on the SFR,  $\psi_{GMC}$ , the SN metal yield,  $p_{SN}$ , and the metallicity of the gas from which the stars were formed,  $Z_g$ , and is given by  $\dot{m}_{inj}^Z = (p_{SN} + R_{SN} Z_g) \psi_{GMC}$ . The term  $p_{SN} \psi_{GMC}$  corresponds to the newly synthesized metals and  $R_{SN} Z_g \psi_{GMC}$  to the metals present in the gas from which stars were made (see Appendix A for a description of how the recycled fraction and yield are calculated).

The rates of change in the mass of metals in the interior of bubbles and in the swept-up gas component are given by

$$\frac{dm_b^Z}{dt} = \dot{m}_{inj}^Z + \frac{dm_{sw}^Z}{dt}, \quad (6)$$

$$\frac{dm_{sw}^Z}{dt} = 4\pi R^2 \rho_d v_s Z_g. \quad (7)$$

Similarly to equation (5), it is possible to isolate the metals that have been incorporated into bubbles from the ISM,  $m_{sw}^Z$ . The internal metallicity of a bubble is therefore  $Z_b = m_b^Z/m_b$ . This way, the enrichment of the ISM will depend on the rate of bubble confinement and break-out.

The high temperature of the interior of bubbles results in a large sound speed,  $c_s \gg v_s$ , which makes the time for a sound wave to cross the interior much shorter than the expansion time. This causes the interior to be isobaric, characterized by a mean pressure  $P_b$ . We calculate the internal bubble pressure, temperature ( $T_b$ ) and cooling time ( $t_{cool}$ ), with the latter two properties defined just behind the shock at  $R$  (see the top panel of Fig. 1), using

$$P_b = \frac{2}{3} u = \frac{E_{th}}{2\pi R^3}, \quad (8)$$

$$T_b(R) = \frac{\mu m_H P_b}{\kappa_B \rho_b(R)}, \quad (9)$$

$$t_{cool}(R) = \frac{3 k_B T_b(R)}{n_b \Lambda(T_b(R), Z_g)}. \quad (10)$$

Here, the internal pressure of a bubble is calculated from its internal energy,  $u$ ,  $k_B$  is Boltzmann's constant, the mean molecular weight of a fully ionized gas (i.e. internal to the bubble) is  $\mu = 0.62$ ,  $m_H$  is the mass of a hydrogen atom,  $\Lambda(T_b, Z_b)$  corresponds to the cooling function and  $n_b = \rho_b(R)/(\mu m_H)$  is the volume number density behind the shock. We adopt the cooling function tables of Sutherland & Dopita (1993).

In order to set the correct initial conditions for the expansion in the adiabatic phase, we use the analytic solutions to the set of equations (1)–(3) given by Weaver et al. (1977). These analytic

solutions are obtained by neglecting the pressure and internal energy of the ambient medium, and the gravity of the stellar plus DM component and by assuming that the injected mass is small compared to the swept-up mass. We do this for an initial short period of time,  $t'$ , which we quantify in terms of the cooling time,  $t' \leq 0.1 t_{cool}$ . At  $t > t'$ , we follow the solution in the adiabatic stage numerically to accurately track the transition to the radiative phase. Our results are insensitive to the precise values of  $t'$ , provided that  $t' < 0.3 t_{cool}$ . The properties of bubbles during this early adiabatic period are

$$R_b(t) = \alpha \left( \frac{\dot{E}_{inj}}{\rho_d} \right)^{1/5} t^{3/5}, \quad (11)$$

$$v_s(t) = \frac{3}{5} \alpha \left( \frac{\dot{E}_{inj}}{\rho_d} \right)^{1/5} t^{-2/5}, \quad (12)$$

$$m_{sw}(t) = \frac{4\pi}{3} \alpha^3 \dot{E}_{inj}^{3/5} \rho_d^{2/5} t^{9/5}, \quad (13)$$

$$m_{sw}^Z(t) = m_{sw}(t) Z_g, \quad (14)$$

$$m_b(t) = m_{sw}(t) + R_{SN} \psi_{GMC} t, \quad (15)$$

$$m_b^Z(t) = m_{sw}^Z(t) + (p_{SN} + R_{SN} Z_g) \psi_{GMC} t, \quad (16)$$

where  $\alpha = 0.86$ . Equations (15) and (16) account for the injected metals and mass from the dying stars.

### 2.1.2 Pressure-driven snowplough expansion

As the temperature of the bubble decreases with time, the cooling time becomes sufficiently short so as to be comparable with the expansion time of the bubble. At this stage, radiative losses from the expanding thick shell can no longer be neglected and the shocked swept-up material quickly becomes thermally unstable and collapses into a thin, dense shell. The shocked mass ejected by SNe in the interior of the thin shell still conserves its energy and the bubble enters a pressure-driven phase. The energy injected by SNe modifies the thermal energy of the shocked interior. We refer to the properties of bubbles in this stage with the label 'pds', denoting pressure-driven snowplough (see the middle panel of Fig. 1).

In this phase, bubbles are characterized by the swept-up mass accumulated in a thin shell,  $m_{sh}$ , and an interior mass,  $m_{int}$ . The interior of the bubble is still isobaric, characterized by a mean pressure,  $P_{int}$ . We consider that the density of the shocked SN injected material is constant and is calculated as  $\rho_{int} = m_{int}/(4/3\pi R^3)$ .

We calculate  $P_{int}$  using equation (8),  $P_{int} = E_{int}/2\pi R^3$ , where  $E_{int}$  is the interior energy of the bubble and is calculated from the energy gained from SNe ( $\dot{E}_{inj}$ ) and the energy loss due to the work done by the interior gas on the expanding shell,

$$\frac{dE_{int}}{dt} = \dot{E}_{inj} - 4\pi R^2 v_s P_{int}. \quad (17)$$

The rates of change of mass and metals in the interior of bubbles are set by the mass and metal injection rates by SNe,  $\dot{m}_{int} = \dot{m}_{inj}$  and  $\dot{m}_{int}^Z = \dot{m}_{inj}^Z$ .

The temperature and cooling time in the interior of the bubble are calculated following equations (9) and (10), but replacing  $\rho(R)$  by  $\rho_{int} = m_{int}/(4/3\pi R^3)$ ,  $P_b$  by  $P_{int}$  and  $Z_b$  by  $Z_{int} = m_{int}^Z/m_{int}$ .

The equations of motion and of the conservation of the total mass and mass in metals for the shell in the pressure-driven stage are

$$\frac{d(m_{\text{sh}} v_s)}{dt} = 4\pi R^2 (P_{\text{int}} - P_{\text{d}}) - \frac{G M_{\text{t}}(R, d)}{R^2} m_{\text{sh}} \quad (18)$$

$$\frac{dm_{\text{sh}}}{dt} = 4\pi R^2 \rho_{\text{d}} v_s, \quad (19)$$

$$\frac{dm_{\text{sh}}^Z}{dt} = 4\pi R^2 \rho_{\text{d}} v_s Z_{\text{g}}. \quad (20)$$

Note that the expansion of the bubbles is driven by the pressure difference ( $P_{\text{int}} - P_{\text{d}}$ ). The gravitational term  $G M_{\text{t}} m_{\text{sh}} / R^2$  comes from integrating  $g \delta M$  over all the mass elements inside a radius that is comoving with the diffuse medium in the equation of motion for an element of fluid of mass  $\delta M$ . We neglect the shell self-gravity, given that  $m_s \ll M_{\text{t}}(R, d)$ .

### 2.1.3 Momentum-driven snowplough expansion

When the expansion time in the pds stage becomes longer than the cooling time of the interior, the bubble enters the momentum-driven phase. The cavity interior to the bubble is composed of low-density cooled gas of total mass  $m_{\text{int}}$ . This interior mass corresponds to the ejected mass from SNe that has not yet had enough time to encounter the shell. The explosions at the centre inject mass and momentum into the shell. The interior density is calculated from the continuity equation

$$\rho_{\text{int}} = \frac{\dot{m}_{\text{inj}}}{4\pi R^2 v_{\text{inj}}}. \quad (21)$$

The density of the ejected material drops with radius and by the time the ejected gas encounters the shell, most of the energy input by SNe has become kinetic energy. Therefore, SN ejected material acts on the shell by increasing the momentum of the shell (see the schematic in the bottom panel of Fig. 1). We therefore consider that  $v_{\text{inj}} = \sqrt{2 \dot{E}_{\text{inj}} / \dot{m}_{\text{inj}}}$ . The equations describing the change of mass and mass in metals of the bubble interior are

$$\frac{dm_{\text{int}}}{dt} = \dot{m}_{\text{inj}} \frac{v_s}{v_{\text{inj}}}, \quad (22)$$

$$\frac{dm_{\text{int}}^Z}{dt} = \dot{m}_{\text{inj}}^Z \frac{v_s}{v_{\text{inj}}}. \quad (23)$$

Here, the amount of injected mass that remains in the interior of the bubble depends on the velocity ratio  $v_s / v_{\text{inj}}$ , which means that if the shell expands slowly, most of the mass injected by SNe quickly reaches the shell. Note that gravity is neglected in the motion of the interior material.

The equations describing the conservation of momentum, total mass and mass in metals for the mds stage are

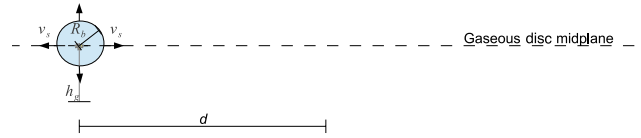
$$\frac{d(m_{\text{sh}} v_s)}{dt} = \dot{m}_{\text{inj}} (v_{\text{inj}} - v_s) - \frac{G M_{\text{t}}(R, d)}{R^2} m_{\text{sh}} - 4\pi R^2 P_{\text{d}}, \quad (24)$$

$$\frac{dm_{\text{sh}}}{dt} = \dot{m}_{\text{inj}} \left(1 - \frac{v_s}{v_{\text{inj}}}\right) + 4\pi R^2 \rho_{\text{d}} v_s, \quad (25)$$

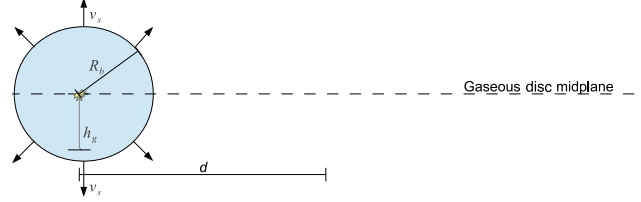
$$\frac{dm_{\text{sh}}^Z}{dt} = \dot{m}_{\text{inj}}^Z \left(1 - \frac{v_s}{v_{\text{inj}}}\right) + 4\pi R^2 \rho_{\text{d}} v_s Z_{\text{d}}. \quad (26)$$

Note that the expansion of the bubbles is driven by the velocity gradient ( $v_{\text{inj}} - v_s$ ).

(1) Early stages of bubble evolution



(2) Bubble at the point of breaking out of the ISM



**Figure 2.** Geometry of the dynamical model for SN feedback. Top panel: the early stages of pressurized bubble growth due to SNe, where the bubble is fully embedded in the ISM, at a distance  $d$  from the galaxy centre, where the disc has a gas scaleheight of  $h_g$ . The bubble radius and expansion velocity are  $R_b$  and  $v_s$ , respectively. Bottom panel: schematic showing the stage of bubble evolution just before breaking out from the ISM. At this stage, the bubble has just exceeded the gas scaleheight.

If the bubble has a radius which exceeds the scaleheight of the galaxy, part of the bubble would be expanding in a lower density medium (see the bottom panel of Fig. 2). We account for this by including a correction factor in the density of the diffuse medium when  $R > h_g$ ,  $\rho_{\text{d}}' = \rho_{\text{d}} (1 - h_g / R)$ , which accounts for the fraction of the surface of the bubble outside the disc. We replace  $\rho_{\text{d}}$  by  $\rho_{\text{d}}'$  in the set of equations describing the evolution of bubbles.

## 2.2 Properties of molecular clouds and the diffuse medium in galaxies

In this section, we describe how we calculate the properties of GMCs and the diffuse medium, and explain the techniques used to follow their evolution throughout the ISM.

### 2.2.1 Molecular cloud properties

The dynamical evolution described above corresponds to a single bubble driven by the SF taking place in one GMC. In order to incorporate this evolution into the galaxy formation context, we consider GMC formation in the ISM of galaxies and subsequent SF in GMCs. For this, it is necessary to define the GMC mass, SF efficiency and the time-scales for the formation and destruction of GMCs. We first define individual GMC properties and then connect them to galaxy properties to estimate their number and radial distribution in Section 2.2.3.

**GMC mass.** Motivated by observations of the Milky Way and nearby galaxies, we consider GMCs to have typical masses of  $m_{\text{GMC}} \approx 10^5 - 10^6 M_{\odot}$  (e.g. Solomon et al. 1987; Williams & McKee 1997; Oka et al. 2001; Rosolowsky & Blitz 2005). We assume that GMCs are fully molecular and that all the molecular gas in galaxies is locked up in GMCs. This is a good approximation for most local galaxies, in which more than 90 per cent of the molecular gas is in gravitationally bound clouds (Ferrière 2001). However, it is important to note that in the densest nearby

starburst (SB) galaxies, some molecular gas is also found in the diffuse component (e.g. M64; Rosolowsky & Blitz 2005).

*The SFR per GMC.*  $\psi_{\text{GMC}}$  depends on the GMC mass and the molecular SFR coefficient,  $\nu_{\text{SF}}$ , as  $\psi_{\text{GMC}} = \nu_{\text{SF}} m_{\text{GMC}}$ . To ensure consistency with the global SF law, we use the same SFR coefficient defined in Section 2. This implies that, as we incorporate the dynamical SN feedback model in the galaxy formation simulation, GMCs forming stars in the disc have different depletion time-scales than those forming stars in the bulge (see Section 3.1 for details). This difference in the SF time-scales of GMCs in normal star-forming galaxies and SBs has been proposed theoretically by Krumholz, McKee & Tumlinson (2009). They argue that in normal galaxies the ambient pressure is negligible compared to the internal pressure of GMCs, and therefore, the properties setting the SF are close to universal. However, in high gas density environments appropriate to SBs, the ambient pressure becomes equal to the typical GMC pressure, and therefore, in order to maintain GMCs as bound objects, their properties need to change according to the ambient pressure. This naturally produces a dichotomy between normal star-forming galaxies and SB galaxies.

*GMC lifetime.* The formation and destruction time-scales of GMCs depend on the properties of the ISM: gas density, convergence flow velocities, magnetic fields, turbulence, etc. (McKee & Ostriker 2007). GMCs can form through large-scale self-gravitating instabilities, which can include Parker, Jeans, magneto-Jeans and/or magnetorotational instabilities (e.g. Chieze 1987; Maloney 1988; Elmegreen 1989; McKee & Holliman 1999; Krumholz & McKee 2005), or through collisions of large-scale gas flows (e.g. Ballesteros-Paredes, Hartmann & Vázquez-Semadeni 1999; Heitsch et al. 2005; Vázquez-Semadeni et al. 2006). GMCs in these formation scenarios tend to last  $\sim 1$ – $3$  crossing times before being destroyed by stellar feedback (i.e. protostellar and stellar winds, and H II regions). Observationally, the lifetime of GMCs is inferred from statistical relations between the location of GMCs and young star clusters and is in the range 10–30 Myr (e.g. Blitz & Shu 1980; Engargiola et al. 2003; Blitz et al. 2007). We therefore restrict the range of the lifetimes of GMCs to  $\tau_{\text{life,GMC}} = 10$ – $30$  Myr.

### 2.2.2 Properties of the pervasive ISM

We assume that the diffuse pervasive medium in the ISM is fully atomic. We define the relevant properties of the diffuse medium (see equations 1–26) as a function of radius for the disc and bulge.

For the gas surface density profiles of the disc and bulge, we assume that both are well described by exponential profiles with half-mass radii,  $r_{50,d}$  and  $r_{50,b}$ , respectively. This is done for simplicity. However, it has been shown that the neutral gas (atomic plus molecular) in nearby spiral galaxies follows an exponential radial profile (Bigiel & Blitz 2012). Davis et al. (2012) found that this is also the case in a large percentage of early-type galaxies in the local Universe. In interacting galaxies and galaxy mergers, Davis et al. show that the gas can have very disturbed kinematics, and in these cases our approximation is no longer valid.

To calculate the H I surface density, we follow Lagos et al. (2011a) and use the Blitz & Rosolowsky (2006) pressure law (Section 3). We assume that this pressure law also holds in higher gas density media, typical of SBs. Hydrodynamic simulations including the formation of H<sub>2</sub> have shown that, for extreme gas densities, the relation between hydrostatic pressure and the  $\Sigma_{\text{H}_2}/\Sigma_{\text{H I}}$  ratio deviates from the empirical pressure law resulting in more H<sub>2</sub> (Pelupessy & Papadopoulos 2009). If the conclusions of Pelupessy et al. are

correct, our assumption that the Blitz & Rosolowsky law holds for SBs would represent an upper limit for the H I mass. The effect of this systematic on the final result of SN feedback is highly non-linear given that having more H I mass makes the expansion of bubbles more difficult, but in the case of escape, more outflow mass is released from the galaxy.

We assume that gas motions in the diffuse medium are dominated by a random component and we choose the vertical velocity dispersion to be  $\sigma_d = 10 \text{ km s}^{-1}$  (Leroy et al. 2008). The source of the motion of the diffuse ISM is not relevant so long as it gives rise to gas dominated by random motions. The assumption of random motions is consistent with turbulence and thermally driven motions (e.g. Wada, Meurer & Norman 2002; Schaye 2004; Dobbs, Burkert & Pringle 2011). We estimate the gaseous disc scaleheight, the volume density and thermal pressure as a function of radius,  $h_g(r_i)$ ,  $\rho_d(r_i)$  and  $P_d(r_i)$ , respectively. The set of equations defining these properties is

$$h_g(r_i) = \frac{\sigma_d^2}{\pi G \left[ \Sigma_g(r_i) + \frac{\sigma_d}{\sigma_*(r_i)} \Sigma_*(r_i) \right]}, \quad (27)$$

$$\rho_d(r) = \frac{\Sigma_{\text{atom}}(r_i)}{2 h_g(r_i)}, \quad (28)$$

$$P_d(r_i) = \rho_d(r_i) \sigma_d^2. \quad (29)$$

Here  $\sigma_*$  is the velocity dispersion of the stars, and  $\Sigma_{\text{atom}}(r_i)$ ,  $\Sigma_g(r_i)$  and  $\Sigma_*(r_i)$  are the atomic, total gas (molecular plus atomic) and stellar surface densities, respectively, at  $r_i$ . In Appendix B1, we describe the calculation of  $\sigma_*$  and the origin of the expression for  $h_g$ . The choice of  $\sigma_d$  fixes the internal energy of the diffuse medium throughout the disc and bulge, so that  $u = 3/2 P_d$ .

Note that we include the contribution of helium in  $\rho_d(r_i)$ . The filling factor of molecular clouds in the ISM is very small, typically  $F_{\text{GMC}} \approx 0.01$  (McKee & Ostriker 2007), so we assume that the filling factor of the diffuse gas is  $F_d = 1$  and therefore we do not include it in equations (27)–(29).

The gas scaleheight includes the gravitational effect of stars through  $\Sigma_*(r_i)$ . The underlying assumption in equation (27) is that the galaxy is in vertical equilibrium and that the diffuse medium is characterized by a uniform pressure.<sup>1</sup> Using equation (27) and for  $\sigma_d = 10 \text{ km s}^{-1}$ , we find that the mean scaleheight of SB galaxies at  $z = 0$  is  $\approx 50$  pc for galaxies with stellar mass in the range  $10^8 < M_{\text{stellar}} < 10^9 M_{\odot}$  and  $\approx 10$  pc for galaxies with  $10^{10} < M_{\text{stellar}} < 10^{11} M_{\odot}$ . At  $z = 7$ , these numbers decrease to  $\approx 5$  and  $\approx 1$  pc, respectively. In the case of quiescent galaxies at  $z = 0$ , the mean  $h_g$  is  $\approx 450$  pc for galaxies with  $10^8 < M_{\text{stellar}} < 10^9 M_{\odot}$  and  $\approx 100$  pc for galaxies with  $10^{10} < M_{\text{stellar}} < 10^{11} M_{\odot}$ . At  $z = 7$ , these numbers decrease to  $\approx 60$  and  $\approx 5$  pc, respectively. Note that  $h_g$  is very sensitive to the velocity dispersion of the gas, and therefore if we assume higher values for  $\sigma_d$  (see Section 4.3.3), we would find scaleheights larger by factors of 20–100.

We warn the reader that observations have shown that local SB galaxies have gas velocity dispersions systematically larger compared to spiral and dwarf galaxies (e.g. Solomon et al. 1997; Downes & Solomon 1998), with values that range between

<sup>1</sup> Shetty & Ostriker (2012) use a set of vertically resolved hydrodynamic simulations to show that vertical equilibrium is reached within a vertical crossing time and Koyama & Ostriker (2009) show that variations in pressure vertically are within a factor of 2.

$\sigma_d = 20$  and  $100 \text{ km s}^{-1}$ , with a median of  $\sigma_d \approx 60 \text{ km s}^{-1}$ . These values of  $\sigma_d$  may drive the typical GMC mass to increase too, as the Jeans mass in a disc scales with the gas velocity dispersion as  $M_J \propto \sigma_d^4 / \Sigma_g$ . In this paper, we analyse the general effect of increasing  $\sigma_d$  and  $M_{\text{GMC}}$  in the mass loading and velocity of the outflow in Section 4.3.3. However, we assume the same velocity dispersion and GMC mass in SBs as quiescent galaxies for simplicity. In a future paper, we investigate the effect of assuming different  $\sigma_d$  and  $M_{\text{GMC}}$  for SBs.

### 2.2.3 Connecting GMCs and galaxy properties

We follow the evolution of bubbles in rings within the disc and the bulge, and assume cylindrical symmetry: all bubbles at a given radius  $r_i$  from the centre are identical, where  $i = 1, \dots, N_r$ . We estimate the number of molecular clouds in the ISM at a given timestep that give rise to a new generation of bubbles. If at a timestep  $t = t_j$  the radial profile of molecular mass is  $\Sigma_{\text{mol}}(r, t_j)$ , the total number of GMCs in an annulus of radius  $r_i$  and width  $\delta r$  is

$$N_{\text{GMCs},i,j} = \frac{2\pi \int_{r_i-\delta r/2}^{r_i+\delta r/2} \Sigma_{\text{mol}}(r, t_j) r \, dr}{m_{\text{GMC}}}. \quad (30)$$

The rate of GMC formation in the annulus  $i$  in a given time  $t_j$  is therefore estimated as

$$\dot{N}_{\text{GMC,new},i,j} = \frac{N_{\text{GMCs},i,j}}{\tau_{\text{life,GMC}}}. \quad (31)$$

Note that by fixing the SFR coefficient,  $\nu_{\text{SF}}$ , and the properties of GMCs, we are implicitly assuming that all GMCs at a given timestep are forming stars.

We performed tests to choose the value of  $N_r$  to ensure convergence in the results presented in this work. These tests suggest  $N_r = 10$ . The spatial extent of each ring  $i$  depends on the total extent of the disc we choose to resolve. We model out to  $5r_{50}$  in disc radius, so the molecular mass enclosed is  $>99.999$  per cent of the total. This defines the extent of the individual annuli,  $\delta r = 5r_{50}/N_r$ .

Note that, at high redshift, galaxies can have large fractions of molecular gas (Lagos et al. 2011b). Due to our assumptions, namely, that the molecular gas is locked up in GMCs and that bubbles do work against the diffuse medium, this large molecular gas content has an effect on the dynamics of bubbles only through its gravitational effect on the mid-plane of the disc and the higher SFRs, which result in more SNe. Although our model can be improved to include other physical effects that are enhanced at the contact surface between the superbubble and high density media, we show in Sections 4.3.2 and 4.3.3 that our predictions for the mass loading and velocity of the winds are currently limited by our choice of parameters describing the ISM and GMCs.

### 2.2.4 Bubble confinement and break-out

**Confinement.** If bubbles are slowed down sufficiently, they are assumed to mix with the surrounding medium. The condition for mixing to take place is obtained by comparing the bubble expansion velocity to the velocity dispersion of the diffuse component of the ISM. Confinement takes place if  $v_s \leq \sigma_d$ . If this happens, we assume instantaneous mixing and add the mass and metals of the bubble to the diffuse medium of the ISM.

**Break-out from the ISM.** If a bubble reaches the edge of the disc or the bulge with an expansion velocity exceeding the sound speed of the diffuse ISM, it is assumed to break out from the ISM. The

edge is defined as a fixed fraction of the gas scaleheight,  $f_r h_g$  (see Section 2.2.2 for the definition of gas scaleheight). The opening angle of the wind at the moment it escapes from the galaxy is given by  $\theta \equiv 2 \arccos(1/f_r)$ , assuming that bubbles are centred at the mid-plane of the disc. A fraction  $f_{\text{bo}}$  of the mass and metals carried away by bubbles will escape from the galaxy. This depends on the choice of  $f_r = R/h_g$  and is given by

$$f_{\text{bo}} = \left(1 - \frac{h_g}{R}\right) = 1 - f_r^{-1}. \quad (32)$$

A fraction  $(1 - f_{\text{bo}})$  of the mass and metals carried away by bubbles is assumed to be confined in the ISM. The physical motivation for this choice is that the gas expanding along the major axis of the disc does not escape and that, in the case of the gas expanding perpendicular to the mid-plane of the disc, Rayleigh–Taylor instabilities grow at the edge of the ambient gas due to the drastic change of density. These instabilities produce fragmentation in the swept-up mass and some of this material is reincorporated into the galaxy. MacLow & McCray (1988) and Mac Low et al. (1989), by means of hydrodynamical simulations, estimated  $f_r \approx 1-2$  for a Milky Way-like galaxy. Mac Low et al. (1989) show that approximately 10 per cent of the mass contained in shells at the point of break-out accelerates upwards and  $\approx 90$  per cent stays in the ISM. Similar values have been obtained by more sophisticated hydrodynamical simulations (e.g. de Avillez & Berry 2001; Fujita et al. 2009). In detail, the break-out radius and the mass in shells escaping the galaxy disc are thought to mainly depend on the density contrast between the disc and halo gas which sets the development of instabilities which fragments the bubble shells. Other hydrodynamical effects, such as weak magnetic fields in the ISM, can inhibit the generation of Rayleigh–Taylor instabilities and/or help accelerate the cool shell gas even further away through magnetic pressure (e.g. Fujita et al. 2009). These effects influence the cold dense gas of bubbles, while the hotter, interior material is shown to escape to the hot halo in all of the simulations. Taking into account these results, we restrict the range of values of  $f_r$  to  $f_r \approx 1.1-2$ , implying that a significant fraction of the swept-up mass in bubbles stays in the ISM. The hot gas contained in the interior of bubbles is assumed to fully escape into the hot halo. In our standard model, we adopt  $f_r = 1.5$ . In Section 4.3.2, we show how the mass outflow rate varies when  $f_r$  takes the lowest and highest values in the range above.

Fig. 2 shows a schematic of the evolution of bubbles in the ISM. We summarize all the parameters needed to characterize GMCs and the ISM of galaxies in Table 1. We give there the reference value used for our standard SN feedback model but also give the ranges motivated by observations and theory, which we also test in Sections 4.3.2 and 4.3.3.

## 3 INCORPORATING DYNAMICAL SN FEEDBACK INTO A GALAXY FORMATION SIMULATION

One of the aims of this paper is to study how the outflow rate depends on galaxy properties in a galaxy population which has a representative set of star formation histories (SFH) and which resembles observed galaxy properties. We achieve this by incorporating the full dynamical model described in Section 2 into the semi-analytic galaxy formation model GALFORM, which is set in the  $\Lambda$  cold dark matter framework.

In Section 3.1, we briefly describe the GALFORM model and in Section 3.2 we give details on how we modify the model to include the dynamical model of SNe presented in Sections 2 and 2.2.1.



**Table 1.** List of parameters in the dynamical SN feedback model. In the right-hand column, theoretical and observational constraints on these parameters are described. The values adopted in our preferred model (referred to as the standard model in the text) are indicated in parentheses.

Symbol	Parameter	Range and value adopted	Constraints from obs. and theory
GMC parameters			
$M_{\text{GMC}}$	Typical mass of a GMC	$M_{\text{GMC}} = 10^5 - 10^8 M_{\odot}$ (std. model $M_{\text{GMC}} = 10^6 M_{\odot}$ )	Estimated by Solomon et al. (1987), Williams & McKee (1997).
$\tau_{\text{life, GMC}}$	Lifetime of a GMC	$t_{\text{life, GMC}} = 10 - 30 \text{ Myr}$ (std. model $t_{\text{life, GMC}} = 10 \text{ Myr}$ )	Observations and theoretical arguments favour values in the range given here (e.g. Blitz & Shu 1980; Dobbs et al. 2011).
Diffuse medium parameters			
$\sigma_{\text{d}}$	Velocity dispersion of the gas in discs	$\sigma_{\text{d}} \approx 5 - 70 \text{ km s}^{-1}$ (std. model $\sigma_{\text{d}} = 10 \text{ km s}^{-1}$ )	van der Kruit & Freeman (2011). Used to calculate $P_{\text{d}}$ , $u_{\text{d}}$ and $h_{\text{g}}$ .
Disc parameters			
$f_*$	Ratio of the scaleradius to the scaleheight of the stellar disc	$f_* = r_s / h_{\text{star}} \approx 7.3$	Kregel, van der Kruit & de Grijs (2002). Used to calculate $P_{\text{ext}}$ and $h_{\text{g}}$ .
$f_r$	Defines the radius at which bubbles are assumed to have escaped the galaxy.	$f_r = 1.1 - 2$ (std. model $f_r = 1.5$ )	In principle $f_r$ is a free parameter. However, we set a range within which we vary $f_r$ as to get a break-out mass fraction consistent with previous theoretical estimates (e.g. MacLow & McCray 1988; Fujita et al. 2009).
SF parameters			
$\nu_{\text{SF}}$	SFR coefficient	$\nu_{\text{SF}} = 0.25 - 1 \text{ Gyr}^{-1}$ (std. model $\nu_{\text{SF}} = 0.5 \text{ Gyr}^{-1}$ )	Determines the SFR per unit molecular mass $\Sigma_{\text{SFR}} = \nu_{\text{SF}} \Sigma_{\text{mol}}$ . Measured by e.g. Leroy et al. (2008).
$P_0$	Pressure normalization	$\log(P_0/k_{\text{B}} [\text{cm}^{-3} \text{ K}]) = 4.19 - 4.54$ (std. model $\log(P_0/k_{\text{B}} [\text{cm}^{-3} \text{ K}]) = 4.54$ )	$\Sigma_{\text{H}_2} / \Sigma_{\text{H}_1} = (P_{\text{ext}}/P_0)^{\alpha_{\text{P}}}$ . Measured by e.g. Wong & Blitz (2002), Blitz & Rosolowsky (2006), Leroy et al. (2008).
$\alpha_{\text{P}}$	Power-law index in the pressure law	$\alpha_{\text{P}} = 0.73 - 0.92$ (std. model $\alpha_{\text{P}} = 0.92$ )	Measured (see authors above).

### 3.1 The GALFORM model

The GALFORM model takes into account the main physical processes that shape the formation and evolution of galaxies (Cole et al. 2000). These are (i) the collapse and merging of DM haloes, (ii) the shock-heating and radiative cooling of gas inside DM haloes, leading to the formation of galactic discs, (iii) quiescent SF in galaxy discs, (iv) feedback from SNe, from AGN and from photoionization of the IGM, (v) chemical enrichment of stars and gas, and (vi) galaxy mergers driven by dynamical friction within common DM haloes, which can trigger bursts of SF and lead to the formation of spheroids (for a review of these ingredients, see Baugh 2006 and Benson 2010). Galaxy luminosities are computed from the predicted SF and chemical enrichment histories using a stellar population synthesis model. Dust extinction at different wavelengths is calculated self-consistently from the gas and metal contents of each galaxy and the predicted scalelengths of the disc and bulge components using a radiative transfer model (see Lacey et al. 2011 and Gonzalez-Perez et al. 2012).

GALFORM uses the formation histories of DM haloes as a starting point to model galaxy formation (see Cole et al. 2000). In this paper, we use halo merger trees extracted from the Millennium  $N$ -body simulation (Springel et al. 2005), which assumes the following cosmological parameters:  $\Omega_{\text{m}} = \Omega_{\text{DM}} + \Omega_{\text{baryons}} = 0.25$  (with a baryon fraction of 0.18),  $\Omega_{\Lambda} = 0.75$ ,  $\sigma_8 = 0.9$  and  $h = 0.73$ . The resolution of the  $N$ -body simulation corresponds to a minimum halo mass of  $1.72 \times 10^{10} h^{-1} M_{\odot}$ , which in the Lagos et al. (2012) model corresponds to a stellar mass limit of  $7 \times 10^7 h^{-1} M_{\odot}$ . This

is sufficient to resolve the haloes that contain most of the  $\text{H}_2$  in the universe at  $z < 8$  (Lagos et al. 2011b). The construction of the merger trees used by GALFORM is described in Merson et al. (2013).

In this paper, we focus on the Lagos et al. (2012, hereafter Lagos12) model, which includes a two-phase description of the ISM, i.e. composed of the atomic and molecular contents of galaxies, and adopt the empirical SF law of Blitz & Rosolowsky (2006). The physical treatment of the ISM in the Lagos et al. model is a key feature affecting the predicted outflow rate of galaxies, as we show in Section 4, which justifies our choice of exploring the full dynamical model of SNe in this model.

The Blitz & Rosolowsky (2006) empirical SF law has the form

$$\Sigma_{\text{SFR}} = \nu_{\text{SF}} f_{\text{mol}} \Sigma_{\text{g}}, \quad (33)$$

where  $\Sigma_{\text{SFR}}$  and  $\Sigma_{\text{g}}$  are the surface densities of the SFR and the total cold gas mass, respectively,  $\nu_{\text{SF}}$  is the inverse of the SF time-scale for the molecular gas,  $\nu_{\text{SF}} = \tau_{\text{SF}}^{-1}$ , and  $f_{\text{mol}} = \Sigma_{\text{mol}}/\Sigma_{\text{g}}$  is the molecular-to-total gas mass surface density ratio. The molecular and total gas contents include the contribution from helium, while the  $\text{H}_1$  and  $\text{H}_2$  masses only include hydrogen (helium accounts for 26 per cent of the overall cold gas mass). The integral of  $\Sigma_{\text{SFR}}$  over the disc corresponds to the instantaneous SFR,  $\psi$ . The ratio  $f_{\text{mol}}$  is assumed to depend on the internal hydrostatic pressure of the disc as (Blitz & Rosolowsky 2006)

$$\frac{\Sigma_{\text{mol}}}{\Sigma_{\text{atom}}} = f_{\text{mol}} / (f_{\text{mol}} - 1) = \left( \frac{P_{\text{ext}}}{P_0} \right)^{\alpha_{\text{P}}}. \quad (34)$$

For a description of how we calculate  $P_{\text{ext}}$ , see Appendix B1. The parameter values we use for  $\nu_{\text{SF}}$ ,  $P_0$  and  $\alpha_P$  are the best fits to observations of nearby spiral and dwarf galaxies,  $\nu_{\text{SF}} = 0.5 \text{ Gyr}^{-1}$ ,  $\alpha_P = 0.92$  and  $\log(P_0/k_B [\text{cm}^{-3} \text{ K}]) = 4.54$  (Blitz & Rosolowsky 2006; Leroy et al. 2008; Bigiel et al. 2011; Rahman et al. 2012).

For SBs the situation is less clear. Observational uncertainties, such as the conversion factor between CO and  $\text{H}_2$  in SBs, and the intrinsic compactness of star-forming regions, have not allowed a clear characterization of the SF law in this case (e.g. Kennicutt 1998; Genzel et al. 2010; Combes et al. 2011; see Ballantyne, Armour & Indergaard 2013 for an analysis of how such uncertainties can bias the inferred SF law). Theoretically, it has been suggested that the SF law in SBs is different from that in normal star-forming galaxies (Pelupessy & Papadopoulos 2009). The ISM of SBs is predicted to always be dominated by  $\text{H}_2$  independently of the exact gas pressure. For these reasons, we choose to apply equation (33) only during quiescent SF (i.e. SF fuelled by the accretion of cooled gas on to galactic discs) and retain the original SF prescription for SBs, which are driven either by galaxy mergers or disc instabilities (see Cole et al. 2000 and Lagos et al. 2011a for details). In the SBs, the SF time-scale is taken to be proportional to the bulge dynamical time-scale above a minimum floor value (which is a model parameter) and involves the whole ISM gas content in the SB, giving  $\text{SFR} = M_{\text{gas}}/\tau_{\text{SF,SB}}$  (see Granato et al. 2000 and Lacey et al. 2008 for details), with

$$\tau_{\text{SF,SB}} = \max(\tau_{\text{min}}, f_{\text{dyn}}\tau_{\text{dyn}}). \quad (35)$$

Here we adopt  $\tau_{\text{min}} = 100 \text{ Myr}$  and  $f_{\text{dyn}} = 50$  following Lagos12.

Throughout the paper we will refer to galaxies as ‘SB galaxies’ if their total SFR is dominated by the SB mode,  $\text{SFR}_{\text{starburst}} > \text{SFR}_{\text{quiescent}}$ , while the remainder of the model galaxies will be referred to as ‘quiescent galaxies’.

### 3.2 Predicting the SFH of galaxies

The GALFORM model includes two gas phases in the ISM of galaxies, an atomic and a molecular phase, which correspond to the warm and cold phases, respectively. By including dynamical modelling of SN feedback, we introduce a new phase into the ISM of galaxies corresponding to the interiors of expanding bubbles (see Section 2).

The equations of SF need to be modified accordingly to include the contribution from the mass and metals in bubbles. The chemical enrichment is also assumed to proceed through the expansion of SN inflated bubbles: stellar winds and SN feedback shock the surrounding medium and inflate bubbles through thermal energy, so the new metals produced by recently made intermediate- and high-mass stars will be contained in the interiors of bubbles. In the case of low-mass stars, recycling of mass and newly synthesized metals feeds the ISM directly. In the case of confinement, metals contained in the thin, dense shell of swept-up gas and the interior of bubbles are mixed instantaneously with the cold and warm ISM. Note that we do not apply any delay to the mixing of metals given that the cooling time for the hotter phases is typically small ( $t_{\text{cool}} = 5 \times 10^2\text{--}10^5 \text{ yr}$ ).

The five mass components of the system are the stellar mass of the disc,  $M_*$ , the total gas mass in the ISM (molecular plus atomic),  $M_{\text{g,ISM}}$ , the mass in bubbles (interior plus shell) in the ISM,  $M_{\text{b,ISM}}$ , the mass of the hot gaseous halo of the galaxy,  $M_{\text{hot}}$ , and the mass escaping the galaxy disc through bubbles,  $M_{\text{eject}}$ . The latter represents all gas that has not yet mixed with the hot halo gas, i.e. that is thermally/kinematically decoupled from the hot halo gas. The underlying assumption is that all gas ejected from the disc ends up

in a reheated gas reservoir. The reincorporation time,  $\tau_{\text{rein}}$ , of the ejected component into the halo is always larger than the timestep over which we perform the integration. We therefore calculate the rate of reincorporation of gas into the hot halo component only with the ejected mass available at the beginning of the timestep,  $M_{\text{eject}}$ . We remind the reader that in this paper we use the standard approach of GALFORM to calculate  $\tau_{\text{rein}}$ . This consists of parametrizing  $\tau_{\text{rein}}$  as depending linearly on the dynamical time-scale of the halo regulated by an efficiency, which is a free parameter of the model,  $\tau_{\text{rein}} = \tau_{\text{dyn}}/\alpha_{\text{reheat}}$  (we retain the value of  $\alpha_{\text{reheat}} = 1.2$  used in Lagos12). In Paper II, we introduce a physical modelling of the reincorporated gas and the time-scale for this process.

Fig. 3 depicts the exchange of mass and metals between the different components of galaxies: the hot halo, ISM, stars and bubbles expanding in the ISM. As in the original model of Cole et al. (2000), we assume that during SF, the inflow rate from the hot halo,  $\dot{M}_{\text{cool}}$ , is constant, implicitly assuming that SN heating plays no role in the inflow rate until the ejected mass and metals are incorporated into the hot halo after time-scale  $\tau_{\text{rein}}$ . The gas mass in the ISM is affected by  $\dot{M}_{\text{cool}}$ , the rate at which mass is recycled from evolved stars (assumed to go straight to the ISM), the rate at which bubbles sweep up mass from the ISM,  $\dot{M}_{\text{sw,ISM}}$ , and the rate of bubble confinement,  $\dot{M}_{\text{conf,ISM}}$ , and break-out,  $\dot{M}_{\text{bo,ISM}}$  (the calculation of each of these is described in detail in Appendix C). At each substep in the numerical solution scheme, we update the values of each of the mass variables. It is therefore possible to replenish the atomic/molecular gas contents and also modify the  $\text{H}_2/\text{H I}$  ratio, as the gas and stellar surface densities change.

The set of equations describing the flow of mass and metals between the different phases are

Mass exchange:

$$\dot{M}_* = (1 - R_{\text{ES}} - R_{\text{SN}})\psi, \quad (36)$$

$$\begin{aligned} \dot{M}_{\text{g,ISM}} = & \dot{M}_{\text{cool}} + (R_{\text{ES}} - 1)\psi - \dot{M}_{\text{sw,ISM}} + \dot{M}_{\text{conf,ISM}} \\ & + (1 - f_{\text{bo}})\dot{M}_{\text{bo,ISM}}, \end{aligned} \quad (37)$$

$$\dot{M}_{\text{b,ISM}} = R_{\text{SN}}\psi + \dot{M}_{\text{sw,ISM}} - \dot{M}_{\text{conf,ISM}} - \dot{M}_{\text{bo,ISM}} \quad (38)$$

$$\dot{M}_{\text{eject}} = f_{\text{bo}}\dot{M}_{\text{bo,ISM}} - \frac{M_{\text{eject}}}{\tau_{\text{rein}}}, \quad (39)$$

$$\dot{M}_{\text{hot}} = -\dot{M}_{\text{cool}} + \frac{M_{\text{eject}}}{\tau_{\text{rein}}}. \quad (40)$$

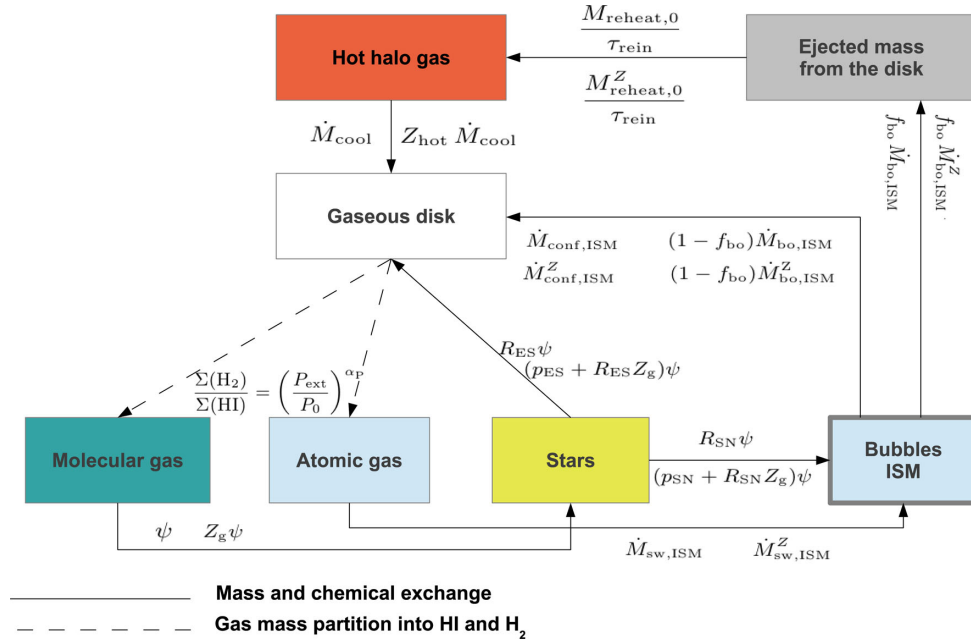
Metallicity exchange:

$$\dot{M}_*^Z = (1 - R_{\text{ES}} - R_{\text{SN}})Z_g\psi, \quad (41)$$

$$\begin{aligned} \dot{M}_{\text{g,ISM}}^Z = & \dot{M}_{\text{cool}}Z_{\text{hot}} + (p_{\text{ES}} + R_{\text{ES}}Z_g)\psi - \dot{M}_{\text{sw,ISM}}^Z \\ & + \dot{M}_{\text{conf,ISM}}^Z + (1 - f_{\text{bo}})\dot{M}_{\text{bo,ISM}}^Z, \end{aligned} \quad (42)$$

$$\dot{M}_{\text{b,ISM}}^Z = (p_{\text{SN}} + R_{\text{SN}}Z_g)\psi + \dot{M}_{\text{sw,ISM}}^Z - \dot{M}_{\text{conf,ISM}}^Z - \dot{M}_{\text{bo,ISM}}^Z \quad (43)$$

$$\dot{M}_{\text{eject}}^Z = f_{\text{bo}}\dot{M}_{\text{bo,ISM}}^Z - \frac{M_{\text{eject}}^Z}{\tau_{\text{rein}}}, \quad (44)$$



**Figure 3.** Schematic of the flow of mass and metals in the dynamical model of SN feedback. The scheme shows the exchange of mass and metals (solid lines) between the hot halo, stars and the three gas phases in the ISM, and the partition of the ISM gas into the atomic and molecular gas components (dashed lines), corresponding to equations (36)–(43) in the text. Note that the same scenario would apply to SBs without the inflow of cooled gas from the hot halo.

$$\dot{M}_{\text{hot}}^Z = -\dot{M}_{\text{cool}} Z_{\text{hot}} + \frac{M_{\text{eject}}^Z}{\tau_{\text{rein}}}. \quad (45)$$

The recycled mass from newly formed stars is specified separately for SNe,  $R_{\text{SN}}$ , and intermediate- and low-mass stars,  $R_{\text{ES}}$  (namely, evolved stars). We calculate the recycled fractions of each stellar mass range following equation (A2). SNe are considered to be all stars with  $m > 8 M_{\odot}$ , and less massive stars in the range  $1 < m/M_{\odot} < 8$  are considered as evolved stars (intermediate- and low-mass stars). Stars less massive than  $1 M_{\odot}$  have lifetimes larger than the age of the Universe and therefore do not recycle mass into the ISM. The yield is also defined separately for SNe and evolved stars in order to inject the metals from SNe into the bubbles, whilst metals from evolved stars go directly into the ISM. We adopt the instantaneous mixing approximations for the metals in the ISM. This implies that the metallicities of the molecular and atomic phases in the ISM are equivalent and equal to  $Z_{\text{g}} = M_{\text{g,disc}}^Z / M_{\text{g,disc}}$ . The metallicity of the hot gas in the halo is  $Z_{\text{hot}} = M_{\text{hot}}^Z / M_{\text{hot}}$ .

The system of SF (equations 36–43) applies for quiescent SF and SBs. In the latter case  $\dot{M}_{\text{cool}} = 0$ . During an SB, we assume that all bubbles expanding in galaxy discs are destroyed, as well as bubbles expanding in the satellite galaxy in the case of a galaxy merger. The new generation of stars made in the SB creates a new generation of inflated bubbles expanding over the bulge.

## 4 PHYSICAL CHARACTERIZATION OF BUBBLES IN THE ISM

In this section, we explore the physical properties of bubbles and the main drivers of their evolution in the ISM of galaxies. In Section 4.1, we focus on individual examples of bubbles in ad hoc galaxies. We explore how the bubble mass depends on different global galaxy properties, such as the gas fraction, gas metallicity and scaleheight, and local properties, such as gas density and surface density. In Sections 4.2–4.4, we focus on the outflow properties of GALFORM

galaxies when the full dynamical model for SN feedback is included (see Section 2.2.2). Comparisons with observations and previous theoretical work are presented and discussed in Section 4.4.

### 4.1 Properties of individual bubbles

We study the dependence of the mass in a single bubble (interior plus shell) on the properties of the diffuse medium with the aim of determining which local properties are the more relevant in setting the mass of bubbles at the point of break-out or confinement (i.e. their maximum mass).

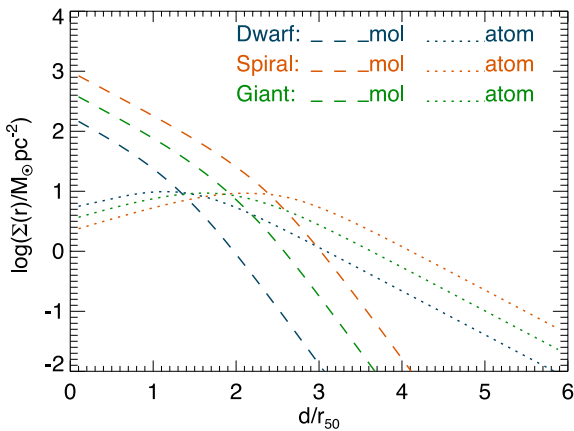
In order to fully characterize a single bubble in the ISM of a galaxy, we need to choose values for the galaxy properties which are required in the dynamical SN feedback model, namely the gas and stellar mass in the disc and the bulge, the half-mass radii of both stellar components, the halo virial mass, radius and concentration, the gas metallicity and the location of the bubble in the galaxy disc. We focus on three example galaxies with properties within a representative range which are listed in Table 2.

To calculate the expansion of a single bubble in the ISM of these galaxies, we use the standard set of parameters in Table 1 to describe GMCs and the ISM. In Fig. 4, we show the radial profiles of the atomic and molecular gas for the three galaxies of Table 2. We construct these profiles using the Blitz & Rosolowsky (2006) relation (equation 34). The three galaxies plotted in Fig. 4 show central regions dominated by molecular gas and atomic gas surface densities which saturate at  $\approx 10 M_{\odot} \text{pc}^{-2}$ , above which the gas is mainly molecular.

In order to study the dependence of the maximum mass of bubbles on galaxy properties, we vary the mass of gas and stars, the gas metallicity and the distance of the bubbles from the galaxy centre for the three galaxies in Table 2. These parameters are expected to have an effect on the expansion of bubbles by varying the gas density, scaleheight, cooling time-scale, gravitational field, etc. The strategy is to vary one property at a time leaving the other ones unchanged, to see how the predictions change. We evolve bubbles

**Table 2.** Properties of the three example galaxies used to study the effect of the different physical parameters on the evolution of bubbles in the ISM. We list the 10 properties we need to characterize the radial profiles of the stellar, gaseous and DM components, disc and bulge half-mass radii,  $r_d$  and  $r_b$ , stellar mass in the disc and the bulge,  $M_{*,d}$  and  $M_{*,b}$ , cold gas mass,  $M_{\text{gas,ISM}}$ , gas metallicity,  $Z_g$ , halo virial mass,  $M_{\text{halo}}$ , radius,  $r_{\text{vir}}$ , and halo concentration,  $c$ . We also fix the distance to the galaxy centre at which the example bubble is located,  $d$ . The properties listed define the local properties of the ISM (see Appendix B). For those parameters which we vary, we give the range chosen to study their effect on the bubble expansion, and in the line below this we give the reference value.

Model	Dwarf	Spiral	Giant
Varying parameters			
$M_{\text{gas,ISM}}/M_{\odot}$	$10^7-10^{9.5}$	$10^8-10^{11}$	$10^9-10^{12}$
Reference value	$5 \times 10^9$	$8 \times 10^{10}$	$1 \times 10^{11}$
$M_{*,d}/M_{\odot}$	$10^7-10^{9.5}$	$10^8-10^{11}$	$10^9-10^{12}$
Reference value	$10^9$	$5 \times 10^{10}$	$10^{11}$
$Z_g/Z_{\odot}$	$10^{-3}-2$	$10^{-3}-2$	$10^{-3}-2$
Reference value	0.1	1	2
$d/r_d$	0–6	0–6	0–6
Reference value	0.5	0.5	0.5
Fixed parameters			
$r_d/\text{kpc}$	2.5	6	10
$M_{*,b}/M_{\odot}$	0	$8 \times 10^9$	$2 \times 10^{11}$
$r_b/\text{kpc}$	0	0.5	3
$M_{\text{halo}}/M_{\odot}$	$5 \times 10^{10}$	$10^{12}$	$10^{14}$
$r_{\text{vir}}/\text{Mpc}$	0.08	0.2	1
$c$	5	5	5



**Figure 4.** Surface density of molecular and atomic gas as a function of the distance from the galactic centre in units of the half-mass radius of the three example galaxies listed in Table 2. Line styles and colours show different components of the gas content in the different galaxies as labelled.

until they become confined or break out from the galaxy disc. When we fix  $d$ , we arbitrarily choose  $d = 0.5 r_{50}$  for illustration. This value of  $d$  typically corresponds to a region where bubbles break out. The four experiments (i.e. changing  $d$ ,  $Z_d$ ,  $M_{\text{gas,ISM}}$  and  $M_{*,d}$ ) are performed for each of the galaxies of Table 2 and the results are shown in the top panel of Fig. 5. The maximum mass of a single bubble shown in Fig. 5 corresponds to the mass at the point of break-out or confinement.

In the central regions of galaxies, bubbles break out from the galaxy disc, while in the outskirts bubbles tend to be confined. In the case of the ‘dwarf’ galaxy, the break-out region is restricted to

$d \lesssim 0.5 r_{50}$ , while in the case of the ‘spiral’ and ‘giant’ galaxies, the region of break-out extends out to  $d > r_{50}$ . In the break-out regions, there is a strong relation between the bubble mass and the distance from the galactic centre. This is driven by an underlying relation between  $m_b$  and the gas scaleheight or gas surface density.

Variations in the gas metallicity have very little effect on the resulting bubble mass. When the gas surface density is high, the metallicity plays only a minor role because the cooling time is already very short and bubbles become radiative very quickly. In the case of low gas surface densities, the cooling time becomes long even for high metallicities, which preserves the energy of the bubbles. In the case that metallicity does have an effect on the bubble mass, the differences found are always less than a factor of  $\sim 2$ .

Strong variations in the maximum mass of the bubble are obtained when varying  $M_{\text{gas,ISM}}$ . In the regime of break-out from the galaxy disc, the bubble mass quickly decreases when increasing  $M_{\text{gas,ISM}}$ . As  $M_{\text{gas,ISM}}$  increases, the surface density of gas also increases. This reduces the gas scaleheight, which reduces the bubble mass. The reason for this is that the radius the bubble needs to reach to escape the galaxy decreases, and therefore also the total mass that it is able to sweep up also decreases, as this is proportional to the bubble volume. The higher  $M_{\text{gas,ISM}}$  results in an overall decrease of the bubble mass by a factor of 100–500.

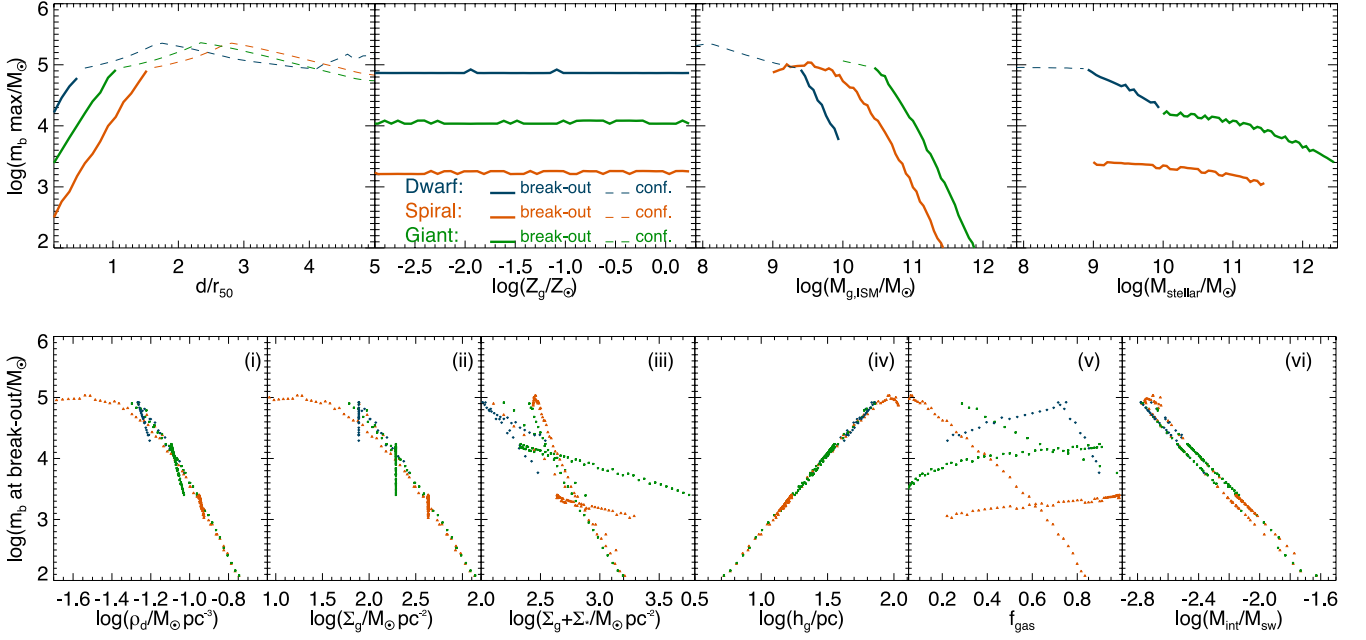
Variations in stellar mass have a non-negligible effect on the bubble mass, particularly at the massive end of the range tested (see the second row of Table 2). There is a trend of decreasing bubble mass with increasing stellar mass in the region of break-out. This happens due to the increasing gravitational field driven by the higher stellar surface densities, which decreases the gas scaleheight of the disc and the radius the bubble needs to reach to break out. The bubble mass obtained when increasing the stellar content of galaxies can be lower by up to a factor of 3. The effect of the more efficient deceleration of bubbles due to the larger gravitational field when the stellar mass increases is secondary to the effect of the stellar surface density on the gas scaleheight, and represents only  $\approx 0.1-5$  per cent of the total effect observed when increasing  $M_{*,d}$ .

The distance to the galactic centre and the gas content of the galaxies shown in Fig. 5 drive the strongest variations in bubble mass. This is due to the dependence of  $m_b$  on the gas density (atomic plus molecular) and the gas scaleheight, which is shown in the bottom-right panel of Fig. 5. We include only those examples in which the bubble breaks out from the galaxy disc. Bubble masses in the cases tested here are always dominated by the swept-up mass (see the bottom-right panel of Fig. 5). However, there is an increasing contribution from  $m_{\text{inj}}$  to  $m_b$  for decreasing  $m_b$ . We give physical insight into the relations between  $m_b$ ,  $h_g$  and  $\Sigma_g$  in the next subsection.

In the case of the gas fraction, we find that there is a complex dependence of  $m_b$  on  $f_{\text{gas}}$ . The gas fraction acts to modify the normalization of the relation between the total outflow rate and  $h_g$  and the power-law index of the relation between the total outflow rate and  $\Sigma_g$ . The gas fraction is also responsible for the dispersion at fixed  $\Sigma_g$  in panel (ii) in the bottom of Fig. 5.

#### 4.1.1 Analytic derivation of the scaling relations of single bubbles

At the point of break-out, the volume of the gas disc occupied by a single bubble is  $V = 2\pi h_g^3 (f_g^2 - 1/3)$ . In the regime where  $m_{\text{inj}} \ll m_{\text{sw}}$ , which is a representative limit for most bubbles (see the bottom-right panel of Fig. 5), and neglecting temporal changes



**Figure 5.** Top panel: bubble mass at the point of break-out or confinement, maximum  $m_b$ , as a function of the distance to the galaxy centre in units of the half-mass radius,  $d/r_{50}$  (left-hand panel), the gas metallicity in units of the solar metallicity,  $Z_g/Z_\odot$  (middle-left panel), the ISM mass (molecular plus atomic),  $M_{g, \text{ISM}}$  (middle-right panel) and the stellar mass,  $M_{\text{stellar}}$  (right-hand panel). The segments of the curves shown with solid lines correspond to those regions of the planes where bubbles end up breaking out from the galactic disc. Those segments shown with dashed lines correspond to regions where bubbles end up confined in the ISM of the galaxy. Bottom panel: bubble mass at the point of break-out as a function of the local properties (i) atomic gas density, (ii) total (molecular plus atomic) gas surface density, (iii) surface density of total gas plus stars, (iv) gas scaleheight, (v) gas fraction and (vi) the ratio between the interior and the swept-up mass of bubbles (the interior mass corresponds to the fraction of the total mass injected by SNe that has not yet cooled down or hit the shell). Individual realizations for each galaxy are shown as points in the colours labelled.

in the gas density of the diffuse medium during the evolution of bubbles in the ISM, one can write the bubble mass as

$$m_b = \rho_d V = (1 - f_{\text{mol}})\pi(f_r^2 - 1/3)\Sigma_g h_g^2. \quad (46)$$

In order to find an expression for  $m_b$  in terms of  $\Sigma_g$  and  $h_g$  alone, we need to express  $f_{\text{mol}}$  as a function of the same variables.

We can write  $f_{\text{mol}}$  in terms of the gas (atomic and molecular) density

$$1 - f_{\text{mol}} = \frac{1}{1 + (P_{\text{ext}}/P_0)^{\alpha_p}} = \frac{1}{1 + \left(\frac{\Sigma_g}{2h_g} \sigma_d^2/P_0\right)^{\alpha_p}}. \quad (47)$$

By introducing the expression for  $f_{\text{mol}}$  into equation (46), we find that

$$m_b \approx \begin{cases} \pi(f_r^2 - 1/3)\Sigma_g h_g^2 & \left(\frac{\Sigma_g}{2h_g} \sigma_d^2/P_0\right) \ll 1 \\ \pi(f_r^2 - 1/3)\left(\frac{2P_0}{\sigma_d^2}\right)^{\alpha_p} & \left(\frac{\Sigma_g}{2h_g} \sigma_d^2/P_0\right) \gg 1 \\ \cdot \Sigma_g^{1-\alpha_p} h_g^{2+\alpha_p}. \end{cases} \quad (48)$$

If we now apply the limit  $\Sigma_g \gg (\sigma_g/\sigma_*)\Sigma_*$ , where gas dominates over stars in the gravity acting on the gas layer, we find that  $h_g \propto \sigma_d^2/\Sigma_g$  and

$$m_b \propto \begin{cases} h_g \propto \Sigma_g^{-1} & f_{\text{mol}} \ll 1 \\ h_g^{1+2\alpha_p} \propto \Sigma_g^{-(1+2\alpha_p)} & f_{\text{mol}} \approx 1. \end{cases} \quad (49)$$

These expressions describe the relations shown in the bottom panel of Fig. 5, where we obtain, in the high-density regime,  $\Sigma_g \gtrsim 70 M_\odot \text{pc}^{-2}$ , the power-law relations  $m_b \propto h_g^{2.5}$  and  $m_b \propto \Sigma_g^{-2.3}$ , and in the lower density regime, we find  $m_b \propto h_g^{0.7}$  and  $m_b \propto \Sigma_g^{-0.8}$ . These power-law relations are approximate as the exact value of the

power-law index changes slightly from case to case. From this analytic derivation of the scaling relations, it is fair to say that the transition from the atomic- to molecule-dominated media has a large impact on the mass of a bubble at the point of break-out.

If we assume a steady state (i.e. the SFR is constant), we can write the outflow rate per annulus as a function of each individual bubble mass as

$$\dot{M}_{\text{eject}} = \frac{f_{\text{bo}} m_b M_{\text{mol}}}{\tau_{\text{life,GMC}} M_{\text{GMC}}}. \quad (50)$$

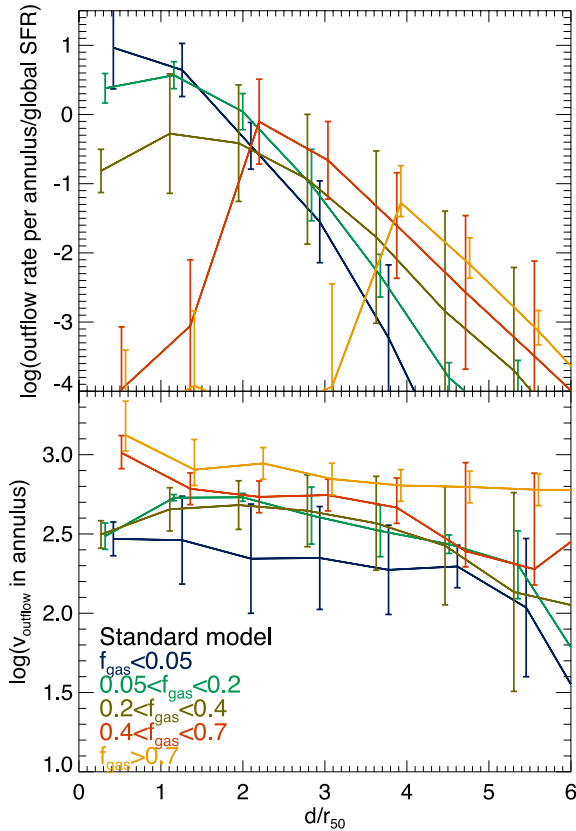
Considering  $\psi = \nu_{\text{SF}} M_{\text{mol}}$ , we can directly write  $\beta$  per annulus in terms of a single bubble mass

$$\beta = \frac{\dot{M}_{\text{eject}}}{\psi} = \frac{f_{\text{bo}}}{\nu_{\text{SF}} \tau_{\text{life,GMC}} M_{\text{GMC}}} m_b. \quad (51)$$

There is a direct relation between  $\beta$  and  $m_b$  in the case of a steady state. We therefore expect to see a similar transition in the relation between the outflow rate and the gas surface density to the one obtained for  $m_b$ : from a steeper relation in galaxies with molecule-dominated ISM to a shallower relation in galaxies with atomic-dominated ISM. From equations (48) and (51), we also see how each of the parameters describing the ISM and GMCs affects individual bubble masses and the global outflow rate.

## 4.2 Radial profile of the mass loading factor and outflow velocity

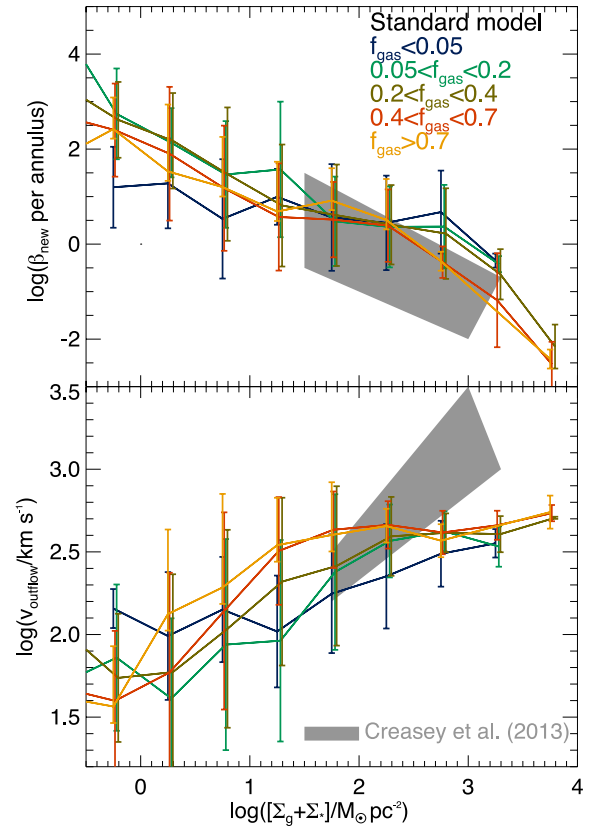
In order to physically characterize the outflow rate in a galaxy population which resembles the observed one, we use the GALFORM semi-analytic model, into which we incorporate the dynamical feedback described in Section 2. The key difference with the analysis of



**Figure 6.** Top panel: the outflow rate contributed by each annulus in units of the global SFR, as a function of the distance from the galactic centre in units of the half-mass radius,  $d/r_{50}$ , for the dynamical model with the standard choice of parameters (see Table 1) and for galaxies at  $z < 0.1$  and with  $M_* > 10^{10} h^{-1} M_{\odot}$ . For quiescent SF, we use  $r_{50}$  of the disc, and for SBs,  $r_{50}$  of the bulge. The solid lines and error bars represent the median and 10–90 per cent range of the distributions. The predictions are plotted for galaxies with different gas fractions, as labelled. Bottom panel: as in the top panel, but here we show the outflow velocity of the gas at the point of break-out as a function of the distance from the galactic centre in units of the half-mass radius.

Section 4.1 is that here we explore the whole galaxy population and the outflow rate with the aim of characterizing (i) a preferred radius from which most of the material escapes and the outflow velocity, and (ii) the scaling relations between the mass loading factor,  $\beta$ , and local properties of the disc, computed in an annulus which is at a distance  $d$  from the galactic centre. The galaxies used in the analysis in this section are selected so that they are close to the break of the stellar mass function at low redshift,  $M_* > 10^{10} M_{\odot} h^{-1}$ , and have  $z < 0.1$ . This selection makes the galaxy properties comparable to those simulated by Creasey et al. (2013).

In order to gain insight into (i), we show in the top panel of Fig. 6 the outflow rate in each radial annulus in units of the global SFR as a function of the distance from the galactic centre. We distinguish between galaxies with different gas fractions,  $f_{\text{gas}} = M_{\text{g,ISM}} / (M_{\text{g,ISM}} + M_*)$ . There is a tendency for gas-rich galaxies to have most of the mass breaking out from the disc at  $d \approx r_{50}$ , while in gas-poor galaxies most of the mass escapes from close to the galactic centre. We calculate the radius inside which half of the global outflow mass escapes,  $\dot{M}_{\text{out}}(d < r_{\text{out}}) = \dot{M}_{\text{eject}}/2$ , where  $\dot{M}_{\text{eject}}$  is the global outflow rate. Galaxies in Fig. 6 with  $f_{\text{gas}} > 0.8$  have  $r_{\text{out}} = 0.8 r_{50}$  and those with  $f_{\text{gas}} < 0.1$  have  $r_{\text{out}} = 0.4 r_{50}$ . This is consistent with the picture presented in Section 4.1, where the gas-poor dwarf galaxy



**Figure 7.** Top panel: the ratio of the outflow rate to SFR per annulus as a function of the surface density of gas plus stars for galaxies at  $z < 0.1$  and with  $M_* > 10^{10} h^{-1} M_{\odot}$  and for different gas fractions, as labelled, in the model with the standard set of parameters (see Table 1). The solid lines and error bars correspond to the median and 10 and 90 percentiles of the distributions. The shaded region corresponds to the predictions of Creasey et al. (2013), and is plotted over the range of surface density of gas plus stars probed by the simulations. Bottom panel: as in the top panel but for the outflow velocity per annulus as a function of  $(\Sigma_{\text{g}} + \Sigma_{*})$ .

has a more centrally concentrated outflow than galaxies that are gas rich.

In the bottom panel of Fig. 6 we show the mass-weighted velocity of the gas escaping the galaxy disc as a function of the distance from the galactic centre,  $d$ , for galaxies with different gas fractions. There is a trend of increasing outflow velocity with increasing  $f_{\text{gas}}$ . Gas-rich galaxies typically have a molecule-dominated ISM. In these galaxies the density of atomic, diffuse gas is lower, resulting in a more inefficient deceleration of bubbles. The predicted values of the outflow velocity are comparable with the observed values. We directly compare with observations of the outflow velocity in Section 4.4.

Concerning the scaling relations of the outflow [listed as (ii) above], we calculate the ratio between the mass outflow rate and the SFR in each annulus,  $\beta_{\text{annulus}}$ , and investigate its dependence on the local properties of the disc, as estimated at the mean radius of each annulus. The top panel of Fig. 7 shows the relation between  $\beta_{\text{annulus}}$  and  $(\Sigma_{\text{g}} + \Sigma_{*})$ , evaluated at  $r_{\text{annulus}}$ , for galaxies with different gas fractions. There is a tight correlation between the two quantities, with only a modest dependence on other galaxy properties, such as the gas fraction. This is expected from the correlation between  $m_b$  and  $(\Sigma_{\text{g}} + \Sigma_{*})$  (Section 4.1). The results of Creasey et al. (2013) (see Section 1 for details) are also shown in Fig. 7 by the shaded

region, plotted over the range of surface densities probed by their simulations. Our predicted relation is similar to what Creasey et al. found using a completely different approach (see Section 1).

The best fit to the relation in Fig. 7 is

$$\beta_{\text{annulus}} = \left[ \frac{\Sigma_g + \Sigma_*}{69 M_\odot \text{pc}^{-2}} \right]^{-1.3}. \quad (52)$$

The bottom panel of Fig. 7 shows the outflow velocity,  $v_{\text{outflow}}$ , as a function of  $(\Sigma_g + \Sigma_*)$ , evaluated at  $r_{\text{annulus}}$ . There is a trend of increasing  $v_{\text{outflow}}$  for increasing  $(\Sigma_g + \Sigma_*)$ . Our predictions for  $v_{\text{outflow}}$  also overlap with those of Creasey et al., although we find that outflow velocities  $> 1000 \text{ km s}^{-1}$  are statistically unlikely. These velocities can occur for SBs in our model (see Section 4.3.1). Note that for a given  $(\Sigma_g + \Sigma_*)$ , there is a trend of  $\beta$  decreasing with and  $v_{\text{outflow}}$  increasing with increasing gas fraction. This prediction is also in agreement with the findings of Creasey et al.

Note that changes in the SN feedback model parameters, which are summarized in Table 1, produce similar deviations to those found for the galaxy-wide  $\beta$  and mass-weighted  $v_{\text{outflow}}$  in Section 4.3.2. We find that the surface density normalization and power-law index in equation (52) increase with increasing redshift, in a similar way that the global  $\beta$  does (Fig. 16). Therefore, the similarity between our predictions and those of Creasey et al. is confined to our low-redshift galaxy sample. Note that the results of Fig. 7 for a fixed gas fraction do not depend on stellar mass or redshift, but the global normalization and power-law index of equation (52) do due to the predominance of gas-poor galaxies at low redshift and of gas-rich galaxies at high redshift.

### 4.3 Statistical properties of the outflow rate and velocity

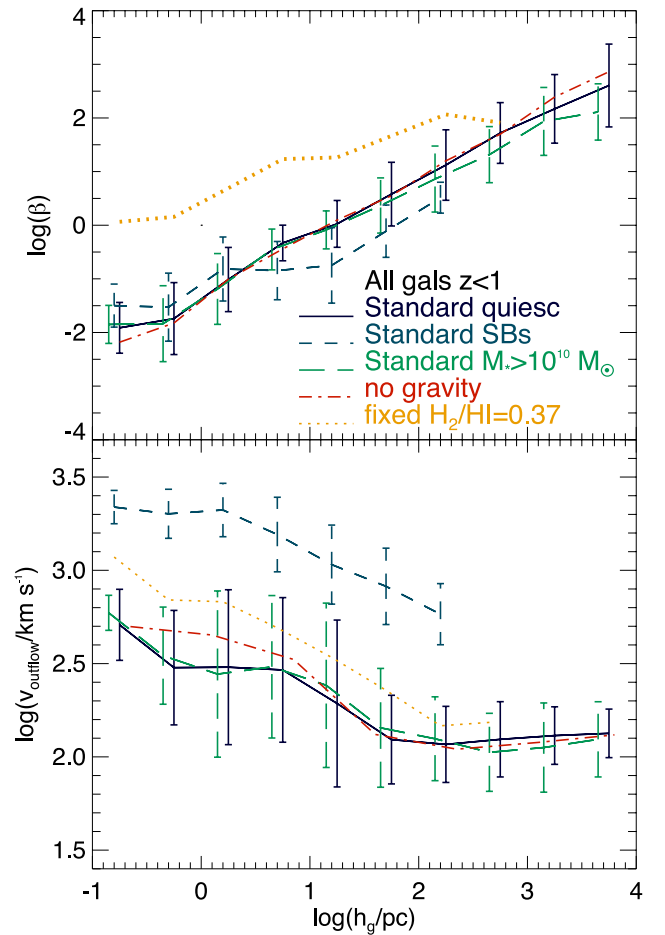
In this section, we attempt to answer three questions: What is the effect of the multiphase treatment of the ISM on  $\beta$ ? What is the overall effect of varying the physical parameters of the ISM and GMCs on the outflow rate? Is the outflow rate dominated by adiabatic or radiative bubbles?

Here we analyse galaxies from GALFORM, after the full dynamical model of SN feedback is included in the calculation. At each redshift we focus on galaxies with  $M_* > 10^8 h^{-1} M_\odot$ , to be safely above the resolution limit of the Millennium simulation (Section 3.1). We consider the total mass loading rate of the outflow,  $\beta$ , which we define as  $\beta = \dot{M}_{\text{eject}}/\psi$ , where  $\dot{M}_{\text{eject}}$  corresponds to the total mass breaking out from the ISM (given by  $f_{\text{bo}} \dot{M}_{\text{bo,ISM}}$  in equations 36–43) and  $\psi$  is the instantaneous SFR. In Section 4.3.5, we analyse the metal loading of the wind, which we define as  $\beta^Z = \dot{M}_{\text{eject}}^Z/Z_g \psi$ . This  $\beta$  differs from the  $\beta_{\text{annulus}}$  of Section 4.2 in two respects: the former is integrated over the galaxy and over longer timesteps.

In Sections 4.3.1–4.3.4, we show the total mass loading  $\beta$  as a function of the gas scaleheight at the half-mass radii of galaxies,  $h_g$ . This can be understood from the strong dependence of  $m_{\text{bo}}$  on  $h_g$  and the small dispersion in this relation (see Section 4.1). In Section 4.3.5, we show how and where  $\beta^Z$  differs from  $\beta$  and the reasons for such differences.

#### 4.3.1 Testing the effect of the multiphase medium and gravity on the outflow properties

The top panel of Fig. 8 shows the correlation between  $\beta$  and  $h_g$  at the half-mass radius obtained with and without considering gravity from stars and DM in equations (1)–(3), (18)–(20) and (24)–(26), and using the standard set of parameters to describe GMCs and the ISM of galaxies (see Table 1). We plot the gas scaleheight at the



**Figure 8.** Top panel: the mass loading,  $\beta$ , as a function of the gas scaleheight at the half-mass radius for quiescent (solid line), SB (dashed line) and a subsample of massive galaxies,  $M_* > 10^{10} h^{-1} M_\odot$  (long-dashed line), in the model with the standard set of parameters (Table 1). In the case of quiescent SF,  $h_g$  is evaluated at  $r_{50}$  of the disc, and for SBs, at  $r_{50}$  of the bulge. We include in the plot all galaxies in GALFORM at  $z < 1$  and with  $M_* > 10^8 h^{-1} M_\odot$ . We also show the effect of suppressing gravity on the expansion of bubbles (dot-dashed line), and of assuming a constant  $\text{H}_2/\text{H}\text{I}$  ratio instead of that derived from the Blitz & Rosolowsky pressure law (dotted line). The solid lines and error bars indicate the median and 10 and 90 per cent ranges of the predictions. For clarity, error bars are shown only for selected cases. Bottom panel: as in the top panel, but here we show the mass-weighted outflow velocity as a function of the gas scaleheight.

half-mass radius in the range from 0.1 to  $10^4$  pc, but galaxies with such extreme half-mass radii are very rare. In fact, the median  $h_g$  for SBs ranges from 50 pc in low-mass galaxies to 10 pc in high-mass galaxies, and for quiescent galaxies it ranges from 450 pc in low-mass galaxies to 80 pc in high-mass galaxies.

We find that  $\beta$  is only slightly affected when gravity is not included. This agrees with what we find for individual bubbles, in which gravity has an effect of at most 5 per cent on the final bubble mass. The effect of including the  $\text{H}_2/\text{H}\text{I}$  ratio calculated from the Blitz & Rosolowsky pressure law in the modelling of the ISM is much larger than the direct gravitational effect, as the dotted line in Fig. 8 shows. The omission of self-consistent multiphase modelling is represented by the results obtained with a fixed  $\text{H}_2/\text{H}\text{I} = 0.37$  ratio, which is the value used in previous work to estimate  $\text{H}\text{I}$  from the total cold gas content (e.g. Power, Baugh & Lacey 2010; Kim et al. 2011). With a fixed  $\text{H}_2/\text{H}\text{I}$  ratio, the mass loading increases by

a factor of up to 100 for galaxies with the smallest gas scaleheights (i.e. highest density regimes). This is due to the anticorrelation between  $H_2/H_1$  and  $h_g$  (Lagos et al. 2011b). Galaxies with very high gas and/or stellar surface densities have smaller  $h_g$  and larger  $H_2/H_1$ , driving a lower overall content of H I and therefore providing less material for bubbles to sweep up, reducing the outflow mass. This effect is very large in more extreme cases, where the pressure law predicts little H I. This is also clear from the single bubble examples of Section 4.1, in which the bubble mass is greatly reduced in molecule-dominated media. This demonstrates the importance of the ISM modelling introduced in Lagos et al. (2011a,b), and also included in some other recent models (e.g. Fu et al. 2010).

In the top panel of Fig. 8, we show the relations for SB and massive galaxies separately. This stresses the similarity between the relations displayed by quiescent and SB galaxies in the  $\beta$ – $h_g$  plane and the fact that massive galaxies follow the same relation as the overall galaxy population, which is dominated in number by lower mass systems. This is because the mass loading  $\beta$  is primarily determined by the gas scaleheight and the gas fraction, as we show later in Section 5.2.

In the bottom panel of Fig. 8, we show the mass-weighted outflow velocity as a function of the gas scaleheight. There is a trend of decreasing velocity for increasing  $h_g$ . SB galaxies exhibit a relation with a similar slope to that of quiescent galaxies but offset by  $\approx 0.5$  dex to larger velocities. This is due to the different SF laws assumed in the model for the SB and quiescent SF modes (see Section 3.1). For a fixed  $h_g$ , an SB galaxy generally has a larger SFR than its quiescent counterpart. This drives larger energy and momentum injection, resulting in larger outflow velocities. The effect of gravity in the outflow velocity is only minor, as is also the case for  $\beta$ . The effect of including the Blitz & Rosolowsky pressure law in the modelling of the ISM on the outflow velocity is more significant, and its omission results in velocities that are larger by a factor of  $\approx 2$  at small  $h_g$ . In Section 4.4, we compare our predicted velocities with observations.

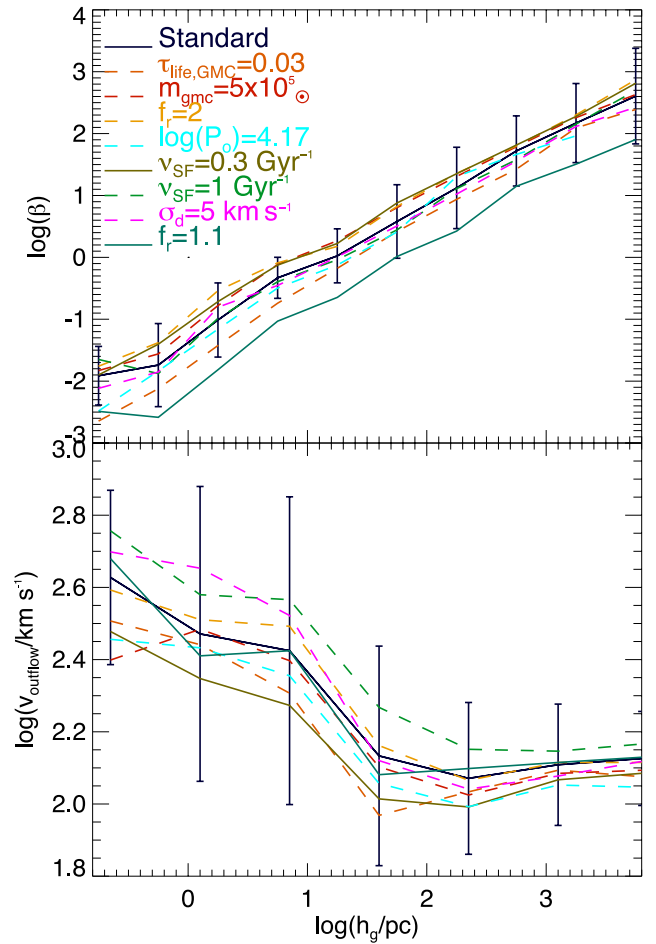
#### 4.3.2 Assessing the impact of ISM and GMC parameters on the outflow properties

The top panel of Fig. 9 shows the predicted mass loading as a function of the gas scaleheight when varying the parameters associated with the modelling of GMCs and the diffuse medium (see Table 1). Changes in the GMC and diffuse medium model parameters drive different normalizations in the  $\beta$ – $h_g$  relation but have a weak impact on the shape of the relation. The variations between the models that produce the smallest and largest  $\beta$  values, which correspond to adopting  $f_r = 1.1$  and  $v_{SF} = 0.3 \text{ Gyr}^{-1}$ , respectively, are at most a factor of  $\approx 10$ . It is reasonable to argue that a better understanding of the multiphase nature of the ISM and the properties of GMCs is very important, even more so than including some of the physical mechanisms in the expansion of bubbles, such as gravity. This was also hinted at in Fig. 8 from the effect of adopting a multiphase ISM description of the outflow rate.

The effect of each of the parameters in Table 1 on  $\beta$  is summarized below.

(i) Smaller values of  $f_r$  result in smaller  $\beta$  values by a factor of  $\approx 3$ –5. This is expected from the role  $f_r$  plays in determining the break-out radius of bubbles and therefore the bubble mass (equation 48).

(ii) Adopting a smaller SF coefficient or a smaller GMC mass drives an increase in  $\beta$  due to the lower SFR predicted by the



**Figure 9.** Top panel: the predicted mass loading,  $\beta$ , as a function of the gas scaleheight. In the case of quiescent SF,  $h_g$  is evaluated at  $r_{50}$  of the disc, and for SBs, at  $r_{50}$  of the bulge. The predictions are shown for different choices of the model parameters, as labelled. We include in the plot all galaxies in GALFORM at  $z < 1$  with  $M_* > 10^8 h^{-1} M_\odot$ . Lines and error bars indicate the median and 10 and 90 percentile ranges of the relations. For clarity, the percentile range is shown only for one model as they are all similar. The solid lines are used for the model with the standard set of parameters and those predicting the lowest and the highest  $\beta$  for a given  $h_g$ . The dashed lines are used for the rest of the models (see Table 1). Bottom panel: as in the top panel, but here we show the mass-weighted outflow velocity as a function of the gas scaleheight.

former and the higher number of GMCs predicted by the latter. The effect of increasing  $v_{SF}$  or  $M_{GMC}$  is therefore a smaller  $\beta$ . Adopting a longer lifetime for GMCs also decreases  $\beta$  due to the anticorrelation between  $\beta$  and  $\tau_{\text{lifc, GMC}}$ .

(iii) A smaller hydrostatic pressure normalization in the Blitz & Rosolowsky law (see Section 3) drives larger  $\beta$  but only in galaxies which have a molecule-dominated ISM, as it only affects this regime (see equation 48). In these cases, the lower  $P_0$  drives smaller individual bubble masses and therefore smaller  $\beta$  (see equation 51). Similarly, the effect of decreasing  $\sigma_d$  is to slightly decrease  $\beta$ , which is also expected from the analysis of Section 4.1.1.

The effect of varying the parameters above on the mass-weighted outflow velocities,  $v_{\text{outflow}}$ , is shown in the bottom panel of Fig. 9. Variations in  $v_{\text{outflow}}$  due to different ISM parameter choices are smaller than in the case of  $\beta$ , with a difference between the minimum and maximum  $v_{\text{outflow}}$  of  $\approx 0.5$  dex. The models predicting the



highest and lowest  $\beta$  are not the same as those predicting the highest and lowest  $v_{\text{outflow}}$ . This is because  $v_{\text{outflow}}$  is more affected by those parameters directly changing the energy injection into the ISM by SNe. Indeed, the parameter that is most important in setting  $v_{\text{outflow}}$  is the SF coefficient,  $\nu_{\text{SF}}$ . The more efficient the conversion from gas to stars, the higher is the outflow velocity. This is consistent with what is shown for quiescent and SB galaxies in Fig. 7.

#### 4.3.3 The outflow rate and velocities in galaxies with extreme ISM conditions

Resolved observations of the ionized gas in star-forming galaxies at  $1 \lesssim z \lesssim 3$  have shown that they have velocity dispersions that are systematically larger than the ones measured for the neutral gas content of local spiral and dwarf galaxies, and that they host star-forming clumps which can be more extended and luminous in H $\alpha$  than local clumps (e.g. Law et al. 2007; Puech et al. 2007; Genzel et al. 2008; Livermore et al. 2012; see Glazebrook 2013 for a recent review), similarly to local SBs (see Section 2.2.2). Galaxies more massive than  $M_{\text{stellar}} \gtrsim 10^{11} M_{\odot}$  built up more than half of their stellar mass at  $z > 1$  (e.g. Pérez-González et al. 2008), and therefore may form most of it in a clumpy, turbulent ISM. However, it is important to bear in mind that the low number of galaxies on the observational samples does not allow us to conclusively determine how representative these are of the overall galaxy population. Another important warning is that the velocity dispersion measured at high redshift corresponds to the ionized component of the ISM, while the relevant quantity for our model is the atomic and molecular gas velocity dispersion. Other systematic effects include the point spread function and the limited spatial resolution that can bias the inferred values towards higher observed velocity dispersion and more extended clumps (e.g. see Glazebrook 2013 for a discussion of systematics).

Given the important role an ‘extreme’ ISM phase could play in galaxy evolution, we investigate in this section the effect on the mass loading and velocity of the outflow of increasing  $\sigma_{\text{d}}$  and  $M_{\text{GMC}}$ . We adopt  $M_{\text{GMC}} = 10^8 M_{\odot}$  and  $\sigma_{\text{d}} = 70 \text{ km s}^{-1}$  as representative values for clumpy galaxies. We also test intermediate values for the GMC mass,  $M_{\text{GMC}} = 10^7 M_{\odot}$ , and for the gas velocity dispersion,  $\sigma_{\text{d}} = 30 \text{ km s}^{-1}$ , to better test the effects of increasing  $M_{\text{GMC}}$  and  $\sigma_{\text{d}}$ .

We ran three simulations with increased  $M_{\text{GMC}}$  or  $\sigma_{\text{d}}$  and one with both quantities increased with respect to the standard choice of ISM and GMC parameters (see Table 1). The results of those runs are shown in Fig. 10 for quiescent and SB galaxies. We focus on galaxies in the redshift range  $1 < z < 3$  to match the redshift range of the surveys described above. The increase in  $M_{\text{GMC}}$  by two orders of magnitude decreases  $\beta$  by  $\approx 1.5$  dex, while the increase in  $\sigma_{\text{d}}$  by a factor of 3 increases  $\beta$  by  $\approx 1$  dex. This is consistent with the variations we expect from our simplified analytic solution for  $\beta$  (Section 4.1.1). When we increase both,  $\sigma_{\text{d}}$  and  $M_{\text{GMC}}$ , the variations in  $\beta$  compensate in a way that adopting  $\sigma_{\text{d}} = 70 \text{ km s}^{-1}$  and  $M_{\text{GMC}} = 10^8 M_{\odot}$  causes  $\beta$  to decrease by at most 0.5 dex with respect to the values obtained in our standard choices for these parameters. From the Jeans mass in a disc,  $M_{\text{J}} \propto \sigma_{\text{d}}^4 / \Sigma_{\text{g}}$ , we expect both quantities to increase together and thus we expect net variations in  $\beta$  of at most a factor of 3 in galaxies with more extreme ISM conditions, which could be representative of the high-redshift population.

In the case of the outflow velocity (lower panels in Fig. 10), we find that the increase in  $\sigma_{\text{d}}$  and  $M_{\text{GMC}}$  drives smaller variations than

in  $\beta$ , in the range of 0.3–0.4 dex. This is consistent with the picture presented in Section 4.3.2, where  $\nu_{\text{SF}}$  drives the largest variations in the outflow velocity. Note that the effect of adopting different values of these parameters is different for quiescent galaxies than it is for SBs. This is driven by the different SF laws assumed in each SF mode (see Section 3.1).

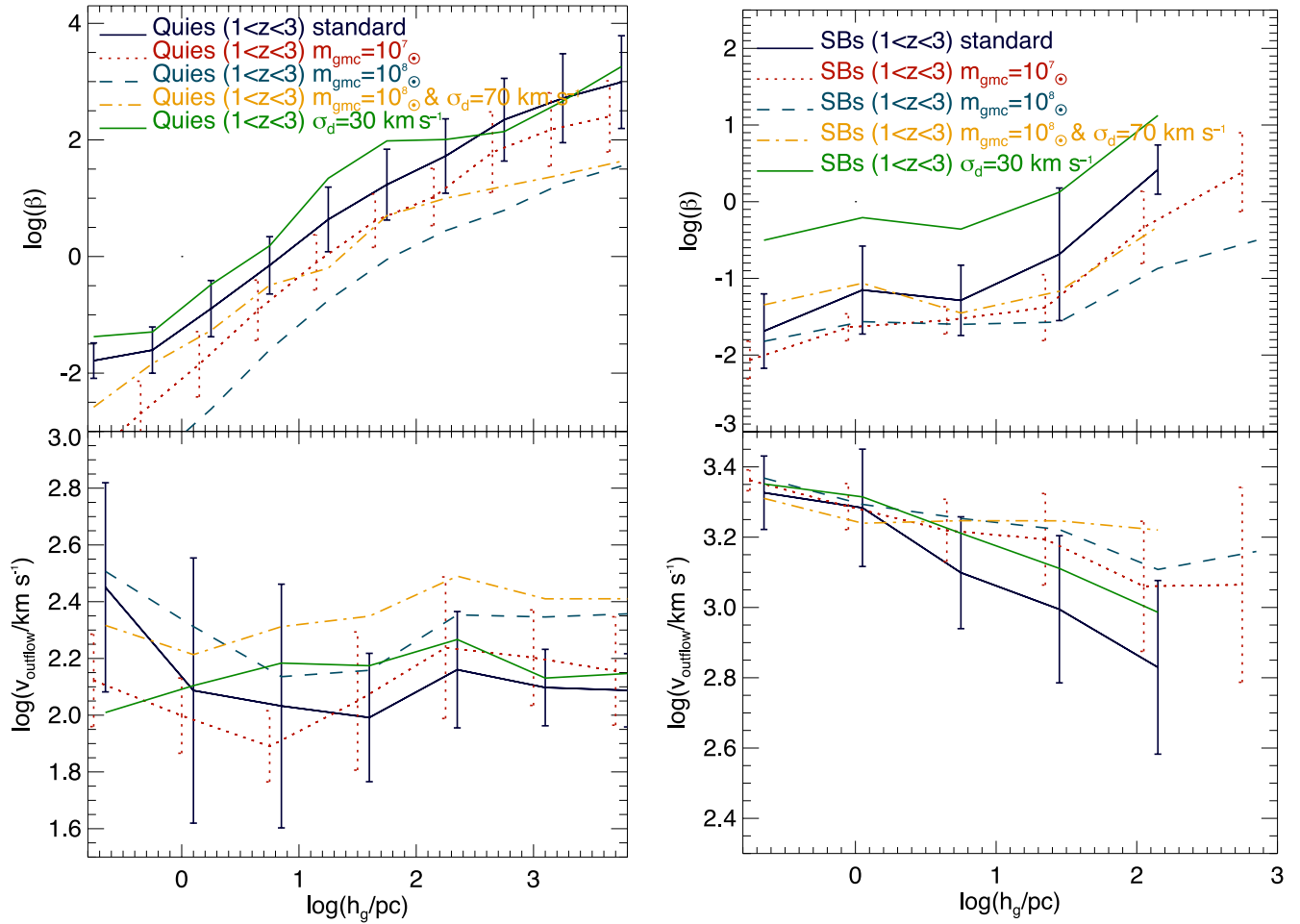
#### 4.3.4 The physical regimes of the outflow

Bubbles inflated by SN feedback can escape the galaxy in any of the three evolutionary stages described in Section 2. We now quantify where and when each of these stages dominates the outflow of material.

Fig. 11 shows the mass loading,  $\beta$ , as a function of the gas scaleheight,  $h_{\text{g}}$ , evaluated at the half-mass radius for the model with the standard set of parameters. We find that at high redshift, most of the outflow in galaxies is produced by bubbles escaping in the momentum-driven stage, while low-redshift galaxies with small gas scaleheights have mass outflow rates dominated by bubbles escaping in the pressure-driven stage. High-redshift galaxies have a gas scaleheight set by the gas surface density with a negligible contribution from the surface density of stars. In the low-redshift regime, galaxies with small gas scaleheights have, by comparison, a more important contribution from the stellar component. In fact, the median gas fraction of the galaxy sample with  $h_{\text{g}} < 10 \text{ pc}$  in the high- and low-redshift samples is 0.98 and 0.18, respectively. Galaxies which have the gas scaleheight set mainly by the stellar surface density have bubbles where the cooling time for the interior gas is large enough for bubbles to escape the disc in the pds stage. In the case of the larger gas scaleheight galaxy population, the scaleheight is set mainly by the gas surface density, so no significant difference with redshift is obtained.

When bubbles escape the ISM in the radiative phase (i.e. pds or mds), this implies that most of the outflow mass is in a cold, dense phase (i.e. molecular or neutral atomic gas) and that the interior mass of the bubbles is only a minor contributor. This qualitatively agrees with what is observed in local galaxies (e.g. Tsai et al. 2009, 2012). A quantitative comparison will be presented in a forthcoming paper (Lagos et al., in preparation).

The adiabatic phase only rarely dominates the outflow rate, since the transition from the ad to the pds stage takes place early on in the evolution of bubbles. This transition almost always takes place on a time-scale of  $\approx 10^3$ – $10^5$  yr. Full confinement due to deceleration of bubbles rarely takes place (i.e. the case in which no bubbles break out from the galaxy disc), and happens mainly in places where the scaleheight is large and the bubble has time to decelerate to the velocity dispersion of the diffuse gas (i.e. at low gas densities). Most of the gas which remains in the ISM therefore corresponds to gas expanding in the direction close to the plane [i.e. the fraction  $(1 - f_{\text{bo}})$  in equations (36)–(43)] rather than to bubbles which are fully confined in the ISM. The tendency we find for bubbles to break out in the radiative phase contrasts with what Monaco (2004a) found, whose model predicts that most bubbles escape during the adiabatic phase. This difference may be due to the assumptions Monaco makes that bubbles expand against the hot phase. In our model, bubbles expand against the warm phase, whose density is typically higher than the hot phase, which results in larger cooling rates. We find that our approach gives answers more similar to fully hydrodynamical simulations in the range where they overlap (see Section 4.2).



**Figure 10.** Top panel: the predicted mass loading,  $\beta$ , as a function of the gas scaleheight,  $h_g$ , for quiescent SF (left-hand panel) and SBs (right-hand panel). In the case of quiescent SF,  $h_g$  is evaluated at  $r_{50}$  of the disc, and for SBs, at  $r_{50}$  of the bulge. The predictions are shown for the standard choice of parameters, and for extreme values of  $M_{\text{GMC}}$  and  $\sigma_d$ , as labelled, which could be representative of the conditions of high-redshift star-forming galaxies. Since we want to investigate these high-redshift galaxies, we include in the plot all galaxies in GALFORM at  $1 < z < 3$  with  $M_* > 10^8 h^{-1} M_{\odot}$ . Lines and error bars indicate the median and 10 and 90 percentile ranges of the relations. For clarity, the percentile range is shown only for two models as they are all similar. Bottom panel: as in the top panel, but here we show the mass-weighted outflow velocity as a function of  $h_g$ .

#### 4.3.5 Outflow rates of mass and metals

We have analysed the physics behind the dependence of  $\beta$  on galaxy properties and gave analytic derivations for such relations. However, a key part of the impact of outflows on galaxy evolution is the fate of the metals carried away by bubbles. In the model, we assume that the metals which flow out from the galaxy accumulate in the ejected mass component, which is later reincorporated into the hot halo gas (see equations 36–45). The amount of metals outflowing from the galaxy therefore has a direct impact on the cooling rate of the hot halo gas and hence on subsequent gas accretion and SF in the galaxy.

Here, we analyse the loading factor of metals defined as  $\beta^Z = \dot{M}_{\text{eject}}^Z / (Z_g \psi)$  (see equation 44). The top panel of Fig. 12 shows the metal loading factor as a function of the mass loading factor for galaxies at different redshifts. Galaxies at  $z < 2$  follow a relation which is close to  $\beta^Z = \beta$ , but which shows a flattening at  $\beta \lesssim 0.5$  (i.e. in the small gas scaleheight regime). However, as the redshift increases, deviations become important and begin at increasingly larger  $\beta$ . At  $z > 6$  there is almost no correlation between  $\beta^Z$  and  $\beta$ , with  $\beta^Z \approx 30$  independent of  $\beta$ , albeit with a large dispersion. This behaviour is due to high-redshift galaxies having intrinsically lower

metallicity gas from which stars form. In the low-metallicity regime, metals in bubbles coming from the swept-up gas are negligible compared to those coming from SN ejecta; in the limit of  $Z_g \ll p_{\text{SN}}$  and  $4\pi R^2 Z_g \rho_d v_s \ll \dot{m}_{\text{inj}}^Z$ , we can write the metal outflow rate due to a single bubble as

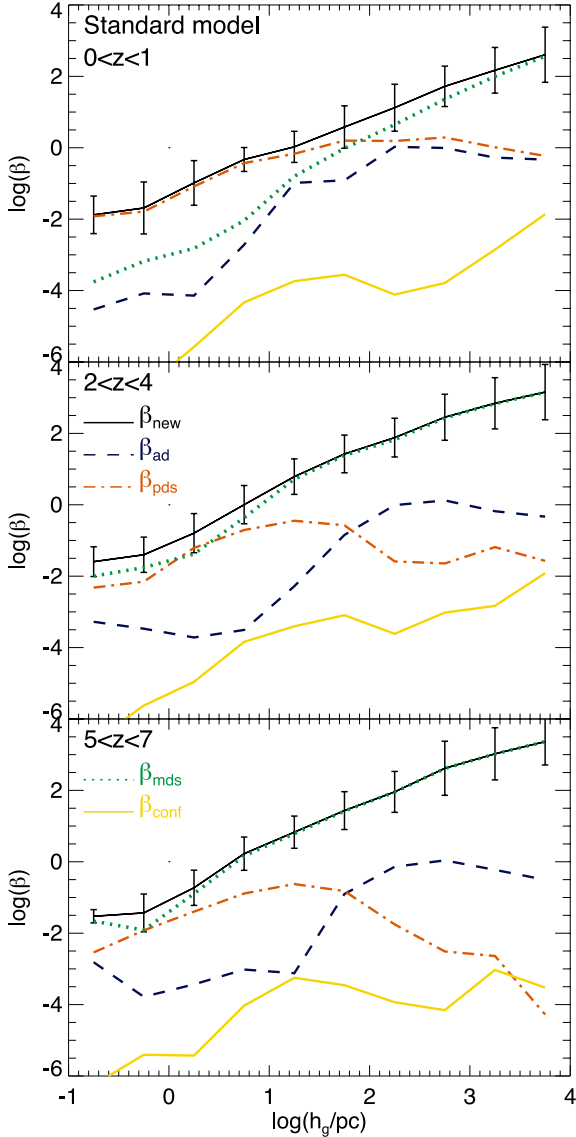
$$\dot{m}_{\text{eject}}^Z = f_{\text{bo}} p_{\text{SN}} \psi_{\text{GMC}} = f_{\text{bo}} p_{\text{SN}} v_{\text{SF}} M_{\text{GMC}}. \quad (53)$$

The rate of metals flowing out from the galaxy in a given annulus is regulated by the number of GMCs in that annulus  $\dot{M}_{\text{eject}}^Z = f_{\text{bo}} p_{\text{SN}} v_{\text{SF}} M_{\text{mol}}$ . We then calculate  $\beta^Z$  per annulus in this regime

$$\beta^Z = \frac{\dot{M}_{\text{eject}}^Z}{Z_g v_{\text{SF}} M_{\text{mol}}} = \frac{f_{\text{bo}} p_{\text{SN}}}{Z_g}. \quad (54)$$

Because we assume instantaneous mixing in GALFORM, this  $\beta^Z$  is representative of the global metal loading factor. In the limit of  $Z_g \ll p_{\text{SN}}$ ,  $\beta^Z$  shows no dependence on  $h_g$ . However, the mass outflow rate has a strong dependence on  $\Sigma_g$ , regardless of the metallicity of the ISM. This results in very little correlation between  $\beta^Z$  and  $\beta$  in this low-metallicity regime.

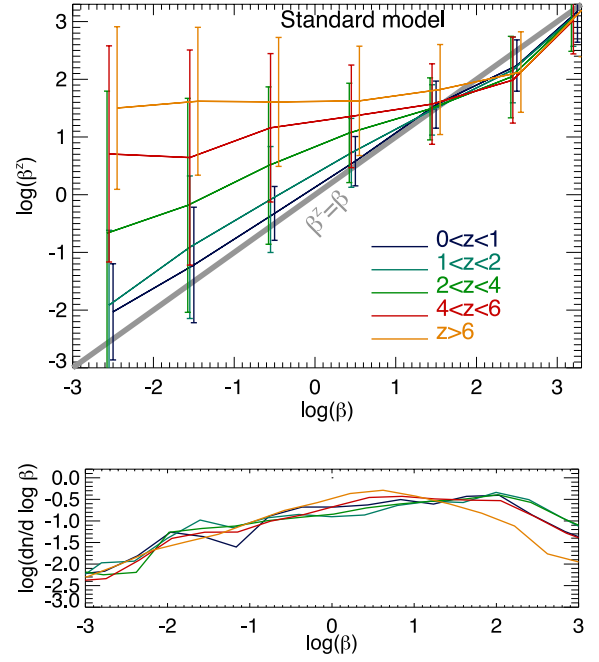
If the ISM is already enriched with some metals, which corresponds to approximately  $Z_{\text{gas}} \gtrsim 0.05\text{--}0.1 Z_{\odot}$ , the density of the



**Figure 11.** The mass loading factor,  $\beta$ , as a function of the gas scaleheight at the half-mass radius for galaxies with  $M_* > 10^8 h^{-1} M_\odot$  in three redshift ranges, as labelled in each panel. In the case of quiescent SF,  $h_g$  is evaluated at  $r_{50}$  of the disc, and for SBs, at  $r_{50}$  of the bulge. The contribution to the total  $\beta$  (solid line) from bubbles escaping in the adiabatic, pressure-driven and momentum-driven snowplough phases is shown as dashed, dot-dashed and dotted lines, respectively. The ratio between the rate of mass confinement and the SFR,  $\beta_{\text{conf}}$ , is shown as the triple-dot-dashed line. Lines represent the medians and the error bars, which are shown for clarity only for the total  $\beta$ , represent the 10–90 percentile range.

gas in the ISM also has an important effect on  $\beta^Z$  given that the term  $4\pi R^2 Z_g \rho_d v_s$  becomes comparable to or larger than the term  $\dot{m}_{\text{inj}}^Z$  in the evolution of single bubbles (see equation 3). In this case, a correlation between  $\beta^Z$  and  $\beta$  arises.

Although a non-linear relation between  $\beta$  and  $\beta^Z$  is predicted, we find that most galaxies in our simulation follow a relation which is close to  $\beta^Z = \beta$ . This can be seen from the distribution of  $\beta$  for different redshifts in the bottom panel of Fig. 12. Quantitatively, at least 75 per cent of galaxies at any redshift have  $\beta > 1$  and at least 50 per cent at  $z < 5$  have  $\beta > 10$ . This puts at least half or more of the galaxies in the regime where  $\beta^Z \sim \beta$ . Galaxies deviating this relation are the most metal-poor ones, which typically

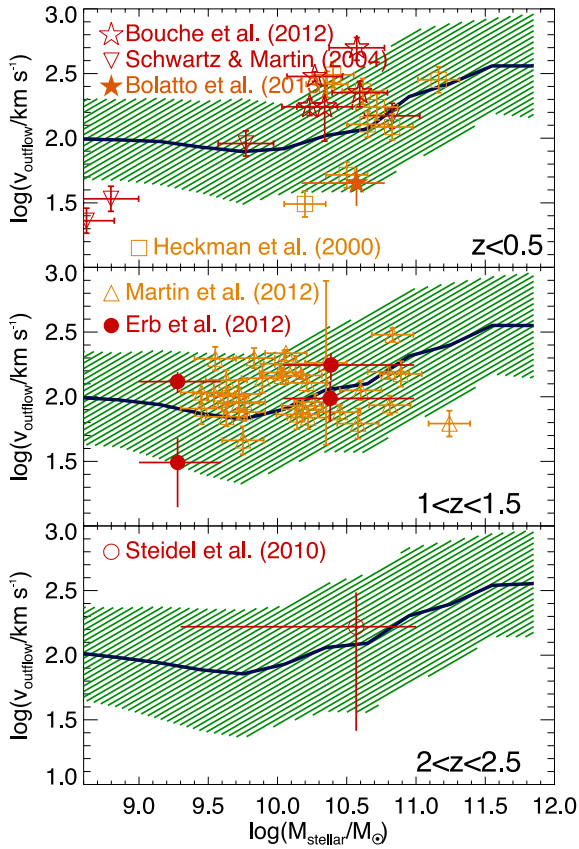


**Figure 12.** Top panel: the metal loading factor,  $\beta^Z = \dot{M}_{\text{eject}}^Z / (Z_g \psi)$ , as a function of the mass loading factor,  $\beta = \dot{M}_{\text{eject}} / \psi$ , for galaxies with  $M_* > 10^8 h^{-1} M_\odot$ . Both quantities are integrated over the galaxy and in the same timesteps. Lines with error bars represent the median and the 10–90 percentile range, respectively, for galaxies at different redshifts, as labelled. The thick, straight line shows  $\beta^Z = \beta$ . Bottom panel: normalized distribution of  $\beta$  for galaxies in the same redshift ranges as in the top panel.

correspond to those with low stellar masses. As we show later in Section 6, the inclusion of a metal loading factor with an independent parametrization from the mass loading factor in GALFORM has a small effect on the luminosity of galaxies. However, if we wish to analyse in detail the gas content of galaxies and the evolution of the mass–metallicity relation, we would need to allow for such variations in the  $\beta^Z$  parametrization included in the model.

#### 4.4 Comparison with observations and non-cosmological hydrodynamical simulations

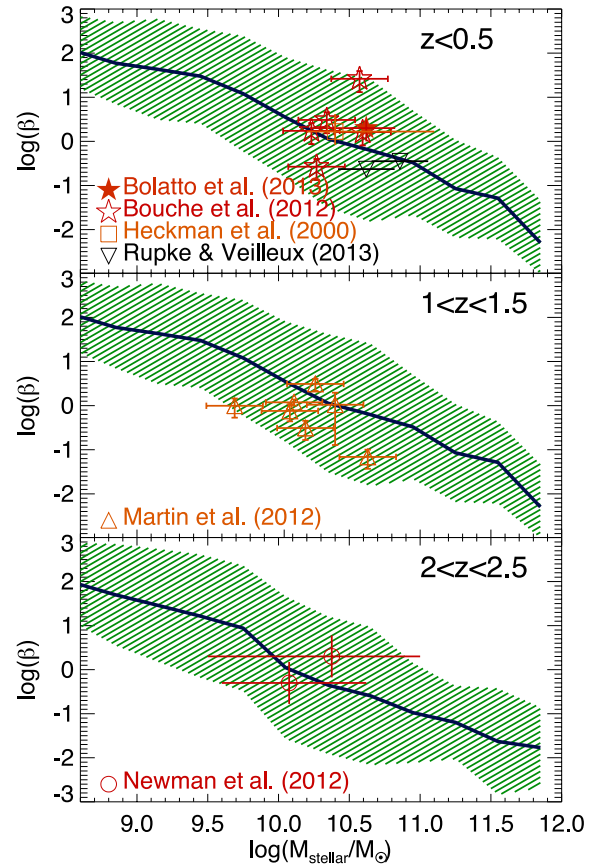
We compare our predictions for the mass loading of the wind,  $\beta$ , with the values inferred from observations by Heckman et al. (2000), Martin et al. (2012), who use absorption features in galaxy spectra, Newman et al. (2012), who use emission line galaxy spectra, Bouché et al. (2012), who use absorption lines in the lines of sight to background quasars (probing the outflow and inflow of gas), Bolatto et al. (2013), who inferred the total outflowing mass from molecular emission, and Rupke & Veilleux (2013), who simultaneously study absorption and emission lines. Heckman et al. (2000) and Bouché et al. (2012) focus on  $L^*$  galaxies at low redshift ( $z \lesssim 0.1$ ), while Martin et al. (2012) focus on galaxies at  $z \approx 1$  and Newman et al. on galaxies at  $z \approx 2$ . Heckman et al., Bolatto et al. and Rupke et al. do not provide stellar masses for their galaxy samples. We therefore use the near-IR photometry available in the NASA/IPAC Extragalactic Database to estimate the stellar mass from the  $K$ -band luminosity. If only the  $H$ -band luminosity is given, we use the colour measurements of Boselli et al. (2000),  $H - K \sim 0.25$ , to convert to a  $K$ -band luminosity. We then use the median  $K$ -band mass-to-light ratio from Bell et al. (2003) to convert to stellar masses. We



**Figure 13.** The mass-weighted outflow velocity,  $v_{\text{outflow}}$ , as a function of stellar mass. The panels and galaxy selections are as in Fig. 14. In the top panel, we show the observationally inferred outflow velocities of individual galaxies from Heckman et al. (2000), Schwartz & Martin (2004), Bolatto et al. (2013) and Bouché et al. (2012). In the middle panel, we show the inferred outflow velocities in individual galaxies from the sample of Martin et al. (2012) and the median velocity of the galaxy samples of Erb et al. (2012). In the bottom panel, we show the median outflow velocity and stellar mass of the sample of Steidel et al. (2010). In the case of Erb et al. and Steidel et al., the error bars in the stellar mass axis correspond to the range of stellar masses in the samples, while the error bars in the y-axis correspond to the standard deviation in the samples. In the case of Erb et al., we plot two different velocity estimates for each stellar mass range, which correspond to two different iron transitions, corresponding to those giving the lowest and highest blueshift velocities. Note that the number of data points in this figure from Martin et al. is larger than in Fig. 14. This is because only a third of the sample had measured SFRs to provide an estimate of  $\beta$ .

apply the same calculation to estimate stellar masses in the sample of Schwartz & Martin (2004), shown in Fig. 13. In the case of Bouché et al. (2012),  $r$ -band absolute magnitudes are given for each galaxy in the sample, so we use the  $r$ -band mass-to-light ratio from Bell et al. (2003) to convert to stellar masses. Finally, we adopt a correction of 0.71 in stellar mass to convert from the adopted IMF in Bell et al., the ‘diet’ Salpeter, to the Kennicutt (1983) IMF. Given the uncertainties in the scalings above, we conclude that we cannot estimate stellar masses to a factor better than 0.2 dex and adopt this number as a typical error (see Mitchell et al. 2013 for a recent discussion on stellar mass estimate uncertainties).

Fig. 14 shows  $\beta$  as a function of stellar mass for our standard model (see Table 1). Symbols show the median stellar mass and the  $\beta$  inferred from observational samples. Our model predicts  $\beta$  values which are in broad agreement with those inferred from ob-



**Figure 14.** The mass loading,  $\beta$ , as a function of stellar mass for galaxies that have an outflow, in three different redshift ranges, as labelled, for the standard set of parameters (Table 1). The solid lines and the shaded regions indicate the median and 10 and 90 per cent ranges of the distributions. The observationally inferred  $\beta$  from Heckman et al. (2000), Martin et al. (2012), Newman et al. (2012), Bouché et al. (2012), Bolatto et al. (2013) and Rupke & Veilleux (2013) are shown using symbols, as labelled. The error bars in the mass axis for Heckman et al. and Newman et al. represent the range of stellar masses of the galaxies in the samples and on the y-axis we show the range of inferred  $\beta$ . In the case of Newman et al., the two samples correspond to a low-SFR sample, which has a lower median stellar mass, and a high-SFR sample. In the cases of Bolatto et al., Bouche et al., Rupke et al. and Martin et al., the errors in the stellar mass and  $\beta$  estimates are shown for individual galaxies. The data from Martin et al. plotted in the middle panel correspond to the subset of galaxies in their sample that have measured SFRs.

servations. However, there are large uncertainties associated with the inference of outflow rates from observations, in addition to the statistical uncertainties arising from the small number of objects sampled. The main uncertainties in the calculation of outflow rates from observations come from the conversion between the ion and hydrogen column densities, which depends on the gas metallicity and ionization factor, the assumed geometry (e.g. Prochaska, Kasen & Rubin 2011) and the still uncertain nature of absorption by low-ionization metal lines, in the case of absorption line studies in quasar sightlines. Note that the error bars plotted in Fig. 14 do not include the systematic errors associated with the modelling assumptions made to derive  $\beta$ , and represent lower limits for the uncertainties.

Fig. 13 shows the mass-weighted outflow velocity as a function of stellar mass. We show the observational estimates from Heckman et al. (2000), Schwartz & Martin (2004), Bouché et al. (2012) and Bolatto et al. (2013) at  $z \approx 0-0.1$ , Martin et al. (2012) and Erb et al. (2012) at  $z \approx 1$  and Steidel et al. (2010) at  $z \approx 2$ . Note that for

Erb et al. and Steidel et al., the error bars correspond to the standard deviation of  $\beta$  in the full sample, while we plot individual errors in the rest of the observational samples. Heckman et al., Schwartz et al., Martin et al., Erb et al. and Steidel et al. use galaxy absorption line spectroscopy to infer an average blueshift of the ionized component with respect to the systemic velocity, Bouche et al. use Mg II absorption lines in the lines of sight to background quasars to infer an outflow velocity, and Bolatto et al. use molecular emission lines to measure the kinematics of the cold gas. The predicted outflow velocities are broadly consistent with those inferred from the observations. The estimates of the velocities and outflow rates from the observations are not straightforward, as the different gas phases of the outflow could have different velocities and mass loadings. This becomes evident in the data points of Erb et al. (2012) shown in Fig. 13; in a given stellar mass range, the two values of the outflow velocity correspond to two different iron line transitions. In the case of the model, the plotted outflow velocities are calculated from the expansion velocities of bubbles at the point of break-out and are dominated by the phase that contributes the most to the outflow mass. We predict that in many cases this corresponds to a warm or cold phase (neutral or molecular). In the case of observations, most of the available data probe warm ionized gas and are corrected to account for the neutral component. Ideally, these data need to be complemented by deep observations at millimetre wavelengths to directly probe the part of the outflow that is in a cold phase.

There are additional selection effects in the observations shown in Figs 13 and 14, which are not taken into account in the comparison with the model. First, almost all of the observational samples are selected to include only highly star-forming galaxies, except for Bouché et al. (2012), which uses quasi stellar object absorption lines. Secondly, the reported outflow velocities correspond only to galaxies in which there was a detectable outflowing component. This biases the measurements against low-mass outflows. These effects need to be properly reproduced in the selection of galaxies in the model before carrying out a detailed comparison with the observations. For instance, the model predictions for the full galaxy population shown in Fig. 13 are only marginally consistent with the velocities inferred by Schwartz & Martin (2004) for three dwarf SB galaxies. We calculate the median outflow velocity of galaxies with stellar masses in the range  $10^8$ – $10^9 M_\odot$  and with  $\text{SFR} > 0.1 M_\odot \text{ yr}^{-1}$ , corresponding to the properties of the Schwartz & Martin sample (Martin et al. 2012), and find  $v_{\text{out}} \approx 70 \text{ km s}^{-1}$ , with a 10 percentile of  $10 \text{ km s}^{-1}$  and a 90 percentile of  $300 \text{ km s}^{-1}$ . The sample of Schwartz & Martin, although not statistical, is broadly consistent with the predictions of the model for dwarf, star-forming galaxies. This supports our conclusion that a careful comparison is needed. In a future paper, we will analyse more fully the outflow mass in different phases and carry out a more detailed comparison with observations (Lagos et al., in preparation).

There are a few examples in which the different phases of the outflow are added to infer a total mass loading. This is the case of the SB galaxies in Sturm et al. (2011) and Rupke & Veilleux (2013). Sturm et al. and Rupke et al. present estimates for the mass loading of the winds of small samples of local SBs from multiphase gas observations and derived  $\beta \sim 0.1$ – $1.1$ , while in our model, we predict a median  $\beta \approx 0.3$  for SB galaxies with stellar masses  $10^{10} < M_*/M_\odot < 10^{11}$ , which overlaps with the stellar mass range of the observations. The predicted  $\beta$  is consistent with the observations within the error bars. The measured outflow velocities in the observational samples range from  $100$  to  $800 \text{ km s}^{-1}$ , again consistent with the predicted mass-weighted velocities of SBs in our model, which for the same stellar masses above range

between  $250$  and  $1500 \text{ km s}^{-1}$ . Observationally inferred outflow velocities vary in a galaxy-to-galaxy basis and with the traced gas phase.

We find that our model agrees better with observationally inferred outflow rates compared to previous theoretical work on SN feedback and mass ejection from the ISM. For example, Efstathiou (2000) implemented a physical model for galaxy evolution in which self-regulation was imposed: energy loss by cloud collisions is compensated by the energy input by SNe. Efstathiou predicted that galaxies with  $M_{\text{stellar}} \approx 5 \times 10^{10} M_\odot$  have a mass loading factor in winds from the ISM of  $\beta \approx 0.2$ , which is a factor of more than 10 lower than the values inferred by Martin (1999) and Bouché et al. (2012). The assumptions in the modelling of Efstathiou are different from ours. An important difference is that we do not assume self-regulation in galaxies but instead we are able to test it. In addition to this, Efstathiou assumes that cooling in the interior of bubbles inflated by SNe is negligible and therefore SN remnants can only contribute to the hot phase of the ISM. In our model we allow the interior of bubbles to cool down, which is a key process to follow, as in most of the cases cooling is efficient and bubbles enter a radiative phase rather quickly.

We find that our predicted outflow rates are similar to those found by Hopkins et al. (2012) in simulations that resolve scales just below the size of GMCs and model SN feedback by injecting thermal energy stochastically into neighbouring particles. However, their outflow rates correspond to the sum of several processes, such as photoevaporation and radiation pressure, and are not exclusively SN-driven outflows. They argue that in dense environments, radiation pressure dominates the overall outflow rate. In those environments, our scheme predicts a larger contribution to the outflow rate from SNe than that predicted by Hopkins et al. Nonetheless, note that we indirectly assume that photoionization takes place due to our assumption of SNe driving bubbles which expand against the warm medium instead of the dense gas from which stars form.

## 5 TOWARDS A NEW PARAMETRIZATION OF THE OUTFLOW RATE

One of the main aims of this paper is to establish if the results of our dynamical model of SN feedback can be reproduced using a simple parametrization cast in terms of global galaxy properties. In this section, we use our dynamical model of SN feedback embedded in GALFORM to assess parametrizations of the mass loading used in the literature (Section 5.1) and search for an improved way of reproducing the mass loading factor (Section 5.2).

### 5.1 Dependence of the outflow rate on circular velocity

As discussed in the introduction, a widely used approach in galaxy formation models is to parametrize the mass loading of the outflow solely in terms of the circular velocity,  $v_{\text{circ}}$ , which is considered as a proxy for the depth of the potential well of the galaxy. Scalings of  $\beta$  with circular velocity can be motivated by invoking momentum-conserving ( $\beta \propto v_{\text{circ}}^{-1}$ ) or energy-conserving ( $\beta \propto v_{\text{circ}}^{-2}$ ) winds, or the power-law index can be treated as a free parameter, as in GALFORM and most other semi-analytic models. Our model has the power to test such assumptions by directly comparing the  $\beta$  calculated for a given timestep with the circular velocity of the galaxy.

Parametrizations of SN feedback that include a direct scaling with the circular velocity of the galaxy can be grouped into two: those

assuming a single scaling relation for both the outflow rate from the galaxy and from the halo, and those which separate them into two different mass loading factors,  $\beta_{\text{ISM}}$  for the mass loading of the galaxy and  $\beta_{\text{halo}}$  for that of the halo. GALFORM is an example of the first type (see also Lagos, Cora & Padilla 2008; Cook et al. 2010). In the second type, we find the models of e.g. Croton et al. (2006), Monaco, Fontanot & Taffoni (2007), Macciò et al. (2010) and Guo et al. (2011). For instance, Croton et al. (2006) assume that the outflow rate from the galaxy scales linearly with the instantaneous SFR, and adopt  $\beta_{\text{ISM}} = 3.5$ . Macciò et al. (2010) and Guo et al. (2011) modified the form of  $\beta_{\text{ISM}}$  so that it makes a transition from a constant value in high circular velocity galaxies to a form in which  $\beta_{\text{ISM}}$  increases as the circular velocity of the galaxies decreases, in order to better reproduce the number density of low-mass galaxies [see (iv) in the list below]. In our model, we calculate  $\beta_{\text{ISM}}$  and compare it with the parametrization from four of the previous models.

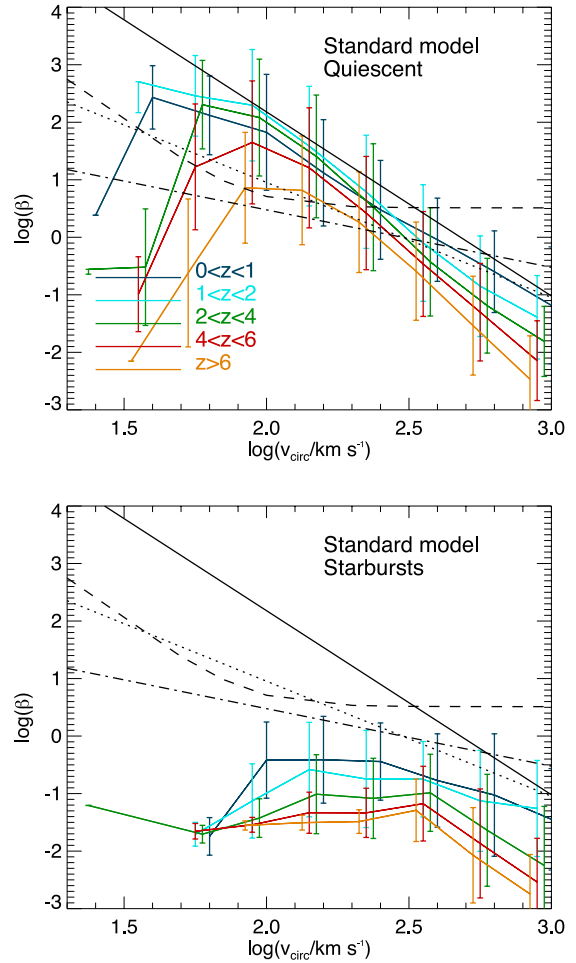
Fig. 15 shows the  $\beta$  predicted by the dynamical SN feedback model after implementing it in the full galaxy formation simulation, plotted as a function of the circular velocity for quiescent (top panel) and SB galaxies (bottom panel). The model shown in Fig. 15 corresponds to the standard choice of model parameters (see Table 1). We overplot for comparison the following parametrizations for the mass loading from the literature.

- (i)  $\beta = (v_{\text{circ}}/300 \text{ km s}^{-1})^{-2}$  from Baugh et al. (2005) (dotted line in Fig. 15).
- (ii)  $\beta = (v_{\text{circ}}/300 \text{ km s}^{-1})^{-1}$  from Dutton, van den Bosch & Dekel (2010) (dot-dashed line in Fig. 15). In the Dutton et al. model, the normalization velocity is calculated from the momentum injected by a single SN that ends up in the outflow, which is  $3.2 \times 10^4 M_{\odot} \text{ km s}^{-1}$  for a Kennicutt IMF.
- (iii)  $\beta = (v_{\text{circ}}/485 \text{ km s}^{-1})^{-3.2}$  from Bower et al. (2006) (solid line in Fig. 15).
- (iv)  $\beta = 6.5 [0.5 + (v_{\text{circ}}/70 \text{ km s}^{-1})^{-3.5}]$  from Guo et al. (2011) (dashed line in Fig. 15), which gives an SN-driven wind with a high mass loading even in galaxies with very high circular velocities, e.g. corresponding to those at the centre of clusters.

There are three key conclusions that can be drawn from Fig. 15: (i) a single power-law fit cannot describe the dependence of  $\beta$  on  $v_{\text{circ}}$ , (ii) there are large variations in the normalization, but also in the slope of the  $\beta$ - $v_{\text{circ}}$  relation with redshift, and (iii) SBs and quiescent galaxies follow different relations.

Regarding the shape of the  $\beta$ - $v_{\text{circ}}$  relation, the top panel of Fig. 15 shows that our dynamical calculations display a trend of  $\beta$  decreasing with increasing  $v_{\text{circ}}$  for galaxies with  $v_{\text{circ}} \gtrsim 80 \text{ km s}^{-1}$ . Below  $v_{\text{circ}} \approx 80 \text{ km s}^{-1}$ , the predicted mass loading shows a flattening or even a turnover followed by a positive  $\beta$ - $v_{\text{circ}}$  relation. The parametrizations used in the literature for the relation between  $\beta$  and  $v_{\text{circ}}$  are a poor description of the relation obtained from our physical model, which does not display a simple power-law behaviour when plotted in this way.

Font et al. (2011) discuss a phenomenological model with a saturation of the SN feedback, which was invoked to reproduce the observed LF and metallicity of the Milky Way's satellites. Font et al. set a ceiling  $\beta = 620$  for  $v_{\text{circ}} < 65 \text{ km s}^{-1}$  to obtain a good match to the properties of the Milky Way's satellites. Our dynamical model of SN feedback predicts a qualitatively similar behaviour to the saturated feedback scheme of Font et al. The peak value of  $\beta$  at  $z = 0$  is similar to the saturation value proposed by Font et al. However, we find that the peak value of the mass loading and the circular velocity at which the peak occurs change with redshift. We also find that saturation



**Figure 15.** Top panel: the mass loading factor,  $\beta = \dot{M}_{\text{eject}}/\psi$ , as a function of the circular velocity of the disc for quiescent galaxies with  $M_* > 10^8 h^{-1} M_{\odot}$  in the model with the standard choice of parameters (see Table 1). The relation is shown for different redshift ranges, as labelled. The solid lines and error bars indicate the median and 10 and 90 percentile ranges of the relations. We also show the parametrizations used in a range of semi-analytic models, corresponding to (i) Baugh et al. (2005, dotted line), (ii) Dutton et al. (2010, dot-dashed line), (iii) Bower et al. (2006, solid line) and (iv) Guo et al. (2011, dashed line) (see the text for details of the models). Bottom panel: the  $\beta$ - $v_{\text{circ}}$  relation in the model with the standard choice of ISM parameters for SB galaxies with  $M_* > 10^8 h^{-1} M_{\odot}$  at different redshifts. In this case, the circular velocity corresponds to that of the bulge. Lines and colours have the same meaning as in the top panel.

velocity varies with the parameters adopted to describe the ISM and molecular clouds, spanning the range  $v_{\text{circ,sat}} \approx 70\text{--}100 \text{ km s}^{-1}$ . In our model, the saturation velocity has no direct connection to the ratio between SN energy and halo potential.

The redshift variation of the mass loading of the wind can be quantified by fitting a power law of the form  $\beta = (v_{\text{circ}}/V_{\text{hot}})^{-\alpha_{\text{hot}}}$  to quiescent galaxies at different redshifts (top panel Fig. 15). For circular velocities in the range  $v_{\text{circ}} > 80 \text{ km s}^{-1}$ , the dependence of  $\alpha_{\text{hot}}$  and  $V_{\text{hot}}$  on redshift is given by

$$\alpha_{\text{hot}} = 2.7 + 2 \log(1 + z), \quad (55)$$

$$V_{\text{hot}} = 425 \text{ km s}^{-1} (1 + z)^{-0.2}. \quad (56)$$

For galaxies with  $v_{\text{circ}}/\text{km s}^{-1} < 80$  and for SBs, the dependence of  $\alpha_{\text{hot}}$  and  $V_{\text{hot}}$  on redshift is more complicated and cannot be

described by simple power-law fits. This behaviour illustrates that the mass loading of the outflow does not have a natural dependence on the circular velocity.

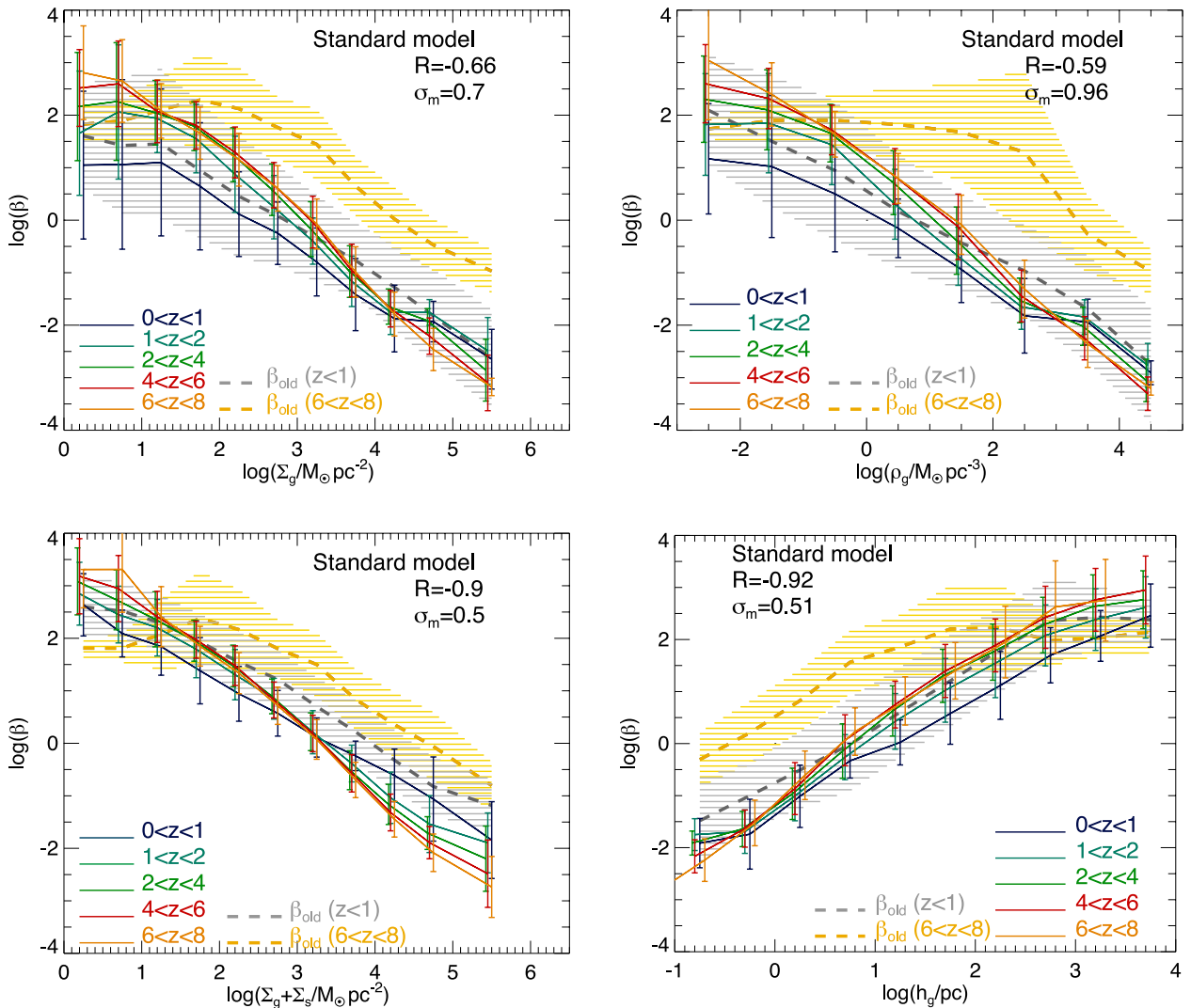
When focusing on SB galaxies only, we find that the dependence of  $\beta$  on  $v_{\text{circ}}$  changes dramatically (see the bottom panel of Fig. 15). This is due to the very different conditions in the ISM in SBs compared to quiescent galaxies, with higher gas surface densities for a given  $v_{\text{circ}}$ . The turnover obtained for quiescent galaxies at  $v_{\text{circ}} \approx 80 \text{ km s}^{-1}$  is also present in SB galaxies at  $z < 2$ . We find that the differences between quiescent and SB galaxies and the turnover at  $v_{\text{circ}} \approx 80 \text{ km s}^{-1}$  can be explained in terms of the more fundamental relation between  $\beta$  and the gas scaleheight,  $h_g$ . For the latter case, both quiescent and SB galaxies follow nearly the same relation (see the top panel of Fig. 8). This explains the nature of the  $\beta$ - $v_{\text{circ}}$  relation: there is a correlation between  $v_{\text{circ}}$  and  $h_g$ , for quiescent

galaxies with  $v_{\text{circ}} > 80 \text{ km s}^{-1}$ , but this is not present at lower  $v_{\text{circ}}$  or in SB galaxies.

## 5.2 A new parametrization of the mass outflow rate

We analyse the dependence of  $\beta$  on the various properties of the disc in order to find the most natural combination of parameters to describe the mass loading. This new way of describing  $\beta$  can therefore be used in semi-analytic galaxy formation models and simulations.

Fig. 16 shows the mass loading factor,  $\beta$ , as a function of (i)  $\Sigma_g$ , (ii)  $\rho_g$ , (iii)  $\Sigma_g + \Sigma_*$  and (iv)  $h_g$ , for the standard set of parameters for GMCs and the diffuse medium (see Table 1). Note that the third of these quantities can be written in terms of the surface density of gas and the gas fraction  $\Sigma_g + \Sigma_* = \Sigma_g/f_{\text{gas}}$ . All



**Figure 16.** The global mass loading factor,  $\beta = \dot{M}_{\text{eject}}/\psi$ , as a function of the gas surface density,  $\Sigma_g$  (top-left panel), gas density,  $\rho_g$  (top-right panel), gas plus stellar surface density,  $\Sigma_g + \Sigma_*$  (bottom-left panel) and the gas scaleheight,  $h_g$  (bottom-right panel), for galaxies with  $M_* > 10^8 h^{-1} M_{\odot}$ . All quantities plotted on the x-axis are calculated at the half-mass radius of the disc in the case of quiescent SF, or the bulge in the case of SBs. The relations are shown for different redshift ranges, as labelled, and correspond to the predictions of the model with the standard choice of parameters (listed in Table 1). The solid lines and error bars indicate the median and 10 and 90 percentile ranges of the relations. For reference, the values of the Pearson correlation coefficient,  $R$ , and the dispersion around the median,  $\sigma_m/\text{dex}$ , calculated for galaxies at  $z < 0.1$  in the new model are written in each panel. We also show the results obtained when using the Bower et al. (2006) choice for the outflow rate,  $\beta_{\text{old}} = (v_{\text{circ}}/485 \text{ km s}^{-1})^{-3.2}$ , for galaxies at  $z < 1$  and  $6 < z < 8$  (dashed lines) in each panel. The horizontal shading represents the 10 and 90 percentile ranges of the relations using the Bower et al. parametrization.

quantities above are evaluated at the half-mass radius of the disc or the bulge,  $r_{50}$  (see Appendix B for the definition of the profiles), and the predictions are shown for all galaxies, quiescent and SB, in different redshift ranges. We decide to study the relation between  $\beta$  and these quantities due to the correlation we find between the mass of a single bubble at the point of break-out from the disc and the local properties  $\rho_g$ ,  $\Sigma_g$ ,  $\Sigma_g + \Sigma_*$  and  $h_g$  (see Fig. 5). We also show the resulting relation between  $\beta$  and the quantity plotted on the  $x$ -axis if we use the old mass loading parametrization [see point (iii) in the list of Section 5.1].

We find that our results can be approximately described by the following fits:

$$\beta = \left[ \frac{\Sigma_g(r_{50})}{1.6 \times 10^3 M_\odot \text{pc}^{-2}} \right]^{-0.6} \quad (57)$$

$$\beta = \left[ \frac{\rho_g(r_{50})}{14 M_\odot \text{pc}^{-3}} \right]^{-0.5} \quad (58)$$

$$\beta = \left[ \frac{\Sigma_g(r_{50}) + \Sigma_*(r_{50})}{2.6 \times 10^3 M_\odot \text{pc}^{-2}} \right]^{-1} \quad (59)$$

$$\beta = \left[ \frac{h_g(r_{50})}{8 \text{pc}} \right]^{1.1} \quad (60)$$

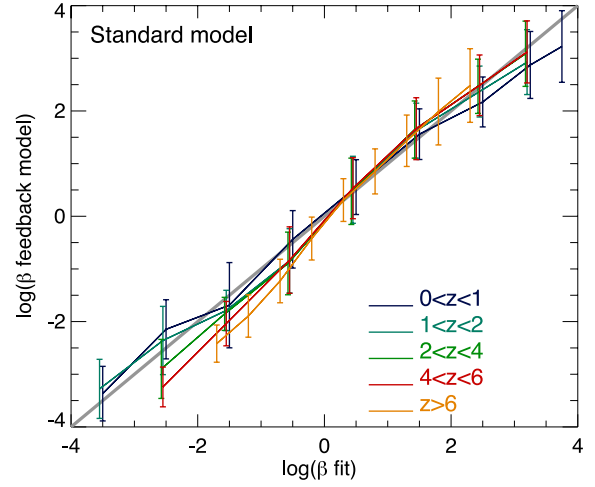
We quantify how good the correlation is by using two statistics, the Pearson correlation coefficient,  $R$ , and an estimate of the dispersion around the median,  $\sigma_m$ . For each  $x$ -axis bin, we calculate a dispersion,  $\sigma_x$ , corresponding to the ratio between the sum of the square of the deviations around the median in the  $y$ -axis and the number of objects in the bin. We then calculate  $\sigma_m$ , which corresponds to the square root of the median value of the distribution of  $\sigma_x$ . We calculate  $\sigma_m$  in the log–log plane, in units of dex. Note that  $R$  and  $\sigma_m$  are independent statistics which can be used to assess how good the correlation is between two quantities. The values for both quantities for galaxies at  $z < 0.1$  are written in each panel of Fig. 16.

In terms of the Pearson correlation factor,  $R$ , and the dispersion,  $\sigma_m$  (shown in Fig. 16), the properties that best describe  $\beta$  are  $\Sigma_g + \Sigma_*$  and  $h_g$ . Fig. 16 shows that the normalization and power-law index of the above relations vary with redshift, with high-redshift galaxies following a steeper relation than low-redshift galaxies. This trend can be understood as being due to high-redshift galaxies having larger gas fractions compared to lower redshift galaxies. Galaxies with a high gas fraction typically have a molecule-dominated ISM, and these are predicted to follow a steeper relation between  $\beta$  and  $h_g$  than those with an atomic-dominated ISM, which are typically gas poor (see Section 4.1.1 for an analytic derivation of such a trend). We find that the redshift trend can be removed by adding an extra dependence on the gas fraction to the expressions for  $\beta$ ,

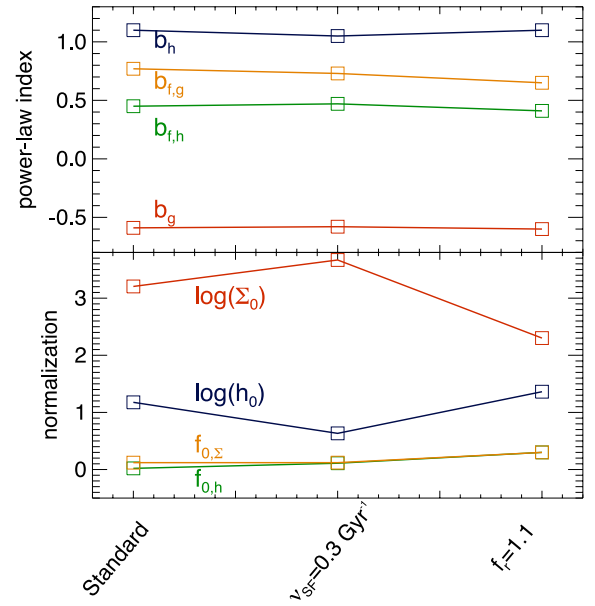
$$\beta = \left[ \frac{\Sigma_g(r_{50})}{1600 M_\odot \text{pc}^{-2}} \right]^{-0.6} \left[ \frac{f_{\text{gas}}}{0.12} \right]^{0.8} \quad (61)$$

$$\beta = \left[ \frac{h_g(r_{50})}{15 \text{pc}} \right]^{1.1} \left[ \frac{f_{\text{gas}}}{0.02} \right]^{0.4}, \quad (62)$$

which both have a Pearson correlation factor of  $R \approx 0.97$  and a dispersion  $\sigma_m \approx 0.3$  dex for galaxies at  $z < 0.1$ . This is shown in Fig. 17, where the fit of equation (62) is compared with the directly calculated  $\beta$ . Most of the redshift evolution seen in Fig. 16 is removed.



**Figure 17.** The predicted mass loading from the full model ( $y$ -axis) plotted against the fit given by equation (62), expressed in terms of the gas scaleheight and gas fraction, for galaxies at different redshifts, as labelled.



**Figure 18.** Top panel: power-law slope of the relations between  $\beta$ ,  $h_g$ ,  $\Sigma_g$  and  $f_{\text{gas}}$ , quantified as  $\beta = (h_g/h_0)^{b_h} (f_{\text{gas}}/f_{g,h})^{b_{f,h}}$  and  $\beta = (\Sigma_g/\Sigma_0)^{b_g} (f_{\text{gas}}/f_{g,\Sigma})^{b_{f,g}}$ . Lines are as labelled in the panel. The results for the model with the standard set of parameters and those predicting the highest and lowest  $\beta$  are shown as symbols. The parameters of the fit correspond to fitting the relations above in subsamples of galaxies at  $z < 8$  with different gas fractions. Bottom panel: normalizations of the relations above for the three models of the top panel. Lines are as labelled in the panel. The plot shows that the power-law slopes are not affected by changes in the parameters describing the ISM and SF but that only the normalizations of the relations change.

Equations (61) and (62) are also useful to characterize the mass loading  $\beta$  obtained in the model when varying the parameters used in the ISM modelling (Table 1). This is shown in Fig. 18, in which the power-law indices and normalizations for the relations, defined as  $\beta = (\Sigma_g/\Sigma_0)^{b_g} (f_{\text{gas}}/f_{0,\Sigma})^{b_{f,g}}$  and  $\beta = (h_g/h_0)^{b_h} (f_{\text{gas}}/f_{0,h})^{b_{f,h}}$ , are shown for three different choices of ISM model parameters. The model using  $f_r = 1.1$  corresponds to the weakest feedback model and that with  $\nu_{\text{SF}} = 0.3 \text{ Gyr}^{-1}$  to the strongest feedback model. The



three choices of model parameters produce very little variation in the power-law indices of the above relations (top panel of Fig. 18). Variations are observed in the normalizations of the relations and represent different feedback strengths (bottom panel of Fig. 18). This means that if we were to include the parametric form given by equations (61) and (62) in the semi-analytic model, we would need to vary the zero-point of these relations to reproduce the results for different parameters for the diffuse ISM and GMCs. Equations (61) and (62) describe our results for the mass loading  $\beta$  in galaxies at any redshift, within the range tested (i.e.  $z < 10$  and  $M_* + M_{\text{gas, ISM}} > 10^8 h^{-1} M_\odot$ ) with very little dependence on redshift or stellar mass.

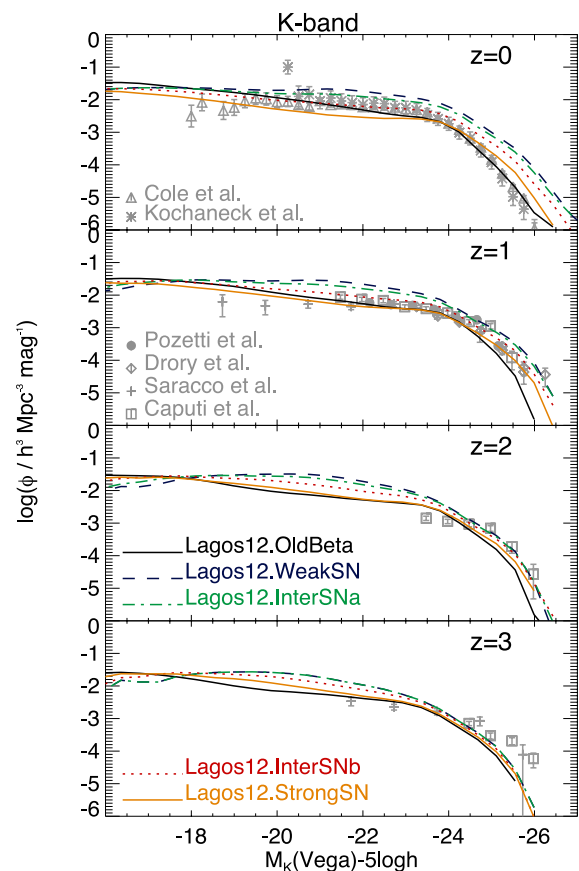
The old parametrization (shown by the dashed lines in Fig. 16) results in a trend of  $\beta$  decreasing with the properties plotted on the  $x$ -axis, given the correlation already discussed between  $v_{\text{circ}}$  and these variables. However,  $\beta_{\text{old}}$  differs from the mass loading  $\beta$  for galaxies with low surface densities of gas by up to a factor of  $\approx 5$  in either direction, and overestimates  $\beta$  at the high surface density regime by up to a factor of  $\approx 100$ , depending on the redshift. In Fig. 16,  $\beta_{\text{old}}$  varies with redshift much more strongly than the new parametrizations, and therefore overestimates the SN feedback in high-redshift galaxies. This reflects the importance of the analysis performed in this paper and the need for a revision of such parametrizations. The largest differences between the predicted  $\beta$  and  $\beta_{\text{old}}$  are obtained at high redshifts.

The difference between SBs and quiescent galaxies apparent in the  $\beta$ – $v_{\text{circ}}$  plane in Fig. 15 is greatly reduced in the  $\beta$ – $h_g$  plane (see the top panel of Fig. 8). This is because SB galaxies of a given  $v_{\text{circ}}$  have much higher densities in stars and gas than their quiescent counterparts. Although the relation is noisier due to the lower numbers of SBs in the model output compared to quiescent galaxies, the  $\beta$ – $h_g$  relation is very similar in slope and normalization to that for quiescent galaxies. This suggests that the dependence of mass loading is fundamental and captures the relevant physics determining  $\beta$ .

## 6 THE IMPACT OF THE NEW OUTFLOW MASS LOADING ON GALAXY FORMATION

In this section, we consider the impact of our dynamical model of SN feedback on galaxy properties and compare with the predictions of the model which uses the old parametrization. We first estimate the error associated with using the parametric form defined in equation (62) instead of performing the full calculation carried out in this paper. Secondly, we analyse the net effect of our dynamical modelling on galaxy properties by focusing on two statistical properties of galaxies: (i) the evolution of the LF in the  $K$  and  $V$  bands, and (ii) the evolution of the global SFR density. An analysis of a complete set of galaxy properties will be presented in a future paper (Lagos, Lacey & Baugh, in preparation). Note that the experiment carried out in this section attempts to identify general trends in the LF and SFR density due to the new SN feedback model rather than predicting exact normalizations of both quantities. The reasons for this are first, that this model does not include a self-consistent treatment of the reincorporation of the gas that has escaped the galaxy, but instead uses the parametrization described in Section 3.2, and secondly, the parameters associated with the AGN feedback treatment have not been modified to recover the agreement with the observations at the bright end of the LF.

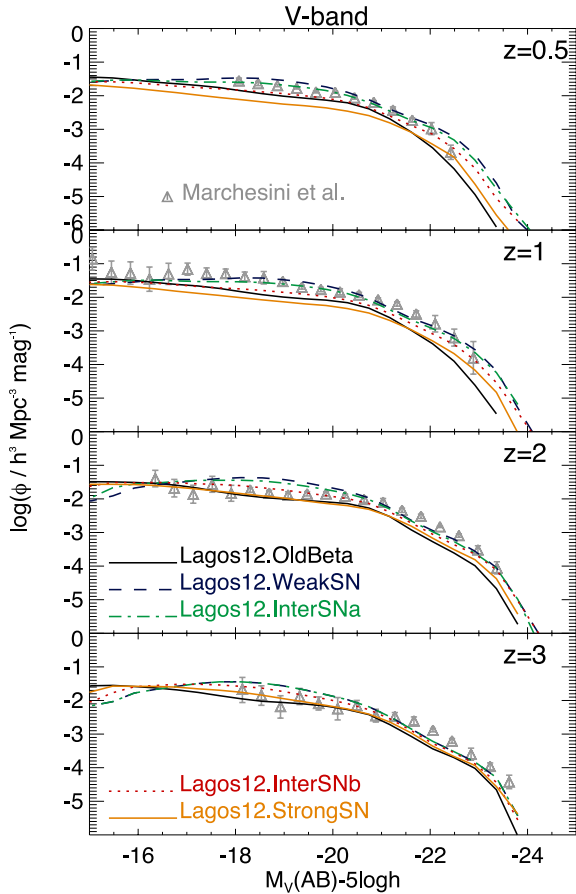
We ran the full dynamical model in which  $\beta$  is calculated self-consistently, and compare with the model using the prescription from equation (62) to calculate  $\beta$ , under the simplifying assump-



**Figure 19.** Rest-frame  $K$ -band galaxy LF for the Lagos12 model with the old and the new SN prescriptions (see Table 3), at various redshifts, as labelled. Observational results from Pozzetti et al. (2003), Drory et al. (2004), Saracco et al. (2006) and Caputi et al. (2006) are shown as grey symbols, identified by the key in the two top panels. Note that the models have not been returned to fit the observed LF.

tion of  $\beta^Z = \beta$ . We compared the LFs predicted by both procedures in the bands 900–1200 Å,  $b_J$ ,  $V$ ,  $K$  and 8  $\mu\text{m}$ . At  $z = 0$ , the largest differences are obtained in the far-UV band, but are at most  $\approx 25$  per cent. The other bands show differences in the range 5–20 per cent. However, at  $z = 6$  these differences can be as large as 80 per cent. The reason for the larger differences at high redshifts is that we currently do not allow for variations in the parametrization of  $\beta^Z$  with respect to  $\beta$ , like those shown in Fig. 12. Such variations have only a minor effect at  $z = 0$ , but they have an effect in  $z \gtrsim 4$  galaxies, where larger differences between  $\beta$  and  $\beta^Z$  are predicted by the dynamical model. The main drivers of the differences seen in the LFs are differences in the cold gas mass and mass in metals in the ISM. The stellar mass and hot gas mass functions are similar to within  $< 40$  per cent at redshifts  $z = 0$ – $6$ . In the redshift range shown in Figs 19 and 20, variations between the self-consistent calculation and the calculation using the  $\beta$  parametrization are not significant. We calculate the best parametrizations using the form of equation (62) for the different ISM parameter choices and present in Table 3 the results for four choices of parameters spanning the full range of feedback strength.<sup>2</sup> We find that using the prescription

<sup>2</sup> Note that the weak SN feedback model of Table 3 gives mass loading factors that are about three times lower than the standard choice of parameters, which is also representative of the predicted  $\beta$  in the case of the extreme ISM conditions analysed in Section 4.3.3.



**Figure 20.** Rest-frame  $V$ -band galaxy LF for the Lagos12 model with the old and the new SN prescriptions (see Table 3), at various redshifts, as labelled. Observational results from Marchesini et al. (2012) are shown as grey symbols. Note that the models have not been returned to fit the observed LF.

**Table 3.** Models shown in Figs 19–21. The first row gives the old parametrization used to describe the outflow. The next four rows show alternative models using the new  $\beta$  parametrization of equation (62). Each parametrization represents different parameter choices for the full SN feedback dynamical model, which is indicated in the parentheses. The parametrization used for each model is shown in the second column.

Model	$\beta$ parametrization
Lagos12.OldBeta	$\left(\frac{V_{\text{circ}}}{485 \text{ km s}^{-1}}\right)^{-3.2}$
Lagos12.WeakSN ( $f_t = 1.1$ )	$\left(\frac{h_g}{23 \text{ pc}}\right)^{1.1} \left(\frac{f_{\text{gas}}}{0.3}\right)^{0.4}$
Lagos12.InterSNa ( $\tau_{\text{life, GMC}} = 0.03 \text{ Gyr}$ )	$\left(\frac{h_g}{17 \text{ pc}}\right)^{1.1} \left(\frac{f_{\text{gas}}}{0.1}\right)^{0.4}$
Lagos12.InterSNb (Std.)	$\left(\frac{h_g}{15 \text{ pc}}\right)^{1.1} \left(\frac{f_{\text{gas}}}{0.02}\right)^{0.4}$
Lagos12.StrongSN ( $\nu_{\text{SF}} = 0.3 \text{ Gyr}^{-1}$ )	$\left(\frac{h_g}{4 \text{ pc}}\right)^{1.1} \left(\frac{f_{\text{gas}}}{0.3}\right)^{0.4}$

for  $\beta$  given in equation (62) gives reliable results that closely follow the behaviour of the full dynamical model at  $z < 4$ , but significantly speeds up the calculation.

In order to analyse the effect of the new dynamical model of SN feedback on galaxy properties, we focus on the Lagos12 model and

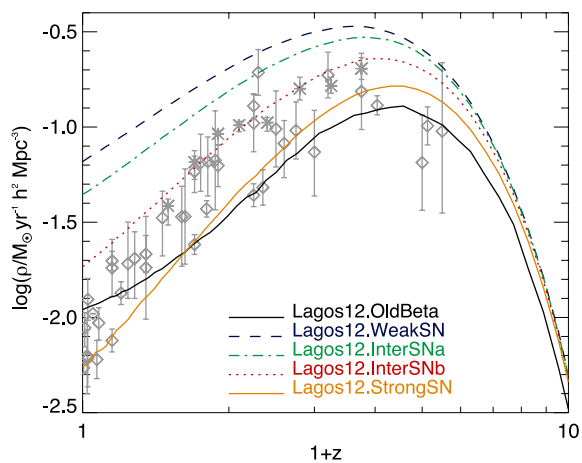
vary the SN feedback prescription. We compare the four alternative models listed in Table 3.

Fig. 19 shows the  $K$ -band LF at various redshifts for the five models listed in Table 3. We remind the reader that we are not trying to fit observations here, but rather we are trying to see the effect the modelling of feedback has on galaxy properties starting from a model which uses a completely different way of calculating  $\beta$ . The most interesting feature in Fig. 19 is that all the models that use the new feedback model developed in this paper give a shallower faint-end slope at  $z < 2.5$ , regardless of the ISM model parameters, but produce a higher overall normalization for the LF. The model with the strongest feedback (Lagos12.StrongSN) shows a faint end that is similar to the original model. There is a trend of a shallower faint end with weaker SN feedback models, although this trend changes with band and redshift. It is also clear that the models predict very weak evolution of the slope of the faint end. The shallower faint-end slope predicted by our new feedback scheme suggests that the problem of the predicted steep faint end of the LF and low-mass end of the stellar mass function could be largely overcome by using the new parametrization of the mass loading (equation 62). The physical reason behind the shallower faint-end slopes obtained by using the new  $\beta$  parametrization is that faint galaxies typically have large  $h_g$  and therefore can reach very large values of  $\beta$ . These faint galaxies do not necessarily correspond to those with the smallest  $v_{\text{circ}}$ , and therefore in these galaxies, the new parametrization drives larger  $\beta$  than that obtained with the  $v_{\text{circ}}$  parametrization.

The bright end of the  $K$ -band LF predicted by the models using the new feedback prescription is higher in all the cases compared to the original model. This is due to the lower  $\beta$  predicted by the dynamical SN feedback model compared to the parametrization adopted in the Lagos12.OldBeta model. This, in addition to the unchanged gas reincorporation time-scale, leads to more bright galaxies. In Paper II, we will model the expansion of bubbles in the halo to remove this process as a free parameter. We will analyse in more detail the effect of SN feedback on the bright end of the LF.

Fig. 20 is equivalent to Fig. 19 but shows the  $V$ -band LF for  $z > 0.5$ . The behaviour of the models in this band is broadly the same as in the near-IR: the new feedback scheme, regardless of the strength of the SN feedback, predicts a shallower faint end of the LF up to  $z \approx 1.5$ . However, above that redshift, the strength of the SN feedback plays an important role in determining whether the faint end is shallower or steeper than predicted by the original model. The slope of the faint end in the  $V$ -band LF varies more strongly with redshift and in a complex way compared to the variations seen in the  $K$ -band LF.

Interestingly, the different SN feedback models of Table 3 converge to similar LFs in both the  $K$  and  $V$  bands at  $z \gtrsim 3$  but evolve differently towards  $z = 0$ . This is because these models predict galaxies with different SFH. Fig. 21 shows the global SFR density evolution predicted by each of the models of Table 3. The models using the new SN feedback scheme predict that the global SFR peaks at slightly lower redshifts compared to the original model, with weaker SN feedback producing a lower redshift for the peak. Note that even the model with the strongest SN feedback produces larger SFR densities at  $z \approx 2-4$  compared to the model using the old  $\beta$  parametrization. Compared to observations, the model with the strongest SN feedback predicts SFR densities that are too low, while that with the weakest SN feedback gives SFR densities that are too high. It is interesting to note that the model with the strongest SN feedback results in the largest decline in the global SFR per unit volume, dropping by a factor of  $\approx 30$  from the peak to the present



**Figure 21.** The evolution of the cosmic SFR per unit volume for the Lagos12 model with the old and the new SN prescriptions which give rise to different strengths of SN feedback (see Table 3), as labelled. The observational estimates of Karim et al. (2011, asterisks) and the data compilation of Hopkins (2004, diamonds) are also shown. Hopkins (2004) assumes a Salpeter IMF and Karim et al. (2011) a Chabrier IMF. Therefore, SFRs have been scaled to a Kennicutt IMF (scaled down by a factor of 2 in the Salpeter case and down by a factor of 1.12 in the Chabrier case).

day. A key physical process to analyse before ruling out any of these models is the reincorporation time-scale of the gas after outflowing from the ISM into the hot gas reservoir of the halo. Also, other galaxy formation parameters may have to be reset, since these were based on the old outflow model. In any case, the fact that the use of the new  $\beta$  parametrization predicts a shallower LF of galaxies points to the need to revise the physics included in galaxy formation models and simulations.

## 7 DISCUSSION AND CONCLUSIONS

We have presented a dynamical model of SN feedback which tracks the evolution of bubbles inflated by SNe into the ISM of galaxies. Our model includes a range of processes which can affect the expansion of bubbles: gravity, radiative energy losses, external pressure from the diffuse medium and temporal changes in the ambient gas. Bubbles inflated by SNe are evolved from the adiabatic to the radiative phases until the point of break-out from the galaxy disc or bulge, or confinement in a multiphase ISM. The multiphase model of the ISM includes a diffuse, atomic phase, a dense, molecular phase and a hot, low-density phase. The hot, low-density phase corresponds to the interior of bubbles. The metal enrichment of the ISM and halo due to SNe takes place through bubbles. The location of star-forming regions, or GMCs, which give rise to bubbles is connected to the radial distribution of molecular gas, which allows us to study both the global outflow rate and the radial profile of galactic outflows. The aims of this work are (i) to test the importance of each of the physical processes included in the expansion of bubbles and to explore the parameter space of the modelling of GMCs and the ISM, (ii) to determine which combinations of galaxy properties the outflow rate best correlates with and (iii) to improve upon widely used parametric forms for the outflow rate used in the literature.

To help us assess these points, we embed our calculations in the GALFORM semi-analytic model, which follows the formation and evolution of galaxies in the framework of hierarchical structure formation. We take advantage of the two-phase medium description introduced into GALFORM by Lagos et al. (2011a,b), to trace SF and

star-forming regions using the cold molecular component of the ISM, while allowing bubbles to sweep up gas only from the diffuse neutral atomic component. In the Lagos et al. model, the molecular-to-atomic mass ratio is calculated from the radial profile of the hydrostatic pressure, and the SFR is calculated from the molecular gas radial profile (e.g. Blitz & Rosolowsky 2006; Leroy et al. 2008). The semi-analytic model provides the initial conditions needed by the dynamical model of SN feedback: the stellar and DM contents, the surface density of atomic and molecular gas, the gas metallicity and the scalelength of each mass component. This modelling allows us to study the relation between the rate at which mass escapes from the galaxy disc or bulge (outflow rate) and the properties of the disc, bulge and halo, over a wide dynamic range. Previous work has focused on hydrodynamical simulations covering a narrow dynamic range, which has been chosen somewhat arbitrarily (Hopkins et al. 2012; Creasey et al. 2013), or which have adopted Sedov analytic solutions for the evolution of bubbles (e.g. Efstathiou 2000; Monaco 2004a). One of our goals is to complement and extend this work by using a more general SN feedback model and the galaxy population and SFH produced by the semi-analytic model.

We summarize our main conclusions below.

(i) We find that the mass loading of the outflow,  $\beta$ , decreases with increasing gas surface density and increases with increasing gas scaleheight. On the other hand, the outflow velocity increases with increasing gas surface density and decreases with increasing gas scaleheight. These trends are seen in both the global and local mass loading and velocity of the wind.

(ii) We find that the multiphase ISM treatment included in our model is essential for reproducing the observed outflow rates of galaxies. When fixing the diffuse-to-cloud mass ratio instead of calculating it from the hydrostatic pressure, we find variations in the predicted mass loading  $\beta$  of up to two orders of magnitude in the highest gas density regimes. This emphasizes the importance of the multiphase ISM included in our modelling. By adopting different, but still plausible parameters in the modelling of GMCs and the diffuse medium, we find variations in  $\beta$  of a factor up to  $\approx 3$  and in  $v_{\text{outflow}}$  of a factor up to  $\approx 1.7$  in either direction. We also find that by the time bubbles escape from the ISM, they are radiative in the majority of the cases.

(iii) When comparing our predicted outflow rates and velocities with those inferred from observations (e.g. Martin 1999; Bouché et al. 2012), we find good agreement. We also find that our predictions are similar to those from the non-cosmological hydrodynamical simulations of Hopkins et al. (2012) and Creasey et al. (2013), in the regimes they were able to probe. Our work therefore confirms the finding that the surface density of gas is an important quantity in determining the mass loading of the outflow.

(iv) The widely used parametric forms describing SN feedback and relating the mass loading  $\beta$  to only the circular velocity of the galaxy do not capture the physics setting the outflow rates from galaxies. For instance, we find that the trend of  $\beta$  decreasing with  $v_{\text{circ}}$  is only valid for galaxies with  $v_{\text{circ}} \gtrsim 80 \text{ km s}^{-1}$ . Below this threshold,  $\beta$  flattens or decreases with decreasing  $v_{\text{circ}}$ . We also find that the relation between  $\beta$  and  $v_{\text{circ}}$  changes substantially with redshift. We find that tighter relations are those between  $\beta$  and the gas scaleheight and gas fraction,  $\beta \propto [h_g(r_{50})]^{-1} [f_{\text{gas}}]^{0.4}$ , and between  $\beta$  and the surface density of gas and the gas fraction,  $\beta \propto [\Sigma_g(r_{50})]^{-0.6} [f_{\text{gas}}]^{0.8}$ . Changing the parameters in the model of GMCs and the diffuse medium can change the normalization of these relations, but does not alter the power-law index. We find that SB and quiescent galaxies follow similar relations, with SBs

being slightly offset to lower  $\beta$  compared to quiescent galaxies. The outflow velocities can also vary between SBs and quiescent galaxies depending on the adopted SF law. A more rapid conversion from gas to stars drives larger velocities due to the higher energy and momentum injection rate from SNe.

(v) We study the effect of the dynamical model of SN feedback developed here on galaxy properties and test the inclusion of the new parametrization of  $\beta$  [see (iv) above]. We find that the faint end of the near-infrared LF becomes shallower in the model using the new feedback scheme compared to the old model. We find that this shallowing of the faint end takes place regardless of the parameters assumed to describe the diffuse ISM and GMCs, with a trend of weaker SN feedback predicting a shallower faint end of the LF.

Our model is subject to simplifications required to model the evolution of bubbles in the ISM of galaxies. A critical simplification we make is to fix the GMC mass. A more sophisticated approach would be to include a distribution of GMC masses and their spatial distribution following a theoretical estimate of the spatial clustering of GMCs of different masses (Hopkins 2012). However, such a description also requires more detailed information about the ISM. Instead, we test our predictions by varying the adopted GMC mass in the range allowed by observations (see Table 1), and find variations in the normalization of the mass loading described in (iv), but with little impact on the power-law indices.

The agreement we find between our model and detailed hydrodynamical simulations (Hopkins et al. 2012; Creasey et al. 2013) suggests that we capture the relevant physics determining the rate at which mass escapes from the ISM of galaxies, despite the simplifications made in our modelling. The advantage of our calculations is that a much wider range of ISM conditions can be explored than is feasible in the more expensive hydrodynamical simulations. We have given predictions for the outflow rate for a very wide range in galaxy properties and cosmic epochs. The method developed in this paper also allows radial profiles of the outflow rate to be obtained. The new generation of integral field spectroscopy instruments, such as Infrared Multi-object Integral Field Spectroscopy in the Very Large Telescope (Sharples et al. 2004) and the Sydney-Anglo-Australian Observatory Multi-object Integral field spectrograph (Croon et al. 2012; Fogarty et al. 2012), will make the observations of outflows routine in local and high-redshift galaxies, and will allow us to constrain our model observationally.

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## APPENDIX A: THE RECYCLE FRACTION AND YIELD OF DIFFERENT STELLAR POPULATIONS

The number of SNe per solar mass of stars formed,  $\eta_{\text{SN}}$ , is calculated from the IMF,  $\phi(m) \propto dN(m)/dm$ , as

$$\eta_{\text{SN}} = \int_{m_{\text{SN}}}^{m_{\text{max}}} \phi(m) dm, \quad (\text{A1})$$

where  $m_{\text{SN}} = 8 M_{\odot}$  and  $m_{\text{max}} = 120 M_{\odot}$ . For the Kennicutt (1983) IMF adopted here,  $\eta_{\text{SN}} = 9.4 \times 10^{-3} M_{\odot}^{-1}$  (in the case of a Salpeter IMF,  $\eta_{\text{SN}} = 7.3 \times 10^{-3} M_{\odot}^{-1}$ ). In Section 2.1.1, we define the mass injection rate from SNe depending on the recycled fraction of massive stars,  $R_{\text{SN}}$ . This recycled fraction also depends on the IMF as

$$R_{\text{SN}} = \int_{m_{\text{SN}}}^{m_{\text{max}}} (m - m_{\text{remn}}) \phi(m) dm, \quad (\text{A2})$$

where  $m_{\text{rem}}$  is the remnant mass. Similarly, we define the yield from SNe as

$$p_{\text{SN}} = \int_{m_{\text{SN}}}^{m_{\text{max}}} m_i(m) \phi(m) dm, \quad (\text{A3})$$

where  $m_i(m)$  is the mass of metals produced by stars of initial mass  $m$ . We use the stellar evolution models of Marigo (2001) and Portinari, Chiosi & Bressan (1998) to calculate the ejected mass from intermediate and massive stars, respectively. For a Kennicutt IMF, we obtain  $R_{\text{SN}} = 0.14$  and  $p_{\text{SN}} = 0.018$ .

## APPENDIX B: RADIAL PROFILES OF THE STELLAR AND DM COMPONENTS AND THE MID-PLANE PRESSURE

An important driver in the evolution of bubbles is the gravitational attraction exerted by the stellar and DM components. We describe here how we calculate the mass enclosed by a sphere of radius  $R$  located at a distance  $d$  from the centre of the galaxy. We perform our calculations of bubble evolution in shells in the disc, which defines  $d$  (see Section 2.2.1).

The total stellar plus DM mass within a sphere of radius  $R$  displaced by  $d$  from the centre of the galaxy corresponds to

$$M_{\text{t}}(R, d) = M_{*}(R, d) + M_{\text{DM}}(R, d), \quad (\text{B1})$$

where  $M_{*}(R, d) = M_{*,\text{disc}}(R, d) + M_{*,\text{bulge}}(R, d)$  is the total stellar mass,  $M_{*,\text{disc}}(R, d)$  and  $M_{*,\text{bulge}}(R, d)$  represent the mass in the disc and the bulge, respectively, and  $M_{\text{DM}}(R, d)$  is the mass in DM, in all cases enclosed in  $R$ . We describe below how we calculate the variables of equation (B1).

*Disc radial profile.* We assume that discs are well described by a radial exponential profile with a scaleradius  $r_s$ , which is related to the half-mass radius as  $r_{50,\text{disc}} = 1.67 r_s$  (Binney & Tremaine 2008). We define the stellar surface density of the disc at a distance  $d$  from the centre as

$$\Sigma_{*,\text{disc}}(d) = \frac{M_{*,\text{disc}}^{\text{t}}}{2\pi r_s^2} e^{-d/r_s}. \quad (\text{B2})$$

Here,  $M_{*,\text{disc}}^{\text{t}}$  is the total stellar mass in the disc. If the relevant sphere of radius  $R$  is at a distance  $d$  from the centre, then the stellar mass in the mid-plane of the disc exerting the gravitational attraction on the bubble is approximately

$$M_{*,\text{disc}}(R, d) \approx \frac{4\pi R^3}{3} \frac{\Sigma_{*,\text{disc}}(d)}{2h_*}. \quad (\text{B3})$$

Here,  $h_*$  is the scaleheight of the stars, which we estimate from the scaleradius of the disc following the empirical results of Kregel et al. (2002),  $r_s/h_* = 7.3$ .

*Bulge radial profile.* The potential well of a galactic bulge,  $\Phi(r)$ , can be well described by a Dehnen profile (Dehnen 1993) with  $\gamma_b = 3/2$  which closely resembles a de Vaucouleurs (1953)  $r^{1/4}$  profile,

$$\Phi(r) = \frac{G M_{*,\text{bulge}}^{\text{t}}}{r_0} \frac{1}{2 - \gamma_b} \left[ 1 - \left( \frac{r}{r + r_0} \right)^{2 - \gamma_b} \right], \quad (\text{B4})$$

where  $r_0$  is the scaleradius and  $M_{*,\text{bulge}}^{\text{t}}$  is the total stellar mass of the bulge. The scaleradius relates to the half-mass radius of the bulge,  $r_{50,\text{b}}$ , as

$$r_{50,\text{b}} = r_0 (2^{1/(3-\gamma_b)} - 1)^{-1}. \quad (\text{B5})$$

In this definition of the potential well, the volume density profile of stars is

$$\rho_{*,\text{bulge}}(r) = \frac{(3 - \gamma_b)}{4\pi} \frac{M_{*,\text{bulge}}^{\text{t}} r_0}{r^{\gamma_b} (r + r_0)^{4 - \gamma_b}}. \quad (\text{B6})$$

Although the stars in the bulge follow a de Vaucouleurs profile, the gas is assumed to be better characterized by an exponential profile,

as has been observed in early-type galaxies (e.g. Crocker et al. 2011; Davis et al. 2011; Serra et al. 2012). This means that the same geometry adopted for the case of discs applies here: bubbles expand in a coordinate system displaced by  $d$  in the  $x$ -axis. However, the difference with the case of the disc is that here the stellar profile has spherical symmetry. With this in mind, we approximate the stellar mass enclosed by a bubble of radius  $R$  displaced by  $d$  from the centre as

$$M_{*,\text{bulge}}(R, d) \approx \frac{4\pi R^3}{3} \rho_{*,\text{bulge}}(d). \quad (\text{B7})$$

We use the equations above to calculate the  $M_*(R, d)$  that goes into equations (1)–(3), (18)–(20) and (24)–(26).

*DM radial profile.* Here we assume that DM haloes are well described by an NFW profile (Navarro, Frenk & White 1997). We follow the description of Cole et al. (2000), where haloes contract in response to the presence of baryons. The galaxy disc, bulge and DM halo adjust to each other adiabatically.

The volume mass density of DM is described in an NFW profile as

$$\rho_{\text{DM}}(r) = \frac{\delta_c \rho_c}{(r/r_s)(1+r/r_s)^2}, \quad (\text{B8})$$

where  $r_s$  is the DM scalerradius,  $\delta_c$  is the characteristic (dimensionless) density and  $\rho_c$  is the critical density of the universe. As before, the mass enclosed within a sphere of radius  $R$  displaced by  $d$  from the centre of the potential well,

$$M_{\text{DM}}(R, d) \approx \frac{4\pi R^3}{3} \rho_{\text{DM}}(d), \quad (\text{B9})$$

assuming that  $\rho_{\text{DM}}(d)$  is approximately constant within the bubble.

Note that equations (B7) and (B9) are accurate in the regime where  $d/R \gg 1$ . In this paper, we neglect the effect of tidal forces on bubbles, which arise from the asymmetric gravitational field, which distort their shape. This would affect the size of bubbles perpendicular to the gaseous disc and therefore the break-out of bubbles.

## B1 The mid-plane hydrostatic pressure of disc galaxies and the gas scaleheight

Under the assumptions of local isothermal stellar and gas layers, and  $\sigma_* > \sigma_{\text{gas}}$ , the mid-plane hydrostatic pressure in discs,  $P_{\text{ext}}$ , can be approximated to within 10 per cent by (Elmegreen 1989)

$$P_{\text{ext}}(r) \approx \frac{\pi}{2} G \Sigma_{\text{gas}}(r) \left[ \Sigma_{\text{gas}}(r) + \left( \frac{\sigma_d}{\sigma_*(r)} \right) \Sigma_*(r) \right], \quad (\text{B10})$$

where  $\Sigma_{\text{gas}}$  and  $\Sigma_*$  are the surface densities of gas and stars at  $r$ , respectively, and  $\sigma_g$  and  $\sigma_*$  give the vertical velocity dispersion of the gas and stars. We assume a constant gas velocity dispersion,  $\sigma_d = 10 \text{ km s}^{-1}$  (Leroy et al. 2008). By assuming that  $\Sigma_* \gg \Sigma_{\text{gas}}$ ,  $\sigma_*(r) = \sqrt{\pi G h_* \Sigma_*(r)}$ , where  $h_*$  is the stellar scaleheight. This approximation could break down for very high redshift galaxies, whose discs are gas dominated. In such cases, we assume a floor of  $\sigma_* \geq \sigma_g$ .

In the case of the gas scaleheight, we simply assume vertical equilibrium, where the gravitational force is balanced by the pressure of the gas,  $P = \sigma_d^2 \rho_g$ , where  $\rho_g = \Sigma_g / 2 h_g$  and  $\Sigma_g$  is the gas surface density (molecular plus atomic gas). Using equation (B10) to define

the pressure on the mid-plane of the disc due to the gravitational force, we can write

$$h_g(r) \approx \frac{\sigma_d^2}{\pi G \left[ \Sigma_{\text{gas}}(r) + \left( \frac{\sigma_d}{\sigma_*(r)} \right) \Sigma_*(r) \right]}. \quad (\text{B11})$$

## APPENDIX C: CALCULATION OF SWEEP-UP, CONFINEMENT AND BREAK-OUT MASS RATES

The contribution from bubbles to the rate of change of the mass and metallicity in the ISM and hot halo gas depends on their evolution. In this appendix, we briefly describe how we calculate the overall contribution from bubbles in different evolutionary stages included in the set of equations (36)–(45).

*The swept-up mass.* Each galaxy has generations of bubbles whose evolution depends on the time they started their expansion and their spatial distribution in the galaxy. Each galaxy has its SFH sampled in a fine grid in time that goes down to the current time,  $t_c$ . Each time interval,  $dt'$ , in the SFH of a galaxy has associated a new generation of  $N_{\text{GMC},i,t'}$  set of bubbles in the annulus  $i$  of the galaxy disc. Each of these bubbles has swept up a mass  $m_{\text{sw}}(r_i, t')$  from the diffuse medium and has a total mass  $m_b(r_i, t')$  at  $t'$ . The number of annuli used to solve the equations of bubble expansion (Section 4.1) is  $N_r$ . The overall rate of swept-up mass is

$$\dot{M}_{\text{sw,ISM}}(t_c) = \int_0^{t_c} \sum_{i=1}^{i=N_r} N_{\text{GMC},i,t'} \dot{m}_{\text{sw}}(r_i, t') (1 - H_{r,h}) (1 - H_{v,\sigma}) dt'. \quad (\text{C1})$$

Here,  $H_{r,h}$  and  $H_{v,\sigma}$  are step functions defined in terms of the radius of bubbles,  $R_s$ , the gas scaleheight,  $h_g$ , the expansion speed of bubbles,  $v_s$ , and the velocity dispersion of the warm gas phase of the ISM,  $\sigma_d$ , as  $H_{r,h} = H[f_r h_g(r_i, t') - R_b(r_i, t')]$  and  $H_{v,\sigma} = H[\sigma_d - v_s(r_i, t')]$ . The quantities  $h_g$ ,  $R_b$  and  $v_s$  depend on time and annulus. Equation (C1) implies that all bubbles contribute to the swept-up mass rate unless they have been confined or broken out from the ISM in previous times. Bubbles at different evolutionary stages can coexist in an annulus.

*Confined bubbles.* Confined bubbles contribute positively to  $\dot{M}_{\text{g,ISM}}$ . The confinement of bubbles depends on whether the expansion velocity of bubbles reaches or exceeds the velocity dispersion of the warm phase in the ISM,  $\sigma_d$ . The rate of mass transferred to the ISM by confinement is

$$\dot{M}_{\text{conf,ISM}}(t_c) = \int_0^{t_c} \sum_{i=1}^{i=N_r} N_{\text{GMC},i,t'} \dot{m}_b(r_i, t') H_{v,\sigma} dt'. \quad (\text{C2})$$

*Break-out of bubbles.* The break-out of bubbles from the ISM contributes positively to the ISM gas due to the fraction of gas mass in the bubbles that stays in the ISM,  $(1 - f_{\text{bo}})$ . The condition for break-out is that the radius of the bubbles reaches a factor  $f_i$  of the gas scaleheight,  $R_b \geq f_i h_g$ . The rate of break-out gas mass in the ISM is

$$\dot{M}_{\text{bo,ISM}}(t_c) = \int_0^{t_c} \sum_{i=1}^{i=N_r} N_{\text{GMC},i,t'} \dot{m}_b(r_i, t') H_{r,h} dt'. \quad (\text{C3})$$