

Inflation and Dark Matter in the Higgs Portal of the Classically Scale Invariant Standard Model

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Abstract

We consider a minimal classically scale-invariant extension of the Standard Model. In this theory, the Higgs mechanism is triggered and the electroweak symmetry breaking is generated radiatively by the Coleman-Weinberg sector which is coupled to the SM Higgs. We extend the Higgs portal interactions of the theory to include an additional singlet which is also non-minimally coupled to gravity. This generates a single-field slow-roll inflation mechanism in the effective field theory formulation which is robust up to Planck scales. Our approach does not require integrating in any additional new physics degrees of freedom to unitarise the theory in the sub-Planckian regime where inflation happens. As a result, no large threshold corrections appear in our approach to inflation so that the electroweak scale and the SM Higgs mass are not affected. The singlet field responsible for inflation also gives a viable dark matter candidate in our model. We also discuss the relation between classical scale-invariance of the effective theory and the possible local scale invariance of the full theory and comment on the naturalness of the electroweak scale.

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1 Introduction

The discovery of the $\simeq 125$ GeV scalar particle [1, 2] with the properties of the Standard Model Higgs boson together with the so far negative results for searches of supersymmetry are pointing to a possibility of a very different model building paradigm based on a minimally extended Standard Model with manifest classical scale invariance. In this paper we will present the first, to the best of our knowledge, realisation of a slow-roll cosmological inflation mechanism in a classically scale-invariant BSM theory.

The rationale for advocating this ‘heretical’ approach to model building based on classical scale invariance is as follows.

(1.) First point in favour of (approximate) scale symmetry is a well-known ‘experimental’ fact that there is just a single occurrence of a non-dynamical scale in the Standard Model (SM) – the Higgs mass-squared input parameter μ_{SM}^2 – appearing in the SM Higgs potential,

$$V_{\text{cl}}^{\text{SM}}(H) = \mu_{\text{SM}}^2 H^\dagger H + \frac{\lambda_{\text{H}}}{2} (H^\dagger H)^2. \quad (1.1)$$

By selecting a particular negative value for it, an expectation value $v \simeq 246$ GeV for the Higgs field, and the Higgs mass $m_h \simeq 125$ GeV are obtained, $\mu_{\text{SM}}^2 = -\frac{1}{2} m_h^2 = -\frac{1}{2} \lambda_{\text{H}} v^2$. Classical scale invariance, broken by μ_{SM}^2 , is recovered by re-interpreting this scale in terms of a vacuum expectation value of a new scalar ϕ , coupled to the SM via the Higgs portal interaction, $\lambda_{\text{P}} |H|^2 |\phi|^2$,

$$V_{\text{cl}}(H, \phi) = \frac{\lambda_{\text{H}}}{2} (H^\dagger H)^2 + \frac{\lambda_{\phi}}{4!} |\phi|^4 - \lambda_{\text{P}} (H^\dagger H) |\phi|^2, \quad \mu_{\text{SM}}^2 = -\lambda_{\text{P}} |\langle \phi \rangle|^2. \quad (1.2)$$

The classical theory described by (1.2) is scale-invariant, and insofar as the appropriate natural value for $\langle \phi \rangle \ll M_{\text{UV}}$ can be generated quantum mechanically, it will trigger the electroweak symmetry breaking.

(2.) Already 40 years ago in Ref. [3] Coleman and Weinberg discovered that a massless scalar field ϕ coupled to a gauge field dynamically generates a vacuum expectation value via dimensional transmutation of the logarithmically running coupling constants. The $\langle \phi \rangle$ is non-vanishing, calculable in a weakly-coupled theory, and its value is naturally small (i.e. exponentially suppressed) relative to the UV cutoff of the theory,

$$\langle \phi \rangle \sim M_{\text{UV}} \times \exp \left[-\frac{\text{const}}{g_{\text{CW}}^2} \right] \ll M_{\text{UV}}, \quad (1.3)$$

where g_{CW} is the gauge coupling of ϕ . In the Appendix we review the Coleman-Weinberg mechanism to recall some of the key expressions for the convenience of the Reader.

In the $\text{SM} \times \text{U}(1)_{\text{CW}}$ theory with the classical scalar potential (1.2), the ϕ -vev in (1.3) then generates the Higgs vev v and the Higgs mass m_h , as dictated by (1.2),

$$|\langle \phi \rangle|^2 = \frac{\lambda_{\text{H}}}{\lambda_{\text{P}}} \frac{v^2}{2} \quad |\langle \phi \rangle|^2 = \frac{1}{\lambda_{\text{P}}} \frac{m_h^2}{2}, \quad (1.4)$$

and one can set $v = 246$ GeV, $m_h = 125$ GeV and $\lambda_{\text{H}} = \frac{m_h^2}{v^2}$. The $\text{U}(1)_{\text{CW}}$ theory coupled to the SM via the Higgs portal with the scalar potential (1.2) was first considered in [4].

Note that no input mass scales are allowed in the Coleman-Weinberg theory and, in the course of UV renormalisation, the subtraction scheme is chosen to set the *renormalised masses* at the origin of the field space to zero,

$$m^2|_{\phi=0} := V''(\phi)\Big|_{\phi=0} = 0, \quad (1.5)$$

where V is the quantum-corrected effective potential. Does the masslessness condition (1.5) amount to a fine-tuning?

A theory with no input mass-scales is classically scale invariant [5]. The scale invariance is not exact, but neither it is broken by an arbitrary amount. The violation of scale invariance is controlled in quantum theory by the anomaly in the trace of the energy-momentum tensor, or in other words, by the logarithmic running of dimensionless coupling constants and their dimensional transmutation scales which, in the weakly coupled perturbation theory, are much smaller than the UV cutoff. Bardeen has argued in [6] that in order to maintain anomalously broken scale invariance, one should select a regularisation scheme which does not explicitly break classical scale invariance (so that it does not introduce positive powers of the UV cutoff scale into renormalised quantities).

In dimensional regularisation, the equation (1.5) is satisfied automatically. Indeed, no power-like dependences on the cutoff scale appear in dimensional regularisation, and in theories like ours, which contain no explicit mass scales at the outset, no finite corrections to mass terms at the origin of the field space can appear either. In this regularisation which preserves classical scale invariance, the theory as it stands is not fine-tuned, at least, not in the technical sense.

If the higher theory above the Planck scale breaks the classical scale invariance of its IR effective theory, through heavy degrees of freedom coupled to the Standard Model,¹ then from the perspective of this microscopic theory, the masslessness condition does amount to a fine-tuning. But from the perspective of our sub-Planckian theory used for particle physics computations, there is no fine-tuning, and no practical problems arise associated e.g. with cancellations between large scales, etc.

Furthermore, once the masslessness condition is enforced at one scale, it holds at all RG scales [7]. To summarise, within dimensional regularisation, the renormalisation condition (1.5) and, more generally, equations,

$$\frac{\partial^2 V(H, \phi)}{\partial H^\dagger \partial H} \Big|_{H=\phi=0} = 0, \quad \frac{\partial^2 V(H, \phi)}{\partial \phi^\dagger \partial \phi} \Big|_{H=\phi=0} = 0, \quad (1.6)$$

are self-consistent, contain no fine-tuning in the theory at hand, and are independent of the RG-scale.

The phenomenology of the $\text{SM} \times \text{U}(1)_{\text{CW}}$ model (1.2) in the context of LHC, future colliders and low energy measurements was analysed recently in [7]. The minimal model has only two remaining free parameters, the portal coupling, λ_{P} and the mass of the second scalar eigenstate m_{h_2} (with m_{h_1} being the observed Higgs mass)², and it was shown that the model is perfectly

¹ Assuming that the higher theory maintains no symmetry principle which could cancel its heavy contributions to the Higgs mass.

²The Higgs field and the CW field both have vevs and mix thus the mass of the second scalar eigenstate is the relevant physical parameter rather than the mass of the CW scalar. This parameter can also be traded for the mass of the CW gauge field Z' .

viable. In particular, the presently available Higgs data constrains the portal coupling to be $\lambda_P \lesssim 10^{-5}$ on the part of the parameter space where the second scalar is in the region between 10^{-4} GeV and $m_{h_1}/2$. For heavier m_{h_2} , the portal coupling is much less constrained experimentally, and has a theoretical upper limit of $\lambda_P \lesssim 10^{-2}$, see Fig. 2 of Ref. [7] for more detail.

Other related studies of classically scale-invariant models can be found in [8–18].

(3.) As the resulting theory has to be valid up to a very large UV scale, the classical scale invariance provides a powerful principle for the BSM model building. No vastly different scales³ can co-exist in the theory: first, it is difficult to generate a large hierarchy of scales from the same $\langle\phi\rangle$. Secondly, the large scales, if they do appear, would ultimately couple to the Higgs and would destabilise its mass. The BSM theory is a minimal extension of the SM which should address all the sub-Planckian shortcomings of the SM, such as the generation of matter-anti-matter asymmetry, inflation, dark matter, stabilisation of the SM Higgs potential, without introducing scales higher than $\langle\phi\rangle$ which itself is not much higher than the electroweak scale.

Can one generate the matter-anti-matter asymmetry of the Universe in these settings?

It was shown in Ref. [17] that the classically-invariant $\text{SM}\times\text{U}(1)_{CW}$ theory (1.2) where the Coleman-Weinberg $\text{U}(1)_{CW}$ sector is interpreted as the gauged $B-L$ symmetry of the SM, can indeed generate the observed value of matter-anti-matter asymmetry. The $B-L$ model [19] is an appealing extension of the SM as it automatically contains three generations of right-handed Majorana neutrinos necessary (needed to cancel the gauge anomaly of $\text{U}(1)_{B-L}$). The standard see-saw mechanism generates masses of visible neutrinos and facilitates neutrino oscillations. The CW field ϕ carries the $B-L$ charge 2 and its vev generates the Majorana neutrino masses and the mass of the $\text{U}(1)_{B-L}$ Z' boson. In the classically conformal settings, these models were first considered in [11, 13]. As well-known, however, the standard thermal leptogenesis formalism [20] requires very heavy Majorana neutrino masses of the order of 10^9 GeV, to generate the observed baryon asymmetry of the Universe. Such heavy neutrino masses go against the grain of our classically scale-invariant approach.

Ref. [17] used a different realisation of the leptogenesis mechanism proposed in [21] and further developed in [22, 23] – the leptogenesis due to flavour oscillations of the right-handed Majorana neutrinos. It was shown in [17] that the classically-invariant $\text{SM}\times\text{U}(1)_{B-L}$ model can generate the observed matter-anti-matter asymmetry on a large portion of the model parameter space without introducing any additional mass scales and without requiring any resonant fine-tuning. The Majorana masses are in the range between 200 MeV and 500 GeV; they, together with and a heavier mass for the Z' boson, are generated by the CW vev $\langle\phi\rangle$.

(4.) *Vacuum stability:* It is well-known that in the Standard Model the Higgs quartic coupling becomes negative rendering the Higgs effective potential unstable at above the scale $\simeq 10^{11}$ GeV. The next-to-next-to leading order analysis of the SM Higgs effective potential carried out in [24] found the best-fit value of the Higgs self-coupling at the Planck scale to be small but negative,

$$\frac{1}{2}\lambda_H(M_{\text{Pl}}) = -0.0144 + 0.0028 \left(\frac{m_h}{\text{GeV}} - 125.66 \right) \pm 0.0047_{M_t} \pm 0.0018_{\alpha_s} \pm 0.0028_{\text{th}}, \quad (1.7)$$

³Apart from the Planck scale M which is treated separately.

with further refinements carried out in [25].

It is remarkable that λ_H and its beta function nearly vanish at the Planck scale, but without new physics, the absolute stability of the Standard Model Higgs potential is ruled out [24, 25].

A minimal and robust way to repair the electroweak vacuum stability is provided by the Higgs portal extension of the SM – and this is provided automatically in the classically scale-invariant theory (1.2). There are two effects helping to stabilise the vacuum: first is the fact that the portal coupling gives a positive contribution to the beta function of the Higgs quartic coupling, $\Delta\beta_{\lambda_H} \sim +\lambda_P^2$. The second effect occurs due to a vev of the second scalar, $\langle\phi\rangle > v$, leading to mixing between ϕ and the Higgs and resulting in a threshold correction lifting the SM Higgs self-coupling [26, 27]. In the present context of classically scale-invariant $\text{SM} \times \text{U}(1)_{CW}$ theory, the vacuum stabilisation was addressed in recent papers Refs. [13, 14, 16, 18]. The values and ranges of the couplings needed to stabilise the Higgs potential depend on the model specific RG-running.⁴ The conclusion we draw from this work is that the vacuum stability up to the Planck scale is effectively restored on large portions of the parameter space for classically conformally invariant extensions of the Standard Model.

We also mention that adding an additional singlet to the theory coupled to the Higgs via a portal will further enhance the stability of the potential.

(5.-6.) *Inflation* and *dark matter* within the classically scale-invariant BSM formulation is the subject of the present paper. In the following section we will extend the Higgs portal model by introducing a single real scalar field $s(x)$ with a non-minimal coupling to gravity. An exponentially flat potential for the singlet will be generated, as required for a successful implementation of a slow-roll cosmological inflation, preserving the classical scale invariance of the model and without the need of perturbative unitarisation at sub-Planckian scales.

Quite pleasingly, the singlet $s(x)$ is protected by a Z_2 symmetry and gives a viable candidate for dark matter. Thus the roles of the inflation and of the scalar dark matter particle are unified and are straightforwardly incorporated in the context of the classically scale-invariant Higgs-portal BSM model.

2 Singlet-field slow-roll inflation in the Higgs portal

In the standard Big Bang cosmology the cosmological inflation was proposed in [28] to solve the flatness, isotropy, homogeneity, horizon and relic problems. Inflation is the leading theory of the early universe, confirmed by observations including the recent data from Planck satellite [29] which favour a simple inflationary scenario with only one slow rolling scalar field. Slow-roll inflation occurs in many models constructed in the past, although, in general, the underlying microscopic particle physics of many inflation models is still unknown, as it involves energy scales far higher than can be probed at colliders.

⁴The authors of [16, 18] considered a model with the non-Abelian $\text{SU}(2)_{CW}$ gauge group. Ref. [16] showed that for the values of the portal coupling $\lambda_p = 2\lambda_P \simeq 4 \times 10^{-3}$ and the non-Abelian CW coupling $g_{CW} \sim 1$, the vacuum stability is recovered (here the second scalar is heavy, $m_{h_2} \simeq 165$ GeV). More generally, in Ref. [18] the allowed regions on the entire parameter space $\{\lambda_p, g_{CW}\}$ consistent with the vacuum stability and perturbativity were determined for $m_{h_2} < m_{h_1}$ and for $m_{h_2} > m_{h_1}$, we refer to their Fig. 1 for more detail. Vacuum stability in the $\text{SM} \times \text{U}(1)_{B-L}$ theory was addressed in [14] and [13] following a more constrained than our approach, where the vanishing (rather than just positivity) of the Higgs potential was requested at M_{Pl} .

We will focus on the approach based on renormalisable QFT Lagrangians, which are then coupled to gravity, and in addition to the usual Einstein-Hilbert term also involve a non-minimal coupling of a scalar field to gravity. By taking the non-minimal coupling ξ to be large, a flat slow-roll potential is generated and inflation takes place, as originally discussed in [30].

An interesting extremely minimal proposal which bridges cosmology with the Standard Model particle physics is that the Higgs itself could play the role of the inflaton [31]. The coupling $\xi \sim 10^4$ however introduces two sub-Planckian scales in the theory, M_{Pl}/ξ and $M_{\text{Pl}}/\sqrt{\xi}$, where the theory appears to violate unitarity, thus spoiling the self-consistency of the inflationary approach, which has led to a considerable debate in the literature as to whether such models make sense quantum mechanically [32–37]. From now and for the rest of this section we will be using the reduced Planck mass $\sim 10^{18}$ GeV and will refer to it simply as M .

In [38] it was shown that the Higgs inflation model can be unitarised by introducing (or integrating in) a new scalar with the mass M/ξ participating in the inflation. This allowed the authors of [38] to raise the unitarity bound up to the Planck scale, well above the inflationary scale $\sim M/\sqrt{\xi}$. This approach however is incompatible with the requirement of classical scale invariance of the theory which we want to maintain. The mass $M/\sqrt{\xi}$ of the new scalar field, which interacts with the Higgs, is a non-dynamical input scale. Upon integrating out this massive degree of freedom the Higgs mass is destabilised and requires fine-tuning.

We take an alternative approach to inflation which is based on a classically *massless* and vev-less real scalar field s . Our inflaton is neither the Higgs itself as in Ref. [31], nor the heavy scalar as in Ref. [38]. Our approach is also different from the singlet-assisted Higgs portal inflation of Refs. [26, 39] where the inflaton was a combination of the Higgs and the singlet with a non-vanishing vev (for earlier and related work see also [40–44] and references therein). With the inflaton (or more precisely the non-minimally coupled to gravity scalar) being a single-component field, we will be able to avoid problems with violations of unitarity at intermediate scales in pure gravity and kinetic sectors, along the same lines as in [34, 36].

We start from the classical scalar potential (1.2) of the SM coupled to the CW sector which we rewrite as

$$V_{\text{cl}}(H, \phi) = \frac{\lambda_H}{2} \left(|H|^2 - \frac{\lambda_P}{\lambda_H} |\phi|^2 \right)^2 + \frac{\tilde{\lambda}_\phi}{4!} |\phi|^4. \quad (2.1)$$

Here the new ϕ -self-coupling $\tilde{\lambda}_\phi$ is defined in terms of the original parameters of (1.2) via a shift $\tilde{\lambda}_\phi = \lambda_\phi - 12\lambda_P^2/\lambda_H$.

We now extend this model by adding a single degree of freedom, a one-component real scalar field $s(x)$. It is a gauge singlet which is coupled to the $\text{SM} \times \text{U}(1)_{\text{CW}}$ theory only via the scalar portal interactions with the Higgs and ϕ ,

$$V_{\text{cl}}(H, \phi, s) = \frac{\lambda_{hs}}{2} |H|^2 s^2 + \frac{\lambda_{\phi s}}{4} |\phi|^2 s^2 + \frac{\lambda_s}{4} s^4 + V_{\text{cl}}(H, \phi), \quad (2.2)$$

Equations (2.1)-(2.2) describe the general renormalisable gauge-invariant scalar potential for the three massless scalars,

$$\left. \frac{\partial^2 V(H, \phi, s)}{\partial s \partial s} \right|_{s=H=\phi=0} = 0, \quad \left. \frac{\partial^2 V(H, \phi, s)}{\partial H^\dagger \partial H} \right|_{s=H=\phi=0} = 0, \quad \left. \frac{\partial^2 V(H, \phi, s)}{\partial \phi^\dagger \partial \phi} \right|_{s=H=\phi=0} = 0. \quad (2.3)$$

as required by the classical scale invariance of the theory.

The coupling constants in the potential (2.2) are taken to be all positive (or at least non-negative), thus the potential is stable and the positivity of λ_{hs} and $\lambda_{\phi s}$ ensure that no vev is generated for the singlet $s(x)$. Instead the CW vev $\langle\phi\rangle$ generates the mass terms for the singlet,

$$m_s^2 = \frac{\lambda_{hs}}{2} v^2 + \frac{\lambda_{\phi ss}}{2} |\langle\phi\rangle|^2, \quad (2.4)$$

in the vacuum $s = 0$, $\phi = \langle\phi\rangle$, $H = \frac{v}{\sqrt{2}} = \sqrt{\frac{\lambda_P}{\lambda_H}} |\langle\phi\rangle|$.

We now proceed to couple our theory to gravity. To achieve the slow-roll inflation we need to introduce large non-minimal couplings to gravity of at least some of the scalars [30, 31]. As noted earlier, the central point of our approach is to non-minimally couple *only* the singlet s , such that the Lagrangian takes the form,

$$\begin{aligned} \mathcal{L}_J = \sqrt{-g_J} \left(-\frac{M^2}{2} R - \frac{\xi_s}{2} s^2 R + \frac{1}{2} g_J^{\mu\nu} \partial_\mu s \partial_\nu s + g_J^{\mu\nu} (D_\mu H)^\dagger D_\nu H + \frac{1}{2} g_J^{\mu\nu} (D_\mu \phi)^\dagger D_\nu \phi \right. \\ \left. - \frac{\lambda_s}{4} s^4 - \frac{\lambda_{hs}}{2} |H|^2 s^2 - \frac{\lambda_{\phi s}}{4} |\phi|^2 s^2 - V(H, \phi) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \text{Fermions} + \text{Yukawas} \right). \end{aligned} \quad (2.5)$$

The term $(\xi_s/2) s^2 R$ is the non-minimal coupling of the singlet s to gravity, with R being the scalar curvature. It will follow that for successful inflation the value of the non-minimal coupling constant ξ_s should be relatively large, $\sim 10^4$. For this reason, we will treat ξ_s (and $\sqrt{\xi_s}$) as large parameters $\gg 1$. In this sense, s is distinguished from the two other scalars, H and ϕ , which in our case have either vanishing or small loop-induced non-minimal gravitational couplings ($\xi_H \sim \xi_\phi \sim 1 \ll \xi_s$ and thus are neglected in (2.5) without loss of generality).

$M \sim 10^{18}$ GeV denotes the reduced Planck mass; in the Lagrangian (2.5) M appears only in the Einstein-Hilbert term and does not couple directly to non-gravitational degrees of freedom. For the time being we will treat it as an explicit scale characterising the classical gravitational background.⁵ Finally, the subscript J in (2.5) defines the so-called Jordan frame, where the scalar-to-gravity coupling is non-minimal and kinetic terms are canonically normalised.

The second line on the *r.h.s.* of (2.5) contains the scalar potential (2.1) and (2.2), plus radiative corrections, plus kinetic terms for gauge and fermion fields and Yukawa interactions.

An immediate question arises concerning the non-minimal coupling of the singlet s to gravity, $\frac{\xi_s}{2} s^2 R$ in (2.5): Will the appearance of the relatively large coupling $\xi_s \sim 10^4$ render the theory ill-defined at an intermediate scale M/ξ_s which is below the high scale M ? If one expands around the flat space, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M$, the non-minimal coupling generates the dimension-5

$$\sim \frac{\xi_s}{M} s^2 \partial^2 h$$

interaction. This interaction generates contributions to e.g. the $2s \rightarrow 2s$ scattering processes mediated by the graviton exchange. However, it was shown in Ref. [36] that when summed over s , t and u channels, the leading order in ξ_s contributions to this process cancel, and the

⁵To keep the scale invariance manifest in the full theory coupled to gravity, M can be recast in terms of the vev of the dilation of the spontaneously broken local scale invariance. In dimensional regularisation, the scale-invariance can be kept manifest in quantum theory. We will return to this point in the Conclusions section.

first non-vanishing effects are suppressed by the high scale (Planck mass M); thus the theory in this sector appears to be safe up to the gravity scale M , and as such is well-behaved at the intermediate scale M/ξ_s . For the cancellation to occur it is however essential that there is only a single scalar non-minimally coupled to gravity [36]. So, the theory coupled to gravity as defined by its Lagrangian (2.5) in the Jordan frame appears to be safe up to the high UV scale M and the appearance of the relatively large non-minimal coupling ξ_s does not immediately invalidate the theory at lower intermediate scales. We will return to this point in what follows.

We now perform a metric transformation to the Einstein frame where the non-minimal scalar to gravity interaction is removed,

$$g_{\mu\nu} \rightarrow \Omega^{-2} g_{\mu\nu}, \quad \Omega^2 := 1 + \frac{\xi_s s^2}{M^2}, \quad (2.6)$$

so that the Lagrangian becomes,

$$\begin{aligned} \mathcal{L}_E = \sqrt{-g_E} \left(-\frac{1}{2} M^2 R + \left(\frac{\Omega^2 + \frac{6\xi_s^2 s^2}{M^2}}{\Omega^4} \right) \frac{g_E^{\mu\nu} \partial_\mu s \partial_\nu s}{2} + \frac{g_E^{\mu\nu} (D_\mu H)^\dagger D_\nu H}{\Omega^2} + \frac{1}{2} \frac{g_E^{\mu\nu} (D_\mu \phi)^\dagger D_\nu \phi}{\Omega^2} \right. \\ \left. - \frac{1}{\Omega^4} \left(\frac{\lambda_s}{4} s^4 + \frac{\lambda_{hs}}{2} |H|^2 s^2 + \frac{\lambda_{\phi s}}{4} |\phi|^2 s^2 + V(H, \phi) \right) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{\text{Fermions}}{\Omega^3} + \frac{\text{Yukawas}}{\Omega^4} \right). \end{aligned} \quad (2.7)$$

As expected, the non-minimal interaction of s with the scalar curvature R has disappeared. The price for this is the non-canonical normalisation of the kinetic term for s . The factor $6\xi_s^2 s^2/M^2$ arises from the transformation of R ,

$$-\sqrt{-g_J} \Omega^2 R(g_J) = -\sqrt{-g_E} \Omega^2 R(g_E) - 6\Omega \partial_\mu (\sqrt{-g_E} g_E^{\mu\nu} \partial_\nu \Omega), \quad (2.8)$$

followed by the integration by parts in the second term. This indeed gives

$$\frac{6\xi_s^2 s^2}{M^2} \times \frac{g_E^{\mu\nu}}{2\Omega^2} \partial_\mu s \partial_\nu s, \quad (2.9)$$

which appears in (2.7).

The expression (2.9) describes a dimension-6 interaction suppressed by the scale M/ξ_s . This appears to be a dangerously low scale for the successful investigation of inflation which as we will see occurs at the larger scale $M/\sqrt{\xi_s}$. Of course this is the Einstein frame reflection of the same phenomenon we have already considered in the Jordan frame (see the paragraph above (2.6)). For the same reason, in the class of models considered here, where the non-minimal coupling to gravity involves a *single* real scalar, the appearance of this scale in the kinetic term (2.9) is harmless. It turns out that the contribution to $2s \rightarrow 2s$ scattering of the 4-point vertex (2.9) is vanishing when the external states are put on-shell [36]. This is true at tree-level and continues to hold to arbitrary loop order. The underlying reason for why the theory continues to be well-behaved at the intermediate scale (at least in the scalar-kinetic sector and in the scalar-gravity sector) is the existence of the field redefinition in the Einstein frame which will render the kinetic term canonical. We will now proceed along these lines.

To bring the kinetic term for our singlet field s into the canonical form we perform the field redefinition $s = s(\sigma)$ following [31] and define the singlet field $\sigma(x)$ via

$$\sigma = \text{sign}(s) \int_0^s ds \sqrt{\frac{1}{\Omega^2} + \frac{6\xi_s s^2}{M^2 \Omega^4}}, \quad (2.10)$$

so that

$$\frac{d\sigma}{ds} = \sqrt{\frac{1}{\Omega^2} + \frac{6\xi_s s^2}{M^2\Omega^4}} = \sqrt{\frac{\Omega^2 + \frac{3}{2}M^2(\partial_s\Omega)^2}{\Omega^4}}. \quad (2.11)$$

The kinetic term for the singlet in (2.6) is now brought to the canonical form,

$$\left(\frac{1}{\Omega^2} + \frac{6\xi_s s^2}{M^2\Omega^4}\right) \frac{g_E^{\mu\nu} \partial_\mu s \partial_\nu s}{2} = \frac{1}{2} g_E^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma. \quad (2.12)$$

We note that as pointed out in [36], this field redefinition is possible and works only for the case of a *single* real scalar field coupled non-minimally to gravity – which is the case we are considering.⁶

At small field values, for example at the electroweak scale and all the way up to $s \lesssim 10^{14}$ GeV, the field redefinition (2.10) amounts to

$$\sigma(x) = s(x), \quad \text{for } s \ll M/\xi_s. \quad (2.13)$$

At higher values of s , it is easy to see that the solution for σ is well approximated by the analytic expression [45]

$$\sigma(x) = \sqrt{\frac{3}{2}} M \log \Omega^2[s(x)], \quad \text{for } s \gg \frac{M}{\xi_s}, \quad (2.14)$$

or

$$s(x) = \frac{M}{\sqrt{\xi_s}} \sqrt{\exp\left(\frac{2\sigma(x)}{\sqrt{6}M}\right) - 1}, \quad \text{for } s \gg \frac{M}{\xi_s}, \quad (2.15)$$

At an even higher scale $s \gg M/\sqrt{\xi_s}$ this gives,

$$s(x) = \frac{M}{\sqrt{\xi_s}} \times \exp\left(\frac{\sigma(x)}{\sqrt{6}M}\right), \quad \text{for } s \gg \frac{M}{\sqrt{\xi_s}}. \quad (2.16)$$

From (2.14) and (2.15) we can compute

$$\frac{s^2(x)}{\Omega^2} = \frac{M^2}{\xi_s} \times \left(1 - \exp\left[-\frac{2\sigma(x)}{\sqrt{6}M}\right]\right) \simeq \frac{M^2}{\xi_s}, \quad \text{for } s \gg \frac{M}{\xi_s}, \quad (2.17)$$

and thus we find that at large field values, the singlet self-interaction potential in (2.7) is exponentially flat when expressed in terms of the canonically normalised $\sigma(x)$, and is well-suited for the slow-roll inflation [31],

$$V_E(s) := \frac{\lambda_s}{4} \frac{s^4(x)}{\Omega^4} = \frac{\lambda_s M^4}{4 \xi_s^2} \left(1 - \exp\left[-\frac{2\sigma(x)}{\sqrt{6}M}\right]\right)^2, \quad \text{for } s \gg \frac{M}{\xi_s}. \quad (2.18)$$

⁶For multi-component scalars, such as for example the Higgs itself (4 real components) or the CW complex scalar ϕ (2 real components), no field redefinition would have amounted to the canonical normalisation of all field components. As a result, the perturbative unitarity would have been lost at the scale M/ξ which is lower than the inflation scale $\sim M/\sqrt{\xi}$ we want to study. The use of the unitary gauge where only the radial component remains and the extra degrees of freedom are gauged away only shifts the unitarity problem to the gauge-Higgs interactions and the longitudinal vector boson sector.

Next we need to take care of the inverse powers of Ω in the Lagrangian (2.7). The appeared because the transformation (2.6) was a pure metric transformation, rather than the full Weyl scaling transformation,

$$\text{Weyl: } g_{\mu\nu} \rightarrow \Omega^{-2} g_{\mu\nu}, \quad \text{scalars} \rightarrow \Omega^1 \text{scalars}, \quad A_\mu \rightarrow A_\mu, \quad \Psi \rightarrow \Omega^{3/2} \Psi. \quad (2.19)$$

We would now like to excise inverse factors of Ω from the Einstein-frame Lagrangian in (2.7) to achieve the canonical normalisation for all the remaining kinetic terms. To do this, we now transform the scalar fields H and ϕ (but not s or σ !) as well as the fermions, as prescribed by the Weyl transformation (2.19). For the kinetic terms of non-singlet scalar fields in (2.7) (here we introduce the simplifying notation $\vec{\varphi} = (H/\sqrt{2}, \phi) = (h_1, h_2, h_3, h_4, \phi_1, \phi_2)$ for all six real components of the Higgs and the CW scalar), with the transformation $\vec{\varphi} \rightarrow \Omega \vec{\varphi}$ we have,

$$\begin{aligned} \frac{1}{2} \frac{g_E^{\mu\nu} \partial_\mu(\Omega \vec{\varphi}) \partial_\nu(\Omega \vec{\varphi})}{\Omega^2} &= \frac{1}{2} g_E^{\mu\nu} (\partial_\mu \vec{\varphi}) (\partial_\nu \vec{\varphi}) \\ &+ \frac{1}{2} \frac{|\vec{\varphi}|^2}{6M^2} g_E^{\mu\nu} (\partial_\mu \sigma) (\partial_\nu \sigma) + \frac{\vec{\varphi} \cdot (\partial_\mu \vec{\varphi}) g_E^{\mu\nu} (\partial_\nu \vec{\varphi})}{\sqrt{6} M} \\ &\simeq \frac{1}{2} g_E^{\mu\nu} (\partial_\mu \vec{\varphi}) (\partial_\nu \vec{\varphi}), \end{aligned} \quad (2.20)$$

where we have used the substitution $\partial_\mu \log \Omega = \partial_\mu \sigma(x)/(\sqrt{6}M)$ valid at the singlet field values $s \gg \frac{M}{\xi_s}$. The key point we want to make is that the two terms on the second line of (2.20) are suppressed by the Planck scale $\sqrt{6} M \gg M/\sqrt{\xi_s} \gg M/\xi_s$. The Planck scale is treated as the *large* scale $>$ than the UV cutoff in our effective field theory in the gravitational background, and we can neglect the terms suppressed by M . The inflation will happen at the parametrically lower scale $\sim M/\sqrt{\xi_s}$ dictated by the exponential fall-off of the potential (2.18), and is under control in the theory with the canonically normalised kinetic terms, given by the last line of (2.20).

The terms in (2.7) involving portal interactions or the singlet s with other scalars, at large values of the singlet field (where $s^2/\Omega^2 = M^2/\xi_s$) become

$$\frac{1}{\Omega^4} \left(\frac{\lambda_{hs}}{2} |\Omega H|^2 s^2 + \frac{\lambda_{\phi s}}{4} |\Omega \phi|^2 s^2 \right) \simeq \frac{\lambda_{hs} M^2}{2\xi_s} |H|^2 + \frac{\lambda_{\phi s} M^2}{4\xi_s} |\phi|^2, \quad (2.21)$$

giving the mass terms for the Higgs and the CW field in this large- s -field regime, with the singlet s being (exponentially) decoupled from the rest of the theory, *c.f.* Eq. (2.17). Finally, the Higgs portal potential $V(H, \phi)$ is the homogeneous degree-4 expression in terms of scalars, given by the classical potential (2.1) plus the radiative corrections also involving $\log(\phi/\langle\phi\rangle)$, such as the 1-loop CW expression, (see the Appendix),

$$V_{CW}(\phi) = V_{\text{cl}} + \frac{3}{4} \alpha_{CW}^2 |\phi|^4 \left[\log \left(\frac{|\phi|^2}{\langle|\phi|^2\rangle} \right) - \frac{25}{6} \right]. \quad (2.22)$$

Since V goes as a scalar field to the 4th power, the factors of Ω cancel,

$$\frac{1}{\Omega^4} V(\Omega H, \Omega \phi) = V(H, \phi). \quad (2.23)$$

To summarise, the Einstein-frame Lagrangian after the Weyl transformation of all the fields, except for the singlet $s(x)$ which is substituted by the field redefinition (2.10), at large values

of the singlet field takes the canonical form. The singlet in this regime is effectively decoupled from the SM degrees of freedom and the exponentially flat potential (2.18),

$$\begin{aligned} \frac{\mathcal{L}_E}{\sqrt{-g_E}} = & -\frac{1}{2} M^2 R + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{\lambda_s M^4}{4 \xi_s^2} \left(1 - \exp \left[-\frac{2\sigma}{\sqrt{6}M} \right] \right)^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \text{Fermions} \\ & + (D^\mu H)^\dagger D_\mu H + \frac{1}{2} (D^\mu \phi)^\dagger D_\mu \phi - \frac{\lambda_{hs} M^2}{2 \xi_s} |H|^2 - \frac{\lambda_{\phi s} M^2}{4 \xi_s} |\phi|^2 - V(H, \phi) + \text{Yukawas}. \end{aligned} \quad (2.24)$$

In the equation above we have used the exponential form potential (2.18) for the canonically normalised σ -singlet which is the correct description at large field values. At the same time, to simplify the formulae, in the terms involving portal interactions between the singlet and other scalars, we used the asymptotic values for s^2/Ω^2 as in (2.21), emphasising the emerging large mass terms for H and ϕ , but neglecting exponentially suppressed corrections involving σ . These are small exponentially suppressed effects which are not going to modify the analysis of inflation.

The inflaton is the σ -field and the inflation is generated by the self-interacting part of the potential in (2.24),

$$V(\sigma) = \frac{\lambda_s M^4}{4 \xi_s^2} \left(1 - \exp \left[-\frac{2\sigma(x)}{\sqrt{6}M} \right] \right)^2. \quad (2.25)$$

It is exponentially flat for large field values and is suitable for the slow-roll inflation [31]. The slow-roll inflation parameter is

$$\epsilon := \frac{M^2}{2} \left(\frac{V(\sigma)/d\sigma}{V(\sigma)} \right)^2 = \frac{4 M^4}{3 \xi_s^2 s^2}. \quad (2.26)$$

Inflation ends when $\epsilon_{\text{end}} = 1$ which corresponds to $s_{\text{end}} = (4/3)^{1/4} M/\sqrt{\xi_s}$ or $\sigma_{\text{end}} \simeq 0.94M$. Inflation starts at the singlet field value [45] $s_0 \simeq 9.14 M/\sqrt{\xi_s}$. The CMB normalisation condition,

$$\frac{V}{\epsilon}(s = s_0) \simeq (0.0276 M)^4, \quad (2.27)$$

which determines the non-minimal singlet coupling to gravity [31, 45],

$$\xi_s \simeq 4.7 \times 10^4 \sqrt{\lambda_s}. \quad (2.28)$$

The spectral index and the tensor-to-scalar perturbation ratios in this model are the same as computed in the Bezrukov-Shaposhnikov Higgs-inflation model [31]. They are found to be in excellent agreement [45] with the latest Planck measurements, [29].

Our realisation of inflation in (2.24) is a one-field slow-roll inflation model. The singlet field σ plays the role of the inflaton, while the other degrees of freedom, such as the Higgs and the CW scalar at the inflation scale being much heavier than the Hubble expansion parameter during inflation, $H = \sqrt{\frac{\lambda_s}{12}} \frac{M}{\xi_s}$.

$$m_h = \sqrt{\frac{\lambda_{hs}}{2}} \frac{M}{\sqrt{\xi_s}} \quad \text{and} \quad m_\phi = \sqrt{\frac{\lambda_{\phi s}}{2}} \frac{M}{\sqrt{\xi_s}} \quad \gg \quad H = \sqrt{\frac{\lambda_s}{12}} \frac{M}{\xi_s}. \quad (2.29)$$

The main technical advantage of the singlet field realisation of inflation presented here, compared to the original Higgs inflation model [31] and the subsequent implementations with other multi-component non-minimally coupled scalars, as in [26, 39], is that our model does not require introduction of new physics effects associated with the ‘low’ M/ξ_s and ‘intermediate’ scale $M/\sqrt{\xi_s}$. Indeed, the appearance of the low-scale dimension-6 operator in (2.9) was rendered harmless by the field redefinition of the one-component field $s(x)$.

At the same time we should note that the field redefinition (2.10)-(2.11) when applied in the regime of *small* field values, $s \ll M/\xi_s$, does introduce higher-dimensional corrections in the Einstein-frame potential for the singlet, *cf* (2.18),

$$V_E(s) = \frac{\lambda_s}{4} \frac{s^4(x)}{\Omega^4} = \frac{\lambda_\sigma}{4} s^4(x) - \frac{\lambda_s}{4} \frac{\xi_s^2}{M^2} \sigma^6(x) + \dots, \quad \text{for } s \ll \frac{M}{\xi_s}. \quad (2.30)$$

If one considers multi-particle (2-to-4 and higher) scattering processes, this appears to break unitarity at scales $\frac{M}{\xi_s}$ in the perturbative regime of the theory when expanding at small values of σ . These scattering processes are relevant for the theory in the ‘collider regime’ i.e. in the regime where we are using perturbation theory around vanishing background field values. However in the collider regime the theory is safe: scales $\sim \frac{M}{\xi_s}$ are of course inaccessibly high for any collider experiments. On the other hand, in order to probe the theory around the scale of inflation, the more appropriate approach is to expand around a non-trivial background. In this case, as argued in Ref. [37], the effective UV-cutoff scale of the theory itself depends on the background field values (i.e. in our case it depends on \bar{s} or $\bar{\sigma}$) and rises with these values appropriately. The (slightly simplified) results of the analysis in [37] imply for the effective UV cut-off of theory, $\Lambda_{\text{eff}}(\bar{s}) \sim \frac{M}{\xi_s}$ for $\bar{s} \ll \frac{M}{\xi_s}$; $\Lambda_{\text{eff}}(\bar{s}) \sim \frac{\xi_s \bar{s}^2}{M}$ for $\frac{M}{\xi_s} \ll \bar{s} \ll \frac{M}{\sqrt{\xi_s}}$, and finally, $\Lambda_{\text{eff}}(\bar{s}) \sim M$ for $\bar{s} \gg \frac{M}{\sqrt{\xi_s}}$ in the inflationary regime. We conclude that the effective potential of the theory is likely to remain safe at all relevant energy scales (below the Planck scale) when evaluated at the appropriate background field value (see Ref. [37] and references therein for the detailed analysis of quantum corrections).

We also note that the fact that all the terms for the non-singlet scalars, H and ϕ , are already canonically normalised in the Lagrangian (2.24) implies that there are no non-renormalisable interactions in the theory involving the sub-Planckian scales. Indeed, the only terms we dropped in (2.20) were suppressed by the high scale M . This is different from the case of the Higgs-driven inflation where the gauge and fermion interactions of the Higgs are modified and have to be cut-off at the intermediate scale $M/\sqrt{\xi_s}$, as pointed out in [38].

It is important to note that if the new physics was required to be included at the scales below the UV cutoff M , this would have destroyed the classical scale invariance of our model and induced large threshold contributions to the masses of the Higgs and the CW scalar, reintroducing the fine-tuning problem into the SM. For this reason the elegant construction of Ref. [38] which integrates in a massive linear sigma-model field at the scale M/ξ_s to restore the perturbative unitarity of the Higgs inflation model does not work in our classically scale-invariant case.

Finally, we would like to briefly comment on the role of loop-induced quantum corrections to the effective potential. As explained in the Introduction, our calculational approach is based

on the use of dimensional regularisation where no power-like divergencies can appear. At the 1-loop level we have the Coleman-Weinberg corrected potential,

$$V_{1\text{-loop}}(\sigma) = V_{\text{cl}}(\sigma) + \frac{1}{64\pi^2} (V_{\text{cl}}''(\sigma))^2 \log (V_{\text{cl}}''(\sigma)/M_{UV}^2)^2, \quad (2.31)$$

which entails the derivatives of the exponentially slowly varying potential in the background of a large σ -field. Higher loop orders will involve higher derivatives of the flat potential enhanced by only a logarithmic dependence on the UV-cut-off. As a result, it is quite likely that quantum corrections computed in the dimensional regularisation scheme in the background perturbation theory around a large σ -field will be small and not affect the picture of inflation at the leading order, see also [37] for a more comprehensive discussion of this point.

3 Dark matter

Singlet scalar dark matter (DM) models [46] are the simplest possible UV-complete models of dark matter containing a weakly interacting massive particle (WIMP) – in this case a scalar interacting with the Higgs via the portal interaction.

In the classically scale-invariant $\text{SM} \times \text{U}(1)_{CW}$ theory with the real singlet $s(x)$ and the scalar potential (2.2), the stability of the singlet is protected by a Z_2 symmetry, $s \rightarrow -s$, giving a natural dark matter candidate. Indeed, the Z_2 symmetry of the potential (2.2) is an automatic consequence of the renormalisability (dimension 4), scale-invariance and gauge invariance (which does not allow odd powers of H and ϕ) of the theory.

As shown in the previous section, at large field values, $s(x) > M/\sqrt{\xi_s}$, the singlet (or more precisely its log in (2.16)) plays the role of the inflaton field which during the inflation slowly rolls in an exponentially flat potential (2.25). In this regime, the inflaton gives large masses (2.29) to the SM fields and otherwise decouples from the rest of the theory.

After inflation is completed, the singlet enters the regime $s(x) \ll M/\xi_s$ where it is canonically normalised, its potential is no longer flat and given by (2.2), and the large masses of the SM fields are no longer there. The singlet now assumes the role of the dark matter in the classically scale-invariant theory.

There are only two phenomenologically relevant parameters of this singlet dark matter model, the coupling λ_{hs} of the dark matter particle to the SM Higgs, and the induced mass of the singlet, $m_s \equiv m_{DM}$ in Eq. (2.4) which gives the dark matter mass. Expressed in terms of these two parameters, the DM phenomenology of this classically scale-invariant model is the same as that of the usual singlet scalar DM model.

Papers [47–52] contain the recent comprehensive studies of the singlet scalar DM, see also references therein. Here we give only a very brief summary of their results. The allowed/excluded portions of the λ_{hs} , m_s parameter space are shown on Figure 1 taken from Ref. [52].

1. The relic density of singlet scalar dark matter, Ω_S , should not exceed the full DM relic density Ω_{DM} . The DM relic density is determined by the (in-)efficiency of the DM particles annihilating into the SM degrees of freedom. These processes are dominated by the Higgs-mediated s-channel. Essentially the constraint $\Omega_S/\Omega_{DM} \leq 1$ amounts to a lower bound on the coupling λ_{hs} . The excluded region is the lower portion of the parameter space shown in dark grey on both plots of Fig. 1.

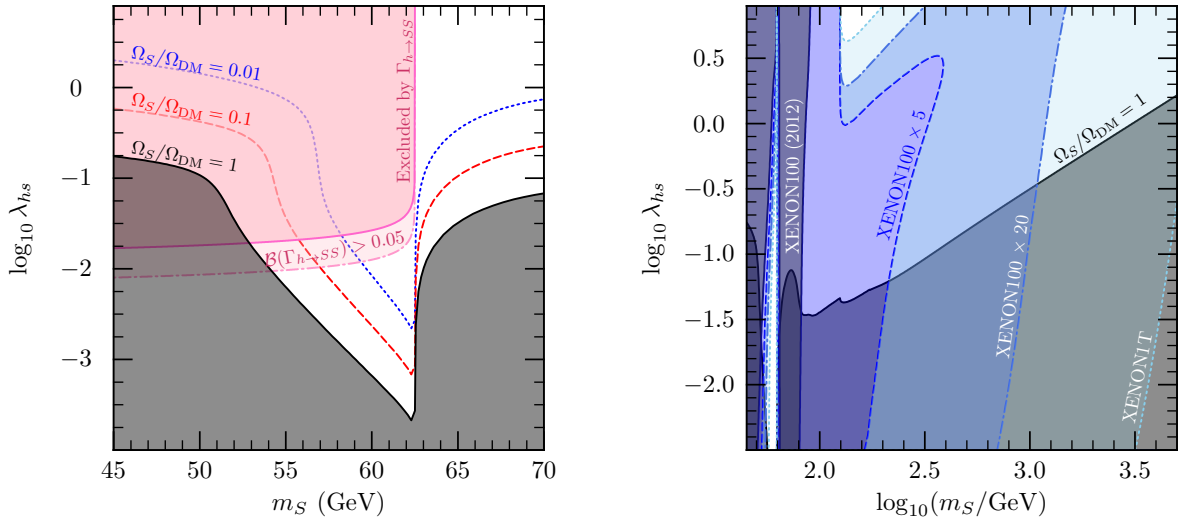


Figure 1: **DM exclusion contours from Ref. [52]** on the λ_{hs} , m_s plane. *Left* plot is a close-up on the region $m_s \lesssim m_h/2$. The dark-shaded lower region is ruled out by the upper bound on the singlet DM relic density, $\Omega_S/\Omega_{\text{DM}} \leq 1$. The region in the upper-left corner is ruled out by constraints on invisible Higgs decays. *Right* plot is over a wide mass range $45 \text{ GeV} \leq m_s \leq 5 \text{ TeV}$. The DM relic density exclusion is shown superimposed with the current bound from XENON100 and bounds from future direct detection experiments (in lighter shades of blue).

2. Indirect detection of dark matter constraints arise from annihilation of DM to SM particles. For singlet scalar DM these constraints work much in the same way as the relic density constraint above, and for $m_s > m_h/2$ provide a slight improvement on the relic density contours.

3. When $m_s < \frac{1}{2} m_h$ the DM can be produced via invisible Higgs decays at the LHC with the Higgs partial decay widths into the singlet DM particles given by

$$\Gamma_{h \rightarrow ss} = \frac{\lambda_{hs}^2 v^2 \sqrt{1 - 4m_s^2/m_h^2}}{16\pi m_h}. \quad (3.1)$$

The upper bounds on the invisible Higgs decays from the LHC give the exclusion contour shown in red on the upper-left corner of the left plot of Fig. 1 which zooms into the singlet mass region between 45 and 70 GeV. Not surprisingly, the bounds from invisible Higgs decays and from the DM relic density work in the opposite way, with the former excluding larger values of the coupling λ_{hs} in the kinematically allowed phase space.

4. Direct detection experiments provide upper bounds on the DM-nucleon cross-sections. Here DM interacts elastically with nuclei through the Higgs boson exchange. The resulting nuclear recoil is then interpreted in terms of the DM mass and DM-nucleon cross section. The spin-independent cross-section is

$$\sigma^{\text{SI}} = \frac{\lambda_{hs}^2}{4\pi m_h^4} \frac{m_N^4 f_N^2}{(m_s + m_N)^2}, \quad (3.2)$$

where f_N parameterises the Higgs-nucleon coupling. These bounds are translated into exclusion contours on the (λ_{hs}, m_s) plane, which essentially are lower bounds on the scalar DM mass m_s .

Constraints from the current XENON100 direct detection experiment [53] are shown in dark blue on the left of second plot in Fig. 1 together with the future reach of XENON upgrades in lighter shades of blue, as computed in Ref. [52].

To summarise, the singlet scalar DM model is perfectly viable at present on a large portion of its parameter space for $m_s > \frac{1}{2} m_h$. At lower values of m_h in the region between 55 GeV and $\frac{1}{2} m_h$, there is only a small triangle (in white) left unexplored, as can be seen from the plot on the left. More generally, the XENON100 upgrade and the XENON1T direct detection experiment should be able to probe the entire parameter space of the singlet DM model [49, 52].

4 Conclusions

In this paper we considered a classically scale-invariant SM theory extended with the Coleman-Weinberg sector and an additional real singlet field, both coupled to the SM via the Higgs portal interactions. The singlet classically massless field $s(x)$ was shown to give rise to the inflaton and to the dark matter candidate.

We have presented for the first time an implementation of the slow-roll inflation mechanism in a BSM theory with classical scale invariance. Our model is a single-real-field inflationary model which does not suffer from a breakdown of unitarity at scales below or comparable to the scale of inflation. The model maintains its classical scale invariance and is self-consistent all the way up to the Planck-scale UV cutoff. The SM Higgs potential is stabilised by the Higgs portal interactions with the Coleman-Weinberg scalar. The Higgs mass and the scale of the electroweak symmetry breaking are unaffected by the inflaton and its interactions. Furthermore, the singlet field responsible for inflation also gives a viable scalar dark matter candidate in our model.

When discussing cosmological inflation in Section 2, our classically scale invariant quantum field theory was coupled to a gravitational background with an explicit dependence on the reduced Planck scale M . How does this fit with classical scale invariance of the non-gravitational theory?

The Planck scale M , as it appears in our effective Lagrangians (2.5) and (2.24) does not cause problems to the effective field theory. At the scale M and above, the theory coupled to gravity of course becomes non-renormalisable, but M is above the UV cutoff scale of our theory and gravity is treated as an external background. One can continue using dimensional regularisation in the curved background for all QFT processes. The interesting for us phenomena, such as the cosmological inflation, happen at large values of the singlet field set by the intermediate scale $M/\sqrt{\xi_s} \ll M$, at this scale the theory is robust, the scalar inflaton $\sigma(x)$ is exponentially decoupled from the SM degrees of freedom (in the relevant for inflation regime of large field values) and there are no heavy degrees of freedom coupled to the SM sector present in the theory.

Still it is desirable to not have an explicit scale M present in the theory which is supposed to become classically scale-less when gravity decouples. One can do this and embed the classical scale-invariance of the theory in the IR into the scale invariance as a fundamental symmetry of nature - including gravity - with the Planck scale being set by the vev of the dilaton. We stress first that this is not a necessary requirement, the classical scale invariance of the IR theory can in principle exist even if the UV-complete theory is not scale-invariant. Secondly, as we will

explain below, the classical scale invariance of the effective QFT does not automatically follow from the (broken) scale invariance of the gravitational theory. It has to be seen as an additional constraint.

To proceed, we impose the scale-invariance on the full theory including gravity and assume that this is a *local* symmetry which is non-linearly realised – i.e. spontaneously broken by the dilaton field with the vev $\langle\varphi_{\text{dil}}\rangle = M$. Very recently there has been a resurgence of interest in such an approach to gravity and cosmology, see Refs. [54, 55] and also [56–58] and references to earlier work therein for other related approaches which assume or utilise full quantum scale invariance of the underlying theory.

Thus our original Lagrangian (2.5) can be written in a manifestly locally scale-invariant (Weyl-invariant) form if we retrofit the Planck scale in terms of the dilaton vev, $\langle\varphi_{\text{dil}}\rangle = M$ and introduce the kinetic term for the dilaton (see section III of [55] for a review of a general construction),

$$\mathcal{L}_J = \sqrt{-g_J} \left(-\frac{\varphi_{\text{dil}}^2 + \xi_s s^2}{2} R - \frac{1}{2} g_J^{\mu\nu} \partial_\mu \varphi_{\text{dil}} \partial_\nu \varphi_{\text{dil}} + \frac{1}{2} g_J^{\mu\nu} \partial_\mu s \partial_\nu s + \dots + V \right), \quad (4.1)$$

where the dots indicate omitted kinetic terms of other fields, and V is the scalar potential which must scale as the 4th power of the scalars, which our potential does (see the discussion above (2.22)). The dilaton can be expressed as,

$$\varphi_{\text{dil}}(x) = M \times \exp\left(\frac{\chi(x)}{M}\right), \quad (4.2)$$

where χ transforms linearly under local scale transformations. Note that the dilaton kinetic term in (4.1) has the ‘wrong’ sign, but this is not a problem [54, 55], since the dilaton is not a physical degree of freedom - it can be gauged away. (This also means that there are no additional massless or light degrees of freedom arising from the dilaton, thus no complications for the SM.) In the unitary gauge we have $\varphi_{\text{dil}} = M$.

The dilaton spontaneously breaks local scale invariance of the gravitational theory at the scale M , and gives the mass gap $\sim M$ to heavy gravitational states. In order to maintain the classical scale invariance of the SM theory, the graviton should not couple to any of the SM states in the potential V in (4.1). This should be seen as an additional requirement (rather than an automatic consequence) to the local scale invariance of the microscopic theory.

When Weyl transforming to the Einstein frame, as in section 2, we now promote M to the dilaton field and maintain the manifest local scale invariance, or gauge fix to the unitary gauge and keep M as in section 2. Quantum effects are computed in dimensional regularisation in the manifestly scale-invariant way.

In the flat background and without scale invariance, the dimensionless coupling constants acquire the dependence on the RG scale μ in $D = 4 + \epsilon$ dimensions, $\lambda \rightarrow \lambda \mu^\epsilon$. In the gravitational background, one treats the metric tensor $g_{\mu\nu}$ as a $(4 + \epsilon) \times (4 + \epsilon)$ matrix, and in the Einstein frame, it introduces the dependence on the dilaton. The result is that in our theory with spontaneously broken (or non-linearly realised scale-invariance), the every appearance of the RG scale μ in the effective potential is now substituted by, see Ref. [59],

$$\mu \rightarrow \mu \times \exp\left(\frac{\chi(x)}{M}\right), \quad (4.3)$$

which now transforms under scale-transformations. This gives a manifestly scale-invariant formulation in quantum theory regularised in dimensional regularisation.

The message we take from the above discussion is that the classical scale invariance of the effective SM theory can be compatible with, and form a natural part of the full local scale invariance of the full theory. This, however, is not automatic and requires an additional constraint that the dilaton vev $\langle\varphi_{\text{dil}}\rangle = M$ does not appear in the SM potential, though φ_{dil}/M can appear in the logarithms. After the gravity is decoupled, together with the heavy states whose masses are set by the dilaton mss gap, the $\text{SM} \times \text{CW} \times s$ theory in the IR is classically scale invariant. This classical scale invariance has its own pseudo-dilaton – the CW field ϕ , which is a physical degree of freedom and has nothing to do with the heavy dilaton of the gravitational theory.

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Appendix: Mass generation in a classically massless theory

The simplest theory where the CW mechanism [3] is realised is the massless scalar QED with the classical potential

$$V_{\text{cl}} = \frac{\lambda_\phi}{4!} |\phi|^4. \quad (\text{A.1})$$

The field ϕ is the Coleman-Weinberg complex scalar, it is charged under the $U(1)_{CW}$ gauge symmetry with the usual covariant derivative coupling, $D_\mu \phi = \partial_\mu \phi + ig_{CW} A_\mu \phi$.

Normalisation conditions for ϕ are chosen to be the same as in [3], $\phi = \phi_1 + i\phi_2$, with canonical kinetic terms for its real and imaginary part components $\frac{1}{2}(\partial_\mu \phi_1 \partial^\mu \phi_1 + \partial_\mu \phi_2 \partial^\mu \phi_2)$. This is related to the canonically normalised complex scalar $\Phi = (\phi_1 + i\phi_2)/\sqrt{2}$ via a simple rescaling, $\Phi = \phi/\sqrt{2}$.

The 1-loop corrected scalar potential is given by the usual Coleman-Weinberg expression [3],

$$V(\phi) = V_{\text{cl}} + \Delta V_{1\text{-loop}} = \frac{\lambda_\phi}{4!} |\phi|^4 + \left(\frac{5\lambda_\phi^2}{1152\pi^2} + \frac{3g_{CW}^4}{64\pi^2} \right) |\phi|^4 \left[\log \left(\frac{|\phi|^2}{\mu^2} \right) - \frac{25}{6} \right], \quad (\text{A.2})$$

where μ is the RG scale. The effective potential above is computed in the massless theory, in the UV subtraction scheme is chosen to set the renormalised mass at the origin of the field space to zero,

$$m^2(\phi = 0; \mu) = V''(\phi) \Big|_{\phi=0} = 0. \quad (\text{A.3})$$

We use dimensional regularisation which preserves classical scale-invariance of the theory. In dimensional regularisation the equation (A.3) is satisfied automatically and is independent of the RG scale μ .

The minimum of the Coleman-Weinberg effective potential occurs at $\phi = \langle \phi \rangle$ where the first derivative of (A.2) vanishes. Validity of weakly-coupled perturbative approach requires that $\lambda_\phi \sim \alpha_{CW}^2$ in the minimum (rather than the more usual relation $\lambda_\phi \sim \alpha_{CW}^1$) where we defined $\alpha_{CW} = g_{CW}^2/(4\pi)$. This is because the minimum of the effective potential (A.2) is determined [3] by ballancing the tree-level contribution $\sim \lambda_\phi$ against the 1-loop term $\sim \alpha_{CW}^2$. Thus one can neglect in the $\lambda_\phi^2 \sim \alpha_{CW}^4$ 1-loop contribution on the right hand side of (A.2) which is subleading relative to the α_{CW}^2 term. Choosing the value of the RG scale $\mu = \langle \phi \rangle$, the minimum of the 1-loop corrected effective potential occurs at [3]

$$\partial_\phi V = \left(\frac{\lambda_\phi}{6} - 11 \alpha_{CW}^2 \right) \langle \phi \rangle^3 = 0 \quad (\text{A.4})$$

and the vev $\langle \phi \rangle$ is determined by the condition on the couplings renormalised at the scale of the vev,

$$\lambda_\phi(\langle \phi \rangle) = 66 \alpha_{CW}^2(\langle \phi \rangle) \equiv \frac{33}{8\pi^2} g_{CW}^4(\langle \phi \rangle). \quad (\text{A.5})$$

An RG point can always be found (in the right theory) where this relation between the two couplings holds. In the CW settings the beta function for λ_ϕ is positive,

$$\frac{d\lambda_\phi}{dt} = \beta_{\lambda_\phi} = \frac{1}{4\pi^2} \left(9g_{CW}^4 - 3g_{CW}^2 \lambda_\phi + \frac{5}{6} \lambda_\phi^2 \right) > 0, \quad t := \log(\mu/M_{UV}). \quad (\text{A.6})$$

Thus starting in the UV at a positive initial value of $\tilde{\lambda}_\phi$ and following the RG running to the IR, one reaches the critical value of the RG scale where λ_ϕ crosses zero. Before this happens, at $\lambda_\phi \ll 1$ and positive, the λ_ϕ trajectory necessarily crosses the α_{CW}^2 trajectory. At the RG point where the constraint (A.5) is satisfied, the vacuum is reached, the condensate $\langle\phi\rangle$ develops and the coupling freezes.

Since the couplings run only logarithmically, the vev fixed by the condition (A.5) depends exponentially on the coupling constants. In fact, in weakly coupled perturbation theory the vev is naturally generated at the scale which is exponentially smaller than the UV cutoff M_{UV} . By solving the leading-order RG-running equation for the coupling g_{CW} ,

$$\frac{dg_{CW}}{dt} = \frac{g_{CW}^3}{48\pi^2}, \quad (\text{A.7})$$

we find

$$\langle\phi\rangle = M_{UV} \times \exp\left[-6\pi\left(\frac{1}{\alpha_{CW}(\langle\phi\rangle)} - \frac{1}{\alpha_{CW}(M_{UV})}\right)\right] \ll M_{UV}. \quad (\text{A.8})$$

The CW gauge group is spontaneously broken by $\langle\phi\rangle$ and the gauge boson acquires the mass, $m_{Z'} = g_{CW}|\langle\phi\rangle|$. The mass of the CW scalar is follows from expanding (A.2) to the second order in $\varphi = \phi - \langle\phi\rangle$,

$$m_\varphi^2 = \frac{3g_{CW}^4}{8\pi^2} |\langle\phi\rangle|^2. \quad (\text{A.9})$$

It is parametrically (1-loop) suppressed relative to the corresponding vector boson mass, $m_\varphi^2 = \frac{3}{2}\alpha_{CW}m_{Z'}^2 \ll m_{Z'}^2$.

To adapt the expressions above to a more general CW theory, e.g. the B-L theory where the CW field ϕ has charge $Q_\phi = 2$ under the $U(1)_{CW}$ gauge group, and other matter fields are coupled to the latter with their $B - L$ charges, or other models, we can re-write (A.2), (A.5) and (A.9) as

$$V(\phi) = \frac{1}{4!} \left(\lambda_\phi(\mu) + \beta_{\lambda_\phi} \left[\log\left(\frac{\phi}{\mu}\right) - \frac{25}{12} \right] \right), \quad (\text{A.10})$$

$$\lambda_\phi(\langle\phi\rangle) = \frac{11}{6} \beta_{\lambda_\phi}(\langle\phi\rangle), \quad (\text{A.11})$$

$$\frac{m_\varphi^2}{m_{Z'}^2} = \frac{3}{8\pi^2} (Q_\phi g_{CW})^2 |\langle\phi\rangle|^2. \quad (\text{A.12})$$

Note that Eq. (A.10) is manifestly RG-invariant but it also incorporates the finite contribution $C = \frac{25}{12}$, as in (A.2). The last two equations take these finite contributions into account, e.g. (A.11) originates from $\lambda_\phi = -\frac{\beta_{\lambda_\phi}}{4} + C\beta_{\lambda_\phi} = \frac{11}{6}\beta_{\lambda_\phi}$ and both, the log-dependent and the finite effects are important.

Generalising now this simple model to the $SM \times U(1)_{CW}$ theory, the classical scalar potential in conventions of [7] reads,

$$V_{\text{cl}}(H, \phi) = \frac{\lambda_H}{2} (H^\dagger H)^2 + \frac{\lambda_\phi}{4!} |\phi|^4 - \lambda_P (H^\dagger H) |\phi|^2. \quad (\text{A.13})$$

Here H is the usual SM Higgs complex doublet, which in the unitary gauge takes the form $H^T(x) = \frac{1}{\sqrt{2}}(0, v + h(x))$. The field CW field ϕ is normalised as before, and the Higgs is canonically normalised, $|D_\mu H|^2$. All scalars have vanishing renormalised masses,

$$\left. \frac{\partial^2 V(H, \phi)}{\partial H^\dagger \partial H} \right|_{H=\phi=0} = 0, \quad \left. \frac{\partial^2 V(H, \phi)}{\partial \phi^\dagger \partial \phi} \right|_{H=\phi=0} = 0. \quad (\text{A.14})$$

Once these conditions are enforced at one scale, they hold [7] at all RG scales in dimensional regularisation maintaining classical scale invariance of the model.

This potential can be re-written as,

$$V_{\text{cl}}(H, \phi) = \frac{\lambda_H}{2} \left(|H|^2 - \frac{\lambda_P}{\lambda_H} |\phi|^2 \right)^2 + \frac{\tilde{\lambda}_\phi}{4!} |\phi|^4, \quad \text{where} \quad \tilde{\lambda}_\phi = \lambda_\phi - 12\lambda_P^2/\lambda_H. \quad (\text{A.15})$$

For small positive portal coupling λ_P , the radiative generation of the CW vev proceeds exactly as before with the substitution $\lambda_\phi \rightarrow \tilde{\lambda}_\phi$.

The value of $\langle \phi \rangle$ is given by (A.8) and it induces the Higgs vev required to minimise the potential (2.1) such that $|\langle H \rangle| := \frac{v}{\sqrt{2}} = \sqrt{\frac{\lambda_P}{\lambda_H}} |\langle \phi \rangle|$ and triggers the electroweak symmetry breaking. The Higgs self-coupling and the portal coupling responsible for the emergence of EWSB are then determined from $\langle \phi \rangle$ through [7],

$$\frac{1}{\lambda_P} \frac{1}{2} (125 \text{ GeV})^2 = \frac{\lambda_H}{\lambda_P} \frac{(246 \text{ GeV})^2}{2} = |\langle \phi \rangle|^2, \quad (\text{A.16})$$

where we have set $m_h = 125 \text{ GeV}$ and $v = 246 \text{ GeV}$.

Of course one can argue that the theory under consideration (or for this matter any theory which does not include quantum gravity or in addition has Landau poles in the non-asymptotically free couplings, e.g. the hyper charge, even if these Landau poles occur at above the Planck scale) must have a UV cutoff above which it breaks down. In this case the unknown more microscopic theory above the cutoff, could (or should(?)) contain heavy $\sim M_{\text{Pl}}$ degrees of freedom which after being integrated out produce large threshold contributions to the Higgs mass. The quadratic cutoff contributions to the Higgs mass in non-dimensional regularisation schemes is an effective realisation of these physical effects and implies that the masslessness requirement (A.15) is the fine-tuning. The vanishing masses in (A.14) in this reading correspond to an exact cancellation of all the $\sim M_{UV}^2$ terms between the bare masses terms and the counterterms.

One can also take the view that the unknown microscopic theory might after all not generate such large corrections to the Higgs in order to preserve the classical scale invariance of its sub-Planckian effective theory.

The more practical outcome of this discussion is that from the perspective of our classically scale invariant effective theory which should be valid below and up to the Planck scale, there are no cancellations between large scales in dimensional regularisation of this theory and no associated problems or uncertainties. Technically the SM \times CW theory is natural and there is no fine-tuning.

The computational approach we take is to first define the maximal scale above which the theory should not be used, effectively the Planck scale, and in this theory regulate all integrals

using the analytic continuation prescribed by the dimensional regularisation. With that, the maximal UV scale becomes irrelevant, if one wishes it can be safely taken to infinity, the integrals are convergent in the UV. No powers of the cutoff scale appear in the integrals, the poles in epsilon are subtracted in the usual way in the $\overline{\text{MS}}$ or $\overline{\text{MS}}$ scheme and only the logarithms of the RG scale remain.

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