Comparative Analysis of Credit Risk Models for Loan Portfolios

Chulwoo Han

University of Durham
chulwoo.han@durham.ac.uk

Abstract

In this article, I compare credit risk models that are used for loan portfolios, both from a theoretical perspective and via simulation studies. My study is distinct from previous studies by including new models, considering sector correlation, and performing comprehensive sensitivity analysis. CreditRisk++, CreditMetrics, Basel II internal rating based method, and Mercer Oliver Wyman's model are considered. Risk factor distribution and the relationship between risk components and risk factors are the key distinguishing characteristics of each model. CreditRisk++, due to its extra degree of freedom, has the highest flexibility to fit various loss distributions. It turns out that sector covariance is the most important risk component for risk management in terms of risk sensitivity. Risk sensitivities not only differ among models but also depend on the input parameters and the quantile at which risk is measured. This implies that risk models can only be judged in terms of the portfolio under consideration, and banks should evaluate them based on their own portfolios.

Keywords: Credit risk model, Sector correlation, Loan portfolio, Comparison analysis.

1 Introduction

The importance of credit risk management cannot be overemphasized, and it is well recognized by the banking industry. Most financial institutions already have a credit risk management system and continue to improve it. The introduction of the Third Basel Capital Accord (Basel III) triggered by the credit crisis in 2008 and other changes in the global financial market have accelerated the need to validate and improve currently deployed credit risk management systems. Despite the well-recognized importance of credit risk management, it is also true that many, especially small, domestic banks use their credit risk management system without clearly understanding the assumptions and characteristics of the underlying model. This is partly because of the complexity behind the credit risk models, but also because banks, without thorough evaluation, tend to rely on the opinion of consultancies or the cases of other banks. Also, there are limited resources that provide information on the credit risk models with comprehensive comparative analysis. Therefore, this article aims to provide useful information

for banks which consider validating or upgrading their credit risk models, by comparing widely used credit risk models through various simulation studies. Instead of repeating original derivation of each model, which can be easily found elsewhere, models are approached from the same perspective so as to reveal similarities and differences more clearly. The focus is on the credit risk models that are appropriate for loan portfolios or other assets with no observable market values. The models considered are: CreditRisk++, CreditMetrics, a model by Mercer Oliver Wyman (MOW), and the internal rating based (IRB) model of Basel II.

Gordy (2000), whose work serves as the basis of this study, compares CreditRisk+ and CreditMetrics, and demonstrates how one can be converted to the other. He shows, via empirical studies, that credit risk is particularly sensitive to the volatility of probability of default, and that CreditRisk+ tends to measure credit risk higher than CreditMetrics. In the same period, Crouhy *et al.* (2000) also perform a comparative research on credit risk models. Their study covers more models including KMV model and McKinsey's CreditPortfolioView. However, they mainly summarize each model and fail to provide new insights. Frey and McNeil (2001) take a different approach in comparing CreditRisk+ and CreditMetrics. They consider various distribution assumptions for each model and analyze the effects of the distributions on credit risk. They demonstrate that different distribution assumptions lead to significantly different credit risks, even under the same default correlation. Diaz and Gemmill (2002), unlike other studies where only two states — default and no default — are assumed, compare CreditRisk+ and CreditMetrics allowing rating migration.

This study distinguishes itself from previous studies in the following aspects. First of all, new credit risk models, such as CreditRisk++, MOW, and Basel IRB, are included in the comparison group. Banks using the internal rating based method of the Basel II are required to calculate regulatory capital using IRB model. Hence, it is important to understand the model and compare it with other models. Secondly, while other comparative studies assume only single risk factor, multiple risk factors are considered in this study. This enables us to take the risk factor correlation into account and investigate its effect on credit risk. This is particularly important since credit risk is known to be very sensitive to the risk factor correlation. Finally, a comprehensive sensitivity analysis is given via various simulations. This analysis identifies the input parameters to which credit risk is sensitive and that need a special attention in credit risk management. There are studies that compare risk of a portfolio using different models, but to my best knowledge, there is no such a study that conducts sensitivity analysis, which is much more informative.

This article is organized as follows. In Section 2, key assumptions and methodologies of each model are briefly described. Key differences among the models are also summarized at the end of this section. Section 3 is devoted to simulation studies, in which each model's risk sensitivities to input parameters are assessed and compared to each other. Concluding remarks and suggestions are given in Section 4.

2 Theoretical Comparison of Credit Risk Models

In this section, I briefly describe the credit risk models with a focus on their key assumptions and main results, and address differences among the models from a theoretical perspective.

2.1 Notations

Following notations are used throughout the article. Any other notations that are used will be described when they first appear. A subset of a portfolio is referred to as either a sub-portfolio or sector. A variable with subscript i is associated with an individual asset, and a variable with p is associated with a portfolio.

X : Risk factors. $X = \{X_1, \cdots, X_K\}$
w_i : Asset i's weights on the risk factors. $w_i = \{w_{i1}, \cdots, w_{iK}\}$
R_i : Return on asset i
P_i : Probability of default (PD) of asset i. $P_i(x) = \{P_i | X = x\}$
PD_i : Unconditional PD of asset i, *i.e.*, $PD_i = E[P_i(X)]$
U_i : Loss given default (LGD) of asset i. $U_i(x) = \{U_i | X = x\}$
LGD_i : Unconditional LGD of asset i, *i.e.*, $LGD_i = E[U_i(X)]$
A_i : Exposure at default (EaD) of asset i. Assumed constant.
Y_i : Default indicator. If i defaults, $Y_i = 1$; otherwise, $Y_i = 0$.
$\quad E[Y_i] = PD_i$
ρ_{ij} : Asset correlation coefficient, *i.e.*, $\rho_{ij} = Cov(R_i, R_j)$
ρ_{ij}^D : Default correlation coefficient, *i.e.*, $\rho_{ij}^D = Cov(Y_i, Y_j)$
N : Number of assets in a portfolio
VaR^α : Value-at-Risk at the probability level, α.

2.2 Systemic Risk

Most credit risk models assume one or several systemic risk factors, and define some of the risk components as a function of the risk factors. The most common practice is to assume PD or return on asset as a linear function of the risk factors, while more complex models may also assume LGD as a function of the risk factors. Care should be taken in the latter case since the usual definition of expected loss, $EL = A \cdot PD \cdot LGD$, may not hold due to the correlation between PD and LGD. Below are expositions of some of commonly used relationships between risk components and risk factors.

CreditRisk+ In CreditRisk+, PD is assumed to be a linear function of the risk factors, *i.e.*,

$$P_i = PD_i \left(\sum_{k=0}^{K} w_{ik} X_k \right) \tag{1}$$

where $X_0 = 1$, and X_k $(k > 0)$ are Gamma distributed independent risk factors with mean 1 and variance σ_k^2. Sum of the weights, $\sum_k w_{ik}$, is equal to 1. The

PD conditional on $X = x$ has the form

$$P_i(x) = PD_i \left(\sum_{k=0}^{K} w_{ik} x_k \right) \qquad (2)$$

CreditMetrics While CreditRisk+ assumes PD as a linear function of the risk factors, many other models assume asset return, or more generally latent variable, as a linear function of the risk factors. CreditMetrics and IRB model are in this category and they assume the following.

$$R_i = w_i X + \psi_i \epsilon_i \qquad (3)$$

The systemic risk factor X is $N(0, \Omega)$, and the idiosyncratic risk factors ϵ_i are i.i.d. $N(0, 1)$. Without loss of generality, R_i can be assumed to follow the standard normal distribution, in which case, we have $\psi_i = \sqrt{1 - w_i' \Omega w_i}$. Default can be defined as the case when the return falls below a certain threshold. Denoting the threshold λ_i, the PD conditional on $X = x$ is given by

$$P_i(x) = \Pr(w_i x + \psi_i \epsilon_i \le \lambda_i) = \Pr\left(\epsilon_i \le \frac{\lambda_i - w_i x}{\psi_i} \right) = \Phi\left(\frac{\lambda_i - w_i x}{\psi_i} \right). \qquad (4)$$

Since PD is the probability that R_i is less than or equal to λ_i and $R_i \sim N(0, 1)$, the following holds.

$$PD_i = \Pr(R_i \le \lambda_i) = \Phi(\lambda_i) \rightarrow \lambda_i = \Phi^{-1}(PD_i) \qquad (5)$$

Thus, equation (4) can be rewritten as

$$P_i(x) = \Phi\left(\frac{\Phi^{-1}(PD_i) - w_i x}{\psi_i} \right). \qquad (6)$$

From equation (2) and (6), we can see that the conditional PD is a linear function of the risk factors in CreditRisk+, while it is a nonlinear function in CreditMetrics. In contrast, asset return is a nonlinear function of the risk factors in CreditRisk+ and has the form (Gordy, 2000)

$$R_i = \left(\sum_{k=0}^{K} w_{ik} X_k \right)^{-1} e_i. \qquad (7)$$

where e_i is i.i.d. and has an exponential distribution with parameter value of 1.

2.3   CreditRisk++

CreditRisk+ As seen earlier in this section, CreditRisk+ assumes that probability of default is governed by several risk factors that are Gamma distributed and mutually independent. Therefore, the default correlation between assets is determined by the weights of each asset on the risk factors. Portfolio loss distribution is defined by the probability generating function (PGF) of the form

$$G(z) = \exp\left[-Q_0(1) + Q_0(z) - \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \ln(1 + \sigma_k^2 Q_k(1) - \sigma_k^2 Q_k(z)) \right] \qquad (8)$$

where

$$Q_k(z) = \sum_{i=1}^{n} w_{ik} PD_i z^{A_i}, \quad k = 0, \ldots, K. \tag{9}$$

By differentiating the PGF, portfolio loss distribution and risk measures such as Value-at-Risk can be obtained. For detailed derivation and numerical calculation of the PGF, readers are referred to CSFP (1997) or Gundlach and Lehrbass (2003) (Ch. 2, 5, 7).

CreditRisk++ Even though it is theoretically possible to incorporate asset correlation in CreditRisk+ by appropriately choosing weights, defining independent risk factors and determining weights on them is difficult and impractical. For this reason, extensions of the original model that explicitly take the correlation into account have been introduced, and one of the latest developments is CreditRisk++ by Han and Kang (2008). Creditrisk++ assumes that a correlated risk factor can be divided into a sector specific term Y_k and a macroeconomic term $\hat{Y}$ that are independent of each other, *i.e.*,

$$X_k = \delta_k Y_k + \gamma_k \hat{Y}, \qquad k = 1, \ldots, K \tag{10}$$

where

$$Y_k \quad \sim \quad Gamma\,(\theta_k, 1) \tag{11}$$
$$\hat{Y} \quad \sim \quad Gamma\,(\hat{\theta}, 1) \tag{12}$$

Then, the probability of default can be rewritten as a linear combination of $K + 1$ Gamma distributed independent risk factors.

$$P_i = PD_i \left(w_{0i} + \sum_{k=1}^{K+1} w_{ki} \hat{X}_k \right) \tag{13}$$

where

$$\hat{X}_k \quad \sim \quad Gamma\,(\theta_k, \delta_k), \quad k = 1, \ldots, K, \tag{14}$$
$$\hat{X}_{K+1} \quad \sim \quad Gamma\,\left(\hat{\theta}, 1\right), \quad \text{and} \tag{15}$$
$$w_{i(K+1)} \quad = \quad \sum_{k=1}^{K} w_{ik} \gamma_k \tag{16}$$

$\hat{X}_k,\ k = 1, \ldots, K$ are sector specific risk factors and $\hat{X}_{K+1}$ is a macroeconomic risk factor that has influence on all sectors. The degree of influence is determined by δ_k and γ_k. The expected values and covariance matrix of the correlated risk factors, X_k, have the form

$$E[X_k] \quad = \quad \delta_k \theta_k + \gamma_k \hat{\theta}, \tag{17}$$
$$V[X_k] \quad = \quad \delta_k^2 \theta_k + \gamma_k^2 \hat{\theta}, \tag{18}$$
$$Cov[X_k, X_l] \quad = \quad \gamma_k \gamma_l \hat{\theta}. \tag{19}$$

Appropriately choosing the parameter values, various covariance structures can be described by CreditRisk++. The parameters can be estimated by minimizing the distance between observed covariance matrix and the covariance matrix

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defined above. The main advantage of CreditRisk++ is that it can incorporate risk factor correlations in a very flexible and intuitive manner, while maintaining the framework of CreditRisk+. Therefore all the numerical algorithms developed for CreditRisk+ can be reused without modification.

2.4 CreditMetrics

I consider a two state - default and no default - version of CreditMetrics (CM2S) that is appropriate for loan portfolios. CM2S assumes homogeneity of assets in the same sector:

1. There are many assets in one sector and the size of each asset is sufficiently small compared to the size of the sector.

2. The assets in one sector are governed by one risk factor that is associated with the sector.

3. All the assets in the same sector have the same PD, LGD, and asset correlation.

4. Risk factors are normally distributed and mutually correlated.

In CM2S, asset return is a linear function of the associated risk factor. If asset i belongs to sector k,

$$R_i = w_{ik} X_k + \psi_i \epsilon_i \tag{20}$$

where X_k and ϵ_i are normally distributed and mutually independent. As derived in Gordy (2002), VaR of each sector has the form

$$VaR^\alpha = \sum_{i \in k} A_i \cdot LGD_i \cdot P_i(X_k^\alpha) \tag{21}$$

where X^α denotes the value of X at a probability level α. The equation indicates that VaR is the sum of the expected losses of the assets conditional on that X has the value at the probability level α. Under the homogeneity assumptions, equation (21) can be rewritten as

$$VaR^\alpha = A_p \cdot LGD \cdot \Phi \left(\frac{\Phi^{-1}(PD) - \sqrt{\rho}\Phi^{-1}(\alpha)}{\sqrt{1-\rho}} \right) \tag{22}$$

If sectors are perfectly positively correlated, VaR of a portfolio becomes the sum of VaR's of the sectors. Otherwise, Monte Carlo simulation should be employed to obtain portfolio loss distribution and VaR.

2.5 Mercer Oliver Wyman

Mercer Oliver Wyman (2004) developed a proprietary credit risk model for their clients and it is being used by several financial institutions. MOW is included because it is distinguished from other models in the way of specifying portfolio distribution: MOW does not specify risk factors but rather explicitly assumes the portfolio loss distribution as a beta distribution. Below is a brief derivation of the model.

Default indicator of asset i, Y_i, has the expected value and variance of the form

$$E[Y_i] = PD_i \tag{23}$$

$$V[Y_i] = PD_i(1 - PD_i) \tag{24}$$

LGD of asset i, U_i, is assumed to have the expected value and variance of the form

$$E[U_i] = LGD_i \tag{25}$$

$$V[U_i] = LGD_i(1 - LGD_i)/2 \tag{26}$$

The variance equation implies that the uncertainty in LGD is relatively small when LGD is either very high or low, and it is maximized when LGD is 0.5. If LGD is independent of default, the loss from unit exposure of asset i, $Y_i U_i$, has the expected value and standard deviation of the form

$$\mu_i = PD_i \cdot LGD_i \tag{27}$$

$$\sigma_i = \sqrt{LGD_i^2 \cdot PD_i(1 - PD_i) + PD_i \cdot LGD_i(1 - LGD_i)/2} \tag{28}$$

If the default correlation between assets is ρ^D and LGD is mutually independent, when there are a large number of assets, the expected value and the standard deviation of the portfolio loss per unit exposure can be approximated as

$$\mu_p = \frac{1}{A_p} \sum_{i=1}^{N} A_i \cdot PD_i \cdot LGD_i \tag{29}$$

$$\sigma_p = \sqrt{\sum_{i=1}^{N} \sigma_i^2 + \sum_{i=1}^{N} \sum_{j \neq i}^{N} \sigma_i \sigma_j \rho^D} = \sqrt{\rho^D} \sum_{i=1}^{N} \sigma_i \tag{30}$$

To calculate risk at a probability level, portfolio loss distribution needs to be defined. MOW explicitly assumes that the portfolio loss is beta distributed, and finds the distribution so that the expected value and the standard deviation of the distribution are equal to those derived above. That is, letting the portfolio loss distribution be $Be(a, b)$,

$$a = \mu_p^2 \frac{1 - \mu_p}{\sigma_p^2} - \mu_p \tag{31}$$

$$b = \frac{a}{\mu_p} - a \tag{32}$$

VaR is then calculated from the equation

$$VaR^\alpha = A_p \cdot L_p^\alpha \tag{33}$$

where, $L_p^\alpha = F_\beta^{-1}(\alpha; a, b)$, and F_β is the cumulative distribution function of the beta distribution.

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2.6 Basel II IRB

Banks adopting the internal rating based method are required to calculate the regulatory capital for credit risk using the following formula

$$K = LGD_i \left(\Phi \left(\frac{\Phi^{-1}(PD_i) + \sqrt{\rho}\Phi^{-1}(0.999)}{\sqrt{1-\rho}} \right) - PD_i \right) \frac{1 + (M_i - 2.5)b(PD_i)}{1 - 1.5b(PD_i)}. \tag{34}$$

$\frac{1+(M_i-2.5)b(PD_i)}{1-1.5b(PD_i)}$ is called maturity adjustment, where M_i is the maturity of the asset and $b(PD_i)$ has the form

$$b(PD_i) = (0.11852 - 0.05478 \log(PD_i))^2 \tag{35}$$

0.999 in (34) indicates that the IRB model calculates the capital based on the risk at 99.9% probability level, and this number can be replaced by another number for VaR at a different probability level. Since K has the meaning of unexpected loss, VaR^α in our context is obtained by adding expected loss to K and summing up over the assets:

$$VaR^\alpha = \sum_{i=1}^{n} A_i \cdot K_i^\alpha, \tag{36}$$

$$K_i^\alpha = LGD_i \cdot \Phi \left(\frac{\Phi^{-1}(PD_i) + \sqrt{\rho}\Phi^{-1}(\alpha)}{\sqrt{1-\rho}} \right) \frac{1 + (M_i - 2.5)b(PD_i)}{1 - 1.5b(PD_i)} \tag{37}$$

This equation is similar to equation (22) because the fundamental assumptions of IRB are identical to those of CreditMetrics. IRB is different from CreditMetrics in that there is only one risk factor, assets are not necessarily homogeneous, and there exists the maturity adjustment term. The maturity adjustment is an increasing function of maturity and has a value of one when the maturity is one year. This reflects, as noted in Basel (2005), the fact that long-term credits are riskier than short-term credits. Since $b(PD_i)$ is inversely proportional to PD, default risk increases faster with the maturity when the PD is lower. This is because low PD assets have more room to deteriorate than high PD ones, which are already risky.

2.7 Summary

The main features of the models are compared in Table 1. The fundamental difference of each model is the distribution assumption of the risk factors and the relationship between default and the risk factors. Other assumptions, *e.g.*, homogeneity of assets in CM2S or LGD variance of MOW, can be easily relaxed without affecting the fundamentals of the models.

3 Simulation Analysis

In this section, I demonstrate the characteristics of each model by comparing the credit risks of various portfolios. In particular, the risk sensitivity of each model to input parameters is addressed. Simulation methods are described first and interpretation of the simulation results follows.

	CR++	CM2S	MOW	IRB
No. of Risk Factors	Many	Many		One
Risk Factors Distribution	Gamma	Normal	Beta (loss distribution)	Normal
Risk Factor Relationship	Linear with PD	Linear with asset return	Not specified	Linear with asset return
Risk Factor-Asset	Many - 1	1 - 1		1 - 1
Risk Calculation	Closed form (Numerical)	Simulation	Closed form	Closed form
Others			Stochastic LGD	Maturity adjustment

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Table 1: Comparison of Credit Risk Models

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3.1 Simulation Methods

Credit risks of the seven model portfolios (PF1 - PF7) in Table 2 are measured using the credit risk models described in Section 2. The first portfolio is chosen as a base loan portfolio and the rest portfolios are constructed so as to address the effect of an input parameter on credit risk. In each portfolio, all assets are assumed to have the same EaD, PD, LGD, and asset correlation.[1] By shifting one of these parameters in each portfolio, the risk sensitivity to each parameter is investigated. One risk factor, or equivalently perfect correlation between the risk factors, is assumed for PF1 to PF5, and two risk factors with correlation coefficient of 0.5 and 0.0 is assumed for PF6 and PF7, respectively. When there are two risk factors, the assets are assumed to be equally distributed into two sectors.

The data provided in Table 2 are sufficient to calculate risk using CM2S, while additional parameters need to be identified for CR++ and MOW: the risk factor weights and the risk factor standard deviations in CR++, and the default correlation in MOW. In CR++, the weights and the standard deviations of the risk factors are obtained from the equation

$$w_k \sigma_k = \sigma(P)/PD \tag{38}$$

where

$$\sigma^2(P) = \text{BIVAR}(\Phi^{-1}(PD), \Phi^{-1}(PD), \rho) - PD^2 \tag{39}$$

As the assets are assumed to be homogeneous, the subscripts are omitted for simplicity. BIVAR denotes the cumulative bivariate normal distribution. As you can see from equation (38), the weights and the standard deviations of the risk factors cannot be determined uniquely. A typical value for w_k used in practice is 1 or a high value above 0.5. Thus, I consider two cases, $w_k = 0.5$ and $w_k = 1.0$, and compare the results. When w_k is higher, PD becomes more dependent upon the risk factors, while, $w_k \sigma_k$ being constant, the volatility of the risk factor becomes smaller. The default correlation in MOW is calculated

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[1]Gordy (2000) shows that exposure variation in a portfolio does not affect the credit risk significantly. It is also a common practice to cluster assets of similar properties into a sector. Therefore, I assume homogeneity of assets and focus on the overall shift of each input parameter.

from the equation

$$\rho^D = \frac{\sigma^2(P)}{PD(1 - PD)}.$$ (40)

Both constant (zero variance) and stochastic LGD are considered under MOW.

IRB is identical to CM2S except for the maturity adjustment for the model portfolios considered here. Since the effect of the maturity adjustment on VaR is obvious, IRB is excluded from simulation analysis. MOW is excluded from the risk calculation of PF6 and PF7, as it is one factor model.

	PF1	PF2	PF3	PF4	PF5	PF6	PF7
N	10,000	10,000	10,000	10,000	10,000	10,000	10,000
EaD	100	100	100	100	100	100	100
PD (%)	0.5	0.1	2.0	0.5	0.5	0.5	0.5
LGD (%)	30.0	30.0	30.0	60.0	30.0	30.0	30.0
Asset corr	0.1	0.1	0.1	0.1	0.2	0.1	0.1
Sector corr						0.5	0.0

Table 2: Portfolios for Simulation

This table presents seven portfolios (PF1 ∼ PF7) considered for simulation. Each portfolio is designed to reveal the risk sensitivity of each model with respect to an input parameter. The assets in each portfolio are assumed to have the same exposure (EaD), probability of default (PD), loss given default (LGD), and asset correlation (Asset corr). The assets in PF1 to PF5 are assumed to be allocated to a single sector, while the assets in PF6 and PF7 are assumed to be allocated equally to two sectors with the sector correlation (Sector corr) given in the table.

3.2 Simulation Results

Simulation results are reported in Table 3. I first focus on the results of the base portfolio PF1. The credit risks by CR++ with $w_k = 1$ and MOW with constant LGD are similar, while the credit risk by CM2S is slightly higher. The highest credit risk is attained from CR++ with $w_k = 0.5$. This is because, as seen in equation (38), a smaller weight implies a larger standard deviation of the risk factor, which results in a higher probability at the tail region. This becomes more evident if we compare two CR++ risk curves in Figure 1, where the credit risks of PF1 at different quantiles are displayed: Credit risk increases more rapidly with the probability level when the factor weight is smaller. Comparison of the two MOW models reveals how uncertainty in LGD can increase the risk. Ignoring the uncertainty can cause significant underestimation of the risk. As noted by Giese (2005), stochastic LGD can be even more influential when it is positively correlated with PD, which is the usual case. The difference among models can be attributed mostly to the distribution assumption of each model; gamma distribution of CR++, normal distribution of CM2S and IRB, and beta distribution of MOW.

PF2 and PF3 are designed to assess the sensitivity of the credit risk to the PD, by setting the PD of PF2 to 0.1% (80% decrease from PF1) and the PD of PF3 to 2.0% (400% increase from PF1), while other input parameters are

| | | CR++ | | CM2S | MOW | |
		$w_k = 1.0$	$w_k = 0.5$		Sto. LGD	Fix. LGD
	99.00%	7,419	8,511	7,862	11,176	7,371
PF1	99.90%	11,649	17,020	13,796	18,698	11,206
	99.99%	16,564	26,702	21,012	26,428	15,045
	99.00%	1,894	2,097	1,960	2,851	1,861
PF2	99.90%	3,357	4,560	3,889	5,089	2,987
	99.99%	5,114	7,373	6,543	7,442	4,139
	99.00%	27,781	27,204	24,707	34,774	23,377
PF3	99.90%	40,138	50,002	38,471	54,543	33,623
	99.99%	52,328	76,293	53,232	74,280	43,640
	99.00%	14,840	17,022	15,724	17,142	14,736
PF4	99.90%	23,300	34,040	27,592	26,911	22,366
	99.99%	33,131	53,404	42,024	36,743	29,968
	99.00%	12,771	13,399	12,905	19,624	12,791
PF5	99.90%	25,220	32,980	27,294	38,296	22,118
	99.99%	39,508	55,733	46,134	58,105	31,784
	99.00%	6,543	7,440	6,633		
PF6	99.90%	10,041	13,281	11,001		
	99.99%	13,683	19,895	16,223		
	99.00%	5,323	6,123	5,686		
PF7	99.90%	7,521	9,675	9,021		
	99.99%	9,669	13,345	12,999		

Table 3: Simulation Results

This table reports the VaR of portfolios PF1-PF7. The second column represents the probability level for risk calculation and the figures are VaR from each risk model.
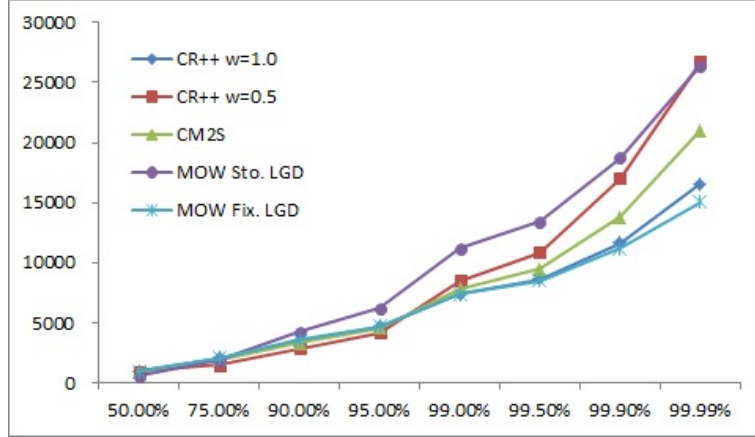
**Figure 1: VaR at Different Probability Levels: PF1**

held constant. The results indicate that the risk sensitivity[2] with respect to the PD is lower than 1. For example, if we compare 99.9% VaR of CM2S, risk decreases by 72% in PF2 and increases by 280% in PF3. The risk sensitivity with respect to the PD is also visualized in Figure 2, where 99.9% VaR of each model is displayed for a range of PD values. The risk curves of CM2S and MOW are concave with respect to the PD, while that of CR++ is almost linear. The linear behavior of CR++ stems from the specification of PD. By specifying PD as a linear function of the risk factors (Equation (1)), default probability only determines the mean of PD but does not affect the overall shape of the loss distribution. It is not clear which model is more compatible with the reality. Given the fact that the expected loss is linear with the PD, it seems reasonable for the unexpected loss to be linear with the PD as well. On the other hand, since an increase of the PD does not necessarily mean an increase of uncertainty, unexpected loss being less sensitive to the PD seems also reasonable.[3]

PF4, while other input parameters being equal, has twice larger LGD than PF1. Since all the models except MOW assume a constant LGD, risk increases linearly with the LGD in these models. As the variance of the LGD in MOW is a parabolic function of the LGD with its maximum at LGD = 0.5, the variance can either increase or decrease depending of the value of LGD. In the sample, the credit risk increases 1.5 times when the LGD doubles from 0.3 to 0.6.

PF5 has twice higher asset correlation than PF1. The asset correlation vs. credit risk diagram is displayed in Figure 3, where 99.9% VaR for different values of the asset correlation is drawn. Obviously, risk becomes larger when the asset correlation increases. What is more intriguing is that, as shown in Figure 3, the risk curves are convex and risk increases sharply as the asset correlation increases. This is contrary to the case of PD. For example, an increase of the asset correlation from 0.1 to 0.2 results in twice larger risk (sensitivity of 1) while increase from 0.1 to 0.9 results in 18 times larger risk (sensitivity of 2) in

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[2]Risk sensitivity is defined as the ratio of the change in credit risk over the change in the input parameter of interest.

[3]Even though the uncertainty in the market remains the same, the variance of PD, $PD(1 - PD)$, increases and the overall risk also increases.
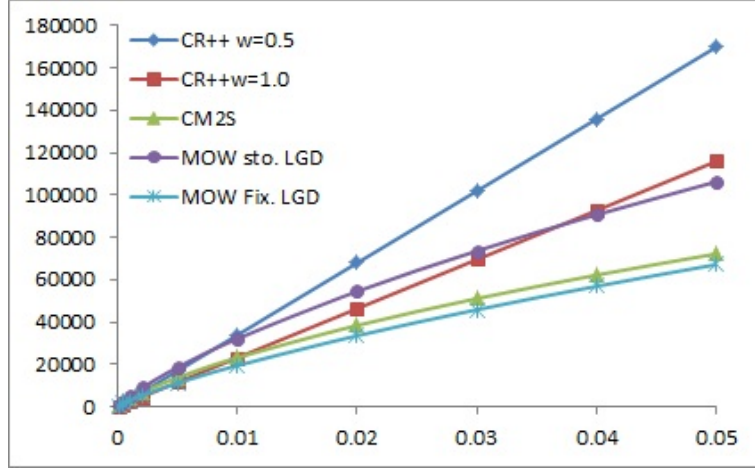
Figure 2: Probability of Default vs. 99.9% VaR

all models except CR++ with $w_k = 0.5$, in which risk increases almost linearly with the asset correlation. Moreover, risk becomes more sensitive to the asset correlation at a higher probability level: From Table 3, the risk sensitivity with respect to the asset correlation is about 1.0 at $\alpha = 99.9\%$, while it ranges from 1.04 to 1.19 at 99.99%. Putting together, the asset correlation becomes a more influential factor at a far tail of the loss distribution, especially when its value is high. This makes the asset correlation a key factor for risk management.
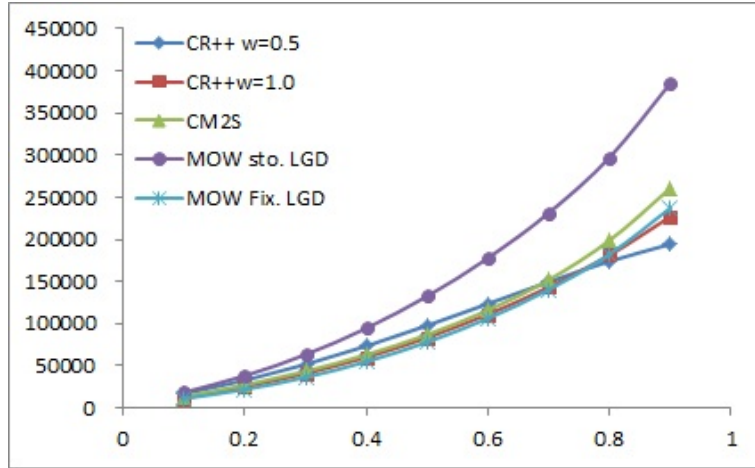
Figure 3: Asset Correlation vs. 99.9% VaR

Finally, PF6 and PF7 are designed to analyze the effect of the sector correlation on risk. Two risk factors are assumed in PF6 and PF7, with the correlation coefficient of 0.5 and 0.0, respectively. PF1 can be regarded as having two perfectly correlated risk factors. The results are reported at the bottom of Table 3.

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The VaR's from multi-factor CM2S are obtained via a Monte Carlo simulation with 1,000,000 iterations. Decrease of the correlation from 1.0 to 0.5 and to 0.0 respectively results in risk reduction of about 20% and 40% in both CR++ and CM2S. For CR++, risk reduction is more apparent when $w_k = 0.5$. Also, risk is reduced more rapidly at a higher probability level. 99.9% VaR's for various sector correlation coefficients are displayed in Figure 4. The risk sensitivity with respect to the sector correlation is more or less similar among the models. To assess the effect of the number of sectors, 99.9% VaR is calculated for the cases of 4, 10, and 50 uncorrelated sectors, and the results are displayed in Figure 5. As expected, risk is reduced as the number of sector increases due to diversification effect. When sectors are highly correlated, diversification benefit becomes much less significant (not reported here). It is remarkable that CR++ with $w_k = 0.5$ enjoys the diversification effect most and the risk changes from the highest when the number of sectors is 1 to lowest when the number of sectors is 50.
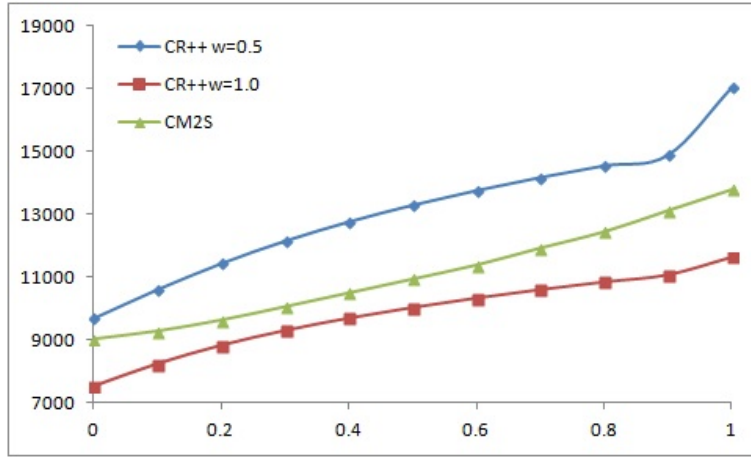

Figure 4: Sector Correlation vs. 99.9% VaR: Two Sectors

Various types of loan portfolios, rare default events, and low credit risk measurement frequency all make it very difficult to evaluate credit risk models and choose one particular model as a standard. Under this circumstance, one important criterion for model selection would be the flexibility of the model to fit various loss distributions. While PD, LGD, and EaD are likely to have little room for adjustment, banks usually have more discretion for defining sectors and their correlations. Thus, these variables could act as control variables to fit the model to actual portfolios. Based on this idea, I calculated risk of each model changing the number of sectors (1 to 50) and sector correlations (0.0 to 1.0) as a measure of flexibility. The results are displayed in Figure 6. CR++ with $w_k = 0.5$ has the widest range of VaR while CR++ with $w_k = 1.0$ has the narrowest range. This indicates that if there are no other dominating criteria to choose another model, CR++ might be a good choice if the control variables are carefully determined.

So far, I have demonstrated the risk sensitivity with respect to the input parameters. Credit risk is most sensitive to the asset correlation, and the sen-
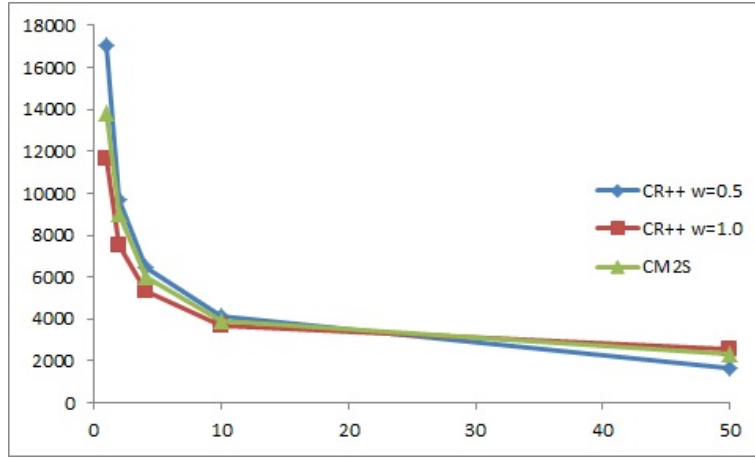
Figure 5: Number of Sectors vs. 99.9% VaR
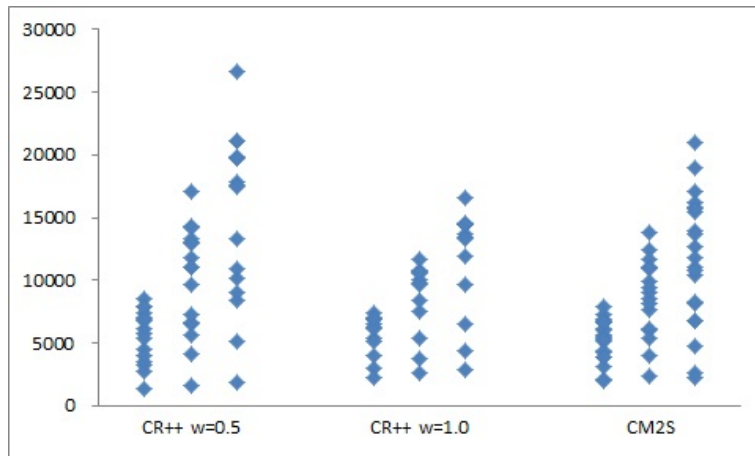
Figure 6: VaR Range
Range of VaR for different numbers of sectors (1 to 50) and sector correlations (0 to 1). For each model, three scattered diagrams are, from left to right, 99%, 99.9%, and 99.99% VaR's.

sitivity becomes even severer when the correlation is higher. Credit risk also appears to be sensitive to the sector correlation. Considering the fact that the asset correlations are normally estimated from the volatility of the sector, sector covariance can be regarded as the most significant risk component in credit risk measurement. Sector covariance is difficult to estimate and can vary widely depending on the economic cycle. Therefore, choosing a proper method for measuring sector covariance is crucial for sound credit risk management. Risk sensitivity varies not only among the models but also with the input parameter values and the quantile at which risk is measured. The difference in the risk sensitivity among the models is most evident with respect to the PD. CR++,

with the additional degree of freedom, has the most flexibility and if flexibility could be considered as a criterion for model selection, CR++ might be worth considering as the first choice.

4   Concluding Remarks

In this article, I compare four widely used credit risk models that are especially suited for loan portfolios: CreditRisk++, CreditMetrics, MOW, and IRB. These models are first compared from a theoretical perspective and then evaluated via various simulation studies. The most important attributes that distinguish one model from another are the risk factor distribution and the relationship between the risk components and the risk factors. These attributes are the determinants of the portfolio loss distribution. For the model portfolios considered in the simulation analysis, risk tends to increase in order of, from low to high, MOW with constant LGD, CR++ with risk factor weight of 1.0, CM2S, CR++ with risk factor weight of 0.5, and MOW with stochastic LGD. However, risk sensitivity is affected by the input parameters values and other factors, and this order cannot be generalized. Risk turns out to be most sensitive to the asset correlation and the sensitivity becomes even higher when the correlation is high. Risk is also very sensitive to the sector correlation. Reflecting the fact that asset correlation is often estimated from the sector volatility, sector covariance is considered to be the most crucial component in risk management. CR++, due to its extra degree of freedom, can have a wide spectrum of loss distribution, even for the same input parameters. This is an advantage of CR++, but at the same time, can cause unexpected results unless the parameters are carefully chosen.

By summarizing the key features of each model and conducting various simulations, this article provides valuable information on credit risk model evaluation for the banks who consider implementing a new model or upgrading the existing one. In particular, by taking multi-factors and their correlations into consideration and performing comprehensive sensitivity analysis with respect to the input parameters, this article exposes subtle differences among models and suggests new insights into them. One model cannot be claimed to be superior to the others in all aspects. Banks should choose the most appropriate model based on their risk preference, management purpose, and own portfolios: If a bank holds a conservative view, it might choose a model that measures risk comparably high; A more stable model might be preferred for calculation of capital buffer, while a more sensitive model might be better suited for internal management and decision making. It would be beneficial for a bank to measure the risk of its own portfolio with several models and compare the results before choosing the final model.

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