



# Bilateral delegation in wage and employment bargaining in monopoly



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## HIGHLIGHTS

- This paper models bilateral delegation in wage bargaining.
- Delegation causes the bargaining pie to shrink severely ruling out mutual gains.
- A player's payoff can be inversely related to his bargaining power.

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## ABSTRACT

We study efficiency and distributional implications of bilateral delegation in wage and employment bargaining in monopoly. Delegation causes underproduction, and the bargaining pie severely contracts rendering mutual gains from delegation impossible. With an increase in the union's bargaining power profit may perversely rise and the union's utility may fall.

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## 1. Introduction

In the strategic delegation literature it is well known that in a Cournot oligopoly firms try to achieve strategic advantage over their rivals by offering sales-oriented incentives to their managers (Vickers, 1985; Fershtman and Judd, 1987; Sklivas, 1987). The consequence is overproduction (relative to the Cournot level). However, if wage is endogenously determined through bargaining, as Szymanski (1994) has shown, overproduction will be accompanied by wage increase, and the sales-oriented scheme will not be always optimal. The managers will face a tension between the task of competing with other firms and the task of negotiating wage with

the union. The optimal incentive scheme reduces this tension by moderating or even reversing emphasis on sales. In a different context, Basu et al. (1997) also showed that a firm would shy away from sales orientation, if it were to face a tax-setting government.

However, it is not well understood what incentive scheme would be optimal if the manager was entrusted with the task of bargaining alone, without having to worry about strategic rivalry. It is also unclear whether efficiency will be in conflict with distribution. Further, one-sided delegation obscures the issue of optimality of delegation itself.

In this paper we address these problems by considering a monopoly setup where both the shareholders and the workers delegate the task of bargaining to a manager and a union leader respectively. The manager is asked to maximize sales or profit, and the union leader is asked to maximize the gross or net wage bill. The bargaining protocol we use is efficient bargaining (McDonald and Solow, 1981). We find that the manager will always be oriented to profit maximization and the union leader to net wage bill maximization. Consequently, there will be underproduction causing the bargaining pie to shrink severely. How much the pie will

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shrink and how it will be distributed depends on the bargaining powers of the players. Starting from a given configuration of bargaining powers, if one party gets stronger his claim over the bargaining pie will be greater, but then his ‘weaker’ opponent will retaliate with harder incentives to reverse his payoff increase. If the second effect overwhelms the first effect, the stronger player’s payoff will fall. We construct examples to illustrate this surprising possibility. We also show that if the players’ decisions to delegate are endogenized, delegation emerges as their dominant strategy.

**2. The model**

We consider a monopoly with labor as the only input, which along with wage is subjected to negotiation. The firm’s sales revenue is denoted as  $s = p(q)q$ , where  $q$  is the output, and  $p(q)$  is the standard inverse demand curve,  $p'(q) < 0$ ,  $p''(q) \leq 0$ . Assuming a concave production function  $q = q(l)$ , we write  $s = s(l)$ ,  $s'(l) < 0$ .

The shareholders of the firm hire a manager and offer an incentive scheme  $z$ , which is, as in Fershtman and Judd (1987), a linear combination of sales  $s$  and profit  $\pi$  as follows:

$$z = \beta\pi + (1 - \beta)s = s(l) - \beta w l. \tag{1}$$

Non-trivial delegation arises if  $\beta \neq 1$ , and there are two types of delegation that can arise: sales oriented delegation (i.e.  $\beta < 1$ ), and profit oriented delegation (i.e.  $\beta > 1$ ). Shareholders maximize profit  $\pi = s - w l$ .

The workers’ union consists of  $N$  identical workers whose reservation wage is  $\theta$  and its objective function is  $u = (w - \theta)l$ . At the worker selection stage,  $l$  members are randomly hired and the remaining  $(N - l)$  members receive the reservation wage from outside. Workers appoint a union leader who is asked to maximize:

$$v = \gamma u(\cdot) + (1 - \gamma)w l = w l - \gamma \theta l. \tag{2}$$

Delegation is captured by  $\gamma \neq 1$ . If  $\gamma > 1$  the union leader is oriented to net wage bill maximization as opposed to  $\gamma < 1$  when he is oriented to gross wage bill maximization. Effectively, the leader is induced to overvalue ( $\gamma > 1$ ) or undervalue ( $\gamma < 1$ ) the opportunity cost of the union.

The firm’s wage and employment are an outcome of bargaining between the firm manager and the union leader. This is a scenario of efficient bargaining between two delegates. The bargaining power of the union leader (and also the union) is exogenously given by  $\alpha$ ,  $0 \leq \alpha \leq 1$ , and the manager’s (and also the shareholders’) bargaining power is  $(1 - \alpha)$ . The reservation payoffs of all parties are zero.

In stage 1 of this simple game the shareholders choose  $\beta$  and simultaneously, the union chooses  $\gamma$ . Then in stage 2 wage and employment (and consequently output) are determined through generalized Nash bargaining. We first consider the stage 2 problem, which is solved by maximizing  $B = [v^\alpha z^{1-\alpha}]$  with respect to  $(w, l)$ .

The solution yields (for the derivation see Chatterjee and Saha, 2013)

$$s'(l) = \beta \gamma \theta, \tag{3}$$

$$w = (1 - \alpha)\gamma \theta + \alpha \frac{s(l)}{\beta l}. \tag{4}$$

Eqs. (3) and (4) give employment and wage respectively. In particular, note that the output choice is not directly affected by the bargaining powers, and  $l$  maximizes  $(s(l) - \beta \gamma \theta)$ . It is easy to check that  $\partial l / \partial \beta < 0$ , and likewise  $\partial l / \partial \gamma < 0$ . Moreover, when  $\beta = \gamma = 1$ , Eq. (3) yields the efficient level of output and Eq. (4) expresses the wage rate as a weighted average of the marginal revenue product and average revenue product of labor.

We now characterize the optimal incentive schemes. In stage 1, the shareholders and the union perfectly anticipate  $l$  and  $w$  (as given implicitly by Eqs. (3) and (4)) and choose  $\beta$  and  $\gamma$  from the

following equations:

$$\pi'(\beta) = \frac{1}{\beta^2} \left[ \beta^2(\beta - 1)\gamma \theta \frac{\partial l}{\partial \beta} + \alpha s(\cdot) \right] = 0, \tag{5}$$

$$u'(\gamma) = \theta \left[ (\gamma - 1) \frac{\partial l}{\partial \gamma} + (1 - \alpha)l \right] = 0. \tag{6}$$

Let  $(\beta^*, \gamma^*)$  be the Nash equilibrium incentives satisfying Eqs. (5) and (6) and assume that the equilibrium is unique and stable. It is obvious that unless  $\beta^* > 1$ , Eq. (5) does not hold for any  $\alpha > 0$ . Likewise, Eq. (6) does not hold, unless  $\gamma^* > 1$  given any  $\alpha < 1$ . That is to say, both sides will incentivize their delegates to cut back on production. In the special case of  $\alpha = 0$ ,  $\beta^* = 1$ ; similarly, when  $\alpha = 1$ ,  $\gamma^* = 1$ . In the Appendix we show as part of the proof of Proposition 1 that  $\beta$  and  $\gamma$  are strategic substitutes to each other. This helps us to compare incentives between bilateral and unilateral delegations. Let us define  $\beta_u$  and  $\gamma_u$  to be the optimal incentives under unilateral delegations. That is,  $\beta_u$  solves Eq. (5) for  $\gamma = 1$ , and  $\gamma_u$  solves Eq. (6) for  $\beta = 1$ . The following proposition characterizes the optimal incentives.

**Proposition 1** (Delegation for Bargaining). *In equilibrium, the shareholders will orient the manager to profit maximization (i.e.  $\beta^* > 1$ ) and the workers will orient their union leader to net wage bill maximization (i.e.  $\gamma^* > 1$ ). However, the higher the bargaining power, the weaker the incentives; that is,  $\beta'(\alpha) > 0$ ,  $\gamma'(\alpha) < 0$ . Thus, in the extreme case, one’s incentive to delegate fully disappears, when one’s bargaining power is maximum. Finally, in comparison to the cases of unilateral delegation, incentives under bilateral delegation will be muted (i.e.  $\beta^* \leq \beta_u$  and  $\gamma^* \leq \gamma_u$ ).*

The combined effects of profit and net wage bill orientations cause severe ‘underproduction’ relative to the no-delegation level (see Eq. (3)). Consequently the bargaining pie will shrink, with uncertain implications for individual payoffs. Greater bargaining power directly increases one’s entitlement to the share of the pie and softens one’s incentives, but simultaneously it hardens the rival’s incentives, unleashing conflicting effects on one’s payoff. To see this we apply the envelope theorem and obtain

$$\pi'(\alpha) = - \underbrace{\left[ \frac{s(l^*)}{l^*} - \beta^* \gamma^* \theta \right]}_{> 0} \frac{l^*}{\beta^*} - \theta \underbrace{\left[ (1 - \alpha)l^* - \gamma^*(\beta^* - 1) \frac{\partial l^*}{\partial \gamma} \right]}_{> 0} \frac{\partial \gamma^*}{\partial \alpha}, \tag{7}$$

$$u'(\alpha) = \underbrace{\left[ \frac{s(l^*)}{l^*} - \beta^* \gamma^* \theta \right]}_{> 0} \frac{l^*}{\beta^*} - \underbrace{\left[ \frac{\alpha s(l^*)}{\beta^{*2}} - (\gamma^* - 1)\theta \frac{\partial l^*}{\partial \beta} \right]}_{> 0} \frac{\partial \beta^*}{\partial \alpha}. \tag{8}$$

The first term of Eq. (7) (inside the bracket) is positive because of concavity of  $s(l)$ . The expression inside the bracket of the second term is also positive due to the fact that  $\partial l^* / \partial \gamma < 0$ . So the overall effect is ambiguous; a similar ambiguity is seen from Eq. (8). Thus, we have a curious possibility—profit can increase and the union’s utility can decrease with an increase in the bargaining power of the union. The ambiguity somewhat diminishes (but does not disappear) if only one side delegates. For example, if only the shareholders delegate,  $\pi'(\alpha) < 0$  (because  $\gamma$  is held constant at 1), but  $u'(\alpha)$  remains ambiguous. Similarly, if only the workers delegate,  $u'(\alpha) > 0$ , but  $\pi'(\alpha)$  is ambiguous. To resolve this ambiguity, we consider an example.

**2.1. An example**

We assume linear demand and constant returns to scale (CRS) technology. Suppose  $p = a - q$  and  $q = l$ . The stage 2 Nash

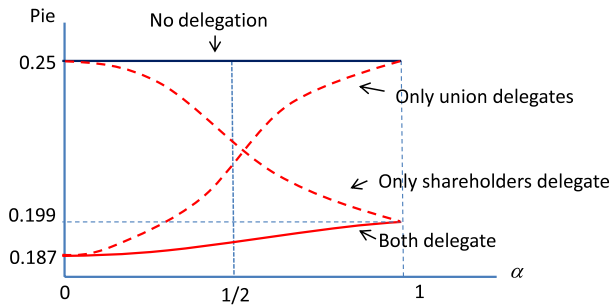


Fig. 1. Bargaining pie.

bargaining employment and wage are:

$$l = \frac{a - \beta\gamma\theta}{2}, \quad w = \frac{\alpha(a + \beta\gamma\theta)}{2\beta} + (1 - \alpha)\gamma\theta.$$

The resultant profit and utility are

$$\pi = \left\{ a - \frac{(a - \beta\gamma\theta)}{2} - \alpha \frac{(a + \beta\gamma\theta)}{2\beta} - (1 - \alpha)\theta\gamma \right\} \times \frac{(a - \beta\gamma\theta)}{2}$$

$$u = \left[ \alpha \frac{(a + \beta\gamma\theta)}{2\beta} + (1 - \alpha)\theta\gamma - \theta \right] \frac{(a - \beta\gamma\theta)}{2}.$$

By maximizing these we obtain the optimal  $\beta$  and  $\gamma$  from the following:

$$-\beta^2\gamma^2\theta^2 [2(\beta - 1) + \alpha] + \alpha a^2 = 0, \tag{9}$$

$$(1 - \alpha)(a - \beta\theta\gamma) + \beta\theta(1 - \gamma) = 0. \tag{10}$$

It is clear that if  $\alpha = 0$ , then  $\beta^* = 1$ , and if  $\alpha > 0$ , then  $\beta^* > 1$  (since  $a > \beta^*\gamma^*\theta$  for positive output). Similarly, if  $\alpha = 1$ ,  $\gamma^* = 1$  and at  $\alpha < 1$ ,  $\gamma^* > 1$ . Further, we can ascertain how the bargaining pie,  $P = \pi + u = s(l) - \theta l$ , behaves with respect to  $\alpha$ . Let  $P^*$  be the equilibrium bargaining pie under bilateral delegation, and let  $P_U$  and  $P_F$  be the same respectively when only the union delegates and when only the shareholders delegate. This is summarized in the following remark.

**Remark 1** (Linear Demand, CRS Technology and Bargaining Pie). Suppose the demand curve is linear and production exhibits constant returns to scale. Then for any  $\alpha \in [0, 1]$ , the bargaining pie under bilateral delegation,  $P^*$ , is bounded above by the smallest of the bargaining pies under unilateral delegation. That is  $P^* \leq \min [P_F, P_U]$ .

*Simulation:* We set  $a = 2$  and  $\theta = 1$  and report the relevant figures in Table 1. Case 0 is the case of no delegation. With  $\beta = \gamma = 1$  we obtain the standard results—employment and the bargaining pie ( $\pi + u$ ) remain invariant to  $\alpha$  and  $\pi'(\alpha) < 0$ .

Case 1 and Case 2 represent the cases of unilateral delegation by shareholders and workers respectively. In both cases employment is lower, and so is the bargaining pie. In Case 1 wage does not grow with  $\alpha$  as much as it does in Case 0, and consequently profit is much higher. In Case 2, both wage and the union’s utility are much higher than the no delegation case. In both cases,  $u'(\alpha) > 0$ ,  $\pi'(\alpha) < 0$ .

Case 3 describes bilateral delegation. Here employment is the lowest, and the pie is the smallest though it is increasing in  $\alpha$ . A visual illustration of the relationship between the bargaining pie and the union’s bargaining power under different scenarios is given in Fig. 1. But the most dramatic finding is the reversal of the relation between  $\pi$  and  $\alpha$ . Here,  $\pi'(\alpha) > 0$  and  $u'(\alpha) < 0$ . The intuition is that with an increase in the union’s bargaining power three effects are unleashed. There is a direct effect leading to a loss in profit.

**Table 1**  
Simulation results.

$a = 2$		$\theta = 1$					
Case 0: No delegation ( $\beta = 1 \gamma = 1$ )							
$\alpha$	$w$	$l$	$u$	$\pi$	$\pi + u$		
1	1.5	0.5	0.25	0	0.25		
0.75	1.375	0.5	0.187	0.063	0.25		
0.55	1.275	0.5	0.137	0.113	0.25		
0.5	1.25	0.5	0.125	0.125	0.25		
0.39	1.195	0.5	0.097	0.153	0.25		
0.25	1.125	0.5	0.063	0.187	0.25		
0	1	0.5	0	0.25	0.25		
Case 1: Only the shareholders delegate ( $\gamma = 1$ )							
$\alpha$	$\beta$	$w$	$l$	$u$	$\pi$	$\pi + u$	
1	1.45	1.19	0.275	0.052	0.15	0.199	
0.75	1.395	1.16	0.30	0.049	0.16	0.21	
0.5	1.322	1.13	0.34	0.043	0.18	0.224	
0.25	1.213	1.08	0.39	0.031	0.206	0.24	
0	1	1	0.5	0	0.25	0.25	
Case 2: Only the union delegates ( $\beta = 1$ )							
$\alpha$	$\gamma$	$w$	$l$	$u$	$\pi$	$\pi + u$	
1	1	1.5	0.5	0.25	0	0.25	
0.75	1.2	1.5	0.4	0.20	0.04	0.24	
0.5	1.33	1.5	0.335	0.167	0.05	0.22	
0.25	1.42	1.5	0.29	0.145	0.061	0.20	
0	1.5	1.5	0.25	0.125	0.062	0.187	
Case 3: Both delegate							
$\alpha$	$\beta^*$	$\gamma^*$	$w$	$l$	$u$	$\pi$	$\pi + u$
1	1.45	1	1.19	0.275	0.052	0.15	0.199
0.75	1.327	1.10	1.25	0.27	0.068	0.13	0.196
0.55	1.235	1.192	1.309	0.264	0.082	0.113	0.195
0.5	1.21	1.217	1.33	0.263	0.086	0.11	0.193
0.39	1.15	1.28	1.369	0.264	0.097	0.096	0.193
0.25	1.10	1.349	1.41	0.256	0.105	0.09	0.191
0	1	1.5	1.5	0.25	0.125	0.062	0.187

Then there is a second and indirect effect occurring through the reduced union incentive  $\gamma$  which helps to increase profit. There is yet another effect occurring through  $\beta$ —adjustment in the firm’s own incentive term. Since  $\beta$  is optimally adjusted (with any change in  $\alpha$ ) its first order effect on profit is zero. Overall, the indirect effect outweighs the direct effect resulting in the reversal of profit and union power relationship.

*Optimality of delegation:* Since  $\pi'(\alpha) > 0$  we see from Table 1 that in equilibrium delegation is profitable for the shareholders (compared to the no delegation case) only after  $\alpha$  exceeds (approximately) 0.55 and for the union only if  $\alpha$  is less than 0.39. Therefore, at all  $\alpha \in (0.39, 0.55)$  both parties are worse off after delegation. Moreover, if we examine the parties’ decision to delegate or not by comparing the equilibrium payoffs from different scenarios of delegation, we see that at all  $\alpha \in (0, 1)$  delegation is the dominant strategy for each party. At  $\alpha = 0$  delegation will still be the dominant strategy of the union, but the shareholders will be indifferent between delegation and no delegation. At  $\alpha = 1$  delegation will be the dominant strategy of the shareholders, but the union will feel indifferent.

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**Appendix**

1. **Proof of Proposition 1.** Most of the claims are confirmed simply by a visual inspection of Eqs. (5) and (6). However, the claim of  $\beta^* \leq \beta_u$  and  $\gamma^* \leq \gamma_u$  relies on the fact that  $\beta$  and  $\gamma$  are

strategic substitutes, which we can establish. Then the comparative static properties of  $\beta^*$  and  $\gamma^*$  also need to be ascertained.

*Strategic substituteness.* We begin by assuming that  $(\beta^*, \gamma^*)$  is a unique Nash equilibrium and it is stable. That is,  $\Delta = \pi''(\beta)u''(\gamma) - \frac{\partial \pi'(\cdot)}{\partial \gamma} \frac{\partial u'(\cdot)}{\partial \beta} > 0$ . To show that  $\beta$  is a strategic substitute to  $\gamma$ , we need to obtain  $\frac{\partial \beta}{\partial \gamma} < 0$  from Eq. (5), and to show that  $\gamma$  is a strategic substitute to  $\beta$  we need to obtain  $\frac{\partial \beta}{\partial \gamma} < 0$  from Eq. (6).

From Eq. (5) derive  $\pi''(\beta) \frac{\partial \beta}{\partial \gamma} + \frac{\partial \pi'(\beta)}{\partial \gamma} = 0$ . Here,  $\pi''(\beta) < 0$  for the profit function to have a maximum. In fact, this can be ensured by assuming  $s'''(l) \leq 0$  (see Chatterjee and Saha, 2013). Next, we show  $\frac{\partial \pi'(\beta)}{\partial \gamma} < 0$ .  $\frac{\partial \pi'(\beta)}{\partial \gamma} = \beta^2(\beta - 1)\theta \frac{\partial l}{\partial \beta} + \beta^2(\beta - 1)\theta \gamma \left[ \frac{\theta}{s''(l)} - \theta \gamma \frac{s'''(l)}{(s''(l))^2} \frac{\partial l}{\partial \gamma} \right]$ . Since  $s'''(l) \leq 0$ , this expression is negative. Hence  $\frac{\partial \beta}{\partial \gamma} < 0$ .

Now from Eq. (6) derive  $u''(\gamma) \frac{\partial \gamma}{\partial \beta} + \frac{\partial u'(\gamma)}{\partial \beta} = 0$ . Again, for  $u(\gamma)$  to have a maximum  $u''(\gamma)$  must be negative at  $\gamma^*$ . Finally, using the fact  $\frac{\partial l}{\partial \gamma} = \frac{\partial l}{\partial \beta} \frac{\beta}{\gamma}$  we write  $\frac{\partial u'(\gamma)}{\partial \beta} = \theta \beta \frac{(\gamma-1)}{\gamma} \frac{\partial^2 l}{\partial \beta^2} + \theta \frac{\partial l}{\partial \beta} \left[ \frac{(\gamma-1)}{\gamma} + (1 - \alpha) \right]$ . We can show that  $\frac{\partial^2 l}{\partial \beta^2} = -\theta \gamma \frac{\partial l}{\partial \beta} \frac{s'''(l)}{(s''(l))^2} \leq 0$  if  $s'''(l) \leq 0$ . Then it is clear that  $\frac{\partial u'(\gamma)}{\partial \beta} < 0$ . This allows us to conclude that  $\frac{\partial \gamma}{\partial \beta} < 0$ .

*Comparative statics.* Differentiating Eqs. (5) and (6), and using the facts that  $\frac{\partial \pi'(\beta)}{\partial \alpha} = \frac{s(l)}{\beta^2} > 0$  and  $\frac{\partial u'(\gamma)}{\partial \alpha} = -\theta l < 0$ , we obtain

$$\begin{aligned} \frac{\partial \beta}{\partial \alpha} &= \frac{1}{\Delta} \left[ -\frac{\partial \pi'(\beta)}{\partial \alpha} u''(\gamma) + \frac{\partial \pi'(\beta)}{\partial \gamma} \frac{\partial u'(\gamma)}{\partial \alpha} \right] \\ &= -\frac{1}{\Delta} \left[ \frac{s(l)}{\beta^2} u''(\gamma) + \theta l \frac{\partial \pi'(\beta)}{\partial \gamma} \right] > 0, \\ \frac{\partial \gamma}{\partial \alpha} &= \frac{1}{\Delta} \left[ -\frac{\partial u'(\gamma)}{\partial \alpha} \pi''(\beta) + \frac{\partial \pi'(\beta)}{\partial \alpha} \frac{\partial u'(\gamma)}{\partial \beta} \right] \\ &= \frac{1}{\Delta} \left[ \theta l \pi''(\beta) + \frac{s(l)}{\beta^2} \frac{\partial u'(\gamma)}{\partial \beta} \right] < 0. \end{aligned}$$

To see  $\beta^* \leq \beta_u$  note that  $\beta^*$  is an optimal response to  $\gamma^* \geq 1$ , whereas  $\beta_u$  is an optimal response to  $\gamma = 1$ . Since  $\beta$  and  $\gamma$  are strategic substitutes, it must be that  $\beta^* \leq \beta_u$ . Analogous reasoning establishes  $\gamma^* \leq \gamma_u$ .  $\square$

**2. Proof of Remark 1.** Suppose  $\alpha \in (0, 1)$  and both sides delegate. From Eq. (9) we obtain  $\beta^* \gamma^* \theta = a[\alpha/\{2(\beta^* - 1) + \alpha\}]^{1/2}$ .

Contrast this with the case of unilateral delegation by the firm (where  $\gamma = 1$  and we write  $\beta = \beta_u$ ):  $\beta_u \theta = a[\alpha/\{2(\beta_u - 1) + \alpha\}]^{1/2}$ .  $\beta_u$  is the optimal response to  $\gamma = 1$ , and  $\beta^*$  is the optimal response to  $\gamma^* > 1$ . Since  $\beta$  and  $\gamma$  are strategic substitutes, it must be that  $\beta_u > \beta^*$ . Hence,  $\beta^* \gamma^* \theta > \beta_u \theta$ .

Now consider Eq. (10). Rewrite it as  $\beta^* \gamma^* \theta = [(1 - \alpha)a + \beta^* \theta]/(2 - \alpha)$ . Contrast this with the case of unilateral delegation by the union. Set  $\beta = 1$  and  $\gamma = \gamma_u$  in the above and it is clearly smaller. Hence, we can say  $\beta^* \gamma^* \theta > \gamma_u \theta$ . This allows us to conclude that for any  $\alpha \in (0, 1)$ ,  $\beta^* \gamma^* > \max[\beta_u, \gamma_u]$  and hence  $P^* < \min[P_U, P_F]$ .

Finally consider  $\alpha = 0$  and  $\alpha = 1$ . At  $\alpha = 0$ ,  $\beta^* = \beta_u = 1$  and  $\gamma^* = \gamma_u > 1$ . Therefore,  $P^* = P_U$ . Alternatively, when  $\alpha = 1$ ,  $\beta_u > 1$  and  $\gamma^* = \gamma_u = 1$ . Hence,  $P^* = P_F$ .

Combining all these observations we write,  $P^* \leq \min[P_F, P_U]$ .  $\square$

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