Risk Premium, Variance Premium and the Maturity Structure of Uncertainty

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Abstract

Expected returns vary when investors face time-varying investment opportunities. Structural long-run risk models (Bansal and Yaron, 2004) and no-arbitrage affine models (Duffie, Pan, and Singleton, 2000) emphasize sources of risk that are not observable to the econometrician. We show that the term structure of risk implicit in option prices can reveal these risk factors. Empirically, we find that the variance term structure reveals two important drivers of the bond premium, the equity premium and the variance premium, jointly. Similarly, two risk factors are sufficient to capture the predictive content of higher-order uncertainty – skewness and kurtosis – but these do not add to the predictive content of the variance factors. The predicted equity premium is counter-cyclical and our results are robust to the inclusion of other known predictors of returns. Overall, our results bode well for our ability to link risk-return trade-offs across different markets, and across horizons, within a parsimonious theoretical framework.

Keywords: Equity Premium, Variance Premium, Risk-neutral Variance, Term Structure of Variance.
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1 Introduction

Expected returns vary when investors face time-varying investment opportunities. For example, in Merton (1973), the premium, EP_t, between equilibrium expected returns on equity and the risk-free rate is proportional to the conditional variance, σ_t^2,

$$EP_t = \gamma \sigma_t^2, \tag{1}$$

where γ is the coefficient of risk aversion. Unfortunately, the *ex-ante* conditional equity premium and variance are not directly observable to the econometrician.[1] Hence, assessment of Equation (1) and inference about the structural parameter γ remains challenging. The more recent theoretical and empirical literature emphasizes that additional risk factors[2] are at play behind expected returns variations but, again, we do not observe expected returns, and typically, we do not observe the relevant measures of risk. Hence, the analysis and quantifications of risk-return trade-offs remain a central question for researchers and practitioners in finance.

This article contributes to the literature in proposing an alternative approach to measure risk factors priced in the option market. We build on the fact that option prices can be used to provide model-free, forward-looking measures of risks. Specifically, we show that the *term structure* of variance implicit in option prices can be used to reveal risk factors. Empirically, we find that (i) the variance term structure reveals two risk factors, (ii) the same two factors exhibit significant predictability for the bond, equity and variance premia, and the predictive content is robust to the inclusion of other known predictors, (iii) the predictive content is strong at short horizons, less than one year, where other popular predictors are relatively less effective and, finally, (iv) the variance risk factors imply counter-cyclical risk premium variations. Improved economic conditions, measured from labor market, capacity utilization or inventory indicators, are associated with counter-cyclical variations in the risk factors which in turn lead to counter-cyclical predicted returns. But, similar to the excess volatility puzzle (Shiller, 1983), less than one-third of the variance risk factor variations can be explained by macroeconomic indicators.

Our main contribution, beyond the predictability results, is to identify risk premium variations that are common across different markets (see (ii) above). In his presidential address to the American Finance Association, John Cochrane refers to a "multivariate

[1] This may explain why the empirical support for the theoretical prediction in Equation (1) is remarkably uneven. French, Schwert, and Stambaugh (1987), Campbell and Hentschel (1992), and Ghysels, Santa-Clara, and Valkanov (2004) find a positive relation between volatility and expected returns. Turner, Startz, and Nelson (1989), Glosten, Jagannathan, and Runkle (1993) and Nelson (1991) find a negative relation. Guo and Savickas (2006) find a positive relation between index volatility and individual stock returns. Ludvigson and Ng (2005) find a strong positive contemporaneous relation between the conditional mean and conditional volatility and a strong negative lag-volatility-in-mean effect.

[2] For instance, volatility risk and jump risk (Bollerslev, Tauchen, and Zhou, 2009; Drechsler and Yaron, 2011).

challenge" to returns predictability (Cochrane, 2011). Specifically, he notes the abundance of empirical results linking one potential risk factor at a time to one type of return at a time. He puts particular emphasis on the fact that there is a strong common element underlying these relationships, and asks "what is the factor structure of time-varying expected returns?" We provide one (partial) response to Cochrane's challenge in that we use the span of option prices to study the factor structure of time-varying expected returns. We find that two factors summarize the compensation for risk implicit in the term structure of option-implied variance and, in addition, the same risk factors are associated with common predictability of the equity, variance and bond risk premia. In other words, the compensation for risks across different financial markets exhibits significant commonalities. Our results also echo results in Cochrane and Piazzesi (2005, 2008) who show that a single combination of forward rates summarizes their predictive content for the risk premium of bonds with different maturities and, moreover, that this fact implies tight restrictions on the price of risk. The cross-market commonalities that we uncover imply similar restrictions and, together with our other results, bode well for the ability of future research to link risk-return trade-offs across different markets, and across horizons, within a parsimonious theoretical framework.

1.1 THE VARIANCE TERM STRUCTURE SPANS RISK FACTORS

Affine no-arbitrage models (Duffie, Pan, and Singleton, 2000) imply that the bond, equity and variance premia at different investment horizons are linear functions of the same risk factors.[3] In other words, risk premia should exhibit a factor structure. But, as in Merton's model noted above, the risk factors are unobservable to the econometrician.[4] Nonetheless, theory also predicts that the risk factors form a basis for the term structure of variance. In other words, a small number of linear combinations from the variance term structure, which can be measured from option prices, should span time variations of expected returns. Note that analog results hold in the class of affine (or approximately affine) long-run risk economies (Eraker, 2008; Bansal and Yaron, 2004).

1.2 THE VARIANCE TERM STRUCTURE PREDICTS THE BOND AND EQUITY PREMIUM

Empirically, we proceed in four steps. First, we measure variance using options on S&P 500 futures across a range of maturities based on the standard model-free measure from Bakshi and Madan (2000) and we show that the variance term structure exhibits a low-dimensional factor structure. Its first three principal components explain close to 97% of

[3] The variance premium is the difference between the expected variance under the historical measure and the risk-neutral measure, $\mathbb{Q}$, which is given by $E_t^{\mathbb{Q}}[\sigma_{t+1}^2] - E_t^{\mathbb{P}}[\sigma_{t+1}^2]$. This is analogous to the definition of the equity premium, $E_t^{\mathbb{P}}[r_{t+1}] - E_t^{\mathbb{Q}}[r_{t+1}]$.

[4] No-arbitrage jump-diffusion models rely on unobservable factors that drive variations in the stochastic drift, volatility and jump intensity of the underlying processes. Similarly, the defining structure of affine long-run risk equilibrium models combines recursive preference with small but persistent stochastic factors in the distribution of consumption growth. The latter are difficult to measure by construction.

the total variation. Each component has a systematic effect across maturities and can be interpreted as level, slope and curvature factors, respectively. Second, we estimate how many factors from the variance term structure are sufficient to summarize its predictive content for bond and equity returns, *jointly*. We use the robust procedure of Cook and Setodji (2003). This dimension-reduction procedure does not focus *a priori* on the leading principal components.[5] The test does not rely on any distributional assumption. It is also robust to departures from linearity. We find that two factors are sufficient to summarize the joint predictability of the bond and equity premia across maturities and across horizons.

In a third step, we estimate these factors via multivariate reduced-rank regressions (RRR) of returns on the variance term structure, where the rank of the coefficient matrix corresponds to the number of linear combinations that are sufficient to summarize the information content.[6] A model with a rank of two – where only two linear combinations from the variance term structure can predict returns – yields R^2s ranging from 5% to 7% for bond returns. The same variance factors predict equity returns with R^2s that range from 3% to 6% at horizons between 1 and 12 months. The predictability is stronger at intermediate horizons and peaks for 3-month returns. Overall, the results confirm the information content of the variance term structure. Moreover, the reduced-rank restriction is supported in the data: allowing for more than two risk factors yields little statistical or economic gain.

We provide several robustness checks. The variance term structure factors exhibit substantial correlation with a battery of other known predictors of equity returns. Notwithstanding this substantial overlap, none of these alternative predictors dent the significant predictability of the variance risk factor. We also document a web of correlations with a rich set of 124 economic and financial variables. This provides further reassurance that these factors capture actual economic risk. In particular, the correlations between the risk factors and economic indicators imply counter-cyclical variations in risk premia. Moreover, we show that the variance risk factors are correlated with the level, slope and curvature of the variance term structure, but that they mix information from other principal components. Cochrane and Piazzesi (2005) obtain a similar result in the bond markets.

In the fourth and final step, we show formally that the variance term structure is also linked to the variance premium. The extant literature sees the variance premium as an important predictor of the equity premium (e.g., Bollerslev, Tauchen, and Zhou 2009). But we do not observe the variance premium directly and one must obtain volatility forecasts under the historical measure to construct a variance premium proxy.[7] We note that both

[5]Cochrane and Piazzesi (2005) provide an example in the context of bond returns where a small principal component of forward rates, which is typically ignored to explain the variations in forward rates themselves, plays an important role in predicting bond excess returns.

[6]Estimation and inference in RRR models is available in closed form. See Anderson (1951) and, more recently, Hansen (2008) as well as Reinsel and Velu (1998), for a textbook treatment.

[7]Estimation of the variance premium, $E_t^{\mathbb{Q}}[\sigma_{t+1}^2] - E_t[\sigma_{t+1}^2]$, often relies on option prices to obtain a model-

the variance term structure and the variance premium reflect a compensation for risk, at least in part, and that one should be informative about the other. We formally test the link between the variance premium and the variance term structure and ask whether the same two variance factors estimated to predict the bond and equity premia *only* can predict the excess variance. Results from excess variance regressions[8] on variance factors yield R^2s reaching up to 10% at the 6-month horizon with an inverted U-shape. Each variance factor plays an important role, but at different horizons. Adding the variance premium proxy from Bollerslev, Tauchen, and Zhou (2009) to the predictive regression yields little predictability improvement – its information content appears to be subsumed in the variance term structure.

1.3 THE FACTOR STRUCTURE EXTENDS TO SKEWNESS AND KURTOSIS

Theory predicts that expected returns should also be linked with variations in higher-order risks, such as asymmetry and tail thickness (Rubinstein 1973; Kraus and Litzenberger 1976). We show that affine no-arbitrage models imply that all the cumulants of multi-horizon returns, including the variance, are affine functions of the same risk factors.[9] Nonetheless, the variance term structure may fail to reveal these higher-order risk factors.[10] For instance, information from measures of higher-order risks implicit in option prices can add to the information from option-implied variance if part of their variations is weakly correlated with variance (relative to the measurement errors in option-implied variance, say). Therefore, we can use the term structure of higher-order risks to discern additional risk factors. Empirically, we construct model-free measures of asymmetry and tail thickness based on cumulants (labeled as skewness and kurtosis hereafter). We find that two factors are still sufficient to summarize the predictive content of the variance, skewness and kurtosis term structures for the bond, equity and variance premia and, in addition, that combining information from higher-order risk measures does not add to the predictability of returns.

As in section 1.2, we proceed in four steps. First, we document the factor structure and estimate the number of factors. Second, we estimate RRR models linking the skewness or

free estimate of the first term but requires specification of a time-series model for the historical dynamics of σ_t for the second term. The extant literature does not discuss whether this approach delivers a precise estimate of the variance premium.

[8] The excess variance, xv^e_{t+1}, is defined relative to the variance premium in a way that is analogous to the definition of excess returns, xr^e_{t+1}, relative to the equity premium. We have that $xr^e_{t+1} = r_{t+1} - E^{\mathbb{Q}}_t[r_{t+1}]$ and $xv^e_{t+1} = E^{\mathbb{Q}}_t[\sigma^2_{t+1}] - \sigma^2_{t+1}$.

[9] The use of the cumulant-generating function to characterize the effect of higher-order cumulants on properties of asset prices is also suggested by Martin (2010). Recall that the first cumulant corresponds to the mean, the second cumulant corresponds to the variance, the third cumulant corresponds to the third central moment and provides a measure of skewness, while the fourth cumulant corresponds to the fourth central moments minus 3 times the squared variance and provides a measure of the tails. The cumulant term structure has been neglected in the literature.

[10] This is yet another similarity with the term structure of interest rates. In principle, yields can reveal all state variables related to the future behavior of the short rate. However, specific cases arise where some factors have a small or no impact on interest rates and remain hidden. See Duffee (2011).

kurtosis term structure, respectively, to the bond and equity returns. We conclude that two factors remain sufficient in every case. Finally, we use the same factors and confirm that the predictability extends to the variance premium. Again, the results are robust to the inclusion of alternative predictors and the factors exhibit the same linkages with other macro and financial indicators. Strikingly, combining factors from the term structures of variance, skewness and kurtosis, together, *does not* provide significant predictability improvements. Instead, two factors are again sufficient to summarize their joint information content. This suggests that the joint distribution of the equity premium, the bond premium and the variance premium could be captured by a model with a small number of factors.

1.4 LITERATURE

Christoffersen, Jacobs, and Chang (2011) review the vast literature that uses option-implied information in forecasting, including for returns predictability.[11] Our approach is most closely related to Bakshi, Panayotov, and Skoulakis (2011). They study the predictive content of the 1-month and 2-month forward variances for S&P 500 and Treasury bill returns.[12] We use a broader range of maturities, as well as higher-order moments, and consider the joint variations of expected returns across markets. Motivated by theory, we uncover the factor structure of option-implied variance and higher-order risk measures.

Leippold, Wu, and Egloff (2007), Amengual (2009), and Carr and Wu (2011) find that two factors are needed to describe the variance premium dynamics. We link these factors to the term structure of risk implicit in option prices. Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2011) ask whether the variance premium can predict the equity premium. Similarly, Zhou (2011) and Mueller, Vedolin, and Zhou (2011) ask whether the variance premium predicts bond returns. Instead, we ask whether the same factors that drive the equity and the bond premium can predict the ex-post excess variance since, like the equity premium, the ex-ante variance premium is not observable. Constantinides and Ghosh (2011) also invert risk factors from a set of financial asset prices: they use the risk-free rate and the price-dividend ratio to reveal risk factors in the context of the long-run risk model of Bansal and Yaron (2004). Our empirical implementation is robust to misspecification of the underlying model.

The rest of the article is organized as follows. Section 2 considers affine no-arbitrage models and derives the multi-horizon cumulant-generating function of excess returns and excess variance. We then show how the term structure of uncertainty can be used to reveal fundamental risk factors. Section 3 introduces the data and measurement of risk from option prices. Section 4 evaluates the information content from the term structure of risk-neutral

[11] In particular, Ang, Hodrick, Xing, and Zhang (2006) shows that option-implied market volatility is priced in the cross-section of equity returns. Chang, Christoffersen, and Jacobs (2011) show that option-implied market skewness is priced in the cross-section of equity returns.

[12] Strictly speaking, they focus on the information content of payoffs contingent on the exponential of future integrated variance.

variance. Section 5 repeats the exercise by extending the information set to include the term structures of skewness and kurtosis. Section 6 concludes.

2 Revealing Risk Factors using the Variance Term Structure

This section motivates the empirical analysis performed in the article. The analysis is mostly based on existing results, but it focuses on the distribution of multi-period excess returns. In particular, we derive expressions for the multi-horizon expected bond and equity *risk premium*. We also derive the multi-horizon *variance premium* and expressions for the conditional variance of returns across investment horizons: the *term structure of variance*. Finally, we show how to use the observed variance term structure to recover the factors driving the risk premium and the variance premium.

2.1 RISK PREMIUM, VARIANCE PREMIUM AND THE VARIANCE TERM STRUCTURE

The one-period (log) excess returns derived from holding some asset are[13]

$$xr_{t+1} \equiv \log \frac{P_{t+1}}{P_t} - r_{f,t},$$

where P_t is the price of that asset at time t and $r_{f,t}$ is the (log) one-period risk-free rate. The excess returns over any horizon τ are given by

$$xr_{t,t+\tau} \equiv \sum_{j=1}^{\tau} xr_{t+j}.$$

Consider an economy with K state variables, X_t, and with the following three properties:

(i) The joint distribution of xr_{t+1} and X_{t+1} belongs to the family of affine jump-diffusion continuous-time (or discretized) models (Duffie, Pan, and Singleton, 2000).

(ii) The risk-free rate, $r_{f,t}$, is an affine function of X_t.

(iii) The stochastic discount factor is an exponential-affine function of X_{t+1} and xr_{t+1} (Gourieroux and Monfort, 2007; Christoffersen, Elkamhi, Feunou, and Jacobs, 2010).

The appendix formalizes these properties and shows that this class of models nests a wide array of discrete-time asset-pricing models. Indeed, the affine long-run risk models with

[13] In section 4, we will consider the excess returns from the broad S&P 500 equity index, xr^e_{t+1}, and the excess returns on a Treasury bond, xr^b_{t+1}.

Epstein-Zin-Weil preferences (Bansal and Yaron, 2004; Eraker, 2008) also fit this description.[14] In this broad class of models, the equity premium and the bond premium over an investment horizon τ, $EP(t,\tau)$ and $BP(t,\tau)$, respectively, are given by

$$\begin{aligned} EP(t,\tau) &\equiv E_t^{\mathbb{P}}\left[xr_{t,t+\tau}^e\right] \\ &= \beta_{ep,0}(\tau) + \beta_{ep}(\tau)^\top X_t, \end{aligned} \tag{2}$$

and

$$\begin{aligned} BP(t,\tau) &\equiv E_t^{\mathbb{P}}\left[xr_{t,t+\tau}^b\right] \\ &= \beta_{b,0}(\tau) + \beta_{b}(\tau)^\top X_t, \end{aligned} \tag{3}$$

where the coefficients $\beta_{ep,0}(\tau)$, $\beta_{ep}(\tau)$, $\beta_{b,0}(\tau)$, $\beta_b(\tau)$ are a function of the underlying model's parameters. In particular, the coefficients $\beta_{ep}(\tau)$ and $\beta_b(\tau)$ characterize the returns that are required by investors to bear the risk associated with variations in the risk factor: the risk-return trade-off. The exposure of xr_{t+1} to a given element of X_{t+1} is zero whenever the corresponding element of $\beta(\tau)$ is zero.

We also use the following definitions,

$$\begin{aligned} \sigma_t^2 &\equiv Var_t\left[xr_{t+1}^e\right] \\ \sigma_{t,t+\tau}^2 &\equiv \sum_{j=1}^{\tau} \sigma_{t+j}^2 \end{aligned}$$

where σ_t^2 is simply the 1-period excess returns variance and $\sigma_{t,t+\tau}^2$ parallels the definition of the integrated variance in continuous-time models. The appendix shows that the variance premium over any investment horizon τ, $VRP\,(t,\tau)$, is affine:

$$\begin{aligned} VRP\,(t,\tau) &\equiv E_t^{\mathbb{Q}}\left[\sigma_{t,t+\tau}^2\right] - E_t^{\mathbb{P}}\left[\sigma_{t,t+\tau}^2\right] \\ &= \beta_{vp,0}(\tau) + \beta_{vp}(\tau)^\top X_t, \end{aligned} \tag{4}$$

where the coefficients, $\beta_{vp,0}(\tau)$ and $\beta_{vp}(\tau)$, depend on the structure of the model and are given in the appendix.[15] Finally, the conditional variance of excess returns under $\mathbb{Q}$ over a

[14] See sections A.1 and A.2 of the appendix. This nests the broad family of asset-pricing models with affine valuation or the Laplace transform introduced by Duffie, Pan, and Singleton (2000) in continuous time and by Darolles, Gourieroux, and Jasiak (2006) in discrete time. See, for example, Piazzesi (2009) for term-structure models, or Christoffersen, Jacobs, Ornthanalai, and Wang (2008) and the references therein for option-pricing models. Chamberlain (1988) provides an alternative argument based on a martingale representation. We thank Nour Meddahi for pointing this out.

[15] In the context of long-run risk models (Bansal and Yaron, 2004), $\beta_{ep}(\tau)$ is not zero whenever X_{t+1} affects the conditional distribution of future consumption growth. Moreover, $\beta_{vp}(\tau)$ is not zero if the consumption volatility is time-varying (Bollerslev, Tauchen, and Zhou, 2009) or if consumption growth is not conditionally Gaussian and can jump (Bakshi and Madan, 2006; Drechsler and Yaron, 2011).

horizon τ is also affine and given by

$$Var_t^{\mathbb{Q}}(\tau) \equiv Var_t^{\mathbb{Q}}[xr_{t,t+\tau}^e] = \beta_{vr,0}(\tau) + \beta_{vr}(\tau)^\top X_t. \tag{5}$$

This implies that measures of variance at different maturities display a factor structure with dimension K. This is similar to interest rate models where yields at different maturities sum the contributions of the real rate, inflation and compensation for risk. In most models, these are determined by a small set of economic variables (e.g., wealth, technology, habits) that are often not observed directly, at least at the desired frequency. But the unobservable economic variables can be revealed via their effects on yields. This important insight is applicable in our context.

2.2 REVEALING RISK FACTORS

Equations (2) and (3) characterize the risk-return trade-offs in a broad class of affine models. Different models emphasize different risk factors, X_t, and imply different patterns of risk loadings, β_{ep}, β_b and β_{vp}, but the risk premium dynamics is linear in each case. The coefficients of that relationship could be estimated directly via ordinary least squares (OLS) if the risk factors, X_t, were observable. This would provide a test to discriminate across different specifications, or serve as guidance to investors. However, the risk factors proposed in the literature are latent or difficult to measure. For example, filtering the factors underlying the volatility dynamics (Christoffersen, Jacobs, Ornthanalai, and Wang, 2008) or the intensity of stochastic jumps (Bates, 2006) poses important econometric challenges and depends on a correct specification of the model.[16]

In contrast, model-free measures of risk-neutral variance are available directly from option prices, and the variance term structure can reveal the risk factors. However, the measured risk-neutral variance differs from the true value, $Var_t^{\mathbb{Q}}(\tau) = \widetilde{Var}_t^{\mathbb{Q}}(\tau) + \nu_t(\tau)$, where we assume that the measurement error, $\nu_t(\tau)$, is uncorrelated with $\widetilde{Var}_t^{\mathbb{Q}}(\tau)$. In other words, in contrast with the computation of bond yields from bond prices, measurement errors cannot be neglected when computing variance from option prices. Stacking measurements across horizons $\tau = \tau_1, \ldots, \tau_q$, and using Equation (5), we have that

$$\widetilde{Var}_t^{\mathbb{Q}} + \nu_t = B_{0,vr} + B_{vr} X_t,$$

where the $q \times 1$ vector, $B_{0,vr}$, stacks the constant $\beta_{vr,0}(\tau)$, and the $q \times K$ matrix B_{vr} stacks the corresponding coefficients $\beta_{vr}(\tau)^\top$. Note that we typically have more observations along

[16]This issue also arises in equilibrium models. For example, the expected consumption growth (Bansal and Yaron, 2004), the volatility of consumption volatility (Bollerslev, Tauchen, and Zhou, 2009) or the time-varying jump intensity (Drechsler and Yaron, 2011; Eraker, 2008) all escape direct measurement.

the term structure than there are underlying factors (i.e., $q > K$). We can then write,

$$X_t = -\bar{B}_{vr} B_{0,vr} + \bar{B}_{vr} \widetilde{Var}_t^{\mathbb{Q}} + \bar{B}_{vr} \nu_t, \tag{6}$$

where the $K \times q$ matrix $\bar{B}_{vr} = (B_{vr}^\top B_{vr})^{-1} B_{vr}^\top$ is the left-inverse of B_{vr}.[17] Equation (6) shows that we can use the variance term structure as a signal for the underlying risk factors.

Stacking Equations (3) and (2) and across horizons, we have that

$$\begin{aligned} BP_t &= \Pi_{bp,0} + \Pi_{bp} \widetilde{Var}_t^{\mathbb{Q}} + \nu_t^{bp} && (7) \\ EP_t &= \Pi_{ep,0} + \Pi_{ep} \widetilde{Var}_t^{\mathbb{Q}} + \nu_t^{ep}, && (8) \end{aligned}$$

respectively, where the components of ν_t^{bp} and ν_t^{ep} are $\nu_t^{bp}(\tau) \equiv \beta_{bp}(\tau)^\top \bar{B}_{vr} \nu_t$ and $\nu_t^{ep}(\tau) \equiv \beta_{ep}(\tau)^\top \bar{B}_{vr} \nu_t$ respectively. In practice, we do not observe the bond premium or the equity premium, but we can only measure ex-post excess returns,

$$\begin{aligned} xr_{t,t+\tau}^e &= EP(t,\tau) + \epsilon_{t,t+\tau}^e \\ xr_{t,t+\tau}^b &= BP(t,\tau) + \epsilon_{t,t+\tau}^b, \end{aligned}$$

which, using Equations (7) and (8), can be rewritten as

$$\begin{aligned} xr_{t+}^b &= \Pi_{bp,0} + \Pi_{bp} \widetilde{Var}_t^{\mathbb{Q}} + (\nu_t^{bp} + \epsilon_{t+}^b) && (9) \\ xr_{t+}^e &= \Pi_{ep,0} + \Pi_{ep} \widetilde{Var}_t^{\mathbb{Q}} + (\nu_t^{ep} + \epsilon_{t+}^e), && (10) \end{aligned}$$

where the xr_{t+} (ϵ_{t+}) notation signals that we have stacked ex-post excess returns (their innovations) at different horizons. Equations (9) and (10) form the basis of our empirical investigation in section 4. Each line of the coefficient vector $\Pi_{ep,0}$ and of the coefficient matrix Π_{ep} is given by

$$\begin{aligned} \Pi_{ep,0}(\tau) &= \beta_{ep,0}(\tau) - \beta_{ep}(\tau)^\top \bar{B}_{vr} B_{0,vr} \\ \Pi_{ep}(\tau) &= \beta_{ep}(\tau)^\top \bar{B}_{vr}, && (11) \end{aligned}$$

respectively, with similar expressions for $\Pi_{bp,0}$ and Π_{bp}, showing that the rank of Π_{ep} and Π_{bp} is at most K, the number of columns of $\bar{B}_{vr}$, irrespective of the number of horizons τ used at estimation. This reduced rank coefficient plays a crucial role in the empirical analysis in section 4.

[17] The left-inverse exists since we consider cases with $q > K$ and B_{vr} has full (column) rank. If the latter condition is not satisfied, then the loadings of the conditional variance, $Var_t^{\mathbb{Q}}$ on the risk factors $X_{t,k}$ are not linearly independent. This implies that less than K linear combinations of the risk factors can be revealed from the variance term structure. In this case, we redefine the risk vector in Equation (6) such that it only contains those linear combinations that are spanned.

2.3 ESTIMATING THE NUMBER OF VARIANCE TERM STRUCTURE FACTORS

We first ask how many linear combinations from the variance term structure summarize its information content for the bond and equity premia. In other words, we want to estimate the rank of the coefficient matrix, Π, in a multivariate regression with the following general form:

$$xr_{t+} = \Pi_0 + \Pi \widetilde{Var}_t^{\mathbb{Q}} + \varepsilon_{t+}, \tag{12}$$

where xr_{t+} is a vector of excess returns, $\widetilde{Var}_t^{\mathbb{Q}}$ is a $q \times 1$ vector of risk-neutral variances. This stacks Equations (9) and (10). The statistical literature on sufficient dimension reduction provides a useful approach to estimating this rank. We follow Cook and Setodji (2003) who introduce a model-free test of the null hypothesis that the rank is r (i.e., $\mathrm{H}_0 : \operatorname{rank} \Pi = r$) against the alternative that the rank is strictly greater than r. The modified Cook and Setodji, test statistic, $\tilde{\Lambda}_r$, is available in closed form and has a χ^2 asymptotic distribution with known degrees of freedom. In particular, this test does not require Gaussian innovations in Equation (12). The test is also robust against departure from linearity, section A.5 of the appendix describes the test formally.

2.4 REDUCED-RANK MULTIVARIATE REGRESSIONS

As stated above, Equations (9) and (10) form the basis of our empirical investigation and, for a given rank, r, they correspond to multivariate RRR for which estimators and the associated inference theory are available since at least Anderson (1951). In particular, for a given estimate of the rank, r, the $p \times q$ matrix, Π, can be rewritten as a product, $\Pi = A\Gamma$, where A and Γ have dimensions $(p \times r)$ and $(r \times q)$, respectively, and where $r < min(p, q)$.[18] Then, we can rewrite Equation (12) as

$$xr_{t+} = \Pi_0 + A\Gamma \widetilde{Var}_t^{\mathbb{Q}} + \varepsilon_{t+}, \tag{13}$$

and the RRR estimators of A, Γ and Π_0 are given from the solution to

$$\arg \min_{A,\Gamma,\Pi_0} \operatorname{trace} \left(\sum_{t=1}^{T} \varepsilon_{t+} \varepsilon_{t+}^{\top} \right), \tag{14}$$

with closed-form expressions given in section A.6 of the appendix. Note that the estimated factors, $\Gamma \widetilde{Var}_t^{\mathbb{Q}}$, can be very different than the leading principal components of $\widetilde{Var}_t^{\mathbb{Q}}$.[19]

[18] See Reinsel and Velu (1998) for a textbook treatment of RRR and a discussion of existing applications in tests of asset-pricing models (e.g., Bekker, Dobbelstein, and Wansbeek, 1996; Zhou, 1995). Anderson (1999) provides a theory of inference under general (e.g., not Gaussian) conditions. Hansen (2008) provides a recent formulation of the estimator. The OLS regression emerges when $r = \min(p; q)$ or, trivially, when $r = 0$ and the regressors are irrelevant.

[19] See, for example, the discussion by Cook in his Fisher Lecture (Cook, 2007) and, in particular, this quote from Cox (1968) "there is no logical reason why the dependent variable should not be closely tied to

Finally, A and Γ are not separately identified, and we choose that rotation which yields orthogonal factors. This is analogous to the standard identification choice in principal component analysis (PCA).

2.5 WHY REDUCED-RANK REGRESSIONS?

Our methodological approach imposes the factor structure predicted by theory but remains agnostic regarding other structural assumptions. This approach is in line with Cochrane (2011), who emphasizes the need to uncover the factor structure behind time-varying expected returns. It is also closely related to Cochrane and Piazzesi (2008), who show that a single factor from forward rates is sufficient to summarize the predictability of bonds with different maturities. In this spirit, we test the joint hypotheses of linearity and reduced-rank structure without any other joint hypotheses about the number and the dynamics of state variables, the conditional distribution of shocks, or the preference of the representative agent. Otherwise, the test will overreject the null hypotheses of a given low number of factors, even if it holds in the data, when these maintained hypotheses are not supported by the data. Similarly, estimation based on the Kalman filter will be severely biased if the maintained structural or distributional assumptions are not supported in the data. In contrast, our approach does need additional hypotheses but, instead, exploits the fundamentally multivariate nature of the problem.[20]

3 Data

3.1 EXCESS RETURNS

We use the Center for Research in Security Prices (CRSP) data set to compute end-of-the-month equity returns on the S&P 500 at horizons of 1, 2, 3, 6, 9 and 12 months. Longer-horizon returns are obtained from summing monthly returns. We use the Fama-Bliss zero-coupon bond prices from CRSP to compute bond excess returns. Excess equity returns are computed using risk-free rates from CRSP.[21]

3.2 EXCESS VARIANCE

As in the case of returns, longer-horizon realized variances are obtained from summing monthly realized variances.[22] We follow Britten-Jones and Neuberger (2000) to compute

the least important principal component [of the predictors].” Cochrane and Piazzesi (2005) provide a case in point in the context of bond returns predictability. Their returns-forecasting factor is a linear combination of forward rates that is only weakly spanned by the leading principal components of forward rates.

[20] In particular, our testing and estimation procedure could not be applied to each line of Equation (12) separately.

[21] The Fama-Bliss T-bill file covers maturities from 1 to 6 months. We use the 1-year rate from the Fama-Bliss zero-coupon files. The 9-month T-bill rate is interpolated when necessary.

[22] We thank Hao Zhou for making end-of-the-month S&P 500 realized variance data available on his web site.

expected integrated variance under the risk-neutral measure (see Equation (4)) from option prices. The excess variance is the difference between the realized variance under the historical measure and the exante measure of conditional variance under the risk-neutral measure. This definition is completely analogous to the definition of excess returns. Explicitly, the excess variance is given by

$$xv^e_{t,t+\tau} \equiv E^{\mathbb{Q}}_t \left[\sigma^2_{t,t+\tau}\right] - \sigma^2_{t,t+\tau}, \tag{15}$$

where the first term is measured exante from option prices and $\sigma^2_{t,t+\tau}$ is the realized variance between t and $t + \tau$.

3.3 RISK-NEUTRAL VARIANCE

We use the OptionMetrics database of European options written on the S&P 500 index. We first construct a weekly sample of closing bid and ask prices observed each Wednesday. This mitigates the impact of intraweekly patterns but includes 328,626 observations. Consistent with the extant literature, we restrict our sample to out-of-the-money call and put options. We also exclude observations with no bid prices (i.e., price is too low), options with less than 10 days to maturity, options with implied volatility above 70% and options with zero transaction volume. Moreover, we exclude observations that violate lower and upper bounds on call and put prices. The OptionMetrics database supplies LIBOR and eurodollar rates. To match an interest rate with each option maturity, we interpolate under the assumption of constant forward rates between available interest rate maturities. We also assume that the current dividend yield on the index is constant through the options' remaining maturities.[23] Finally, we restrict our attention to a monthly sample (see section A.7 of the appendix). This yields 85,385 observations covering the period from January 1996 to October 2008. Table I contains the number of option contracts across maturity and moneyness groups. The sample provides a broad coverage of the moneyness spectrum at each maturity.

3.4 SUMMARY STATISTICS

We then rely on the non-parametric approach of Bakshi and Madan (2000) to measure the conditional variance implicit in option prices at maturities of 1, 2, 3, 6, 9, 12 and 18 months. These correspond to the maturity categories available for trading (see section A.8 of the appendix).[24] Table II provides summary statistics of variance across maturities. Risk-neutral variance is persistent with autocorrelation coefficients between 0.73 and 0.87 across maturities. The average term structure has an inverted U-shape. However, the volatility of

[23] See OptionMetrics documentation on the computation of the index dividend yield.

[24] We originally included the 24-month maturity category. However, its summary statistics contrast with the broad patterns drawn in other categories. For this maturity, risk-neutral variance is more skewed to the right, has fatter tails and is less persistent. Moreover, it is less correlated with other maturities. We consider these results a reflection of higher measurement errors and exclude this category in the following.

risk-neutral variance peaks at two months and then gradually declines with maturity. The risk-neutral variance is also more symmetric and has smoother tails for longer horizons.

3.5 PRINCIPAL COMPONENTS

Variance measures are highly correlated across maturities (not reported). For example, the correlation between 1-month- and 2-month-ahead risk-neutral variances (i.e., $Var^{\mathbb{Q}}(t,1)$ and $Var^{\mathbb{Q}}(t,2)$) is 0.88, while the correlation between 1-month- and 1-year-ahead variances is 0.69. This suggests that a few systematic factors can explain most of the variation across maturities. Panel B of Table II reports the results from a PCA, which is a simple way to summarize this factor structure. The first three principal components explain 88%, 6% and 3% of the term structure of the risk-neutral variance, respectively, and together explain 97.4% of the total variation.

These components reflect systematic variation across the variance term structure. The first component's loadings range from 0.31 to 0.44, with an inverted U-shape across maturities. In other words, most of the variation in the risk-neutral variance can be summarized by a change in the level and curvature of its term structure. Next, the second component is similar to the negative of a slope factor. Its loadings decrease, from 0.49 to -0.57, and pivot around zero near the 6-month maturity. The third component's loadings draw a curvature pattern. The correlation between the first component and a measure of the level, $L_t = \widetilde{Var}^{\mathbb{Q}}(t,6)$, is 0.98, the correlation between the second component and a measure of the slope, $S_t = \widetilde{Var}^{\mathbb{Q}}(t,18) - \widetilde{Var}^{\mathbb{Q}}(t,1)$, is -0.90, and the correlation between the third component and a measure of the curvature, $C_t = 2\widetilde{Var}^{\mathbb{Q}}(t,6) - \widetilde{Var}^{\mathbb{Q}}(t,18) - \widetilde{Var}^{\mathbb{Q}}(t,1)$, is 0.80.

4 Results – Variance Term Structure

Section 2 shows that a broad family of no-arbitrage models contains at its core the implication that a few linear combinations from the term structure of variance can be used to predict returns. Consistent with theory, section 3 shows that the term structure of variance can be summarized by its leading principal components. This section estimates the rank of the matrix Π in Equation (12), which summarizes the risk-return trade-offs between returns and the risk factors contained in the variance term structure. We then estimate the corresponding reduced-rank predictability regression. We find that two risk factors from the variance term structure summarize its information content for the equity premium, the bond premium and the variance premium, jointly. We also compare their predictive content with that of other predictors commonly found in the literature. Finally, we interpret the risk factors via their relationship with a broad range of financial and economic indicators. Together, these results provide reassurance that the estimated factors proxy for actual economic risks.

4.1 RANK ESTIMATES

Formally, we consider different versions of a joint model for the bond and equity premium,

$$xr_{t+} = \Pi_0 + A\Gamma\, \widetilde{Var}_t^{\mathbb{Q}} + \varepsilon_{t+}, \tag{16}$$

where we stack Equations (9) and (10). Line-by-line estimation of A and Γ is not feasible when $A\Gamma$ does not have full rank. Panel A of Table III reports the p-values associated with the Cook-Setodji statistics, $\tilde{\Lambda}_r$, for different ranks ranging from 0 to 6. The tests reject that $\operatorname{rank}\Pi = 0$ and $\operatorname{rank}\Pi = 1$. But we do not reject that $\operatorname{rank}\Pi = 2$.

Estimation of unrestricted univariate return regressions on seven variance measures uses 98 parameters. Instead, the results suggest that two risk factors are sufficient to summarize the predictive content of the variance term structure. Estimation of the multivariate system with only two linear combinations of variance is parsimonious, since it reduces the number of parameters to 42. Standard OLS inference, based on F-statistics, rejects the null hypothesis that the variance term structure is irrelevant (unreported), but misses the factor structure in expected returns. Estimation of the rank in expected returns also leads to a rejection that $r = 0$, but concludes that two factors are sufficient. In other words, the increased predictive power of unrestricted regressions (i.e., going from $r = 2$ to $r = 7$) can be attributed to sampling variability.

The estimated risk factors can be related to the principal components of variance. The first factor is highly correlated with the second principal component (-0.93) as well as with the fourth principal component (0.27). Hence, its variations correspond to changes in the slope of the variance term structure but also combine information from higher-order components. The second risk factor is correlated with the first component (0.24) but also mix information from several higher-order components. Its effect on the variance term structure is hard to interpret but mixes variations in the level and the curvature of the variance term structure.[25]

4.2 EXCESS RETURNS PREDICTABILITY

Panel B of Table III reports the R^2s of predictability regressions of bond excess returns across different rank hypotheses. In particular, the R^2s in the case where the rank is $r = 2$ are 7.3%, 6.6%, 5.9% and 5.5% for annual returns on bonds with 2, 3, 4 and 5 years to maturity, respectively.[26] Compare this with the case $r = 7$ (which corresponds to the standard OLS predictive regressions), where the R^2s are 11.5%, 10.1%, 8.8% and 7.9%, respectively. Similarly, Panel C reports R^2s for equity return predictability. For $r = 2$,

[25] The correlation with the level, $L_t = \widetilde{Var}^{\mathbb{Q}}(t, 6)$, is 0.35 and the correlation with the curvature, $C_t = 2\widetilde{Var}^{\mathbb{Q}}(t, 6) - \widetilde{Var}^{\mathbb{Q}}(t, 18) - \widetilde{Var}^{\mathbb{Q}}(t, 1)$, is 0.42.

[26] Mueller, Vedolin, and Zhou (2011) uses the variance premium and obtains R^2s ranging from 1% to 2% but the sample dates do not coincide.

the R^2s are 3.1% and 6.3% for 1-month and 2-month excess returns. The R^2s then decline smoothly to 3.6% at the 12-month horizon. In all cases, there is little gain from increasing the rank from $r = 2$ to $r = 7$, given the large increase in the number of parameters.[27]

Next, we assess the predictive content of the variance risk factors for equity returns in relation with other known predictors in the literature.[28] We find that none of the alternative predictors captures the significant equity return predictability exhibited by the variance term structure in Panel C of Table III. To check this, we repeat the predictability regressions but where we combine both variance risk factors with one of the alternate predictors, in turn. We test the null hypothesis that variance risk factors can be excluded in a regression that includes one alternative predictor. Panel A of Table IV reports the p-value for the null that variance risk factors are jointly uninformative based on the F-statistics. The statistical evidence is overwhelming at other maturities, especially at intermediate maturities where the variance risk factor remains significant even at the 1% level. However, the variance risk factors are marginally significant (at the 10% level) for 1-month returns.

To gauge the economic significance, we project each variance risk factor on a given alternative predictor in a first stage, to obtain the orthogonalized risk factors (i.e., the residuals). Then, in a second stage, we use these orthogonal components in equity return predictability regressions. This projection measures what part of the predictability can be attributed to the variance risk factors, but weighting the evidence in favor of the alternative predictors. We focus on the R^2 statistics, since the individual factors are identified only up to a rotation. Panel B of Table IV reports the results. The rightmost column provides the corresponding R^2s with the original factors (i.e., from Panel C of Table III). The conclusion is clear: the predictive content of the variance risk factor is broadly unchanged and the overlap with that of the alternative predictors is small.

Interestingly, orthogonalizing the variance factors with respect to the default returns spread produces higher R^2.[29] This implies that some sources of risk that attract compensation in the bond or equity markets are not revealed by the variance term structure. In other words, some linear combinations of the risk factors are unspanned by the variance term structure. A similar issue arises in the interest rate literature and has been discussed in Duffee (2011).

The links with common predictors are not negligible. The first two lines of Table IV)

[27] We do not report estimates of A and Γ, since the orthonormal rotation used for estimation has no special economic meaning.

[28] We consider a set of equity valuation indicators: the S&P 500 index dividend-price ratio, d/p, the index dividend yield, d/y, and the index earning-price ratio, e/p; the following variance measures: the risk-neutral variance from index options, $rnsvar$, the index realized variance, $svar$, the variance premium, vrp; the following yields and bond spread indicators: the 3-month Treasury bill yield, tbl, the 10-year Treasury bond yields, ltr, the term spread, tms, the default spread, dfy, the default returns spread, dfr, and the inflation rate, $infl$.

[29] This arises because the correlation between the default returns spread and the variance risk factors has a sign opposite that of the correlation between the default returns spread and excess returns.

Panel C report the R^2s from univariate regressions of the alternative return predictors on each variance risk factor and the last line reports the sum of these R^2s which corresponds to a regression of each alternative predictor on both risk factors, jointly.[30] This measures the common variations between each predictor and the variance factors. The risk-neutral variance and the variance risk premium exhibit substantial correlations with the variance risk factors (total R^2s of 12% and 15%, respectively). This is not surprising, since the information content of these predictors comes from the option market as well. Predictors obtained from fixed-income markets also correlate with the variance risk factors. Regressions of the T-bill rate, the 10-year bond rate and the term spread result in R^2s of 12%, 13% and 5%, respectively. Finally, regressions of the dividend-price ratio and of the dividend yield, measured from the equity market, result in R^2s of 12% and 11%, respectively.

4.3 EXCESS VARIANCE PREDICTABILITY

Bollerslev, Tauchen, and Zhou (2009) argue that the compensation for risk implicit in the one-month ahead variance premium $VRP(t,1)$ is driven by a factor that also drives the equity premium and that the variance premium predicts equity excess returns. But the variance premium is not observable (see Footnote 7). In contrast, Equation (4) shows that the same risk factors underlying the observable variance term structure also determine the variance premium. Therefore, the risk factors extracted from the variance term structure should also forecast the excess variance (in addition to the equity premium). We can test this prediction formally via regressions of excess variance on the term structure factors,

$$xv^e_{t+} = \Pi_{vp,0} + \Pi_{vp}\ \widetilde{Var}^{\mathbb{Q}}_t + (\nu^{vp}_t + \epsilon^{vp}_{t+}), \tag{17}$$

where, as above, xv^e_{t+} signals that we have stacked excess variance for different horizons, where $\nu^{vp}_t(\tau) \equiv \beta_{vp}(\tau)^\top \bar{B}_{vr}\nu_t$, and where $\epsilon^{vp}_{t+}(\tau) \equiv xv^e_{t,t+\tau} - VRP(t,\tau)$, respectively. The definitions of $\Pi_{vp,0}$ and Π_{vp} are analogous to those given in Equation (11) for excess returns.

We first estimate a benchmark excess variance predictability regression based on the BTZ_t proxy,

$$xv^e_{t+}(\tau) = a_0(\tau) + a_{btz}(\tau)BTZ_t + \varepsilon_{t+}(\tau), \tag{18}$$

for different horizons τ and where BTZ_t is the proxy used by Bollerslev, Tauchen, and Zhou (2009). They use realized volatility from the previous period, σ^2_t, as an estimate of $E^{\mathbb{P}}_t\left(\sigma^2_{t+1}\right)$.[31] The results, reported in Panel A of Table V, show that BTZ_t has no predictive power for the 1-month excess variance, even though the BTZ_t was designed to proxy for the 1-month variance premium. Nonetheless, BTZ_t predicts excess variance at longer horizons

[30] These bivariate R^2s correspond to the sum of the individual R^2s, since the risk factors are orthogonal by construction.

[31] This approach is consistent with a random-walk assumption for the dynamics of σ^2_{t+1}. The realized volatility from last period is then combined with option-implied measure of $E^{\mathbb{Q}}_t\left(\sigma^2_{t+1}\right)$ from the current period.

with R^2s up to 7% at the 6-month horizons.

Panel B of Table V reports OLS estimates of the following regressions:

$$xv^e_{t+}(\tau) = \Pi_{vp,0}(\tau) + A_{vp}\hat{\Gamma}\ \widetilde{Var}^{\mathbb{Q}}_t + \varepsilon_{t+}(\tau). \quad (19)$$

where we use the estimated factors, $\hat{\Gamma}\widetilde{Var}^{\mathbb{Q}}_t$, for $r = 2$ Together, these two risk factors that were estimated to predict the bond and equity premia also predict the variance premium. The adjusted R^2s is 4.9% for the 1-month variance premium. The R^2s then range between 7% and 9% at horizons between 1 and 9 months, respectively, which is more than the predictability obtained using BTZ_t. A look at individual coefficients reveals that each of the estimated linear combinations plays an important role but at different horizons.[32] Panel C of Table V combines the term structure factors, $\hat{\Gamma}\widetilde{Var}^{\mathbb{Q}}_t$, with the BTZ_t factor. The results show that combining predictors yields little predictability improvement. The information content from the BTZ_t factor appears to be subsumed in the variance term structure via the second variance risk factor (its correlation with BTZ_t is -0.38).

Our finding that the variance term structure can be used to predict the variance premium is a novel and significant result. Note that the key insight from Bollerslev, Tauchen, and Zhou (2009) still holds: the variance premium is linked to the equity premium since the risk factors also predict the equity risk premium. In addition, the ability to forecast excess variance is akin to an out-of-sample check since the excess variance was not used to extract risk factors from the term structure. Of course, one potential concern is that both the predictor and the excess variance use information from risk-neutral variance. On the other hand, the BTZ_t proxy, and the first principal component of the variance term structure (unreported results), also use information from the risk-neutral variance, but do *not* predict the 1-month variance premium. Finally, it may be tempting to use Equation (17) along with bond and equity returns in a RRR. However, the measurement errors in excess variance that arise because we measure $E^{\mathbb{Q}}_t\left[\sigma^2_{t,t+\tau}\right]$ from option prices are correlated with the measurement errors in $\widetilde{Var}^{\mathbb{Q}}_t$, which is also obtained from option prices.[33]

4.4 COUNTER-CYCLICAL RISK FACTORS

Following Ludvigson and Ng (2009), we survey the correlations between the risk factors and 124 economic and financial indicators.[34] Figure 1 shows a bar graph where each bar adds the R^2s from the univariate regressions of each factor, in turn, on one of the economic or

[32] Constant terms are not reported for parsimony.

[33] This correlation would distort the rank tests upward. Stambaugh (1988) provides an example where measurement errors due to bid-ask spreads in bond prices lead to overrejection of small factor structure and wrongly favor larger factor structure (his section 4.4, p. 58). Cochrane and Piazzesi (2005) provide a similar example. They use a single factor from forward rates to study bond returns. They show that the single-factor restriction is rejected statistically, but that deviations from a single-factor structure are economically insignificant.

[34] These variables are classified in the following broad categories: real output and income, employment and income, real consumption, real inventories and orders, housing starts, foreign exchange, price index, stock

financial variables. The question we ask is to what extent are these risk factors spanned by each of these indicators. We focus on R^2, since regression coefficients depend on the identification assumptions (i.e., orthonormal factors).

The results show the overlap between the risk factors recovered from options and standard macroeconomic indicators. In particular, both factors are linked to macro variables. The first factor is correlated with inventory indicators – the National Association of Purchasing Management (NAPM) inventory quantity index, NAPM inventory prices index, NAPM employment index– and with the saving rate. The second risk factor exhibits strong commonalities with labor market indicators – the ratio of help-wanted ads to the number of unemployed, and the number of unemployed by duration categories – as well as with the capacity utilization rate. The R^2s hover around 10% for variables related to hours worked, 15% for payroll variables and as high as 20% for the help-wanted index. The coefficient estimates (not reported) imply that the relationship between the estimated expected equity returns and the macro indicator is counter-cyclical. Improved economic conditions – lower unemployment or a higher help-wanted index – lead to lower expected returns. Finally, Boivin, Giannoni, and Stevanovic (2010) construct 10 factors that summarize the evolution of these 124 variables. Together, these 10 factors explain up to 24% and 29% of the risk factor variations. Corradi, Distaso, and Mele (2012) also report that macro variables do not span the volatility and the risk premium implicit in the VIX index.

The results also confirm the overlap between the variance risk factors and common financial variables, such as S&P 500 stock index variables, long-term yields and, importantly, Moody's corporate bond indices. No data from Moody's were used to estimate the factor. Moreover, the chain of correlations between each financial indicator and the variance risk factors, first, and between the risk factors and expected returns, second, implies that the predicted effects on changes in the financial indicators on expected returns have the expected signs. For instance, combining the estimate's sign from the univariate regressions on each financial indicator, and the sign of the loadings in the estimate of the loading matrix $\hat{A}$ above (not reported), implies that expected returns predicted by the variance risk premium rise with a lower valuation on the stock market, higher long-term interest rates and higher corporate bond yields.

5 Term Structure of Higher-Order Cumulants

We show that measures of higher-order risks can also be used to reveal risk factors. Empirically, we find that the skewness and kurtosis term structures predict the bond premium, the equity premium and the variance premium. Their predictive content is similar to that of the variance term structure and can be summarized by two risk factors. Consistent with

price, money and credit aggregates, interest rates, and bonds. See Table VIII for a detailed description. We thank Dalibor Stevanovic for sharing this data.

theory, combining measures of variance, skewness and kurtosis improves predictability only marginally and, strikingly, the predictive content of this broad information set can still be summarized by two factors.

5.1 HIGHER-ORDER CUMULANTS IN EQUILIBRIUM

The variance term structure may fail to reveal all risk factors. This may arise if some factors do not affect the variance, or if their effects are small relative to the measurement errors in the variance or to the innovations in returns. It may be possible to increase the efficiency of our estimates and parse the variance term structure to find additional factors. But this neglects low-hanging fruit. An alternative way is to broaden the information set to include other measurements where the effect of other risk factors may be more easily measured. Section A.3 of the appendix shows that every cumulant of returns is affine in the state vector,

$$M_{t,n}^{\mathbb{Q}}(\tau) = \beta_{n,0}(\tau) + X_t^{\top}\beta_{n,X}(\tau),$$

for any returns horizon τ, and where coefficients depend on the underlying model.[35] Then, an argument similar to that of section 2.2 shows that higher-order cumulants can also be used to reveal X_t:

$$X_t = -\bar{B}_n B_{0,n} + \bar{B}_n \widetilde{M_{t,n}}^{\mathbb{Q}} + \bar{B}_n \nu_{n,t}. \tag{20}$$

Following a path parallel to the previous section, we construct model-free measures of returns cumulants of order 3 and 4 (see section A.8 of the appendix). We also slightly misapply the terminology for ease of exposition and label these cumulants skewness and kurtosis, respectively.

5.2 SUMMARY STATISTICS AND FACTOR STRUCTURE

Panels A and B of Table VI provide summary statistics of the conditional skewness and kurtosis of returns, respectively. The average distribution of returns implicit in the index option is left-skewed and has fat tails. The average skewness lies below zero and slopes downward with the horizon. On the other hand, the average tail is fatter at longer horizons. Skewness and kurtosis are persistent, especially at intermediate horizons.

The correlation matrices (Panels C and D of Table VI) suggest a low-dimensional factor structure, as in the case of risk-neutral variance. Panels E and F report PCA results for the term structures of skewness and kurtosis, respectively. The first three principal components of skewness explain 67%, 15% and 12% of the total variation, respectively, and together explain 93%. Similarly, the first three principal components of kurtosis explain 65%, 19%

[35] See section A.3 of the appendix. Recall that the first cumulant corresponds to the mean, the second cumulant corresponds to the variance, the third cumulant corresponds to the third central moment and provides a measure of skewness, while the fourth cumulant corresponds to the fourth central moments minus 3 times the squared variance and provides a measure of the tails. The conventional measures of skewness and kurtosis are not affine in the risk factors.

and 12% of the total variation, respectively. As for the variance, the loadings reveal that the leading components of skewness and kurtosis have a systematic effect on their respective term structures.

5.3 PREDICTABILITY RESULTS

We estimate different variations of the following multivariate regression:

$$xr_{t+} = \Pi_0 + A\Gamma\ F_t + \varepsilon_{t+}, \tag{21}$$

where, as above, xr_{t+} stacks four excess bond returns and six excess equity returns. We consider different combinations of the variance, skewness and kurtosis term structures to construct the regressors, F_t.

5.3.1 Excess Returns with Skewness or Kurtosis

We first consider each term structure separately. Panel A of Table VII reports the results. First, model $V(2)$ uses the term structure of variance as predictors (i.e., $F_t = \widetilde{Var}_t^{\mathbb{Q}}$). This reproduces a subset of the results reported above (Table III) and provides a point of comparison for models using skewness or kurtosis as predictors. Second, model $S(2)$ includes only the term structure of skewness (i.e., $F_t = \widetilde{Skew}_t^{\mathbb{Q}}$). Third, model $K(2)$ includes only the term structure of kurtosis (i.e., $F_t = \widetilde{Kurt}_t^{\mathbb{Q}}$). In model $S(2)$, the p-value is 6.1% for the null that $r = 1$ and 38.2% for the null that $r = 2$. Similarly, for the $K(2)$ model, the p-value is 7.9% for the null that $r = 1$ and 32.2% for the null that $r = 2$. Hence, the test based on each of these higher moments comes close to rejecting the rank-one restriction in favor of a higher rank, while the rank-two restriction is clearly not rejected. Nonetheless, we report estimation results based on $r = 2$ for comparison, because more general models combining information from different term structures consistently reject the case $r = 1$ (see section 5.3.2). The results show that the ability to predict bond and equity excess returns, as measured by the R^2s, is strikingly similar whether we use any one of the variance, skewness or kurtosis term structures. This is consistent with theory. If anything, skewness and kurtosis appear to be slightly more informative about bond returns, while variance appears to be slightly more informative about equity returns. We stress that this does not imply that only the variance matters. The term structure of risk-neutral variance combines information about historical variance, skewness and kurtosis (Bakshi and Madan, 2006), and changes in the prices of risk.

5.3.2 Combining Variance, Skewness and Kurtosis Term Structure

The VSK(2,2) model combines the two risk factors estimated separately from each of the variance, skewness and kurtosis term structures. Hence, this uses six predictors and asks whether these risk factors add up to more than two factors when combined in the same model. The evidence is unambiguous. The p-value is 1.1% for the null that $r = 1$

and 32.6% for the null that $r = 2$. Again, this is consistent with theory. The predictive content available from the term structure of different risk measures is broadly overlapping. As expected, estimation in the case $r = 2$ yields R^2s that are very close to the highest value obtained above. Of course, we could (at least) reach these values by setting $r = 6$. What is unexpected is that we can summarize these six risk factors into two with little loss of predictive ability.

The VSK(2,2) model is a second-stage estimation that uses factors obtained in a first-stage procedure. Next, we introduce model VSK(7,2), which combines the entire variance, skewness and kurtosis term structures in a single RRR step. This is an alternative way to ask whether the risk factors measured from different term structures add up to more than two factors. Model VSK(7,2) is estimated in one step but, on the other hand, it is more exposed to overfitting given the large number of regressors. Nonetheless, these models yield consistent evidence. The p-value is 1.4% for the null that $r = 1$ and 9.7% for the null that $r = 2$. The p-value has decreased substantially, but Cook and Setodji (2003) report that this test tends to overreject when the number of predictors and regressors is particularly high, as in model VSK(7,2). This biases our result toward concluding in favor of a greater number of factors. Nonetheless, there is a substantial increase in predictability in the case of $r = 2$ when we combine all the risk measures. R^2s now range from 17% to 22% in the case of bond returns (compared to the 9%-10% of more parsimonious models) and from 6% to 18% in the case of equity returns (compare to the 3%-8%). The next section uses the variance premium as an out-of-sample check.

5.3.3 Robustness and Excess Variance Predictability

As in section 4, we check that the risk factors extracted from the variance, skewness and kurtosis term structures survive the inclusion of alternative risk factors (unreported results). We also check that the in-sample predictability obtained from bond and stock returns extends to the variance premium. Panel B of Table VII reports results of excess variance predictability regressions. The results are broadly consistent across all models: the R^2s have an inverted U-shape across horizons, reaching a maximum close to 10% at intermediate horizons between 3 and 6 months. This holds whether the risk factors were extracted from the variance, skewness or kurtosis term structures. Again, the theoretical prediction is supported in the data. In particular, there is no improvement in excess variance predictability for the VSK(7,2) model. Hence, this out-of-sample exercise suggests that some of the increased excess returns predictability obtained above for the VSK(7,2) model is due to in-sample overfitting. Finally, as in section 4.4, we survey the correlations between the risk factors and 124 economic and financial indicators. Similarly to Figure 1, Figure 2 shows a bar graph where each bar adds the R^2s from the univariate regressions of each factor, in turn, on one of the economic or financial variables. Conclusions drawn in section 4.4 remain as both figures display very similar patterns. Economic and financial variables exhibit similar relationships with risk factors extracted from the variance, the skewness and the kurtosis term structures, respectively. This add to the evidence in the previous section suggesting that the the variance, skewness and kurtosis term structure share a similar information content.

6 Conclusion

Affine no-arbitrage models of the stock returns process introduce latent variations in stochastic volatility or jump intensity. But, almost by construction, these factors are difficult to measure and the risk-return trade-offs are difficult to measure. On the other hand, model-free measures of risk-neutral variance, and higher-order moments, are available from option prices. We find that the term structure of risks can be used to reveal risk factors that are important drivers of bond premium, equity premium and variance premium variations. Consistent with theory, we find that a small number of factors, two, summarize the relationship between the equity premium, the bond premium and the variance implicit in option prices.

Our results open several avenues for future research. First, does the predictive content from the term structure of option prices extend to other markets? In particular, is the valuation of individual firms' corporate bonds and equities related to the same risk factors? Similarly, are the risk premia implicit in other derivatives markets (e.g., interest rate or FX derivatives markets) related to the risk factors from index options? Second, how can we reconcile the factor structure common to the variance, skewness and kurtosis term structures with its predictive content for returns within a reduced-form asset-pricing specification? Finally, given an appropriate reduced-form specification that matches the stylized facts uncovered here, what equilibrium model can relate these facts to preferences and economic fundamentals?

A Appendix

A.1 Affine Reduced-Form Models

Discrete-time affine specifications of the return process have the following general Laplace transform of excess returns, Darolles, Gourieroux, and Jasiak (2006):

$$E_t^{\mathbb{P}}\left[\exp\left(u\ xr_{t+1}^e + v^\top X_{t+1}\right)\right] = \exp\left(F_{r,X}^{\mathbb{P}}\left(u,v\right)^\top X_t + F_{r,0}^{\mathbb{P}}\left(u,v\right)\right), \tag{22}$$

and the risk-free interest rate is defined as

$$r_{f,t} = B_0 + B_X^\top X_t. \tag{23}$$

Similarly, under the risk-neutral measure, $\mathbb{Q}$, affine models have the following general representation:

$$E_t^{\mathbb{Q}}\left[\exp\left(u\ xr_{t+1}^e + v^\top X_{t+1}\right)\right] = \exp\left(F_{r,X}^{\mathbb{Q}}\left(u,v\right)^\top X_t + F_{r,0}^{\mathbb{Q}}\left(u,v\right)\right), \tag{24}$$

where

$$\begin{aligned} F_{r,X}^{\mathbb{Q}}\left(u,v\right) &= F_{r,X}^{\mathbb{P}}(u+\gamma, v+\Gamma) - F_{r,X}^{\mathbb{P}}(\gamma,\Gamma) \\ F_{r,0}^{\mathbb{Q}}\left(u,v\right) &= F_{r,0}^{\mathbb{P}}(u+\gamma, v+\Gamma) - F_{r,0}^{\mathbb{P}}(\gamma,\Gamma). \end{aligned}$$

The parameters γ and Γ characterize the conditional stochastic discount factor, $M_{t,t+1}$,

$$M_{t,t+1} = \exp\left(\gamma\, xr_{t+1}^e + \Gamma^\top X_{t+1} + \theta_t\right), \tag{25}$$

and $M_{t,t+1}$ must satisfy

$$\begin{aligned} E_t\left[M_{t,t+1}\right] &= \exp(-r_{f,t}) \\ E_t\left[M_{t,t+1}\exp(xr_{t+1}^e)\right] &= \exp(-r_{f,t}), \end{aligned} \tag{26}$$

which together imply that

$$\begin{aligned} \theta_t &= -F_{r,X}^{\mathbb{P}}(\gamma,\Gamma)^\top X_t - F_{r,0}^{\mathbb{P}}(\gamma,\Gamma) - r_{f,t} \\ 0 &= F_{r,X}^{\mathbb{P}}(1+\gamma,\Gamma)^\top X_t + F_{r,0}^{\mathbb{P}}(1+\gamma,\Gamma) - F_{r,X}^{\mathbb{P}}(\gamma,\Gamma)^\top X_t - F_{r,0}^{\mathbb{P}}(\gamma,\Gamma). \end{aligned} \tag{27}$$

It follows easily that the multi-horizon return $\sum_{j=1}^{\tau} xr_{t+j}^e$ denoted by $xr_{t,t+\tau}^e$ has the following cumulant-generating function under the risk-neutral world:

$$E_t^{\mathbb{Q}}\left[\exp\left(u\ xr_{t,t+\tau}^e\right)\right] = \exp\left(F_{r,0}^{\mathbb{Q}}\left(u;\tau\right) + X_t^\top F_{r,X}^{\mathbb{Q}}\left(u;\tau\right)\right)$$

where the sequence of functions $F_{r,0}^{\mathbb{Q}}\left(u;\tau\right)$ and $F_{r,X}^{\mathbb{Q}}\left(u;\tau\right)$ satisfies the following recursions:

$$\begin{aligned} F_{r,0}^{\mathbb{Q}}\left(u;\tau\right) &= F_{r,0}^{\mathbb{Q}}\left(u;\tau-1\right) + F_{r,0}^{\mathbb{Q}}\left(u, F_{r,X}^{\mathbb{Q}}\left(u;\tau-1\right)\right) \\ F_{r,X}^{\mathbb{Q}}\left(u;\tau\right) &= F_{r,X}^{\mathbb{Q}}\left(u, F_{r,X}^{\mathbb{Q}}\left(u;\tau-1\right)\right), \end{aligned} \tag{28}$$

with initial conditions $F_{r,0}^{\mathbb{Q}}\left(u;1\right) = F_{r,0}^{\mathbb{Q}}\left(u,0\right)$ and $F_{r,X}^{\mathbb{Q}}\left(u;1\right) = F_{r,X}^{\mathbb{Q}}\left(u,0\right)$.

A.2 Affine General-Equilibrium Models

In section A.1, we reported all the steps that led to the main ingredients needed in this article: equity premium, bond premium, equity variance premium and all the risk-neutral cumulants are affine in the factor. The assumptions made to derive these results are the following:

- The physical conditional moment-generating function of $\left(xr_{t+1}^e, X_{t+1}^\top\right)^\top$, is exponential affine in X_t, as postulated in Equation (22)

- The conditional stochastic discount factor, $M_{t,t+1}$, is exponential affine in both xr^e_{t+1} and X_{t+1}, as postulated in Equation (25)
- And, finally, the risk-free ($r_{f,t}$) rate is affine in X_t, as postulated in Equation (23).

In this section, we consider the general class of affine equilibrium models, and show that it satisfies all three assumptions. We consider an affine general equilibrium model (AGEM) similar to Eraker (2008). Suppose that the state of the economy can be summarized by a Markov process $Y_{t+1} \equiv \left(\Delta c_{t+1}, X_{t+1}^\top\right)^\top$, where Δc_{t+1} is the consumption growth process and X_{t+1} is a vector of K (observed and unobserved) state variables independent of consumption growth. The moment-generating function of this state vector under the physical measure is given by

$$E_t\left[\exp\left(u\Delta c_{t+1} + v^\top X_{t+1}\right)\right] = \exp\left(F_0\left(u,v\right) + X_t^\top F_X\left(u,v\right)\right),$$

where the scalar function $F_0\left(u,v\right)$ and the vector function $F_X\left(u,v\right)$ describe the exogenous dynamics of the vector process Z_{t+1}. Assume, further, that the representative agent has recursive preferences of the Epstein-Zin-Weil type. Consequently, the logarithm of the intertemporal marginal rate of substitution is given by

$$s_{t,t+1} = \theta \ln \delta - \frac{\theta}{\psi}\Delta c_{t+1} - \left(1-\theta\right) r_{t+1},$$

where r_{t+1} is the return to the aggregate consumption claim. Using the standard Campbell-Shiller approximation, $r_{t+1} = \kappa_0 + \kappa_1 w_{t+1} - w_t + \Delta c_{t+1}$, the log price-consumption ratio w_t can be well-approximated by an affine function of the vector state variable X_t as

$$w_t = A_0 + A_X^\top X_t,$$

where the scalar coefficient A_0, and the vector coefficient A_X depend on model and preference parameters. Solving for these coefficients is standard in the literature. The (log) stochastic discount factor can then be rewritten as

$$\begin{aligned} s_{t,t+1} = {} & \theta \ln \delta - \left(1-\theta\right)\left(\kappa_0 + \left(\kappa_1 - 1\right) A_0 - A_X^\top X_t\right) \\ & - \gamma \Delta c_{t+1} - \left(1-\theta\right)\kappa_1 A_X^\top X_{t+1}, \end{aligned} \tag{29}$$

and the model-implied log risk-free rate is given by

$$r_{f,t} = B_0 + B_X^\top X_t,$$

where the scalar coefficient B_0 and the vector coefficient B_X depend on the exogenous dynamics and preference parameters,

$$B_0 = -\theta \ln \delta + \left(1-\theta\right)\left(\kappa_0 + \left(\kappa_1 - 1\right) A_0\right) - F_0\left(-\gamma, -\left(1-\theta\right)\kappa_1 A_X\right) \tag{30}$$

$$B_X = -\left(1-\theta\right) A_X - F_X\left(-\gamma, -\left(1-\theta\right)\kappa_1 A_X\right), \tag{31}$$

hence the risk-free rate is affine in the factor X_t, which shows that the general equilibrium satisfies the third requirement enumerated earlier.

It follows that, in this economy, the change-of-measure from the historical probability to the risk-neutral probability is given by

$$M_{t,t+1} = \exp\left(s_{t,t+1} + r_{f,t}\right) = \exp\left(H_0 + H_X^\top X_t - \gamma \Delta c_{t+1} - p_X^\top X_{t+1}\right), \tag{32}$$

where

$$H_0 = -F_0\left(-\gamma, -p_X\right), \quad H_X = -F_X\left(-\gamma, -p_X\right) \quad \text{and} \quad p_X = \left(1-\theta\right)\kappa_1 A_X. \tag{33}$$

Moreover, this implies that the bond premium and the equity premium are linear in the state variables whenever $\theta \neq 1$ and $A_X \neq 0$. These conditions imply that $p_X \neq 0$ and that the pricing kernel varies with X_t. Intuitively, the first condition implies that the agent has preference over the intertemporal resolution of uncertainty (i.e., $\gamma \neq \psi$). The second condition implies that X_{t+1} affects the conditional distribution of future consumption growth.[36]

[36]Strictly speaking, the prices of risk associated with innovations to X_{t+1} may differ from zero, with $\gamma \neq \psi$, but with a constant wealth-consumption ratio (and risk premium) if U_t/c_t varies with X_{t+1}. This arises in the knife-edge case where $\psi = 1$.

Excess return from the claim on aggregate consumption denoted by xr_{t+1}^{e} is computed as

$$xr_{t+1}^{e} = r_{t+1} - \mu_t^{\mathbb{Q}} = -G_0 - G_X^{\top} X_t + \Delta c_{t+1} + \kappa_1 A_X^{\top} X_{t+1}, \tag{34}$$

where $\mu_t^{\mathbb{Q}} \equiv E_t^{\mathbb{Q}}[r_{t+1}]$ is given by

$$\mu_t^{\mathbb{Q}} = \kappa_0 + (\kappa_1 - 1) A_0 + G_0 + (G_X - A_X)^{\top} X_t, \tag{35}$$

with coefficients,

$$G_0 = \mathcal{D}F_0(-\gamma, -p_X) \begin{pmatrix} 1 \\ \kappa_1 A_X \end{pmatrix} \quad \text{and} \quad G_X = \mathcal{D}F_X(-\gamma, -p_X) \begin{pmatrix} 1 \\ \kappa_1 A_X \end{pmatrix}. \tag{36}$$

As a result, we can find the expression of Δc_{t+1} in function of xr_{t+1}^{e} as follows:

$$\Delta c_{t+1} = xr_{t+1}^{e} + G_0 + G_X^{\top} X_t - \kappa_1 A_X^{\top} X_{t+1}. \tag{37}$$

substituting Δc_{t+1} by itS value (see Equation (37) in (32)) implies that

$$\begin{aligned} M_{t,t+1} &= \exp\left(H_0 + H_X^{\top} X_t - \gamma\left(xr_{t+1}^{e} + G_0 + G_X^{\top} X_t - \kappa_1 A_X^{\top} X_{t+1}\right) - p_X^{\top} X_{t+1}\right) \\ &\equiv \exp\left(\gamma\, xr_{t+1}^{e} + \Gamma^{\top} X_{t+1} + \theta_t\right). \end{aligned}$$

Hence the stochastic discount factor is exponential affine in both xr_{t+1}^{e} and X_{t+1}; thus the second requirement enumerated at the beginning of this section is satisfied. Finally, consider the physical conditional moment-generating function of $\left(xr_{t+1}^{e}, X_{t+1}^{\top}\right)^{\top}$:

$$\begin{aligned} E_t\left[\exp\left(u xr_{t+1}^{e} + v^{\top} X_{t+1}\right)\right] &= E_t\left[\exp\left(u\left(-G_0 - G_X^{\top} X_t + \Delta c_{t+1} + \kappa_1 A_X^{\top} X_{t+1}\right) + v^{\top} X_{t+1}\right)\right] \\ &= \exp\left(u\left(-G_0 - G_X^{\top} X_t\right)\right) E_t\left[\exp\left(u \Delta c_{t+1} + (v + \kappa_1 A_X)^{\top} X_{t+1}\right)\right] \\ &= \exp\left(F_0(u, v + \kappa_1 A_X) - G_0 + X_t^{\top}\left(F_X(u, v + \kappa_1 A_X) - G_X\right)\right) \\ &\equiv \exp\left(F_{r,X}^{\mathbb{P}}(u, v)^{\top} X_t + F_{r,0}^{\mathbb{P}}(u, v)\right). \end{aligned}$$

Hence the physical conditional moment-generating function of $\left(xr_{t+1}^{e}, X_{t+1}^{\top}\right)^{\top}$ is exponential affine in X_t as postulated in (22); therefore, our approach is well motivated within the general class of an affine general-equilibrium model.

A.3 Risk-Neutral Cumulants

Then, the nth order cumulants of excess returns denoted, $M_n^{\mathbb{Q}}(t, \tau)$, are the derivative of the log moment-generating function of aggregate returns with respect to u, and evaluated at $u = 0$,

$$M_n^{\mathbb{Q}}(t, \tau) = \beta_{n,0}(\tau) + X_t^{\top} \beta_{n,X}(\tau),$$

where

$$\beta_{n,0}(\tau) = \mathcal{D}^n F_{r,0}^{\mathbb{Q}}(0; \tau) \quad \text{and} \quad \beta_{n,X}(\tau) = \mathcal{D}^n F_{r,X}^{\mathbb{Q}}(0; \tau).$$

The operator $\mathcal{D}$ defines the Jacobian matrix of a real matrix function of a matrix of real variables.[37] Formally, for a given function Υ defined over $\mathbb{R}^m \times \mathbb{R}^n$ and with values in $\mathbb{R}^p \times \mathbb{R}^q$, which associates with the $m \times n$ matrix ξ the $p \times q$ matrix $\Upsilon(\xi)$, we have that $\mathcal{D}\Upsilon(\xi)$ is the $pq \times mn$ matrix defined by

$$\mathcal{D}\Upsilon(\xi) = \frac{\partial vec(\Upsilon(\xi))}{\partial vec(\xi)^{\top}} \quad \text{and} \quad \mathcal{D}\Upsilon(\xi^*) = \left.\frac{\partial vec(\Upsilon(\xi))}{\partial vec(\xi)^{\top}}\right|_{\xi=\xi^*}, \tag{38}$$

and we also define the operator $\mathcal{D}_i$ for which the derivative is taken with respect to the ith argument of the function Υ. In

[37] See Magnus and Neudecker (1988, p. 173).

particular, when $n = 2$, we have the conditional risk-neutral variance

$$Var_t^{\mathbb{Q}}(\tau) \equiv Var_t^{\mathbb{Q}}\left[xr_{t,t+\tau}^e\right] = \beta_{vr,0}\left(\tau\right) + \beta_{vr}(\tau)^\top X_t,$$

where $\beta_{vr,0}\left(\tau\right) = \beta_{2,0}\left(\tau\right)$ and $\beta_{vr}(\tau) = \beta_{2,X}\left(\tau\right)$. Similarly, for $n = 3$ and 4 we get the equation for the risk-neutral skewness and kurtosis.

A.4 Equity, Bond and Variance Premium

Similar to the risk-neutral dynamic, Equation (22) implies that the multi-horizon return $xr_{t,t+\tau}^e$ has the following cumulant-generating function under the physical world:

$$E_t^{\mathbb{P}}\left[\exp\left(u\ xr_{t,t+\tau}^e\right)\right] = \exp\left(F_{r,0}^{\mathbb{P}}\left(u;\tau\right) + X_t^\top F_{r,X}^{\mathbb{P}}\left(u;\tau\right)\right), \tag{39}$$

where the sequence of functions $F_{r,0}^{\mathbb{P}}\left(u;\tau\right)$ and $F_{r,X}^{\mathbb{P}}\left(u;\tau\right)$ satisfies the following recursions:

$$\begin{aligned} F_{r,0}^{\mathbb{P}}\left(u;\tau\right) &= F_{r,0}^{\mathbb{P}}\left(u;\tau-1\right) + F_{r,0}^{\mathbb{P}}\left(u, F_{r,X}^{\mathbb{P}}\left(u;\tau-1\right)\right) \\ F_{r,X}^{\mathbb{P}}\left(u;\tau\right) &= F_{r,X}^{\mathbb{P}}\left(u, F_{r,X}^{\mathbb{P}}\left(u;\tau-1\right)\right) \end{aligned} \tag{40}$$

with initial conditions $F_{r,0}^{\mathbb{P}}\left(u;1\right) = F_{r,0}^{\mathbb{P}}\left(u,0\right)$ and $F_{r,X}^{\mathbb{P}}\left(u;1\right) = F_{r,X}^{\mathbb{P}}\left(u,0\right)$.

A.4.1 Equity premium

The equity premium is, by definition,

$$EP(t,\tau) \equiv E_t^{\mathbb{P}}\left[xr_{t,t+\tau}^e\right].$$

Hence $EP(t,\tau)$ is the derivative of the log moment-generating function of aggregate excess returns (given in Equation (39)) with respect to u, and evaluated at $u = 0$; thus

$$EP(t,\tau) = \beta_{ep,0}(\tau) + \beta_{ep}(\tau)^\top X_t,$$

where

$$\beta_{ep,0}(\tau) = \mathcal{D}^1 F_{r,0}^{\mathbb{P}}\left(0;\tau\right) \quad \text{and} \quad \beta_{ep}(\tau) = \mathcal{D}^1 F_{r,X}^{\mathbb{P}}\left(0;\tau\right).$$

A.4.2 Variance premium

The Variance Premium over any investment horizon τ, $VRP\left(t,\tau\right)$ is defined as

$$VRP\left(t,\tau\right) \equiv E_t^{\mathbb{Q}}\left[\sum_{j=1}^{\tau}\sigma_{t+j}^2\right] - E_t^{\mathbb{P}}\left[\sum_{j=1}^{\tau}\sigma_{t+j}^2\right],$$

where $\sigma_t^2 = Var_t(xr_{t+1}^e)$. σ_t^2 is the order 2 derivative with respect to u of the log moment-generating function given in Equation (22); this implies that

$$\sigma_t^2 = \frac{\partial^2 F_{r,X}^{\mathbb{P}}}{\partial u^2}\left(0,0\right)^\top X_t + \frac{\partial^2 F_{r,0}^{\mathbb{P}}}{\partial u^2}\left(0,0\right),$$

hence

$$\begin{aligned} VRP\left(t,\tau\right) &= \frac{\partial^2 F_{r,X}^{\mathbb{P}}}{\partial u^2}\left(0,0\right)^\top \sum_{j=1}^{\tau}\left[E_t^{\mathbb{Q}}(X_{t+j}) - E_t^{\mathbb{P}}(X_{t+j})\right] \\ &= \frac{\partial^2 F_{r,X}^{\mathbb{P}}}{\partial u^2}\left(0,0\right)^\top \sum_{j=1}^{\tau}\left[\mu_X^{\mathbb{Q}} - \mu_X^{\mathbb{P}} + \left(\phi_X^{\mathbb{Q}}\right)^j\left(X_t - \mu_X^{\mathbb{Q}}\right) - \left(\phi_X^{\mathbb{P}}\right)^j\left(X_t - \mu_X^{\mathbb{P}}\right)\right] \\ &\equiv \beta_{vp,0}(\tau) + \beta_{vp}(\tau)^\top X_t, \end{aligned}$$

with $\phi_X^{\mathbb{Q}} = \frac{\partial F_{r,X}^{\mathbb{Q}}}{\partial v}(0,0)$, $\phi_X^{\mathbb{P}} = \frac{\partial F_{r,X}^{\mathbb{P}}}{\partial v}(0,0)$, $\mu_X^{\mathbb{Q}} = E^{\mathbb{Q}}[X_t] = \left(I - \phi_X^{\mathbb{Q}}\right)^{-1}\frac{\partial F_{r,0}^{\mathbb{Q}}}{\partial v}(0,0)$ and $\mu_X^{\mathbb{P}} = E^{\mathbb{P}}[X_t] = \left(I - \phi_X^{\mathbb{P}}\right)^{-1}\frac{\partial F_{r,0}^{\mathbb{P}}}{\partial v}(0,0)$.

A.4.3 Bond premium

To compute bond premium, it will be useful to compute the term structure of interest rates in closed form. The price of an n-period discount bond at time t is

$$P_t^{(n)} = E_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{n-1} r_{f,t+j}\right)\right] \equiv \exp\left(-nB_{0,n} - nB_{X,n}^{\top}X_t,\right),$$

where the sequence of functions $B_{0,n}$ and $B_{X,n}$ satisfies the following recursions:

$$\begin{aligned}(n+1)B_{0,n+1} &= nB_{0,n} + B_0 - F_{r,0}^{\mathbb{Q}}\left(0, -nB_{X,n}\right)\\ (n+1)B_{X,n+1} &= B_X - F_{r,X}^{\mathbb{Q}}\left(0, -nB_{X,n}\right),\end{aligned}$$

with initial conditions $B_{0,1} = B_0$ and $B_{X,1} = B_X$. We denote excess log returns over the one-period rate by

$$\begin{aligned} xr_{t+1}^{b,n} &\equiv \ln\left(P_{t+1}^{(n-1)}\right) - \ln\left(P_t^{(n)}\right) - r_{f,t}\\ &= nB_{0,n} - (n-1)B_{0,n-1} - B_0 + \left(nB_{X,n} - B_X\right)^{\top} X_t - (n-1)B_{X,n-1}^{\top}X_{t+1}.\end{aligned}$$

Hence the multi-horizon return $\sum_{j=1}^{\tau} xr_{t+j}^{b,n}$ denoted by $xr_{t,t+\tau}^{b,n}$ is given by

$$xr_{t,t+\tau}^{b,n} = \tau\left(nB_{0,n} - (n-1)B_{0,n-1} - B_0\right) + \left(nB_{X,n} - B_X\right)^{\top}\sum_{j=0}^{\tau-1} X_{t+j} - (n-1)B_{X,n-1}^{\top}\sum_{j=1}^{\tau} X_{t+j}.$$

The bond premium is, by definition,

$$\begin{aligned} BP(t,\tau) &\equiv E_t^{\mathbb{P}}\left[xr_{t,t+\tau}^{b,n}\right]\\ &= \tau\left(nB_{0,n} - (n-1)B_{0,n-1} - B_0\right) + \left(nB_{X,n} - B_X\right)^{\top}\sum_{j=0}^{\tau-1}\left(\mu_X^{\mathbb{P}} + \left(\phi_X^{\mathbb{P}}\right)^j\left(X_t - \mu_X^{\mathbb{P}}\right)\right)\\ &\quad -(n-1)B_{X,n-1}^{\top}\sum_{j=1}^{\tau}\left(\mu_X^{\mathbb{P}} + \left(\phi_X^{\mathbb{P}}\right)^j\left(X_t - \mu_X^{\mathbb{P}}\right)\right)\\ &\equiv \beta_{bp,0}(\tau) + \beta_{bp}(\tau)^{\top}X_t.\end{aligned}$$

A.5 Cook and Setodji Test Procedure

Cook and Setodji (2003) propose the following iterated algorithm as an estimator for the rank of Π:

1. Initialize the null hypothesis with $\mathrm{H}_0^{(0)}$: $\operatorname{rank}\Pi = r^{(0)} = 0$.
2. For the hypothesis $\mathrm{H}_0^{(i)}$, compare the $\tilde{\Lambda}_{r^{(i)}}$ statistics with the chosen cut-off from the χ_g^2 distribution; e.g., 5%.
3. If the probability of observing $\tilde{\Lambda}_{r^{(i)}}$ is lower than the cut-off, then reject the null, conclude that $\operatorname{rank}\Pi > r^{(i)}$, and repeat the test under a new null hypothesis where the rank is incremented; i.e., $r^{(i+1)} = r^{(i)} + 1$.
4. Otherwise, conclude that $\operatorname{rank}\Pi = r^{(i)}$. That is, there is insufficient evidence against $\operatorname{rank}\Pi = r^{(i)}$, yet we have rejected $\operatorname{rank}\Pi < r^{(i)}$.

The test is also robust against departure from linearity. Indeed, if $E[Y_t|X_t]$ is not linear in X_t, in contrast with Equation (12), then inference about the rank of Π from estimates of Equation (12) may still be used to form inference about the dimension of the central mean subspace (CMS) of $Y_t|X_t$. A subspace $\mathcal{M}$ of $\mathbb{R}^q$ is a mean subspace of $Y_t|X_t$ if $E[Y_t|X_t]$ is a function of $M^{\top}X_t$ where the $q \times r$ matrix M is a basis for $\mathcal{M}$. The CMS is the intersection of all mean subspaces, see Cook and Setodji (2003).

A.6 Reduced-Rank Regressions

A multivariate reduced-rank regression model can be written as

$$Y_t = A\Gamma^\top F_t + \Psi Z_t + \epsilon_t \quad t = 1, \ldots, T, \tag{41}$$

where A and Γ have size $(p \times K)$ and $(q \times K)$, respectively. The RRR estimators are given from the solution to

$$\min_{A,\Gamma,\Psi} \left| \sum_{t=1}^{T} \epsilon_t \epsilon_t' \right|, \tag{42}$$

and closed-form expressions are given in Theorem 5 of Hansen (2008). In his notation, define the moment matrix,

$$M_{yf} = T^{-1} \sum_{t=1}^{T} Y_t F_t^\top, \tag{43}$$

and define the matrices M_{yy}, M_{yz}, M_{ff} similarly. Also, define

$$\begin{aligned} S_{yy} &= M_{yy} - M_{yz} M_{zz}^{-1} M_{zy} \\ S_{yf} &= M_{yf} - M_{yz} M_{zz}^{-1} M_{zf}, \end{aligned} \tag{44}$$

and define S_{ff} and $S_{yf} = S_{fy}^\top$ similarly. Then, the estimator of A, Γ and of Ψ are given by

$$\hat{\Gamma}\top = [\hat{v}_1, \ldots, \hat{v}_K]\phi \tag{45}$$

$$\hat{A} = S_{y,f} \hat{B} (\hat{B}^\top S_{ff} \hat{B})^{-1}$$

$$\hat{\Psi} = M_{yz} M_{zz}^{-1} - \hat{A}\hat{B} M_{fz} M_{zz}^{-1}, \tag{46}$$

where $[\hat{v}_1, \ldots, \hat{v}_K]$ are the eigenvectors corresponding to the largest K eigenvalues of

$$|\lambda S_{ff} - S_{fy} S_{yy}^{-1} S_{yf}| = 0, \tag{47}$$

and ϕ is an arbitrary $(K \times K)$ matrix with full rank. This is a normalization device and it corresponds to the choice of a particular basis for the subspace spanned by the rows of $\hat{\Gamma}$.

A.7 Constructing a Monthly Sample

Option settlement dates follow a regular pattern through time: contracts are available for 3 successive months, then for the next 3 months in the March, June, September, December cycle and, finally, for the next two months in the June and December semi-annual cycle. This leads to maturity groups with 1, 2 or 3 months remaining to settlement, and then between 3 and 6, 6 and 9, 9 and 12, 12 and 18 and 18 and 24 months remaining to settlement. We group option prices at the monthly frequency using their maturity date, so that enough observations are available within each group to construct non-parametric measures. To see why this is a natural strategy, note first that each contract settles on the third Friday of a month. Consider, then, all observations intervening between two successive (monthly) settlement dates. Each of these observations can be unambiguously attributed to one maturity date. Moreover, within that period, each contract will be attributed to the same maturity group.[38] While a higher number of observations reduce sampling errors in our estimates of risk-neutral moments, it may also increase noise if there is a large within-month time-variation in the distribution of stock returns at given maturities. To mitigate this effect, we always use the most recent observation when the same contract (i.e., same maturity and strike price) is observed more than once.

[38]Take any contract, on any observation date. This contract is assigned to the 1-month maturity group if its settlement date occurs on the following third Friday, to the 2-month group if it occurs on the next-to-following third Friday, etc. This grouping does not change until we reach the next settlement date.

A.8 Cumulants

We rely on the non-parametric approach of Bakshi and Madan (2000) to measure the conditional variance implicit in option prices. Any twice-differentiable payoff, $H(S(t+\tau))$, contingent on the future stock price, $S(t+\tau)$, can be replicated by a portfolio of stock options. The portfolio allocations across option strikes are specific to each payoff H and given by derivatives of the payoff function evaluated at the corresponding strike price. Following Bakshi and Madan, we take

$$H(S(t+\tau)) \equiv (r^e_{t,t+\tau})^n = \ln\left(\left(\frac{S(t+\tau)}{(S(t)}\right)^n\right),$$

so that the fair value, at time t, of a contract paying the second moments of returns over the next τ periods ahead, $V_2^{\mathbb{Q}}(t,\tau) \equiv E_t^{\mathbb{Q}}[e^{-r\tau}\left(r^e_{t,t+\tau}\right)^2]$, is given by

$$\frac{V_2^{\mathbb{Q}}(t,\tau)}{2} = \int_0^{S(t)} \frac{1-\ln(K/S(t))}{K^2} P(t,\tau,K)dK + \int_{S(t)}^{\infty} \frac{1-\log(K/S(t))}{K^2} C(t,\tau,K)dK,$$

and can be directly computed from the relevant European call and put option prices, $C(t,\tau,K)$ and $P(t,\tau,K)$, with maturity τ and strike price K. Finally, the risk-neutral variance at maturity τ is given by

$$Var^{\mathbb{Q}}(t,\tau) = e^{r\tau}V_2^{\mathbb{Q}}(t,\tau) - \mu^{\mathbb{Q}}(t,\tau)^2,$$

where we follow Bakshi, Kapadia, and Madan (2003) to compute $\mu^{\mathbb{Q}}(t,\tau)$. Similarly, option-implied risk-neutral return cumulants are given by

$$\begin{aligned}
M_1^{\mathbb{Q}}(t,\tau) &\equiv \mu^{\mathbb{Q}}(t,\tau) \approx e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V_2^{\mathbb{Q}}(t,\tau) - \frac{e^{r\tau}}{6}V_3^{\mathbb{Q}}(t,\tau) - \frac{e^{r\tau}}{24}V_4^{\mathbb{Q}}(t,\tau)\\
M_2^{\mathbb{Q}}(t,\tau) &\equiv Var^{\mathbb{Q}}(t,\tau) = e^{r\tau}V_2^{\mathbb{Q}}(t,\tau) - \mu^{\mathbb{Q}}(t,\tau)^2\\
M_3^{\mathbb{Q}}(t,\tau) &= e^{r\tau}V_3^{\mathbb{Q}}(t,\tau) - 3\mu^{\mathbb{Q}}(t,\tau)e^{r\tau}V_2^{\mathbb{Q}}(t,\tau) + 2\mu^{\mathbb{Q}}(t,\tau)^3\\
M_4^{\mathbb{Q}}(t,\tau) &= e^{r\tau}V_4^{\mathbb{Q}}(t,\tau) - 4\mu^{\mathbb{Q}}(t,\tau)e^{r\tau}V_3^{\mathbb{Q}}(t,\tau) + 6\mu^{\mathbb{Q}}(t,\tau)^2e^{r\tau}V_2^{\mathbb{Q}}(t,\tau) - 3\mu^{\mathbb{Q}}(t,\tau)^4,
\end{aligned}$$

where we closely follow Bakshi, Kapadia, and Madan (2003) in the computation of $\mu^{\mathbb{Q}}$. Recall that the first cumulant is the mean, the second cumulant is the variance, the third cumulant is the third centered moment, and the fourth cumulant is the fourth centered moment minus 3 times the squared variance.

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Table I: **Option Sample Summary Statistics**

Number of observations (out-of-the-money puts and calls) in each maturity (months) and moneyness (K/S) group. S&P 500 futures option data are from January 1996 to October 2008.

	Moneyness						
Maturity	< 0.90	$0.90 - 0.95$	$0.95 - 0.975$	$0.975 - 1$	$1 - 1.025$	$1.025 - 1.05$	> 1.05
1	3173	3498	2229	2435	2429	2178	2638
2	4849	3350	2115	2423	2435	2098	3938
3	3077	1789	1151	1423	1371	1029	2649
6	4248	1694	987	1056	917	789	2957
9	2679	1020	635	645	484	405	2049
12	1621	598	368	417	375	264	1507
18	1504	500	279	313	267	169	1107
24	890	259	176	235	149	103	703

Table II: **Risk-Neutral Variance Summary Statistics**

Summary statistics of conditional risk-neutral variance across maturities from 1 to 18 months (Panel A) and loadings from principal component analysis of risk-neutral variance (Panel B). Risk-neutral variance measures at each maturity constructed using the model-free method of Bakshi and Madan (2000). Option data are from January 1996 to October 2008.

Panel A - Summary Statistics

	1	**2**	**3**	**6**	**9**	**12**	**18**
Mean	0.037	0.045	0.046	0.049	0.047	0.044	0.044
Std. Dev.	0.024	0.027	0.027	0.026	0.022	0.021	0.022
Skewness	1.484	1.193	1.047	0.888	0.549	0.847	0.478
Kurtosis	5.332	4.066	3.725	3.579	2.497	3.559	2.932
$\rho(1)$	0.738	0.730	0.788	0.820	0.871	0.812	0.809

Panel B - Principal Components

	1	**2**	**3**	**4**	**5**	**6**	**7**
	0.36	0.49	-0.75	-0.23	0.10	-0.05	-0.03
	0.44	0.38	0.33	0.16	-0.41	-0.06	0.60
	0.43	0.20	0.28	0.12	-0.07	0.52	-0.63
Loadings	0.42	-0.06	0.26	0.02	0.32	-0.76	-0.27
	0.35	-0.28	-0.01	0.15	0.70	0.37	0.40
	0.31	-0.42	0.08	-0.81	-0.23	0.09	0.06
	0.31	-0.57	-0.41	0.48	-0.42	-0.07	-0.06
R^2	0.88	0.06	0.03	0.02	0.01	0.00	0.00
Cum. R^2	0.88	0.94	0.97	0.99	0.99	1.00	1.00

Table III: **Excess Return and the Variance Term Structure**

Rank test p-values and R^2s in multivariate regressions, $Y_t = \Pi_0 + \Pi F_t + \epsilon_t$, where each component of Y_t is an excess bond or equity returns, $xr_{t,t+\tau}$, and where $F_t = \{\hat{Var}^{\mathbb{Q}}(t,\tau)\}_{\tau=1,\ldots,q}$ is a $q \times 1$ vector of risk-neutral variance measures. We consider annual excess returns for bonds with maturities of 2, 3, 4 and 5 years, and S&P 500 excess returns at horizons 1, 3, 6, 9 and 12 months. Panel A shows p-values associated with the Cook and Setodji modified statistics, $\tilde{\Lambda}_r$, in a test of the null hypothesis that the rank of the matrix Π is r. Panel B shows the R^2 associated with each of the individual bond return predictability regressions obtained via multivariate reduced-rank regression (RRR) estimation, but for different hypotheses on the rank of the matrix Π. Panel C shows the R^2 associated with each of the individual equity return predictability regressions. Risk-neutral variance measures at each maturity are constructed using the model-free method of Bakshi and Madan (2000). Monthly returns and option data are from January 1996 to October 2008.

Panel A - Rank Test p-values

	$H_0:\ r=0$	$H_0:\ r=1$	$H_0:\ r=2$	$H_0:\ r=3$	$H_0:\ r=4$	$H_0:\ r=5$	$H_0:\ r=6$
p-val	0.0	4.3	22.9	64.8	82.5	81.4	73.0

Panel B - Bond Returns R^2s

	$r=1$	$r=2$	$r=3$	$r=4$	$r=5$	$r=6$	$r=7$
2	7.3	7.3	9.2	11.1	11.4	11.4	11.5
3	6.6	6.6	7.8	9.6	9.9	10.0	10.1
4	5.7	5.9	6.6	8.2	8.7	8.7	8.8
5	5.0	5.5	5.8	7.3	7.8	7.9	8.0

Panel C - Equity Returns R^2s

	$r=1$	$r=2$	$r=3$	$r=4$	$r=5$	$r=6$	$r=7$
1	1.9	3.1	3.1	3.3	3.4	3.5	3.7
2	4.0	6.3	7.2	8.8	9.2	9.2	9.2
3	5.4	6.3	7.5	10.7	11.1	11.1	11.3
6	3.3	5.3	7.6	9.0	9.0	9.1	9.6
9	3.5	4.2	7.9	10.1	10.1	10.1	10.3
12	3.5	3.6	10.5	11.0	11.0	11.0	11.1

Table IV: **Variance Factors and Alternative Equity Return Predictors**

Variance term structure risk factors and alternative equity return predictors. Stock valuation indicators: d/p is the index dividend-price ratio, d/y is the index dividend yield, e/p is the index earning-price ratio. Variance measures: $rnsvar$ is the risk-neutral variance from index options, $svar$ is the index realized variance, vrp is the variance premium. Yields and spread indicators: tbl is the 3-month Treasury bill yield, ltr is the 10-year Treasury bond yields, tms is the term spread, dfy is the default spread, dfr is the default returns spread. Panel A reports the p-value for the null that variance risk factors are jointly uninformative based on the F-statistics associated with the restricted regressions excluding the variance factors. Panel B reports the R^2s from equity excess return predictability regressions on two risk factors of the variance term structure, but orthogonalized with respect to each alternative predictor in turn. The first two lines of Panel C report R^2s from the regression of each of the alternative predictors on the variance risk factor, and the last line of Panel C reports the R^2s from the regressions on both variance risk factors, jointly.

Panel A - F test p-values in Regressions on Variance Risk Factors and One Alternative Predictor

	Stock Valuation				Variance Measures			Yields and Spreads					
Horizon	d/p	d/y	e/p	d/e	rnsvar	svar	vrp	tbl	ltr	tms	dfy	dfr	infl
1	0.10	0.10	0.10	0.09	0.11	0.08	0.20	0.13	0.11	0.09	0.11	0.08	0.10
2	0.01	0.01	0.01	0.01	0.01	0.00	0.04	0.01	0.01	0.01	0.01	0.00	0.01
3	0.01	0.01	0.01	0.01	0.02	0.01	0.03	0.01	0.01	0.01	0.01	0.00	0.01
6	0.02	0.02	0.02	0.02	0.02	0.01	0.07	0.03	0.02	0.02	0.02	0.01	0.02
9	0.04	0.04	0.04	0.04	0.05	0.02	0.10	0.07	0.05	0.04	0.05	0.02	0.05
12	0.06	0.06	0.06	0.06	0.08	0.03	0.10	0.09	0.08	0.05	0.07	0.03	0.07

Panel B - Predictive Content of Orthogonalized Variance Risk Factors

	Stock Valuation				Variance Measures			Yields and Spreads						
Horizon	d/p	d/y	e/p	d/e	rnsvar	svar	vrp	tbl	ltr	tms	dfy	dfr	infl	Table IIIC
1	3.33	3.36	3.88	3.56	3.00	3.96	1.70	2.33	2.30	2.77	1.23	4.98	1.84	3.1
2	6.68	6.78	7.70	7.21	5.48	8.08	2.99	4.91	4.96	5.70	2.93	9.61	3.90	6.3
3	6.20	6.30	7.21	7.61	5.23	8.18	3.98	4.84	4.63	5.79	3.11	9.86	3.64	6.3
6	5.69	5.88	6.92	6.28	5.49	7.45	2.54	3.13	2.52	4.57	1.27	9.70	2.41	5.3
9	4.29	4.52	5.09	5.05	4.51	6.48	2.61	2.76	1.85	3.92	1.48	8.29	1.63	4.2
12	3.32	3.55	3.91	4.01	4.03	6.06	2.85	3.03	1.96	3.76	2.02	7.28	1.59	3.6

Panel C - Regression of Alternative Predictors on Variance Risk Factors

	Stock Valuation				Variance Measures			Yields and Spreads					
	d/p	d/y	e/p	d/e	rnsvar	svar	vrp	tbl	ltr	tms	dfy	dfr	infl
Factor 1	3.19	2.38	0.11	1.37	6.93	6.55	1.29	2.33	5.23	0.17	1.96	1.26	3.08
Factor 2	9.34	8.84	3.73	0.10	4.65	0.11	14.03	9.28	7.67	5.05	4.35	1.33	1.98
Both	12.52	11.22	3.84	1.47	11.58	6.66	15.31	11.61	12.89	5.22	6.31	2.59	5.05

Table V: **Excess Variance Predictability**

Results from multi-horizon predictability regressions of the excess variance over a horizon of τ, $xv_{t,t+\tau}$, with $\tau = 1, 2, 3, 6, 9$ and 12 months, respectively. The predictors include a constant and $\hat{\Gamma}F_t$, the risk factors obtained from the multivariate reduced-rank regression of bond and equity excess returns on the variance term structure (see Table III, $\hat{\Pi} = \hat{A}\hat{\Gamma}$), and BTZ_t, the variance premium proxy from Bollerslev, Tauchen, and Zhou (2009). Newey-West t-statistics with lags corresponding to the investment horizon plus 3 months in parentheses and R^2 reported in percentage. Risk-neutral variance measures at each maturity are constructed using the model-free method of Bakshi and Madan (2000). Monthly variance and option data are from January 1996 to October 2008.

Panel A - BTZ_t

	1	2	3	6	9	12
BTZ_t	2.28	4.84	5.24	6.32	4.59	3.39
	(0.75)	(2.87)	(2.10)	(4.28)	(2.92)	(1.70)
$\bar{R}^2$	-0.1	2.7	3.5	7.0	4.5	2.2

Panel B - $\hat{\Gamma}F_t$

	1	2	3	6	9	12
$\hat{\Gamma}_1 \tilde{Var}_t^{\mathbb{Q}}$	0.011	0.012	0.010	0.009	0.003	0.003
	(2.15)	(1.94)	(1.55)	(1.35)	(0.49)	(0.42)
$\hat{\Gamma}_2 \tilde{Var}_t^{\mathbb{Q}}$	-0.005	-0.007	-0.008	-0.008	-0.009	0.005
	(-1.23)	(-1.78)	(-1.74)	(-2.21)	(-2.51)	(1.56)
$\bar{R}^2$	4.9	8.3	7.8	8.9	7.5	1.4

Panel C - $\hat{\Gamma}F_t$ and BTZ_t

	1	2	3	6	9	12
BTZ_t	0.52	3.01	3.32	4.62	2.67	5.09
	(0.20)	(1.49)	(1.55)	(3.59)	(2.18)	(2.50)
$\hat{\Gamma}_1 \tilde{Var}_t^{\mathbb{Q}}$	0.011	0.012	0.010	0.008	0.002	0.002
	(2.22)	(1.91)	(1.49)	(1.25)	(0.51)	(0.28)
$\hat{\Gamma}_2 \tilde{Var}_t^{\mathbb{Q}}$	-0.004	-0.005	-0.008	-0.005	-0.008	0.008
	(-1.34)	(-1.29)	(-1.74)	(-1.48)	(-1.99)	(2.56)
$\bar{R}^2$	4.3	8.8	8.6	11.8	8.3	6.3

Table VI: **Summary Statistics - Term Structure of Higher-Order Moments**

Panels A and B report summary statistics of risk-neutral cumulants 3 and 4, respectively, across maturities from 1 to 24 months. Panels C and D report the corresponding correlation matrix. Panels E and F report the loadings and explanatory power of each component from a principal component analysis (PCA). Cumulant measures at each maturity are constructed using the model-free method of Bakshi and Madan (2000). Option data are from January 1996 to October 2008.

Panel A - Skewness Summary Statistics

	1	**2**	**3**	**6**	**9**	**12**	**18**
Mean	-0.003	-0.005	-0.007	-0.011	-0.012	-0.011	-0.015
Std. Dev.	0.002	0.004	0.005	0.007	0.007	0.008	0.010
Skewness	-2.036	-1.555	-1.184	-1.083	-1.164	-1.866	-1.677
Kurtosis	8.009	5.307	3.745	3.575	4.279	8.786	7.234
$\rho(1)$	0.408	0.690	0.659	0.804	0.742	0.615	0.577

Panel B - Kurtosis Summary Statistics

	1	**2**	**3**	**6**	**9**	**12**	**18**
Mean	0.001	0.002	0.003	0.006	0.008	0.008	0.012
Std. Dev.	0.001	0.002	0.003	0.005	0.006	0.008	0.010
Skewness	1.626	1.662	1.292	1.151	1.245	2.847	2.483
Kurtosis	5.811	5.550	3.942	3.637	4.705	14.979	11.668
$\rho(1)$	0.509	0.670	0.622	0.785	0.711	0.608	0.472

Panel C - Skewness Correlations

1	**2**	**3**	**6**	**9**	**12**	**18**
1.00	0.76	0.66	0.62	0.51	0.20	0.18
0.76	1.00	0.87	0.87	0.66	0.42	0.37
0.66	0.87	1.00	0.83	0.69	0.40	0.45
0.62	0.87	0.83	1.00	0.79	0.62	0.57
0.51	0.66	0.69	0.79	1.00	0.45	0.66
0.20	0.42	0.40	0.62	0.45	1.00	0.49
0.18	0.37	0.45	0.57	0.66	0.49	1.00

Panel D - Kurtosis Correlations

1	**2**	**3**	**6**	**9**	**12**	**18**
1.00	0.75	0.65	0.62	0.54	0.25	0.22
0.75	1.00	0.81	0.83	0.61	0.38	0.34
0.65	0.81	1.00	0.78	0.65	0.35	0.41
0.62	0.83	0.78	1.00	0.73	0.61	0.53
0.54	0.61	0.65	0.73	1.00	0.30	0.57
0.25	0.38	0.35	0.61	0.30	1.00	0.40
0.22	0.34	0.41	0.53	0.57	0.40	1.00

Panel E - Skewness PCA

	1	**2**	**3**	**4**	**5**	**6**	**7**
	0.08	-0.18	0.11	-0.09	0.11	-0.79	0.55
	0.21	-0.33	0.14	-0.29	0.01	-0.39	-0.77
	0.26	-0.31	0.21	-0.34	0.70	0.39	0.18
Loadings	0.46	-0.41	0.06	-0.20	-0.67	0.26	0.25
	0.43	-0.12	0.33	0.82	0.12	-0.01	-0.07
	0.38	-0.10	-0.89	0.11	0.18	-0.07	-0.02
	0.58	0.76	0.14	-0.25	-0.00	-0.07	-0.01
R^2	0.67	0.15	0.12	0.04	0.01	0.01	0.00
Cum. R^2	0.67	0.82	0.93	0.97	0.99	1.00	1.00

Panel F - Kurtosis PCA

	1	**2**	**3**	**4**	**5**	**6**	**7**
	0.03	-0.02	0.08	0.07	0.10	0.39	-0.91
	0.08	-0.07	0.20	0.29	0.13	0.83	0.41
	0.12	-0.06	0.26	0.35	0.82	-0.33	0.01
Loadings	0.32	-0.26	0.43	0.55	-0.53	-0.23	-0.06
	0.33	0.04	0.67	-0.66	0.03	0.02	0.03
	0.40	-0.81	-0.36	-0.21	0.10	0.03	0.01
	0.78	0.52	-0.34	0.08	0.01	0.02	-0.00
R^2	0.65	0.19	0.12	0.03	0.01	0.00	0.00
Cum. R^2	0.65	0.84	0.96	0.99	1.00	1.00	1.00

Table VII: **Predictive Content of Higher-Order Moments**

Panel A reports p-values of rank tests and R^2s in multivariate regressions, $Y_t = \Pi_0 + \Pi F_t + \epsilon_t$, where each component of Y_t is an excess bond or equity return, $xr_{t,t+\tau}$, and where F_t are different combinations of risk-neutral variance, skewness and kurtosis at horizons 1, 3, 6, 9, 12 and 18 months. Model $V(2)$ assumes $r = 2$ and includes the term structure of variance in F_t. Models $S(2)$ and $K(2)$ also assume $r = 2$. Model $S(2)$ includes the term structure of skewness, and model $K(2)$ includes the term structure of kurtosis. Model VSK(2,2) combines the two risk factors estimated from each of the variance, skewness and kurtosis term structures and assumes $r = 2$. Model VSK(7,2) assumes $r = 2$ and combines all measures from the variance, skewness and kurtosis term structures. We report p-values associated with the Cook and Setodji modified statistics, $\tilde{\Lambda}_r$, for tests of the null hypothesis that the rank of the matrix Π is $r = 1$ and for tests of the null that the rank is $r = 2$. Panel B displays R^2s from multi-horizon predictability regressions of the excess variance, $xv_{t,t+\tau}$ on a constant and $\hat{\Gamma} F_t$, the risk factors obtained from the multivariate reduced-rank regression. Annual excess returns for bonds with maturities of 2, 3, 4 and 5 years, S&P 500 excess returns at horizons 1, 3, 6, 9 and 12 months, and excess variance at horizons 1, 3, 6, 9 and 12 months. Risk-neutral variance, skewness and kurtosis measures at each maturity are constructed using the model-free method of Bakshi and Madan (2000). Monthly returns, realized variance and option data are from January 1996 to October 2008.

Panel A - Bond and Equity Premium

Model	Rank Test H0		Bond Maturity				Equity Return Horizons					
	$H0: r = 1$	$H0: r = 2$	2	3	4	5	1	2	3	6	9	12
$V(2)$	4.3	22.9	7.3	6.6	5.9	5.5	3.1	6.3	6.3	5.3	4.2	3.6
$S(2)$	6.1	38.2	9.1	9.8	9.7	9.5	2.8	6.3	4.9	2.9	2.3	1.1
$K(2)$	7.9	32.2	10.2	11.0	10.8	10.6	2.3	4.6	5.4	6.0	4.1	3.6
$VSK(2,2)$	1.1	32.6	10.1	10.0	9.4	9.0	3.7	7.9	6.5	6.4	4.6	3.4
$VSK(7,2)$	1.4	9.7	21.6	20.3	18.2	16.5	5.8	16.4	16.6	17.4	18.9	17.9

Panel B - Variance Premium

Model	Variance Horizons					
	1	2	3	6	9	12
$V(2)$	6.2	9.5	9.0	10.1	9.7	2.7
$S(2)$	1.1	6.0	6.5	9.8	7.5	0.2
$K(2)$	2.0	5.6	5.5	9.0	5.7	0.1
$VSK(2,2)$	5.8	9.8	9.5	11.2	9.5	1.2
$VSK(7,2)$	1.7	4.3	4.7	9.7	8.8	2.6

Table VIII: **Description of Economic and Financial Variables**

No.	Series Code	
		Real Output and Income
1	IPS10	INDUSTRIAL PRODUCTION INDEX - TOTAL INDEX
2	IPS11	INDUSTRIAL PRODUCTION INDEX - PRODUCTS, TOTAL
3	IPS12	INDUSTRIAL PRODUCTION INDEX - CONSUMER GOODS
4	IPS13	INDUSTRIAL PRODUCTION INDEX - DURABLE CONSUMER GOODS
5	IPS14	INDUSTRIAL PRODUCTION INDEX - AUTOMOTIVE PRODUCTS
6	IPS18	INDUSTRIAL PRODUCTION INDEX - NONDURABLE CONSUMER GOODS
7	IPS25	INDUSTRIAL PRODUCTION INDEX - BUSINESS EQUIPMENT
8	IPS29	INDUSTRIAL PRODUCTION INDEX - DEFENSE AND SPACE EQUIPMENT
9	IPS299	INDUSTRIAL PRODUCTION INDEX - FINAL PRODUCTS
10	IPS306	INDUSTRIAL PRODUCTION INDEX - FUELS
11	IPS32	INDUSTRIAL PRODUCTION INDEX - MATERIALS
12	IPS34	INDUSTRIAL PRODUCTION INDEX - DURABLE GOODS MATERIALS
13	IPS38	INDUSTRIAL PRODUCTION INDEX - NONDURABLE GOODS MATERIALS
14	IPS43	INDUSTRIAL PRODUCTION INDEX - MANUFACTURING (SIC)
15	PMP	NAPM PRODUCTION INDEX (PERCENT)
16	PMI	PURCHASING MANAGERS' INDEX (SA)
17	UTL11	CAPACITY UTILIZATION - MANUFACTURING (SIC)
18	YPR	PERS INCOME CH 2000 $,SA-US
19	YPDR	DISP PERS INCOME,BILLIONS OF CH (2000) $,SAAR-US
20	YP@V00C	PERS INCOME LESS TRSF PMT CH 2000 $,SA-US
21	SAVPER	PERS SAVING,BILLIONS OF $,SAAR-US
22	SAVPRATE	PERS SAVING AS PERCENTAGE OF DISP PERS INCOME,PERCENT,SAAR-US
		Employment and Hours
23	LHEL	INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS (1967=100;SA)
24	LHELX	EMPLOYMENT: RATIO; HELP-WANTED ADS:NO. UNEMPLOYED CLF
25	LHEM	CIVILIAN LABOR FORCE: EMPLOYED, TOTAL (THOUS.,SA)
26	LHNAG	CIVILIAN LABOR FORCE: EMPLOYED, NONAGRIC.INDUSTRIES (THOUS.,SA)
27	LHTUR	UNEMPLOYMENT RATE: BOTH SEXES, 16-19 YEARS (%,SA)
28	LHU14	UNEMPLOY.BY DURATION: PERSONS UNEMPL.5 TO 14 WKS (THOUS.,SA)
29	LHU15	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 WKS + (THOUS.,SA)
30	LHU26	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 TO 26 WKS (THOUS.,SA)
31	LHU27	UNEMPLOY.BY DURATION: PERSONS UNEMPL.27 WKS + (THOUS,SA)
32	LHU5	UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5 WKS (THOUS.,SA)
33	LHU680	UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA)
34	LHUEM	CIVILIAN LABOR FORCE: UNEMPLOYED, TOTAL (THOUS.,SA)
35	AHPCON	AVG HR EARNINGS OF PROD WKRS: CONSTRUCTION ($,SA)
36	AHPMF	AVG HR EARNINGS OF PROD WKRS: MANUFACTURING ($,SA)
37	PMEMP	NAPM EMPLOYMENT INDEX (PERCENT)
38	CES002	EMPLOYEES ON NONFARM PAYROLLS - TOTAL PRIVATE
39	CES003	EMPLOYEES ON NONFARM PAYROLLS - GOODS-PRODUCING
40	CES004	EMPLOYEES ON NONFARM PAYROLLS - NATURAL RESOURCES AND MINING
41	CES011	EMPLOYEES ON NONFARM PAYROLLS - CONSTRUCTION
42	CES015	EMPLOYEES ON NONFARM PAYROLLS - MANUFACTURING
43	CES017	EMPLOYEES ON NONFARM PAYROLLS - DURABLE GOODS
44	CES033	EMPLOYEES ON NONFARM PAYROLLS - NONDURABLE GOODS
45	CES046	EMPLOYEES ON NONFARM PAYROLLS - SERVICE-PROVIDING
46	CES048	EMPLOYEES ON NONFARM PAYROLLS - TRADE, TRANSPORTATION, AND UTILITIES
47	CES049	EMPLOYEES ON NONFARM PAYROLLS - WHOLESALE TRADE
48	CES053	EMPLOYEES ON NONFARM PAYROLLS - RETAIL TRADE
49	CES088	EMPLOYEES ON NONFARM PAYROLLS - FINANCIAL ACTIVITIES
50	CES140	EMPLOYEES ON NONFARM PAYROLLS - GOVERNMENT
51	CES151	AVERAGE WEEKLY HOURS OF PRIVATE NONFARM PAYROLLS - GOODS-PRODUCING
52	CES153	AVERAGE WEEKLY HOURS OF WORKERS ON PRIVATE NONFARM PAYROLLS - CONSTRUCTION
53	CES154	AVERAGE WEEKLY HOURS OF WORKERS ON PRIVATE NONFARM PAYROLLS - MANUFACTURING
54	CES155	AVERAGE WEEKLY HOURS OF WORKERS ON PRIVATE NONFARM PAYROLLS - MANUFACTURING OVERTIME HOURS
55	CES156	AVERAGE WEEKLY HOURS OF PRIVATE NONFARM PAYROLLS - DURABLE GOODS
56	CES275	AVERAGE HOURLY EARNINGS OF PRIVATE NONFARM PAYROLLS - GOODS-PRODUCING
57	CES277	AVERAGE HOURLY EARNINGS OF PRIVATE NONFARM PAYROLLS - CONSTRUCTION

No.	Series Code	
58	CES278	AVERAGE HOURLY EARNINGS OF PRIVATE NONFARM PAYROLLS - MANUFACTURING
		Real Consumption
59	JQCR	REAL PERSONAL CONS EXP QUANTITY INDEX (200=100), SAAR
60	JQCNR	REAL PERSONAL CONS EXP-NONDURABLE GOODS QUANTITY INDEX (200=100), SAAR
61	JQCDR	REAL PERSONAL CONS EXP-DURABLE GOODS QUANTITY INDEX (200=100), SAAR
62	JQCSVR	REAL PERSONAL CONS EXP-SERVICES QUANTITY INDEX (200=100), SAAR
		Real Inventories and Orders
63	MOCMQ	NEW ORDERS (NET) - CONSUMER GOODS & MATERIALS, 1996 DOLLARS (BCI)
64	MSONDQ	NEW ORDERS, NONDEFENSE CAPITAL GOODS, IN 1996 DOLLARS (BCI)
65	PMDEL	NAPM VENDOR DELIVERIES INDEX (PERCENT)
66	PMNO	NAPM NEW ORDERS INDEX (PERCENT)
67	PMNV	NAPM INVENTORIES INDEX (PERCENT)
		Housing Starts
68	HUSTSZ	HOUSING STARTS: TOTAL NEW PRIV HOUSING UNITS (THOUS.,SAAR)
69	HSFR	HOUSING STARTS:NONFARM(1947-58);TOTAL FARM & NON- FARM(1959-)(THOUS.,SA
70	HSMW	HOUSING STARTS:MIDWEST(THOUS.U.)S.A.
71	HSNE	HOUSING STARTS:NORTHEAST (THOUS.U.)S.A.
72	HSSOU	HOUSING STARTS:SOUTH (THOUS.U.)S.A.
73	HSWST	HOUSING STARTS:WEST (THOUS.U.)S.A.
		Foreign Exchange
74	EXRCAN	FOREIGN EXCHANGE RATE: CANADA (CANADIAN $ PER U.S.$)
75	EXRUK	FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND)
76	EXRUS	UNITED STATES;EFFECTIVE EXCHANGE RATE(MERM)(INDEX NO.)
		Price Index
77	PMCP	NAPM COMMODITY PRICES INDEX (PERCENT)
78	PW561	PRODUCER PRICE INDEX: CRUDE PETROLEUM (82=100,NSA)
79	PWCMSA	PRODUCER PRICE INDEX:CRUDE MATERIALS (82=100,SA)
80	PWFCSA	PRODUCER PRICE INDEX:FINISHED CONSUMER GOODS (82=100,SA)
81	PWFSA	PRODUCER PRICE INDEX: FINISHED GOODS (82=100,SA)
82	PWIMSA	PRODUCER PRICE INDEX:INTERMED MAT.SUPPLIES & COMPONENTS(82=100,SA)
83	PUNEW	CPI-U: ALL ITEMS (82-84=100,SA)
84	PUS	CPI-U: SERVICES (82-84=100,SA)
85	PUXF	CPI-U: ALL ITEMS LESS FOOD (82-84=100,SA)
86	PUXHS	CPI-U: ALL ITEMS LESS SHELTER (82-84=100,SA)
87	PUXM	CPI-U: ALL ITEMS LESS MIDICAL CARE (82-84=100,SA)
88	PUXX	CPI-U: ALL ITEMS LESS FOOD AND ENERGY (82-84=100,SA)
89	PUC	CPI-U: COMMODITIES (82-84=100,SA)
90	PUCD	CPI-U: DURABLES (82-84=100,SA)
91	PU83	CPI-U: APPAREL & UPKEEP (82-84=100,SA)
92	PU84	CPI-U: TRANSPORTATION (82-84=100,SA)
93	PU85	CPI-U: MEDICAL CARE (82-84=100,SA)
		Stock Price
94	FSDJ	COMMON STOCK PRICES: DOW JONES INDUSTRIAL AVERAGE
95	FSDXP	S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD (% PER ANNUM)
96	FSPCOM	S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10)
97	FSPIN	S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-43=10)
98	FSPXE	S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO (%,NSA)
		Money and Credit Aggregate
99	FM1	MONEY STOCK: M1(CURR,TRAV.CKS,DEM DEP,OTHER CK'ABLE DEP)(BIL$,SA)
100	FM2	MONEY STOCK:M2(M1+O'NITE RPS,EURO$,G/P&B/D MMMFS&SAV&SM TIME DEP(BIL$,
101	CCINRV	CONSUMER CREDIT OUTSTANDING - NONREVOLVING(G19)

No.	Series Code	
		Miscellaneous
102	UOMO83	LEADING INDEX COMPONENT OF CONSUMER EXPECTATIONS UNITS: NSA, CONFBOARD AND U.MICH.
		Interest Rates and Bonds
103	FYGM3	INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,3-MO.(% PER ANN,NSA)
104	FYGM6	INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,6-MO.(% PER ANN,NSA)
105	FYGT1	INTEREST RATE: U.S.TREASURY CONST MATURITIES,1-YR.(% PER ANN,NSA)
106	FYGT10	INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR.(% PER ANN,NSA)
107	FYGT20	INTEREST RATE: U.S.TREASURY CONST MATURITIES,20-YR.(% PER ANN,NSA)
108	FYGT3	INTEREST RATE: U.S.TREASURY CONST MATURITIES,3-YR.(% PER ANN,NSA)
109	FYGT5	INTEREST RATE: U.S.TREASURY CONST MATURITIES,5-YR.(% PER ANN,NSA)
110	FYPR	PRIME RATE CHG BY BANKS ON SHORT-TERM BUSINESS LOANS(% PER ANN,NSA)
111	FYAAAC	BOND YIELD: MOODY'S AAA CORPORATE (% PER ANNUM)
112	FYAAAM	BOND YIELD: MOODY'S AAA MUNICIPAL (% PER ANNUM)
113	FYAC	BOND YIELD: MOODY'S A CORPORATE (% PER ANNUM,NSA)
114	FYAVG	BOND YIELD: MOODY'S AVERAGE CORPORATE (% PER ANNUM)
115	FYBAAC	BOND YIELD: MOODY'S BAA CORPORATE (% PER ANNUM)
116	SFYGM3	FYGM3-FYFF
117	SFYGM6	FYGM6-FYFF
118	SFYGT1	FYGT1-FYFF
119	SFYGT5	FYGT5-FYFF
120	SFYGT10	FYGT10-FYFF
121	SFYAAAC	FYAAAC-FYFF
122	SFYBAAC	FYBAAC-FYFF
123	FYFF	INTEREST RATE: FEDERAL FUNDS (EFFECTIVE) (% PER ANNUM,NSA)
124	Bspread10Y	FYBAAC-FYGT10

Figure 1: **Variance Term Structure and Economic Variables**

R-squares from univariate regressions of the risk factors from the variance term structure on 124 macroeconomic and financial variables. See Table VIII for a description of each series. Each bar sums the R^2 of the univariate regression of each risk factor on one of the economic or financial indicators. The factors are estimated using end-of-month data from January 1996 to July 2008.

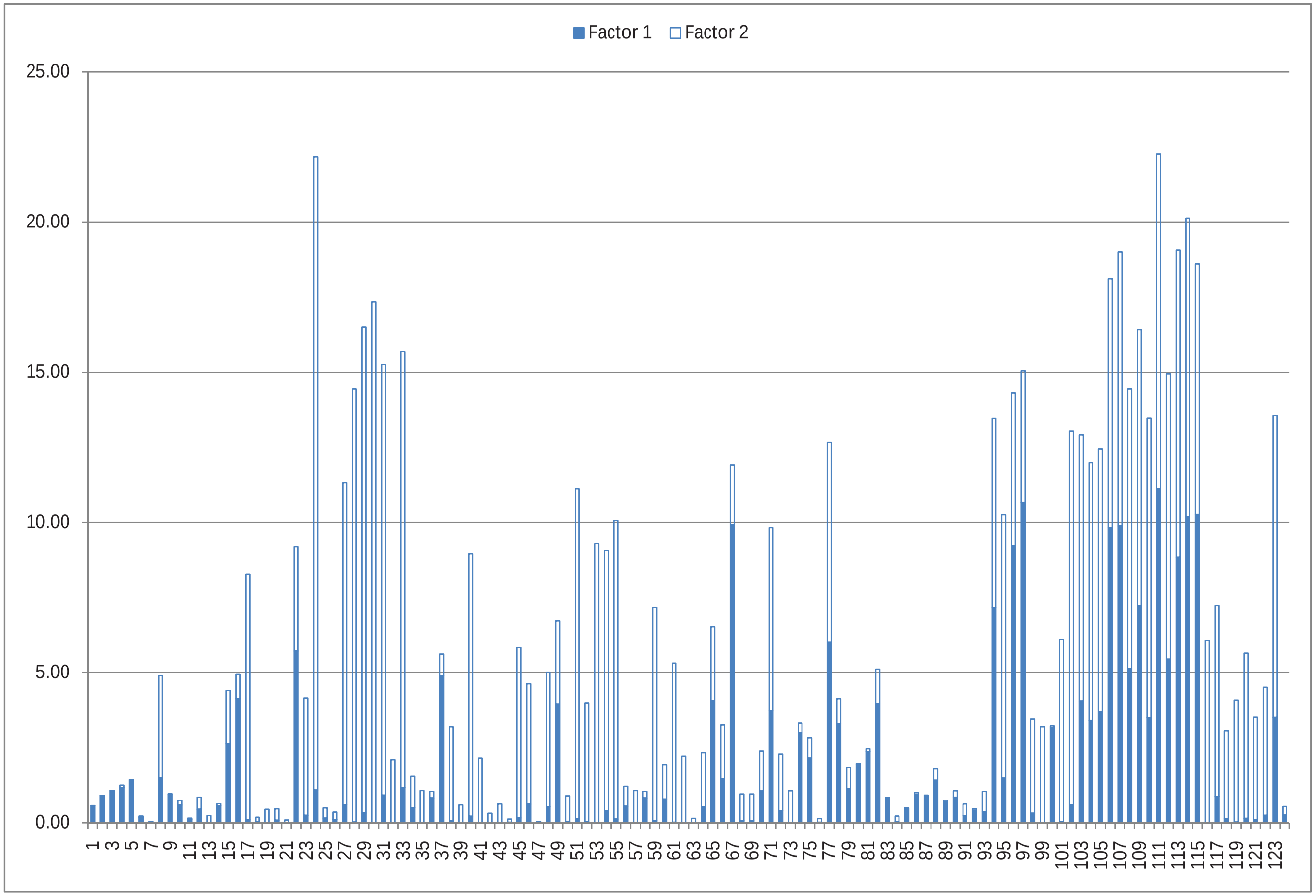

Figure 2: **Variance, Skewness and Kurtosis Term Structures and Economic Variables**

R-squares from univariate regressions of two risk factors from the term structures of variance, skewness and kurtosis on 124 macroeconomic and financial variables. See Table VIII for a description of each series. Each bar sums the R^2 of the univariate regression of each risk factor on one of the economic or financial indicators. The factors are estimated using end-of-month data from January 1996 to July 2008.