A Multiple-Goal Investment Strategy for Sovereign Wealth Funds: The Case of China

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ABSTRACT

This paper develops a multiple-goal investment strategy for sovereign wealth funds with particularly emphasis on the case of China. In an optimizing framework for policy response, we embed the Black-Litterman (B-L) model into the Mean Variance Mental Accounting (MVMA) approach. The B-L method provides a means of modelling return expectations, while the MVMA framework helps achieve the multiple goals. Our framework is applied to the empirical derivation of the optimal asset allocation for China's sovereign wealth fund.

Key Words: Sovereign Wealth Fund; Multiple-goal investment strategy; Mental accounting; Black-Litterman models; Strategic asset allocation

JEL Codes: G11, G02, E58, C11, C61.

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1. Introduction

Sovereign Wealth Funds (SWFs), or the public funds that manage foreign assets on behalf of sovereign with a relatively longer investment horizon, have emerged as prominent institutional investors in global capital markets in recent years. The number and size of SWFs have experienced rapid growth since the turn of the new century, of which the SWFs from China has been a particular example of impressive development. China's first SWF on record, the China Investment Corporation (CIC), was established in September 2007, with an initial capital injection of US\$ 200 billion. Its founding is largely prompted by the intention of the Chinese government to mitigate the costs of carrying the massive and ever-growing foreign reserves by pursuing higher returns from investing the excessive reserves. At the end of 2012, the external assets under CIC management stood at US\$ 482 billion, one of the largest in the world's SWFs (TheCityUK, 2013).

SWF investment in recent years has exhibited two critical changes. First, the recent global financial crisis prompts SWFs to play some role in reaching out for financial stability. Many countries have injected sizable capital into systemically important banks to help them deal with financial distress. This shifts the role of SWFs from being an investor who transfers wealth from now to the future to serve as a promoter of financial stability who provides contingent liquidity

to domestic financial systems when necessary. IMF (2008b) confirms that SWF's capital injections can facilitate stability of the recipients' share prices by reducing share price volatility in the short run, and thus SWFs are able to fulfil a shock-absorbing function.

Second, with the growing size of SWF assets, many countries have established more than one SWFs or assigned different mandates to their one SWF,¹ to achieve varying policy objectives such as providing liquidity needs in the short run and transfer wealth in the long run. According to Kunzel et al. (2010) and IMF (2011), Chile has founded two SWFs: a Social and Economic Stabilization Fund and a Pension Reserve Fund, to meet its short-term and long-term macroeconomic objectives. Singapore has also created two SWFs, i.e. Temasek Holdings and Government of Singapore Investment Corporation, with the former being a saving fund and the latter a foreign reserve investment fund. The Kuwait Investment Authority as a singular institute serves two SWF functions: a stabilization fund and a savings fund. In Norway's case, its Government Pension Fund Global performs three functions in one: a stabilization fund, a savings fund, and a pension reserve fund. In all, it is common for SWFs to pursue a multiple-goal strategy for their investments.

In the context of China, despite the remarkable economic growth it has achieved in recent decades, the country remains faced with considerable uninsured risks, particularly in the financial area. Allen et al. (2012) suggest that China's current financial system is vulnerable both to traditional financial crises and to the simultaneous currency, banking and stock market crises, which can seriously disrupt the economy and undermine social stability. Other serious risks are

¹ According to IMF (2012), SWFs can be categorized into four types based on their distinct mandates and policy objectives: Stabilization Funds, Saving Funds, Pension Reserve Funds, and Reserve Investment Funds.

also present in the economy. For example, Qiu (2013) maintains that the rapidly ageing population and the potential funding crisis of China's National Social Security Fund represent a severe social challenge to China's continuing economic prosperity. Against this backdrop, it would be not entirely pertinent for China to use foreign reserves, which are huge, only for reaping higher financial yields. It is necessary and desirable for China to mandate its SWFs multiple tasks ranging from seeking higher returns, to providing support to financial stability, and to supporting provision of social security, to list just a few. In this light, the Chinese government may administer its SWF by setting up various types of fund to meet the needs of achieving multiple policy objective, which calls for the formulation of a multiple-goal investment strategy.

However, the existing literature has been relatively spars on formulating such a strategy. This paper aspires to fullfill towards this void by proposing a multiple-goal SWF investment framework for China. The modelling attempt will, featur the embedding of the Black-Litterman model (Black and Litterman, 1992) into the Mean Variance Mental Accounting framework introduced by Das, Markowitz, Scheid, and Statman (2010). The Black-Litterman (B-L) model is an approach of formulating forward-looking return forecasts to overcome the error-maximizing of the mean-variance optimization (Best and Grauer, 1991). It uses the implied equilibrium excess returns as the starting point and combines subjective investors' views, thus forming the posterior expected returns estimates based on the Bayesian estimation method.²

 $^{^{2}}$ Lucid discussion of the Bayesian analysis and the B-L model can be found in Lee (2000) and Christodoulakis (2002). Meucci (2010) and Walters (2011) survey the original B-L model and its various extensions.

Das et al. (2010) combine the mean-variance theory of portfolio management (MVT) by Markowitz (1952) with the behavioural portfolio theory (BPT) by Shefrin and Statman (2000). The BPT is a goal-based theory in which investors divide their wealth into a variety of mental accounts of a set of portfolios corresponding to various goals.³ A central feature of BPT is that investors take their portfolios not as a whole, but as distinct mental accounts in a set of assets, where mental accounts are connected with particular goals and where attitudes towards risk vary across mental accounts (Statman, 2008).

According to Das et al. (2010), the MVMA approach allows to take investment portfolios as collections of Mental Accounting (MA) sub-portfolios. Each sub-portfolio is associated not only with the specified risk-aversion coefficient in the MVT problem, but also with a goal in which there are a threshold return level and a probability in the MA problem. Risk tolerance in each sub-portfolio can be measured by the probability of failing to reach the threshold return level by means of Value at Risk (VaR). They also demonstrate that the MVT problem is mathematically equivalent to the MA problem. As a result, investors in this framework can choose the portfolio with maximum expected returns subject to the VaR constraint capturing the account's motive. In line with Markowitz (1952), optimal portfolios within various accounts are located on the efficient frontier. As a result of combining all sub-portfolios, the aggregate portfolio is also located on the efficient frontier. Various extensions to Das et al. (2010) have been attempted as in for example Alexander and Baptista (2011) and Baptista (2012).

From a normative perspective, our multiple-goal SWF investment framework can guide sovereign wealth managers on how to efficiently operate their SWFs. We propose that the SWF

³ For an introduction to mental accounting, see for example Thaler (1999) and Nofsinger (2011, Chapters 6 and 7).

comprises three mental accounts, or sub-portfolios. The first is a 'liquidity sub-portfolio', in which the managers specify higher risk-aversion coefficients and invest in a short investment horizon for providing contingent liquidity supports to both internal and external banking sectors to cushion against the possible negative effects triggered by traditional financial crises or some 'twin crisis'. The second is an 'investment sub-portfolio', where the managers specify medium risk-aversion parameters, and invest in a medium-term investment horizon for funding contingent domestic liabilities, e.g. contingent pension payments. The third is a 'bequest subportfolio', where the managers with lower risk-aversion parameters invest in a long-term investment horizon, attempting to transfer their wealth across generations. As a result, according to different types of SWFs, the managers can construct their distinct aggregate portfolios by allocating their total investable wealth across the three sub-portfolios in a variety of proportions. While the optimal ratio of each sub-portfolio to the total investable wealth can be investigated in a separate work in future research, in the current paper, we employ the B-L model to obtain both the equilibrium total returns and the B-L total returns as our improved forward-looking return forecasts, and therefore to derive the two sets of optimal asset allocations for SWFs. With the MVMA framework, we run an empirical analysis based on China's SWFs by selecting 16 indices, including cash equivalents, fixed income, equity, and alternative asset.

The rest of the paper is organized as follows. In section 2, we briefly review the literature on SWFs. In section 3, we outline the growth pattern and the potential risks facing China's economy and its sovereign wealth funds. In section 4, we put forward our multiple-goal investment framework for China's SWF. In section 5, we apply our approach to an empirical analysis of China's case. Conclusions are presented in section 6.

2. Related Literature on SWFs

The research on various issues of SWFs has been well documented. Some studies analyze general issues. For example, the IWG⁴ (2008) launched the famous 'Santiago Principles', a set of 24 principles for guiding all arrangements and practices of SWF governance. According to the IWG (2008), the Santiago Principles are in three distinct parts, covering: (1) legal framework, objectives, and macroeconomic policies; (2) institutional framework and governance structure; and (3) investment and risk management framework. Hammer, Kunzel, and Petrova (2008) summarize institutional and operational practices of SWFs by surveying 21 members of the IWG based on the Santiago Principles. Their survey findings suggest that operational practices of SWFs vary significantly, due to their distinct natures and legal personalities. They also find that SWFs within the same type have similar practices, and that the SWF group as a whole shares some broader common practices.

Mitchell, Piggott, and Kumru (2008) probe three different large publicly-held funds, i.e. foreign exchange reserve funds, SWFs, and public pension funds, to point out their similarities and differences, and show that all three play an increasingly significant role in international financial markets. Aizenman and Glick (2009) conduct a statistical analysis of the stylized facts of SWFs by evaluating what factors trigger the establishment of SWFs and affect their size, and investigating the relationship between the transparency and governance of SWFs and domestic and global governance practices. Das, Lu, Mulder, and Sy (2009) provide a guideline to policymakers on the establishment of SWFs, covering various issues from macroeconomic policy objectives to the institutional structure and specific operational considerations. Ang (2010)

⁴ IWG is short for International Working Group of Sovereign Wealth Funds, an institution established in 2008 and made up of 24 countries collectively owning 26 SWFs.

proposes four benchmarks, which form the basis for designing, implementing, and measuring SWFs: (1) the Benchmark of Legitimacy; (2) the Benchmark of Integrated Policy and Liabilities; (3) The Governance Structure and Performance Benchmark; and (4) the Long-Run Equilibrium Benchmark.

Some studies investigate the impact of Sovereign Wealth Fund (SWF) investments on their target firms. For instance, Dewenter, Han, and Malatesta (2010) examine the relationship between the announcement of both investments and divestments of SWFs and the corresponding changes in the values of the firms in which they invest. Kotter and Lel (2011) examine how investment strategies of SWFs and their impact on the value of the target firm are related to the extent of accountability and transparency of SWFs. Knill, Lee and Mauck (2012) explore the return and risk performances of SWF target firms after SWF investment, to find out whether the performances of SWF target firms are more closely analogous to those of state-owned firms or firms invested in by institutional investors.

Some studies investigate the investment strategies of SWFs. For example, Bernstein, Lerner, and Schoar (2009) probe the SWF direct investments towards private equity and their relationship to the organizational structures of SWFs. Chhaochharia and Laeven (2009) find that SWFs tend to invest in countries that share similar cultural traits with them, based on a dataset of 30,000 equity investments by SWFs. Dyck and Morse (2011) explore the behaviour of SWF investments in public equities, private firms, and real estate, based on a novel, hand-collected dataset.

Some studies engage in theoretical modelling on SWFs. For example, Aizenman and Glick (2011) develop a model in which they compare the optimal asset diversification into safe assets and risky assets of a central bank with that of a SWF, showing their distinct investment behaviour. Using two countries and two asset classes, Sa and Viani (2011) develop a dynamic general equilibrium model to explore how a shift in portfolio preferences of foreign investors, i.e. SWFs, may affect various economic variables such as asset prices, consumption, the exchange rate and net debt. Van Den Bremer and Van Der Ploeg (2012) develop an optimal asset management framework for oil-rich economies, in which it is necessary to manage three funds: (1) liquidity funds for dealing with oil price volatility; (2) investment funds for managing domestic investment, and (3) intergenerational funds for smoothing out the benefits across generations. Other studies estimate the risk management of SWFs. For example, Bodie and Briere (2011) develop a new approach to the risk management issue of SWFs, with the help of contingent claim analysis (CCA) originally proposed by Gray, Merton, and Bodie (2007), and apply this approach to the case of Chile.

3. China's Economy and its Sovereign Wealth Fund

3.1. The Growth Pattern of the Chinese Economy

Since launching its first programme of economic reforms in December 1978, China has experienced fast economic growth, with a remarkable 9.9% annual real GDP growth on average from 1979 to 2011. Particularly since its WTO accession in 2001, China has witnessed both fast foreign reserve accumulation and rapid economic growth. According to Morrison (2013), there are two main factors explaining this rapid economic growth in China: the first is large-scale capital investment, financed by hefty domestic savings and foreign investment; the second is rapid productivity growth. Figure 1 shows China's GDP growth and the domestic savings/GDP ratio from 2001 to 2012.

<Figure 1 about here>

According to Figure 1, since 2001 the size of China's GDP has increased dramatically, from 1,325 billion USD in 2001 to 8,227 billion USD in 2012. Meanwhile, the domestic savings/GDP ratio has remained at a relatively high level, more than 40% from 2002 to 2012, peaking at 53.35% in 2008. Such a high level of saving rate provides a stable source for domestic investment. Figure 2 illustrates China's total foreign reserves and the total reserves/GDP ratio from 2001 to 2011.

<Figure 2 about here>

As we can see from Figure 2, the size of total foreign reserves augmented from 216 billion USD at the end of 2001 to 3,203 billion USD at the end of 2011, indicating huge current account

surpluses and foreign direct investment (FDI) during this period. At the same time, the total reserves to GDP ratio gradually increased, peaking at 48.40% in 2009, then declining slightly to 43.76% in 2011.

In addition, China's economic development has followed a trend of increasing dependence on the imports of raw materials. Figure 3 depicts China's total merchandise imports and the raw material imports/total imports ratio.

<Figure 3 about here>

As shown in Figure 3, since 2001, the size of China's total imports grew from 243.5 billion USD in 2001 to 1,743.5 billion USD in 2011. Meanwhile, the raw material imports/total imports ratio also expanded, from 13.17% in 2001 to 31.52% in 2011.

3.2. Large Risks Facing the Chinese Economy

In spite of China's huge economic achievement during the last decade, there are large risks facing China's economy. First of all, the current underdevelopment of China's financial system may suffer from potentially uninsured risks. For example, Prasad and Wei (2005) argue that China's underdeveloped banking system is subject to external shocks due to the reported Non-Performing Loans (NPLs) problem. Prasad (2009) indicates that 'loss of confidence in the banking system' is potentially one of the most severe risks facing China's economy. Allen et al. (2012) suggest that China's current financial system is vulnerable to traditional financial crises and to the 'twin crisis' of simultaneous currency and banking and stock market crises, which can seriously disrupt the economy and social stability. Second, the current pension reform in China

exerts a negative effect on its sustainable economic growth. Due to the rapidly ageing population and the potential funding crisis of China's National Social Security Fund, there is a severe social challenge facing China's continuing economic prosperity (Qiu, 2013).

3.3. The Investment Pattern of CIC

In September 2007, China's State Council invested 200 billion USD out of China's then 1.4 trillion in foreign reserves to establish the China Investment Corporation (CIC), China's first sovereign wealth fund on record, for the purpose of reducing reserve holding costs and pursuing higher returns. According to the estimation of TheCityUK (2013), at the end of 2012 the CIC held under management foreign financial assets of 482 billion USD. The CIC has invested in a wide range of assets, containing bonds, equities, and alternative assets. Table 1 shows the CIC's asset allocations from 2008 to 2011, according to the annual reports for those four years.

<Table 1 about here>

As reported in Table 1, in 2008 the CIC invested most of its holdings (87.40%) in cash funds and 9.00% in fixed-income securities, a total of 96.40% in safe assets. However, in the subsequent three years, taking its global investment losses in the 07-08 crisis as a lesson, the CIC changed its asset allocation by shifting its portfolio from cash funds and others to alternative investment, including direct investments in non-public companies, private equity, hedge funds, real estate and infrastructure. At the end of 2011, 43% of CIC holdings were in alternative investment and 25% in equities, resulting in 68% of holdings in risky assets.

The Central Huijin Investment Corporation (Central Huijin), a CIC subsidiary, has injected substantial amounts of capital into several of China's large state-owned and systemically important banks, hence providing liquidity support. Table 2 reports the top five portfolio holdings of Central Huijin in China's large state-owned banks. As can be seen from Table 2, at the end of 2011, Central Huijin had a 47.60% share of ownership in the China Development Bank (CDB), 35.40% in the Industrial and Commercial Bank of China (ICBC), 40.10% in the Agricultural Bank of China (ABC), 67.60% in the Bank of China (BOC), and 57.10% in the China Construction Bank (CCB).

<Table 2 about here>

As a global institutional investor, the CIC has played a significant role in providing financial stability by injecting its capital into important Western financial institutions. For example, as reported by Pistor (2009), in May 2007 the CIC injected US\$ 3 billion, a 9.9% stake, into Blackstone (US), and in Dec 2007 US\$ 5 billion into Morgan Stanley (US).

4. The Multiple-Goal SWF Investment Framework

In this section, we propose a multiple-goal investment framework for China's SWF to formulate strategic asset allocation and thus to construct the benchmark portfolio, by embedding the Black-Litterman model (Black and Litterman, 1992) into the Mean Variance Mental Accounting framework by Das, Markowitz, Scheid, and Statman (2010). We first delineate the MVMA framework to show the multiple-goal investment mechanism, then employ the B-L model as a means to form forward-looking return forecasts, and finally derive the multiple-goal investment strategy for China's SWF.

4.1. The MVMA Optimization

In our model setting, the problem faced by the sovereign wealth managers is to select portfolio weights $\mathbf{w} = [w_1, ..., w_n]'$ for N assets, in which the assets have an expected return vector $\boldsymbol{\mu} \in R^n$ and a return covariance matrix $\boldsymbol{\Sigma} \in R^{n \times n}$. The standard MV problem is a trade-off between the portfolio return and its variance:

$$\max_{\mathbf{w}} \mathbf{w}' \mathbf{\mu} - \frac{\gamma}{2} \mathbf{w}' \mathbf{\Sigma} \mathbf{w} \qquad (1)$$

subject to the full-investment constraint

$$w'1 = 1$$
 (2)

where $\mathbf{1} = [1, 1, ..., 1]' \in \mathbb{R}^n$, and γ is the risk aversion coefficient, which balances the trade-offs in the mean-variance space.

Based on equations (1) and (2), using the Lagrange-multiplier method, the solution to optimal portfolio weights in closed form is⁵

$$\mathbf{w}(\gamma) = \frac{1}{\gamma} \Sigma^{-1} \left[\boldsymbol{\mu} - \left(\frac{\mathbf{1}' \Sigma^{-1} \boldsymbol{\mu} - \gamma}{\mathbf{1}' \Sigma^{-1} \mathbf{1}} \right) \mathbf{1} \right] \in \mathbb{R}^n \qquad (3)$$

Given the expected return vector μ and the covariance matrix Σ , equation (3) shows that the optimal portfolio weights \mathbf{w} are a function of the risk aversion coefficient γ . According to this solution, the wealth managers can specify γ by choosing distinct values for $\gamma > 0$, and then solve the problem (1) in terms of solution (3). With a collection of different risk-aversion values in hand, they can maximize mean-variance utility to find corresponding points on the efficient frontier.

Meanwhile, the wealth managers in behavioural portfolio theory take their overall portfolio as collections of Mental Accounting (MA) sub-portfolios, in which each sub-portfolio, i.e. each mental account, is mapped onto a goal. Following Das et al. (2010), we assume that the sovereign wealth managers always have difficulty in stating their precise risk-aversion coefficient(γ), but are comfortable to state the threshold levels for each mental account (goal) and their corresponding maximum probabilities of failing to reach them. As a result, the MA problem indicates that the sovereign wealth managers consider a threshold level of return *H* for portfolio *p* in a certain mental account, and regard the maximum probability of the portfolio failing to reach portfolio return r(p) as α . Thus, they have

$$\operatorname{Prob}\left[r\left(p\right) \le H\right] \le \alpha \qquad (4)$$

⁵ The detailed derivation of this solution can be found in the Appendix of Das et al. (2010).

Portfolio returns are assumed to be normally distributed. In terms of Value at Risk (VaR), inequality (4) implies the following inequality:

$$H \le \mathbf{w}' \mathbf{\mu} + \Phi^{-1} (\alpha) [\mathbf{w}' \mathbf{\Sigma} \mathbf{w}]^{1/2} \qquad (5)$$

where $\Phi(\Box)$ is the cumulative standard normal distribution function.

Ultimately, the wealth managers in the MVMA framework act as if they have different risk preferences in each of the mental accounts. Thus, solving the MA problem is equivalent to solving a standard MV problem with a specific 'implied' risk-aversion coefficient. The wealth managers' aim is to derive optimal portfolio weights from equation (3) subject to the constraint (5). Optimization cannot be achieved unless the constraint (5) is an equality. In consequence, the solution to the wealth managers' implied risk aversion γ is formulated by the following equation:

$$H = \mathbf{w}(\gamma)' \mathbf{\mu} + \Phi^{-1}(\alpha) \left[\mathbf{w}'(\gamma) \mathbf{\Sigma} \mathbf{w}(\gamma) \right]^{1/2}$$
(6)

where the solution of $\mathbf{w}(\gamma)$ is provided from equation (3). Plugging equation (3) into equation (6), it is straightforward to find the solution to equation (6), based on which one can obtain different values of the risk preference γ .

As a result, the MVMA framework suggests that the portfolio optimization problem for the wealth managers is specified by a threshold level of return H and a probability value α . When the managers specify their MA preferences for each sub-portfolio through the parameter pair (H, α) , they implicitly denote what their risk preferences (γ) are over the given portfolio choice

set (μ, Σ) . With the risk aversion coefficient (γ) , the wealth managers can derive their optimal portfolio weights.

However, because SWFs are unleveraged positions, we need to resort to Quadratic Programming (QP) optimizers to derive optimal portfolio weights under short-selling constraints. Following Das et al. (2010), the sovereign wealth managers can employ Value at Risk (VaR) as their risk management framework, which can be expressed by the MVMA problem as:

Solve_{$$\gamma$$} **w**(γ)'**µ** + $\Phi^{-1}(\alpha)\sqrt{\mathbf{w}(\gamma)'\Sigma\mathbf{w}(\gamma)} = H$ (7)

where $\mathbf{w}(\gamma)$ is the first order condition to the following MV problem:

$$\max_{w} w' \boldsymbol{\mu} - \frac{\gamma}{2} w' \boldsymbol{\Sigma} w \qquad (8)$$

subject to the full invested constraint and short-selling constraints

$$\mathbf{w}'\mathbf{1} = 1$$
, $\mathbf{w} \ge 0$ and $\mathbf{w} \le 1$ (9)

According to equations (7) to (9), for each sub-portfolio, each VaR constraint which is specified by a threshold level H and a probability value α in the MA problem corresponds to a particular implied coefficient of risk aversion γ in the MV problem. Thus, the wealth managers solve the nonlinear equation (7) based on a specified γ , i.e. a specified sub-portfolio, and thus derive the optimal portfolio weights $\mathbf{w}(\gamma)$ by solving the QP in equations (8) and (9). For the specified γ or sub-portfolio, the managers need to check whether the solution $\mathbf{w}(\gamma)$ can make equation (7) hold. If not, they must change γ accordingly and then solve the QP until equation (7) holds.

4.2. The Black-Litterman Model

We use the Black-Litterman model to improve our input forecast, i.e. the expected returns. This model employs the equilibrium returns as the starting point for its estimation. Equilibrium returns are inferred from the market capitalization weights, using a 'reverse optimization process'. Black and Litterman (1992) argue that this process, based on market capitalization weights, can derive consensus excess returns, which are consistent with the tangency portfolio of the Capital Asset Pricing Model. With the market forces of supply and demand in equilibrium, the weight allocation across the investment universe is expected to be optimal and the optimal weight can therefore act as the basis for asset allocation.

We follow Satchell and Scowcroft (2000) and Idzorek (2005) to depict the Black-Litterman model. In this model, given the risk aversion coefficient δ that indicates the level of risk against returns of the market portfolio, the historical variance covariance matrix Σ , and the vector of market capitalization weights \mathbf{w}_M , the reverse optimization process can provide the vector of implied equilibrium returns $\boldsymbol{\mu}_M$ in excess of the risk-free rate as

$$\boldsymbol{\mu}_{M} = \delta \boldsymbol{\Sigma} \mathbf{W}_{M} \quad (10)$$

If the wealth managers do not agree with the implied equilibrium excess returns, they can introduce their own views. Specifically, they may take the implied equilibrium returns as the prior distribution and regard the corresponding forecasted returns as forward-looking views-based returns, to form the posterior Black-Litterman returns. For example, assume there are k views, which can be either relative or absolute and are represented in the $k \times 1$ vector \mathbf{Q} . The $k \times n$ matrix \mathbf{P} is then used to define these views: $\mathbf{Q} = \mathbf{P} \cdot \mathbf{r}_a$. The first view is represented as a linear combination of expected returns denoted by the first row of \mathbf{P} . A confidence level is associated with each of the views implied by \mathbf{Q} . Thus, the investor's beliefs can be described by

a normal view distribution: $\mathbf{P} \cdot \mathbf{r}_a \sim N(\mathbf{Q}, \mathbf{\Omega})$, where $\mathbf{\Omega}$ is a $k \times k$ diagonal covariance matrix. In the same vein, the confidence in the equilibrium model and the derived implied returns can be defined. Consequently, we obtain the prior equilibrium distribution: $\mathbf{r}_a \sim N(\mathbf{\mu}_M, \tau \mathbf{\Sigma})$, where τ is a known quantity indicating the uncertainty level to scale the historical covariance matrix $\mathbf{\Sigma}$.

Following the Bayesian estimation method, the wealth managers can generate the posterior Black-Litterman returns as follows:

$$E(\mathbf{r}_{BL}) = \left[\left(\tau \boldsymbol{\Sigma} \right)^{-1} + \mathbf{P}' \boldsymbol{\Omega} \mathbf{P} \right]^{-1} \times \left[\left(\tau \boldsymbol{\Sigma} \right)^{-1} \boldsymbol{\mu}_{M} + \mathbf{P}' \boldsymbol{\Omega} \mathbf{Q} \right]$$
(11)

As a result, with the implied equilibrium excess returns $\boldsymbol{\mu}_{M}$ and the B-L excess returns $E(\mathbf{r}_{BL})$ in hand, we can obtain the implied equilibrium total returns $\boldsymbol{\mu}_{M}^{T}$ and the B-L total returns $E(\mathbf{r}_{BL}^{T})$ by adding to each of them the risk-free rate.

4.3. The Multiple-Goal Investment Strategy for China's SWF

Embedding the B-L model into the MVMA framework, our multiple-goal investment strategy for China's SWF can be accomplished through three steps. First, to meet various macroeconomic policies such as providing liquidity support and transferring wealth across generations, sovereign wealth managers take their portfolios as a collection of three sub-portfolios. Table 3 displays the profile of our designed three sub-portfolios, including their policy objectives, risk tolerance, and investment horizon.

<Table 3 about here>

The first is a 'liquidity sub-portfolio', where the managers specify higher risk-aversion coefficients, showing lower risk tolerance; and they invest in a short investment horizon for providing contingent liquidity support to both internal and external banking sectors to cushion against the possible negative effects triggered by traditional financial crises or 'twin crisis'. The second is an 'investment sub-portfolio', in which the managers specify medium risk-aversion parameters, implying modest risk tolerance; and they invest in a medium-term investment horizon for funding contingent domestic liabilities, e.g. contingent pension payment. The third is a 'bequest sub-portfolio', in which the managers with lower risk-aversion parameters invest in a long-term investment horizon, attempting to transfer such national wealth from now to the future and thus to benefit subsequent generations. As a result, according to different types of funds, the managers can construct their distinct aggregate portfolios by allocating their total investable wealth across the three sub-portfolios in a variety of proportions. Generally, for a 'conservative SWF' (e.g. Stabilization Fund), most of the total investable wealth (more than 50%) should be allocated to the liquidity sub-portfolio, and the remainder into the other two, aiming mainly to meet large liquidity needs; whereas, for a 'progressive SWF' (e.g. Saving Fund, Pension Reserve Fund, or Reserve Investment Fund), most of the wealth should be allocated to the bequest subportfolio, and the remainder into the other two, due to their limited liquidity needs.

Before entering into their three sub-portfolios, the managers first choose their investment classes out of the available investment universe. They derive the implied equilibrium total return $\boldsymbol{\mu}_{M}^{T}$ in the light of market capitalization weights, and the B-L total returns $E(\mathbf{r}_{BL}^{T})$ in the light of their forward-looking investment views. Finally, using $\boldsymbol{\mu}_{M}^{T}$ and $E(\mathbf{r}_{BL}^{T})$ respectively, the managers figure out the two sub-groups of optimal asset allocation for the three sub-portfolios by solving equations (7) to (9), and construct their specified aggregate portfolios based on their overall policy objectives.

5. Empirical Analysis

5.1. Investment Universe and Selection of Asset Classes for China's SWF

It is essential for the sovereign wealth managers to first delineate their investment universe, i.e. the set of asset classes which will be selected for portfolio construction. This is a pivotal step towards forming the basis for the investment policy of China's SWF. Some recent studies have probed various investment patterns for various SWFs, revealing important insights into their investment universe.

The IMF (2012) investigated the observed asset allocations of the four types of SWF (i.e. Stabilization Fund, Saving Fund, Pension Reserve Fund, and Reserve Investment Fund) at the end of 2010 based on the selected 30 SWFs, and suggests that four asset types are used for SWF investment: cash, fixed income, equities, and alternative asset. By and large, stabilization funds are highly risk-averse institutional investors, while the other three types of funds are investors with relatively higher risk tolerance. The IMF (2011) also examined the investment patterns of all four types of SWF based on these four asset types, and emphasizes that specific factors such as the age of the SWF, its funding source, and its investment horizon could give rise to differences in asset allocations even between SWFs that have analogous objectives.

Kunzel et al. (2011) compare the observed asset allocations of some SWFs based on these four asset types before and after the 2008 financial crisis. They suggest that the recent crisis has affected the asset allocations of SWFs in different ways. For example, some stabilization funds, such as Trinidad and Tobago's, have decreased their holdings of cash in favour of fixed income, whereas Ireland's National Pension Reserve Fund has increased its holdings of cash. Some saving funds, such as those of Norway and Canada, have increased their holdings of equities. The Korea Investment Corporation (KIC), a reserve investment fund, has introduced alternative asset investment and increased its holdings of equities. The authors conclude that these shifts are fund-specific and reflect individual circumstances.

In line with the recent observation of SWFs' asset allocations, we select sixteen asset classes covering the four asset types as our investment opportunity set. Half of these are safe assets, including the long-term government bonds of four developed countries, US corporate bonds, US Agencies, US Asset Backed Securities and US 3-month Treasury Bills. The rest are risky assets, comprising equities of four advanced economies, and four alternative assets.

5.2. Data and Implementation

For empirical analysis, we employ 16 indices, comprising bonds, equities and alternative assets, to mimic various market risk factors at the long-term investment horizon. For bonds, we use the long-term government bond indices of four advanced countries (USA, UK, Germany, and Australia), one US Agency bond index, one US asset backed security index, one 3-month US Treasury Bill index, and one US corporate bond index, all of which come from Bank of America Merrill Lynch. For equities, S&P 500, S&P EURO, S&P UK, and S&P ASX300 are employed

as the proxies for US, Eurozone, UK, and Australia equities, respectively. For the four alternative assets, S&P GLOBAL REIT⁶, S&P GSCI Commodity, UBS North American Infrastructure & Utilities, and UBS UK Infrastructure & Utilities are used as our proxies of China's SWF global investment in alternative assets. Monthly total return indices are employed over the sample period from January 1995 to January 2013, with a total of 217 observations. All total return indices are calculated in a log-return style based on a US-dollar denomination. The 3-month US T-Bill is approximated as the risk-free rate.

Table 4 reports the descriptive statistics for all asset classes considered. Among all government bonds, the Australia government bond has the best mean return with the highest standard deviation, while the US government bond has the lowest standard deviation. The German government bond has a slightly higher return than the US government bond, but has a relatively higher standard deviation. For all risk assets including equities and alternatives, UK Infrastructure & Utilities has highest mean return with 13.13%, while Euro equity has highest standard deviation with 22.93%. US T-Bill 3 Month has both lowest mean return and lowest standard deviation in all selected asset classes, showing the quality of highest safety.

<Table 4 about here>

Before running the MVMA framework, we improve our expected return forecasts using the B-L model. Using the implied equilibrium excess returns as the starting point, sovereign wealth managers formulate their forward-looking investment views. Studies such as IMF (2012) suggest that the recent financial crises have caused a rise in the demand for safe assets, boosting their

⁶ REIT stands for Real Estate Investment Trust, one type of Alternative Asset in the US.

price in global markets based on their limited availability. Consequently, taking equilibrium excess returns as a reference, the managers infer that government bonds are more favourable, and make three conservative assumptions: (1) US equity will outperform US government bonds by only 5.5%; (2) UK equity will exceed UK government bonds by only 3.05%; and (3) Australia equity will exceed Australia government bonds by only 3%. In addition, the managers expect that the US and Canada economies will perform better than the UK economy, forming the fourth view that NA INFRA & UTIL will exceed UK INFRA & UTIL by 2.8%. The confidence levels of all four investment views are equal to 50%.

Table 5 shows market weights of the selected asset classes and their two estimates, i.e. equilibrium total returns and the B-L total returns. Concerning market weights, US equity has the largest market capitalization of all the selected asset classes; US government bonds and US agencies have the second and the third largest, respectively; while the UK Infrastructure & Utilities has the least market capitalization. Due to the three conservative assumptions and the one investment view favouring NA INFRA & UTIL, with the exception of North American Infrastructure & Utilities, B-L total returns of all risky assets are slightly less than the equilibrium total returns. For safe assets, only B-L total returns of US government bonds, US agencies, and US T-Bill 3M are not less than those of the equilibrium total returns.

<Table 5 about here>

With equilibrium and the B-L total return, we now use the MVMA framework to implement our multiple-goal investment strategy for China's SWF. By working out equations (7) to (9), we

obtain the two sets of optimal portfolio weights for the selected asset classes. According to each of these sets of optimal weights, we construct our three sub-portfolios, i.e. the liquidity, the investment, and the bequest sub-portfolios, by specifying three distinct risk-aversion parameters from high to low, to achieve our various macroeconomic policy objectives. We also construct distinct aggregate portfolios by allocating the total investable wealth into the three sub-portfolios in a variety of proportions. Specifically, we construct two different aggregate portfolios: the first, which is to mimic a 'conservative SWF', is based on an 50:30:20 division across the three sub-portfolios (50% of the total investable wealth from the liquidity sub-portfolio, 30% from the investment sub-portfolio, and 20% from the bequest sub-portfolio); the second, which is to mimic a 'progressive SWF', is based on a 10:40:50 division. Finally, we investigate the MA problem for all portfolios based on equation (7). Within each portfolio, a VaR constraint makes various threshold levels of returns map into their maximum probabilities of not reaching those threshold return levels.

5.3. Main Results

5.3.1. The results based on equilibrium returns

Table 6 shows the holdings of efficient portfolios for the three sub-portfolios and two aggregate portfolios for all asset classes under the equilibrium return estimates. Although Mehra and Prescott (1985) suggest that the range of risk aversion coefficient should be within the interval from 0 to 10, many studies, such as Ait-Sahalia and Brandt (2001), employ the range of risk aversion coefficient from 0 to 20. We argue that the value in risk aversion coefficient only conveys the degree of risk aversion for investors. As a result, our selection of risk aversion coefficient is within the range from 0 to 20.

<Table 6 about here>

According to Table 6, for the liquidity sub-portfolio, with the highest risk-aversion parameter out of the three sub-portfolios, i.e., $\gamma = 12.725$, its expected return gains 4.98%, with standard deviation of 3.87%. As a result of its lower risk tolerance, the liquidity sub-portfolio holds 77.99% in safe assets, comprising 54.53% in cash equivalents and 23.46% in bonds; and 22.01% in risky assets, made up of 17.68% in equities and 4.33% in alternatives. For the investment sub-portfolio, with $\gamma = 5.526$, the largest holding would be US government bonds (27.48%) and the second largest would be US equity (27.30%). Because of its medium risk tolerance, this portfolio holds 48.55% in safe assets (0.05% in cash equivalent and 48.50% in bonds) and 51.45% in risky assets (41.40% in equities and 10.05% in alternatives). For the bequest sub-portfolio, with $\gamma = 3.312$, the largest holding would be US equity (42.87%) and the second largest would be Euro equity (12.55%). As a consequence of its having the highest risk tolerance out of the three, this portfolio holds 15.21% in safe assets (0.01% in cash equivalents and 15.20% in bonds) and 84.79% in risky assets (67.28% in equities and 17.51% in alternatives). The aggregate portfolio 1 (50:30:20 mix) holds 56.60% in safe assets and 43.40% in risky assets, implying its relatively higher risk averse attitude, while the aggregate portfolio 2 (10:40:50 mix) holds 65.18% in risky assets and the remainder in safe assets, indicating its higher risk tolerance.

Table 7 depicts the threshold levels of return and the corresponding maximum probabilities of not reaching them for the three sub-portfolios and the two aggregate portfolios under equilibrium return estimates. The results in this Table probe the Mental Accounting problem by solving a VaR constraint, i.e. equation (7), to measure the risk tolerance of each portfolio.

<Table 7 about here>

In Table 7, we can observe that the maximum probabilities that sovereign wealth managers would have negative returns are 9.91%, 20.07%, and 24.44% for the liquidity, the investment, and the bequest sub-portfolios, respectively, and for the aggregate portfolios 1 and 2 they are 18.49% and 22.40%, respectively. On the other hand, the maximum probabilities of not reaching 10% threshold return levels are 90.27%, 61.59%, and 49.54% for the liquidity, the investment, and the bequest sub-portfolios, respectively, and for the aggregate portfolios 1 and 2 they are 66.76% and 55.15%, respectively. These results suggest that the liquidity sub-portfolio offers the best way of flight to safety among the three sub-portfolios, while the bequest sub-portfolio has the highest probability of earning higher return; and that holdings in the aggregate portfolio 1 are relatively safer than those in the aggregate portfolio 2.

5.3.2. The results based on the B-L returns

In this sub-section, all the results are according to the B-L return estimates. After the wealth managers state their three conservative investment views and the one investment view favouring NA INFRA & UTIL, Table 8 and Table 9 correspond to and convey the same information as Tables 6 and 7, respectively.

<Table 8 about here>

<Table 9 about here>

Compared with the results in Table 6, Table 8 illustrates that, for the liquidity sub-portfolios, with the exception of the Australia government bond, holdings of all other government bonds have increased slightly, which indicates that those conservative investment views shift the portfolio towards government bonds. For each portfolio in Table 8, due to the view favouring North American Infrastructure & Utilities, holdings of the NA INFRA & UTIL increase, which causes a shift in the composition of risky assets from equities to alternatives. However, because of the overall conservative investment views, for each portfolio, both the expected returns and standard deviations under the B-L return estimates are lower than those under equilibrium return estimates.

For the MA problem, Table 9 implies that, for the liquidity sub-portfolio, the maximum probability of having a negative threshold return level is 9.34%, slightly less than the probability reported in Table 7, due to the fact that the performances in this sub-portfolio, i.e. the expected portfolio return and standard deviation, decline to some degree, compared with the results in Table 7.

6. Conclusions

In this paper, we propose a multiple-goal investment framework for China's SWF by embedding the Black-Litterman model into the Mean Variance Mental Accounting framework. The B-L model is used as a means of forming forward-looking return forecasts, while the MVMA framework helps us to achieve multiple goals. In our proposed framework, we assume that there are three sub-portfolios, i.e. goals: the liquidity, the investment, and the bequest sub-portfolios. Our investment framework and risk management framework can converge to equation (7), in which, for each sub-portfolio, each VaR constraint which is specified by a threshold level H and a probability value α in the MA problem corresponds to a particular implied coefficient of risk aversion γ in the MV problem. Risk tolerance can be measured by VaR. We can construct distinct aggregate portfolios by allocating their total investable wealth across the three subportfolios in a variety of proportions, according to different SWFs. For example, for a 'conservative SWF', most of the total investable wealth (more than 50%) should be allocated to the precautionary sub-portfolio, and the remainder to the other two, mainly aiming to meet large liquidity needs. Conversely, for a 'progressive SWF', most of the wealth should be allocated to the bequest sub-portfolio, and the remainder to the other two, due to their limited liquidity needs.

Investigation as to the optimal ratio of each sub-portfolio of China's SWF to its total investable wealth is beyond the scope of this paper. However, this is a very important issue, especially given that China's SWF has unique risk exposures. Therefore, we suggest that future research should undertake to explore the optimal size of each sub-portfolio for China's SWF, conditional on its unique economic environment.

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Figure 1 China's GDP Growth and the Domestic Savings/GDP Ratio Source: CEIC 2013



Figure 2 China's Total Foreign Reserves and the Total Reserves/GDP Ratio Source: World Bank Database: World Development Indicators (WDI) 2013



Figure 3 China's Total Merchandise Imports and the Raw Material Imports/Total Imports Ratio Source: World Bank Database: World Development Indicators (WDI) 2013 Note: The Raw Material Imports include crude oil, gas, metals, and resources.

Type of Investment	2008	2009	2010	2011
Cash Funds and Others	87.40%	32%	4%	11%
Fixed-Income Securities	9.00%	26%	27%	21%
Equities	3.20%	36%	48%	25%
Alternative Investment	0.40%	6%	21%	43%
Safe Assets	96.40%	58.00%	31.00%	32.00%
Risky Assets	3.60%	42.00%	69.00%	68.00%

Table 1 The CIC's Asset Allocations

Source: The CIC's Annual Reports from 2008 to 2011.

Note: This table shows the global asset allocations of the CIC from 2008 to 2011. Alternative Investments include direct investments into non-public companies, private equity, hedge funds, real estate and infrastructure. Safe Assets include Cash Funds and Others and Equities, while Risky Assets include Fixed-Income Securities and Alternative Investment.

Financial Institute	2008	2009	2010	2011
China Development Bank	48.70%	48.70%	48.70%	47.60%
Industrial and Commercial Bank of China	35.40%	35.30%	35.40%	35.40%
Agricultural Bank of China	50.00%	50.00%	40.00%	40.10%
Bank of China	67.50%	67.50%	67.60%	67.60%
China Construction Bank	48.20%	57.00%	57.10%	57.10%

Table 2 Top Five Portfolio Holdings of Central Huijin

Source: The CIC's Annual Reports from 2008 to 2011.

Note: This table shows the top five portfolio holdings of Central Huijin from 2008 to 2011.

Sub-Portfolio	Policy Objective	Risk Tolerance	Investment Horizon
Liquidity	Provide contingent liquidity supports to both internal and external banking sectors to cushion the possible negative effects caused by traditional financial crises and a "twin crisis"	Lower	Short
Investment	Invest in a medium-term goal to fund contingent domestic liabilities	Modest	Medium
Bequest	Transfer national wealth from now to the future and benefit next generations	Higher	Long

Table 3 Objective, Risk Tolerance, and Investment Horizon of the three Sub-Portfolios

Notes: This table shows the profile of the designed three sub-portfolios for our multiple-goal SWF investment strategy.

		•		
Name	Market	Instrument Type	Mean	Standard Deviation
US GVT	USA	Long-term Bonds	5.72%	3.14%
UK GVT	UK	Long-term Bonds	7.09%	8.71%
GERMAN GVT	Germany	Long-term Bonds	5.90%	10.59%
AUSTRALIA GVT	Australia	Long-term Bonds	9.22%	12.07%
US CORP	USA	Corporate	7.41%	5.49%
US AGENCIES	USA	Agencies	5.74%	2.74%
US ABS	USA	Asset Backed Securities	5.31%	2.36%
US T-BLL 3M	USA	Cash Equivalents	3.15%	0.65%
US EQUITY	USA	Equities	8.48%	16.80%
UK EQUITY	UK	Equities	7.91%	17.65%
EURO EQUITY	Euro Zone	Equities	7.70%	22.93%
AUSTRALIA EQUITY	Australia	Equities	11.32%	22.39%
GLOBAL REIT	International	Alternatives	10.61%	20.68%
GLOBAL COMMODITY	International	Alternatives	4.50%	23.71%
NA INFRA & UTIL	North America	Alternatives	9.24%	15.47%
UK INFRA & UTIL	UK	Alternatives	13.13%	16.39%

Table 4 Descriptive Statistics

Notes: This table shows descriptive statistics of all considered asset classes. Our calculations employ monthly data. The mean returns and standard deviations are reported annually.

Name	Market Weights	Equilibrium Total Returns	The B-L Total Returns
US GVT	23.72%	3.02%	3.12%
UK GVT	2.73%	4.94%	4.77%
GERMAN GVT	3.29%	5.26%	5.02%
AUSTRALIA GVT	0.82%	7.24%	6.63%
US CORP	6.14%	4.23%	4.21%
US AGENCIES	13.60%	3.21%	3.26%
US ABS	0.33%	3.40%	3.39%
US T-BLL 3M	3.00%	3.12%	3.12%
US EQUITY	24.12%	10.67%	9.70%
UK EQUITY	4.40%	10.99%	9.55%
EURO EQUITY	6.57%	13.08%	11.29%
AUSTRALIA EQUITY	2.25%	12.45%	10.73%
GLOBAL REIT	2.15%	10.87%	9.69%
GLOBAL COMMODITY	5.37%	9.54%	8.95%
NA INFRA & UTIL	1.32%	7.64%	7.78%
UK INFRA & UTIL	0.21%	7.93%	6.52%

Table 5 Market Weights and Return Estimates

Sources: Market capitalization data of all safe assets are from BIS Securities Statistics on the BIS official website. The data of all risky assets are from DataStream.

Notes: Market weights are obtained by using market capitalization data of all asset classes. Equilibrium total returns and the B-L total returns are derived by adding the risk-free rate to equilibrium excess return and the B-L excess returns, respectively.

Risk Aversion:	γ = 12.725	γ = 5.526	γ = 3.312	50:30:20 Mix	10:40:50 Mix
Asset Classes	Liquidity Sub- portfolio	Investment Sub-portfolio	Bequest Sub- portfolio	Aggregate Portfolio 1	Aggregate Portfolio 2
US GVT	10.90%	27.48%	0.03%	13.70%	12.10%
UK GVT	0.89%	2.86%	0.29%	1.36%	1.38%
GERMAN GVT	1.92%	4.53%	9.71%	4.26%	6.86%
AUSTRALIA GVT	1.36%	1.23%	4.35%	1.92%	2.80%
US CORP	4.10%	9.23%	0.79%	4.98%	4.50%
US AGENCIES	2.22%	3.08%	0.02%	2.04%	1.47%
US ABS	2.06%	0.09%	0.02%	1.06%	0.25%
US T-BLL 3M	54.53%	0.05%	0.01%	27.28%	5.48%
US EQUITY	11.59%	27.30%	42.87%	22.56%	33.51%
UK EQUITY	2.24%	4.39%	6.34%	3.70%	5.15%
EURO EQUITY	2.90%	7.33%	12.55%	6.16%	9.50%
AUSTRALIA EQUITY	0.96%	2.39%	5.51%	2.30%	3.81%
GLOBAL REIT	1.02%	2.33%	5.84%	2.38%	3.96%
GLOBAL COMMODITY	2.39%	5.88%	8.66%	4.69%	6.92%
NA INFRA & UTIL	0.73%	1.34%	2.42%	1.25%	1.82%
UK INFRA & UTIL	0.20%	0.50%	0.58%	0.37%	0.51%
Cash Equivalents	54.53%	0.05%	0.01%	27.28%	5.48%
Bonds	23.46%	48.50%	15.20%	29.32%	29.35%
Equities	17.68%	41.40%	67.28%	34.72%	51.97%
Alternatives	4.33%	10.05%	17.51%	8.68%	13.21%
Safe Assets	77.99%	48.55%	15.21%	56.60%	34.82%
Risky Assets	22.01%	51.45%	84.79%	43.40%	65.18%
Total Weights	100%	100%	100%	100%	100%
Expected Returns	4.98%	7.40%	10.17%	6.74%	8.54%
Std. Dev.	3.87%	8.82%	14.69%	7.52%	11.26%

Table 6 (Equilibrium Returns)Holdings of Efficient Portfolios for the three Sub-Portfolios and the two Aggregate Portfolios

Notes: The portfolio weights for all portfolios are obtained using the solutions in equations (7) to (9). The expected returns and standard deviations of all portfolios are presented at the bottom of the table.

Risk Aversion:	γ = 12.725	γ = 5.526	γ = 3.312	50:30:20 Mix	10:40:50 Mix
Asset Classes	Liquidity Sub- Portfolio	Investment Sub-Portfolio	Bequest Sub- portfolio	Aggregate Portfolio 1	Aggregate Portfolio 2
-10.00%	0.01%	2.43%	8.49%	1.30%	4.98%
-5.00%	0.50%	7.99%	15.09%	5.91%	11.46%
-3.00%	1.96%	11.92%	18.50%	9.75%	15.27%
0.00%	9.91%	20.07%	24.44%	18.49%	22.40%
3.00%	30.45%	30.89%	31.27%	30.93%	31.13%
5.00%	50.21%	39.28%	36.24%	40.83%	37.66%
10.00%	90.27%	61.59%	49.54%	66.76%	55.15%
Expected Returns	4.98%	7.40%	10.17%	6.74%	8.54%
Std. Dev.	3.87%	8.82%	14.69%	7.52%	11.26%

 Table 7 (Equilibrium Returns)

 Threshold Return Levels and Corresponding Maximum Probabilities of Not Reaching Them

Notes: The results are computed using equation (7) after obtaining portfolio returns and standard deviations for each portfolio.

Table 8 (The B-L Returns)Holdings of Efficient Portfolios for the three Sub-Portfolios and the two Aggregate Portfolios

Risk Aversion:	γ = 12.725	γ = 5.526	γ = 3.312	50:30:20 Mix	10:40:50 Mix
Asset Classes	Liquidity Sub- portfolio	Investment Sub-portfolio	Bequest Sub- portfolio	Aggregate Portfolio 1	Aggregate Portfolio 2
US GVT	12.91%	27.42%	0.05%	14.69%	12.28%
UK GVT	1.20%	2.96%	0.19%	1.53%	1.40%
GERMAN GVT	3.35%	8.28%	11.16%	6.39%	9.23%
AUSTRALIA GVT	1.27%	1.31%	4.23%	1.87%	2.77%
US CORP	4.74%	10.13%	0.13%	5.43%	4.59%
US AGENCIES	0.31%	0.16%	0.04%	0.21%	0.12%
US ABS	0.59%	0.06%	0.04%	0.32%	0.10%
US T-BLL 3M	54.41%	0.04%	0.03%	27.22%	5.47%
US EQUITY	13.74%	32.22%	50.06%	26.55%	39.29%
UK EQUITY	0.01%	0.01%	0.09%	0.03%	0.05%
EURO EQUITY	0.49%	1.32%	4.38%	1.51%	2.76%
AUSTRALIA EQUITY	0.03%	0.02%	0.95%	0.21%	0.48%
GLOBAL REIT	0.05%	0.03%	1.90%	0.41%	0.97%
GLOBAL COMMODITY	2.66%	6.38%	10.20%	5.28%	7.92%
NA INFRA & UTIL	4.26%	9.66%	16.55%	8.34%	12.57%
UK INFRA & UTIL	0.00%	0.00%	0.02%	0.01%	0.01%
Cash Equivalents	54.41%	0.04%	0.03%	27.22%	5.47%
Bonds	24.36%	50.31%	15.84%	30.44%	30.48%
Equities	14.27%	33.57%	55.47%	28.30%	42.59%
Alternatives	6.97%	16.08%	28.66%	14.04%	21.46%
Safe Assets	78.77%	50.36%	15.87%	57.66%	35.95%
Risky Assets	21.23%	49.64%	84.13%	42.34%	64.05%
Total Weights	100%	100%	100%	100%	100%
Expected Returns	4.60%	6.54%	8.70%	6.00%	7.43%
Std. Dev.	3.49%	7.93%	13.21%	6.77%	10.13%

Notes: The portfolio weights for all portfolios are obtained using the solutions in equations (7) to (9). The expected returns and standard deviations of all portfolios are presented at the bottom of the table.

Risk Aversion:	γ = 12.725	γ = 5.526	γ = 3.312	50:30:20 Mix	10:40:50 Mix
Asset Classes	Liquidity Sub- Portfolio	Investment Sub-Portfolio	Bequest Sub- portfolio	Aggregate Portfolio 1	Aggregate Portfolio 2
-10.00%	0.00%	1.86%	7.85%	0.90%	4.27%
-5.00%	0.29%	7.30%	14.99%	5.20%	11.00%
-3.00%	1.46%	11.47%	18.79%	9.17%	15.17%
0.00%	9.34%	20.51%	25.51%	18.75%	23.17%
3.00%	32.27%	32.80%	33.30%	32.86%	33.11%
5.00%	54.51%	42.33%	38.96%	44.11%	40.54%
10.00%	93.91%	66.88%	53.91%	72.26%	60.03%
Expected Returns	4.60%	6.54%	8.70%	6.00%	7.43%
Std. Dev.	3.49%	7.93%	13.21%	6.77%	10.13%

 Table 9 (The B-L Returns)

 Threshold Return Levels and Corresponding Maximum Probabilities of Not Reaching Them

Notes: The results are computed using equation (7) after obtaining portfolio returns and standard deviations for each portfolio.