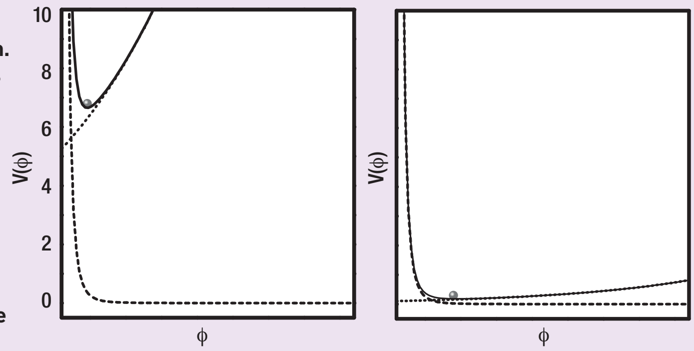
Testing gravity on cosmological scales

Baojiu Li reviews theories of modified gravity, outlining the features needed to satisfy both existing rigorous tests at solar system scales and cosmological theory.

Understanding the nature of gravity is one of the frontiers of theoretical physics. Although Einstein's theory of general relativity has been tested to high accuracy in various ways, these tests have so far been restricted to rather small scales, such as within the solar system. To apply this theory on cosmological scales is a remarkable extrapolation of what is known, yet this is usually not questioned: after all, its successful predictions about the Hubble expansion, cosmic microwave background (CMB) radiation, synthesis of primordial nuclei and formation of the large-scale cosmic structures are simply overwhelming. Even though an as yet unknown dark matter ingredient is needed in most of these predictions, it is commonly believed to be a new particle species.

The discovery of the accelerated expansion of the universe (Riess *et al.* 1998, Perlmutter *et al.* 1999) more than a decade ago has raised some serious questions about our understanding of fundamental physics. Is this due to some mysterious dark energy component which, together with dark matter, comprises more than 95% of the total energy content in the universe, or because general relativity breaks down and must be modified on cosmological scales? The past few years have seen a lot of proposals to modify gravity (Clifton *et al.* 2011), and to distinguish dark energy from modified gravity has become one of the primary goals of large observational projects such as ESA's recently approved EUCLID mission (launch due 2019, http://sci.esa.int/euclid). The aim of this article is to review briefly some recent progress on the theoretical side of this field.



1: Illustration of the chameleon mechanism. The sphere represents the scalaron field and the solid curve is the potential well it moves in, which is the sum of two parts: the self-interaction part (dashed curve) and the matter-interaction part (dotted curve). The amplitude (or height) of the matter-interaction part is proportional to the local matter density. In high-density regions (left) the potential well is very steep and its minimum pushed to very small ϕ; in low-density regions (right) the potential well becomes shallower and its minimum driven to larger ϕ.

Screening in modified gravity

The fundamental degrees of freedom (dofs) in general relativity are the metric field, which is a symmetric rank-2 tensor. In general, modified gravity would involve either extra dofs or completely new ones. These new dofs mediate new forces between matter particles, thereby modifying the standard force law. In what follows, we shall consider only that the extra dofs be scalar (i.e. spin-1), as in the majority of modified gravity theories. Following the convention in the literature, we call a general scalar dof the "scalaron".

A new force is usually not the reason why, in modified gravity theories, the expansion of the universe is accelerated, but it is a by-product and source of problems. Given that general relativity has been tested extensively in the solar system, any such new force would be a bad feature indicating that the theory is automatically ruled out.

An exception would arise if the new force is suppressed by some mechanism and is therefore undetectable in those places where gravity tests have been performed. To see how this can be achieved, let us remember that the force mediated by a scalaron is of Yukawa type, namely its potential produced by a point particle is proportional to $-\beta G e^{-mr}/r$, where G is Newton's gravitational constant, m is the mass of the scalaron and β a dimensionless parameter specifying the strength of the new force. This is in contrast to the Newtonian potential, which is proportional to $-G/r$. Now, to suppress the new force, one could have m and/or β dependent on the scalaron (denoted by ϕ) itself, and use the dynamics of ϕ to drive $m(\phi)$ to be very large and/or $\beta(\phi)$ very close to zero.

A simple way to understand the dynamics of ϕ is to consider it as a particle moving in a one-dimensional potential well. Figure 1 shows such a situation. The potential well (solid curve) of the particle comprises two parts: the first part (dashed curve) comes from the self-interaction of ϕ and is fixed, while the second part (dotted curve) comes from the interaction of ϕ with matter and depends on the environmental matter density. The total potential well has a minimum where ϕ oscillates, and the mass $m(\phi)$ is equal to the frequency of the oscillation. In high matter density regions (left panel) such as our solar system, the potential well is very steep near the minimum and $m(\phi)$ is very heavy, so that the new force is suppressed; while in low-density regions (right panel) the potential well becomes shallow and $m(\phi)$ light, so that the new force can be as strong as the standard gravity. This mechanism to suppress the new force is known as the chameleon mechanism (Khoury and Weltman 2004) for its strong environmental dependence of $m(\phi)$.

Another example is the symmetron mechanism (Hinterbichler and Khoury 2011), as illustrated in figure 2. Here the self-interaction part (dashed curve) of the potential well takes the Mexican-hat shape, while the matter-interaction part (dotted curve) is a parabola, the steepness of which is controlled by the environmental matter density. If the density is high (left panel), the total potential well is dominated by the matter-interaction part and has the same minimum, where $\beta(\phi)$ and therefore the new force vanish; on the other hand, if the density is low (right panel), the total potential well follows the self-interaction part and the particle dynamically moves to one of the two minima, where $\beta(\phi)$ does not vanish.

Other modified gravity theories, such as the dilaton (Brax *et al.* 2010) and Galileon models, rely on other mechanisms to suppress the new force in high-density regions. In general, modified gravity theories have to exploit such mechanisms of screening the new force to pass the rigorous solar system tests.

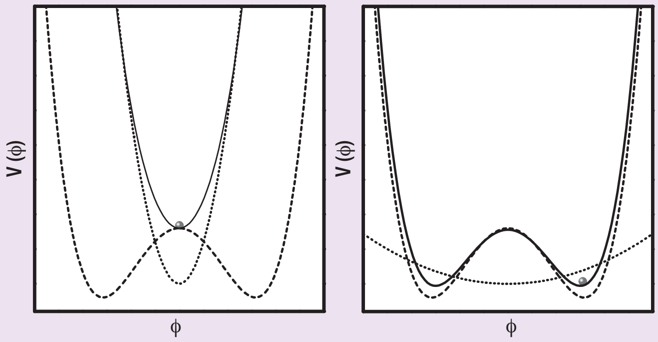
A unified description of screening in modified gravity

In general, the details of the screening depend on the shapes of both the self-interaction and matter-interaction parts of the potential well. By varying the functional forms of these two parts one can, in principle, obtain an infinite number of models. Obviously, this is not the most efficient way to study modified gravity theories: what we are most interested in are the generic imprints of the modified gravity models (which pass the solar system tests by virtue of some screening mechanism) on the evolution of the universe, and for this ideally we would like to describe a large number of models using a few parameters, by varying which we can tune the modified gravity effects.

We (Brax, Davis and Li 2011, Brax *et al.* 2012a) notice that such a generic description can be achieved by parameterizing m and β as functions of the cosmic time, or the scale factor a. This shows that, in the background cosmology, by parameterizing $m(a)$ and $\beta(a)$ one can find ϕ as a function of a as well; as a result, m and β can be expressed in terms of ϕ, and the full theory is reconstructed.

The method shows that, by choosing m and β as simple power-law functions of a, one reproduces almost all the existing modified gravity models with chameleon, symmetron and dilaton screening mechanisms. For example, most models of $f(R)$ gravity (see below) can be reconstructed by letting $\beta = 1/\sqrt{6}$ and $m/m_0 = a^{-r}$, and therefore described by two constant parameters m_0 and r.

We have used such a unified parameterization (Brax *et al.* 2012b, 2012c) to run systematic N-body simulations of modified gravity theories (discussed below) and in principle it allows for general observational constraints on the low-dimensional parameter space.



2: Illustration of the symmetron mechanism. Notations are the same as in figure 1. Here the self-interaction part of the potential well has the Mexican-hat shape, while the matter-interaction part is a parabola. In high-density regions (left) the matter-interaction part dominates so that the potential well has minimum at ϕ=0; in low-density regions (right) the self-interaction part dominates and the potential well develops two non-zero minima.

Cosmological laboratories for gravity

Cosmologists are interested in the universe on very large scales, beyond that of typical galaxies. These scales can roughly be divided into three regimes, from the largest to the smallest: the background universe, which is assumed to be homogeneous; the linear perturbation regime, in which the fluctuations of the density field are assumed to be very small so that linear perturbation theory applies; nonlinear regime (scales of typical galaxy clusters), where the density field fluctuates strongly and the equations which describe the large-scale structure evolution become nonlinear.

Despite the complexity in modified gravity theories, their predictions on the largest scales are often very simple. For example, the cosmic expansion rate in a modified gravity universe can mimic and be practically indistinguishable from that in a standard model universe with a cosmological constant (ΛCDM). The expansion rate is therefore not a powerful tool with which to test gravity.

The intermediate length scales that are well described by linear perturbation theory are not the ideal place to look for signals of modified gravity either. This is because the new force, of Yukawa nature, decays exponentially beyond a finite range m^{-1}, which is often smaller than those length scales and therefore its effects cannot be felt there. Furthermore, as discussed below, linear perturbation theory is usually a bad approximation in modified gravity and its predictions are usually quantitatively inaccurate.

The nonlinear regime corresponds to scales which are often of the same order as the range of the new force predicted by modified gravity theories. So it is the best place to make cosmological tests of gravity and has become the main focus of research over the past few years. However, this regime is also the most difficult to study quantitatively: depending on the local and environmental matter densities, the new force may or may not be screened, and to determine this would necessarily require people to solve accurately the nonlinear partial differential equation that governs the dynamics of the scalaron.

Note that this nonlinearity is an inherent property of the modified gravity theories and at the very heart of the various screening mechanisms. This is in addition to the usual nonlinearity in the governing equations of structure formation, and it means that linearizing the equation of ϕ will cause (perhaps the most important) part of the information to be lost. This is why linear perturbation theory often does not work well for modified gravity theories.

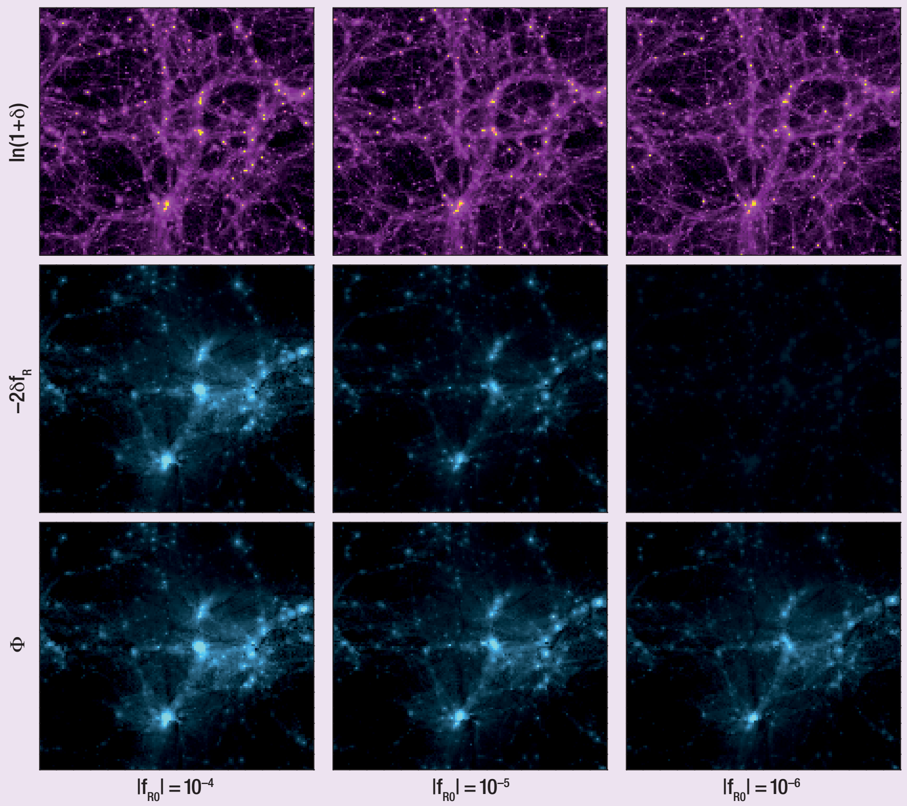
Numerical simulations in modified gravity

A common tool to study the large-scale structure evolution of the universe is numerical simulation. Often, the density field is discretized into a number of "particles", the masses of which can be millions to billions times the solar mass depending on the resolution of the simulation. The computer distributes these particles and moves them in discrete time steps according to their mutual gravitational forces. The particles finally clump together and form objects such as clusters, filaments and walls, much like structures in the observed universe. In the simplest set-up, only gravity is included and other known interactions between luminous matter are neglected; this is usually referred to as N-body simulation.

Generalizing the N-body simulations to modified gravity theories is straightforward in theory; one needs only to solve for the scalaron and therefore the new force mediated by it. In practice, this means that one has to solve numerically the nonlinear differential equation for ϕ on a set of meshes that cover the whole simulation box. There are two main challenges here: first, the equation is highly nonlinear and it needs more computer operations to solve it; second, ϕ, and therefore the new force, becomes very small in high-density regions, where one has to refine the meshes to achieve higher resolution.

Oyaizu *et al.* (2008a, 2008b) and Schmidt *et al.* (2009a) pioneered this approach by running a series of N-body simulations for the $f(R)$ gravity, which is one of the most well-studied modified gravity theories. In their simulations, the nonlinear equation of the scalaron was solved using the multigrid relaxation method on a regular mesh (i.e. no refinements). Later, we modified the publicly available N-body code MLAPM (Knebe *et al.* 2001) in a number of works covering the different modified gravity models: chameleon (Li and Zhao 2009, 2010), $f(R)$ gravity (Zhao *et al.* 2011a), dilaton (Brax *et al.* 2011) and symmetron (Davis *et al.* 2012), in which we incorporated self-adaptive refinement of the meshes to allow high resolutions.

To make the modified gravity simulations state-of-the-art, we also modified the publicly available N-body and hydrodynamical simulation code RAMSES (Teyssier 2002) to include



3 (Top panels): The density fields from the different $f(R)$ gravity simulations: bright/dark regions have high/low matter densities; δ is the density contrast. (Bottom panels): The Newtonian potential Φ from the same simulations – bright/dark regions have deep/shallow potentials. (Middle panels): The scalaron field from the same simulation. Here we have plotted $-2\delta f_R$, where δf_R is f_R minus its background value; note that bright regions have larger $|\delta f_R|$ and therefore smaller scalaron field f_R, as is expected from the chameleon effect. The three columns are for three different $f(R)$ models, the chameleon effect increasing from left to right.

modified gravity solutions. The resulting code, ECOSMOG (Li *et al.* 2012a, Brax *et al.* 2012b, 2012c), not only enables self-adaptive mesh refinements, but also is efficiently parallelized using MPI. It can, therefore, perform high-resolution, large-box and systematic simulations for modified gravity, which are required to cope with the recent progresses in the theoretical modelling and generic parameterization of the theories.

The $f(R)$ gravity

Let us take the $f(R)$ gravity as an example to show some numerical results. The $f(R)$ gravity is proposed as a straightforward generalization of general relativity, by replacing the Ricci scalar R in the Einstein–Hilbert action of gravity,

$$S_{\rm EH} = \int \sqrt{-g}\,\frac{R}{16\pi G}\,d^4x \qquad (1)$$

with an algebraic function $R + f(R)$ (Carroll *et al.* 2005). In this theory, the scalaron is the variable $f_R = df(R)/dR$, and its dynamical equation (in the weak-field limit) is given by

$$\nabla^2 f_R = -\tfrac{1}{3}a^2[R(f_R) - \bar{R} + 8\pi G(\rho_m - \bar{\rho}_m)] \qquad (2)$$

where ∇ denotes the spatial gradient, a is the cosmic scale factor and ρ_m the matter density; an overbar denotes the background value of a quantity, and $R(f_R)$ is obtained by reversing $f_R(R)$. Meanwhile, the Poisson equation is modified to

$$\nabla^2\Phi = \frac{16\pi G}{3}a^2(\rho_m - \bar{\rho}_m) + \tfrac{1}{6}a^2[R(f_R) - \bar{R}] \qquad (3)$$

where Φ is the Newtonian potential.

The chameleon mechanism in the $f(R)$ gravity (Navarro and van Acoleyen 2007, Li and Barrow 2007, Hu and Sawicki 2007, Brax *et al.* 2008) works as follows: if the matter density is high, f_R is driven to very small values (see figure 1) and from equation (2) we can see that $R = \rho_m$ and so equation (3) reduces to the Poisson equation in standard gravity

$$\nabla^2\Phi = 4\pi G a^2(\rho_m - \bar{\rho}_m) \qquad (4)$$

i.e. the new force is effectively screened. On the other hand, if the matter density is low, then the terms involving R in equations (2) and (3) are negligible. In this case, we find $f_R = -\Phi/2$ and equation (3) is then reduced to the Poisson equation in standard gravity but with the Newton constant rescaled by 4/3: the new force is then unsuppressed and can leave significant imprints on the large-scale structure.

Figure 3 is plotted to give an illustration of the behaviour of the new force in the $f(R)$ gravity. We have simulated three models with different background values of f_R today, with $|f_{R0}| = 10^{-4}$ (left column), 10^{-5} (middle column) and 10^{-6} (right column) respectively, using the same initial conditions. We then take the same slice of each simulation box and look at the projected density (top row), scalaron f_R (middle row) and Newtonian potential Φ (bottom row) fields. Note that instead of plotting the forces, which are 3-dimensional vectors, we have plotted their potentials, δf_R for the new force and Φ for the standard gravity. One can see that the relation $\delta f_R = -\Phi/2$ is almost perfectly satisfied for the case of $|f_{R0}| = 10^{-4}$, showing that the screening of the new force is inefficient here. In the case of $|f_{R0}| = 10^{-6}$, however, $|\delta f_R|$ is much smaller than $|\Phi|$, indicating an efficient screening of the new force.

Observational effects of modified gravity

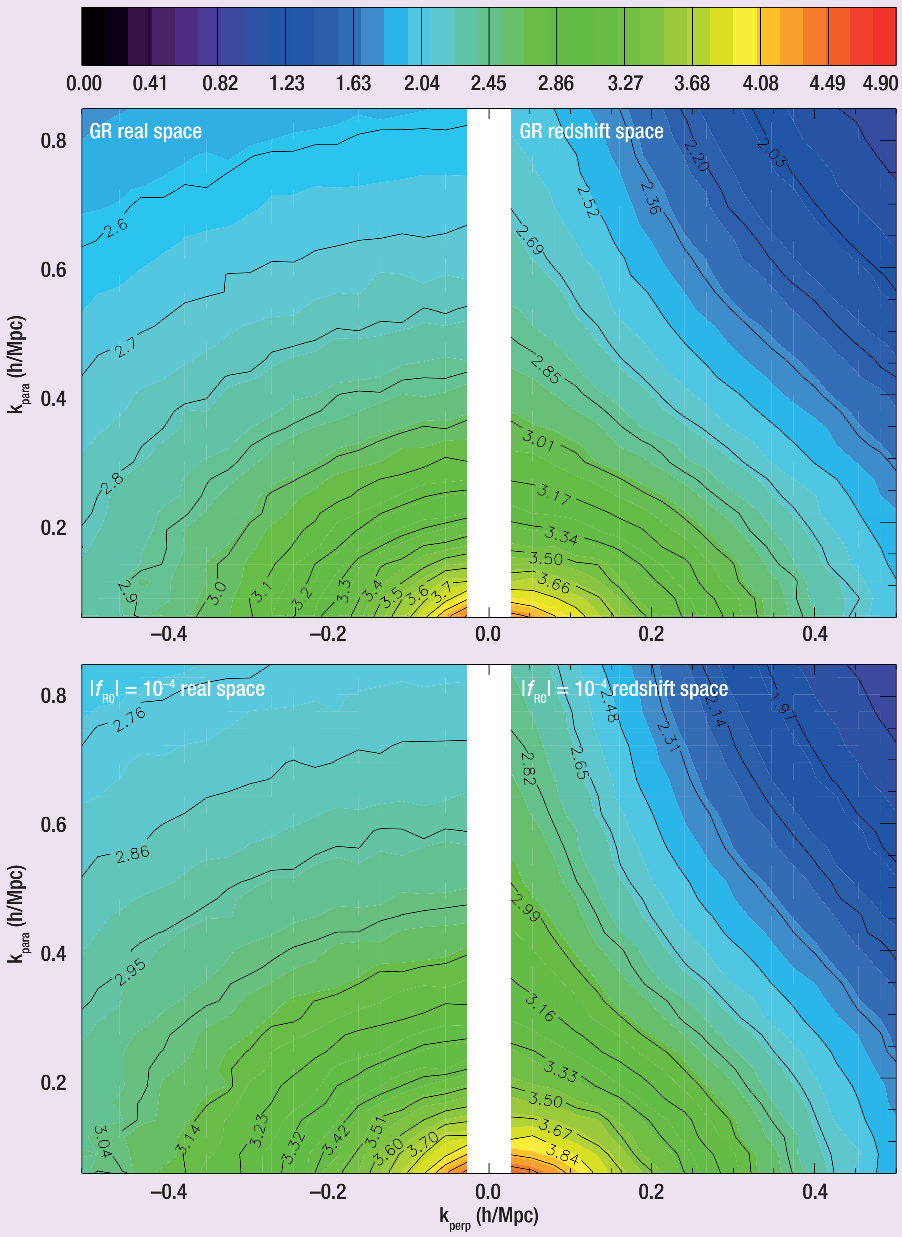
The new force in modified gravity theories implies that matter particles would experience an additional attractive force, and therefore cluster together more strongly than they do in general relativity. This means that more and larger clusters would form, the empty regions would become emptier, and particles would move faster overall. All these effects are, in principle, observable.

The top row of figure 3 shows the density fields in three different $f(R)$ models, where one can see that the effect of the new force is barely recognizable by eye. However, the statistical property of these density fields, quantified by the matter power spectrum $P_{\delta\delta}(k)$, where δ denotes the matter density contrast field and k is reversely proportional to the length scale that one looks at, can be significantly different in these models. We find that the relative difference of the $P_{\delta\delta}(k)$ in the $f(R)$ models shown in figure 3 from that in the standard ΛCDM model can be up to 50% in the case where the new force is not efficiently suppressed ($|f_{R0}| = 10^{-4}$). One can of course argue that the matter power spectrum measured here is different from the observed galaxy power spectrum because of the galaxy bias; nevertheless such a strong difference from the standard ΛCDM model is in principle detectable by other cosmological probes, such as weak gravitational lensing (Beynon *et al.* 2012).

Interestingly, the relative difference of $P_{\delta\delta}(k)$ from the ΛCDM result (as a function of k) can have not only different magnitudes, but also very different shapes for the different $f(R)$ models. As argued by Li *et al.* (2012b), the value of $|f_{R0}|$ roughly determines the time from when the new force becomes unscreened – the smaller it is, the longer this force is screened (figure 3). Essentially, at a given time the models with different $|f_{R0}|$ are just at different stages of a single evolution path. This picture is supported by the observation that, if one goes back in time for the model with $|f_{R0}| = 10^{-4}$, then the shape becomes similar firstly to that of $|f_{R0}| = 10^{-5}$ and then to that of $|f_{R0}| = 10^{-6}$ (both at the present time).

Perhaps more interesting is the velocity field in the modified gravity theories. Jennings *et al.* (2012) find that in these theories the power spectrum $P_{\theta\theta}(k)$ of the velocity divergence field $\theta = \nabla \cdot v$ shows stronger deviations from the ΛCDM prediction than $P_{\delta\delta}(k)$ does. Indeed, for the $f(R)$ model with $|f_{R0}| = 10^{-4}$ the difference of $P_{\theta\theta}(k)$ from ΛCDM can be up to 100% on small scales. The velocity field is an important quantity in cosmology, because the large-scale motion of matter creates an additional redshift effect (to that due to the Hubble expansion) which distorts what we observe in galaxy surveys. More specifically, on large scales, objects (galaxies) fall towards the overdensities and are blue or redshifted along the line-of-sight if they are respectively closer to or further from or closer to the observer. The result is that the distribution of these objects appears squashed along the line-of-sight and the clustering signal is enhanced; on small scales, the opposite happens and the distribution appears elongated along the line-of-sight, causing the "finger-of-God" effect (Kaiser 1987). This is known as the redshift space distortion. Figure 4 shows the redshift space distortion in the standard (top panel) and $f(R)$ gravity (bottom panel, the case of $|f_{R0}| = 10^{-4}$) theories, from which we can see that the latter predicts stronger clustering on large scales (small k) and more elongated finger-of-God on small scales (large k). In particular, inside virialized structures the particles move faster in the $f(R)$ gravity, causing more "damping" of the matter power spectrum on small scales in the redshift space.

The matter power spectrum and redshift space distortion, as well as the cluster number counts studied in Schmidt *et al.* (2009b) and Zhao *et al.* (2011a), mainly look at signals from regions of strong matter clumping, where the effect of new force is weakened by the screening. As a result, the modified gravity effects on these observables depend sensitively on the value of $|f_{R0}|$; for example, with $|f_{R0}| = 10^{-6}$ the matter power spectrum differs from that of ΛCDM by less than 10%, which is hardly detectable in reality. To look for signals of modified gravity, it is therefore sensible to look at empty regions of the universe, i.e. the voids, where the new force is unscreened.

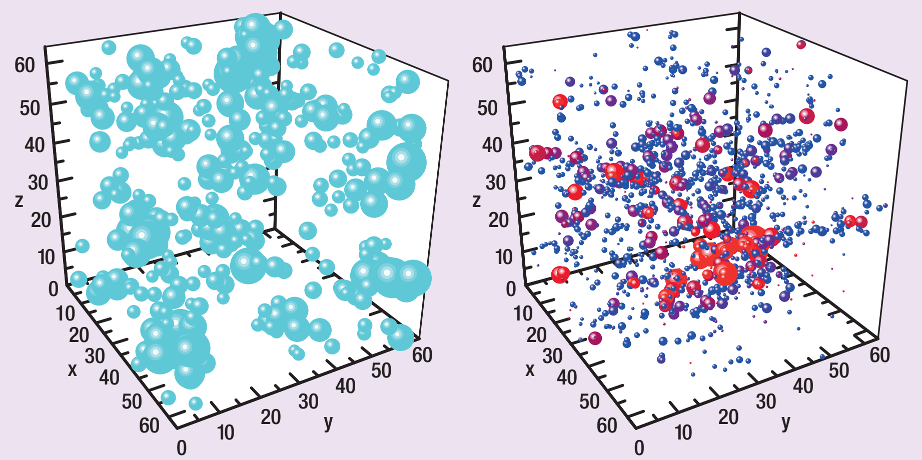


4: The matter power spectra in the real space (left panels) and redshift space (right panels) for the standard ΛCDM model (top panels) and the $f(R)$ model with $|f_{R0}| = 10^{-4}$ (bottom panels). The vertical axis is the length scale in the line-of-sight direction while the horizontal axis is that in the perpendicular direction. The finger-of-God can be seen clearly in the redshift space, and it is more elongated in the $f(R)$ gravity.

Examining voids

Li (2011) and Li *et al.* (2011) look at the void statistics (number density etc) in the chameleon and $f(R)$ gravity theories, and they find that this is strongly affected by the new force, which is unscreened in low-density regions even for the case of $|f_{R0}| = 10^{-6}$. In particular, the new force makes the voids grow faster and larger and end up emptier than they do in the standard ΛCDM model. Lee *et al.* (2012) find that galactic-sized dark matter haloes in voids spin significantly faster than they do in the standard ΛCDM model, because the new force makes halo merger events more frequent. As the spins of the dark matter haloes are strongly correlated to the properties of the low surface-brightness galaxies (LSBGs) hosted by these haloes (Jimemez *et al.* 1998), and the LSBGs occupy a significant fraction of the galaxy population, this suggests the use of the abundance of the LSBGs as a potential discriminator of modified gravity.

One of the important features of many modified gravity theories, such as the $f(R)$ gravity, is that the new force is only felt by massive particles but not by photons. This implies that the dynamical mass felt by massive particles can be 4/3 times the lensing mass felt by photons in $f(R)$ gravity, if the new force is unscreened (Schmidt 2010). As a result, different methods of measuring the mass of a cluster could give results that differ by up to 33%, depending on how efficient the cluster is screened. Zhao *et al.* (2011b) have recently studied this possibility using high-resolution simulations. Because the efficiency of screening depends on the environmental density, they adopt a definition of the "environment" for a given galaxy depending on the number and size of nearby galaxies. They find that there is a strong correlation between the relative difference of the dynamical and lensing masses and the environment. Figure 5 illustrates this point clearly: the spheres on the left are voids in a given simulation box, and those on the right are haloes; in both cases the size of a sphere represents that of the void/halo; the halo is in red if it is efficiently screened and blue if not screened. The dependence of screening on



5: Illustration of the screening of dark matter haloes in the $f(R)$ simulation. (Left): The voids (regions in which the density is less than 0.2 times the average matter density in the universe; represented by bubbles) identified in the simulation box. (Right): The dark matter haloes identified in the same simulation box. The bubble sizes are proportional to the volume of the voids (left) or mass of haloes (right); the colour for the haloes denotes the level of screening – from highly screened (red) to essentially unscreened (blue).

the environment is clear. The idea and results of Zhao *et al.* (2011b) can be easily implemented in observations: Cabre *et al.* (2012), for example, have recently used the *N*-body simulations to calibrate simple approximations to determine the level of screening in galaxy catalogues, and generated screening "maps" out of this.

Finally, it has been noticed that the screening of the new force in modified gravity theories not only affects cosmological observables, but can also have non-negligible effects on much smaller astrophysical scales. For example, Sakstein and collaborators (Davis *et al.* 2011, Jain *et al.* 2012) have shown that if a galaxy is unscreened, then the stellar evolution inside it could be significantly modified by the new force, because the hydrostatic equilibrium in stars is achieved by balancing gravity with the thermal pressure. This could in principle lead to strong constraints on modified gravity, but more detailed quantitative studies are still needed due to the challenges in simulating the galaxy evolution and determining the level of screening.

The future

This article briefly reviews the recent progress in some of most popular modified gravity theories in use in the cosmological community. This is a quickly growing field and this review does not aim for completeness. Indeed, we have not mentioned the DGP (Dvali *et al.* 2000) and Galileon (Nicolis *et al.* 2009, Deffayet *et al.* 2009) theories, which are also hot topics of recent studies. Instead, we want to give the readers an idea about how gravity might be modified on large scales and how this can be probed.

Besides the numerical simulations, (semi)analytic methods in studying the structure formation in modified gravity theories have also been developed recently. These methods are non-trivial generalizations of their counterparts in the standard ΛCDM model, and can be used to gain better understandings of and insights into the underlying gravitational physics. There has been some work in this direction recently (e.g. Koyama *et al.* 2009, Brax and Valageas 2012, Li and Efstathiou 2012, Li and Lam 2012, Lam and Li 2012), which attempt to predict the nonlinear matter power spectrum, bi-spectrum and the cluster abundance for the modified gravity theories semi-analytically. These works can be improved and their results should be compared to the simulations. It is hoped that we will be able to get some analytic fitting formulae from future studies along these lines, which can then be calibrated using the numerical simulations.

In the near future, higher resolution simulations will be made possible. These simulations are essential for us to understand the role of the new force in galaxy formation and evolution, as well as the screening of it inside the galaxies which, as we discussed above, is crucial in making correct predictions about the stellar evolution. The current largest simulation box used in modified gravity simulations is $1.5\,h^{-1}$ Gpc (Jennings *et al.* 2012, Li *et al.* 2012b), and this can be pushed to even larger sizes to match the volume to be covered by future galaxy surveys. To test gravity using these future surveys, it is then necessary to generate mock galaxy catalogues from the *N*-body simulations and possibly make more updated screening maps. Meanwhile, with the recently proposed generic parameterization of modified gravity and the ongoing systematic numerical simulations based on this, one would be able to constrain the parameter space for general scalaron modified gravity theories using current and future observational data.

In brief, the test of gravity is always an important topic in theoretical physics, and now cosmology provides us with a number of ways and opportunities to do this independently from the local tests. If the search for a deviation from the standard general relativity turns out to be positive, it will be a revolution in the development of fundamental physics; if it turns out to be negative, then general relativity is confirmed on very large scales and there leaves only the possibility of having some kind of dark energy. Either way, this would be a conceptually important area, and it is hoped that the efforts that have been made here, both theoretical and observational, will bring some exciting results in the next decade. ●

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