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The fermion content of the Standard Model from a simple world-line theory

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ABSTRACT

We describe a simple model that automatically generates the sum over gauge group representations and chiralities of a single generation of fermions in the Standard Model, augmented by a sterile neutrino. The model is a modification of the world-line approach to chiral fermions.

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1. Introduction

The natural language of high energy physics is second quantisation, or quantum field theory, providing a simple formulation of the processes of particle creation and annihilation. Its triumphs are the astonishing accuracy of QED and the succinctness and elegance of the Standard Model. However, first quantisation can also offer significant insights, which is perhaps not surprising given the success of the approach in studying string theory. For example, Strassler [1] showed that the world-line formalism could be used to derive the Bern–Kosower rules that streamline perturbative calculations. For a more recent application of the formalism see [2]. In this paper we will find that a simple modification offers the possibility of unifying quarks and leptons into a single mathematical structure. In particular we will show that the sum over gauge group representations and chiralities of a generation of the Standard Model arises automatically. We begin by describing a world-line approach to chiral fermions and then set out our model.

2. World-line description of chiral fermions

The action for a left-handed fermion moving in a background gauge-field A is $S=\int d^4x\, i\xi^\dagger\bar\sigma\cdot D\xi$ in the representation [3] of the Dirac matrices

$$(\gamma^\mu)=\begin{pmatrix}0 & \sigma^\mu\\ \bar\sigma^\mu & 0\end{pmatrix},\quad (\sigma^\mu)=(1_2,\sigma^i),\quad (\bar\sigma^\mu)=(1_2,-\sigma^i). \tag{1}$$

$D=\partial+A$ and the coupling has been absorbed into the gauge-field. The effective action obtained by integrating out the fermions is a functional of the background. Rather than study this directly it has often been found more convenient to consider its variation under a change of A, as in [4,5]:

$$\delta\log\int\mathscr{D}(\bar\xi,\xi)\,e^{iS}=\mathrm{Tr}\left((\bar\sigma\cdot D)^{-1}\bar\sigma\cdot\delta A\right) \tag{2}$$

which can be written in terms of the full Dirac matrices as:

$$\mathrm{Tr}\left((\sigma\cdot D\,\bar\sigma\cdot D)^{-1}\sigma\cdot D\,\bar\sigma\cdot\delta A\right)$$
$$=-\int_0^\infty dT\,\mathrm{Tr}\left(P_L\exp\left(T(\gamma\cdot D)^2\right)\gamma\cdot D\,\gamma\cdot\delta A\right) \tag{3}$$

with $P_L=(1-\gamma_5)/2$. γ-matrices can be represented by functional integrals over anti-commuting variables ψ:

$$\int\mathscr{D}\psi\,e^{-\int_0^{2\pi}dt\left(\frac{1}{2}\psi\cdot\dot\psi-\sqrt{2}\eta\cdot\psi\right)}\propto\mathcal{P}\,\mathrm{tr}\,e^{\int_0^{2\pi}dt\,\eta\cdot\gamma}, \tag{4}$$

where η is an anti-commuting source, and we impose anti-periodic boundary conditions $\psi(2\pi)=-\psi(0)$. We write $\propto$ as the

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two sides differ by a (possibly divergent) normalisation constant. Periodic boundary conditions produce a γ_5 insertion resulting in $\mathcal{P}\,\mathrm{tr}\,\gamma_5 e^{\int_0^{2\pi} dt\,\eta\cdot\gamma}$. Now $(\gamma\cdot D)^2 = D^2 + F_{\mu\nu}\gamma^\mu\gamma^\nu/2$. Using this, together with the Wick-rotated path integral representation of the heat kernel [6] $\exp\left(T(\gamma\cdot D)^2\right)$ leads to the world-line representation of (3) which sums over closed paths $w^\mu(t)$:

$$-\int_0^\infty dT \int_{\mathrm{ap-p}} \mathscr{D}(w,\psi)\,\mathcal{N}\,e^{-\frac{1}{2}\int_0^{2\pi} dt\left(\frac{\dot{w}^2}{T}+\psi\cdot\dot{\psi}\right)} \times \mathcal{P}\,\mathrm{tr}\left(g(2\pi)\int_0^{2\pi} dt\,\psi\cdot\dot{w}\,\psi\cdot\delta A\right), \tag{5}$$

where

$$g(t') = \mathcal{P}\exp\left(-\int_0^{t'} \mathcal{A}^R T_R\,dt\right), \quad \mathcal{A} = \dot{w}\cdot A - \frac{T}{2}F_{\mu\nu}\psi^\mu\psi^\nu, \tag{6}$$

and $\{T_R\}$ are anti-Hermitian Lie algebra generators. $\mathrm{tr}\,g(2\pi)$ is the (super)-Wilson loop. $\mathcal{N}$ is a normalisation constant. The action results from the reparametrisation invariant expression

$$\int dt\,\frac{1}{2}\left(\frac{\dot{w}^2}{e} + \psi\cdot\dot{\psi} + \frac{\chi\,\dot{w}\cdot\psi}{e}\right) \tag{7}$$

when the local supersymmetry

$$\delta w = \epsilon\psi, \quad \delta\psi = -\epsilon\left(\frac{\dot{w}}{e} + \frac{\chi\psi}{2e}\right), \quad \delta e = \epsilon\chi, \quad \delta\chi = -2\dot{\epsilon}, \tag{8}$$

and reparametrisation invariance are gauge fixed by the conditions on the einbein $e = T$ and its super-partner $\chi = 0$ [7]. The ap − p subscript on $\int$ denotes that the integral with periodic boundary conditions on the ψ should be subtracted (with appropriate choices of $\mathcal{N}$) from that with anti-periodic conditions so as to generate the P_L insertion. Summing the two contributions instead generates the expression for a right-handed fermion. This representation can be interpreted as the contribution of a particle with world-line $w^\mu(t)$ traced out in the direction of increasing t, but, corresponding to rewriting the Lagrangian in the form $(\xi^t\sigma_2)i\sigma\cdot(\partial - A^t)(\sigma_2\xi^*)$, it can equally be interpreted as the contribution of an anti-particle moving around the closed path in the opposite sense because the effect of changing direction on $g(2\pi)$ is to change the sign of A and replace path-ordering by anti-path-ordering, i.e. $g \to g^\dagger$, and interchange chiralities.

To compute the full one-loop effective action for the fermions in the Standard Model we would have to sum (5) over the appropriate representations of $SU(2)$ and $SU(3)$ and weak hypercharges and take correlated combinations of periodic and anti-periodic boundary conditions on the ψ to pick out the corresponding chiralities. For each generation this consists of ten pieces: the leptons form a left-handed $SU(2)$ doublet, $E_L = (\nu, l)^t$ (in the notation of [18]) and a right-handed $SU(2)$ singlet, l_R, both of which are $SU(3)$ singlets. The quarks form a left-handed $SU(2)$ doublet, $Q_L = (U, D)^t$, and two right-handed singlets U_R and D_R, all being in the fundamental representation of $SU(3)$. To these can be added the anti-particles, transforming in conjugate representations of $SU(3)$ and with opposite chiralities. The $U(1)$ charges associated to these species are −1/2, −1, 1/6, 2/3 and −1/3 respectively with the anti-particles taking the opposite $U(1)$ charges. We denote anti-particles by bars: $\bar{E}_L$, $\bar{l}_R$, etc. The point of this paper is to demonstrate that this somewhat complicated sum can be written very simply as an integral over additional fermionic variables on the world-line.

3. The model

It is well known that if $\tilde{\phi}_r$ and ϕ_s are a set of anti-commuting operators with $\{\tilde{\phi}_r, \phi_s\} = \delta_{rs}$ then the operators $\hat{T}^R \equiv \tilde{\phi}_r T^R_{rs}\phi_s$ satisfy the Lie algebra. These anti-commutation relations follow from a Lagrangian $\tilde{\phi}\cdot\dot{\phi}$, which leads to a propagator containing the step-function $\theta(t_1 - t_2)$ (plus other terms depending on the boundary conditions) which is just what is needed to build the path-ordering in (6) [8]. These two connections between fermionic variables and Lie algebras suggest we consider, instead of the functional integral in (5), the simpler

$$-\int_0^\infty dT \int \mathscr{D}(w,\psi,\tilde{\varphi},\varphi)\,\mathcal{N}\,e^{-\frac{1}{2}\int_0^{2\pi} dt\left(\frac{\dot{w}^2}{T}+\psi\cdot\dot{\psi}+\tilde{\varphi}\cdot\dot{\varphi}+\tilde{\varphi}\mathcal{A}\varphi\right)} \times \int_0^{2\pi} dt\,\tilde{\varphi}\,\psi\cdot\dot{w}\,\psi\cdot\delta A\,\varphi. \tag{9}$$

The path-ordered exponential in (5) can be picked out by a particular choice of boundary conditions, an operator insertion and a choice of normalisation, or, as in [9] and [10], by projecting out a piece with appropriate $U(1)$-charge. We shall not follow these paths, but rather we will find it more useful to consider the integral (9) as it stands. The Lagrangian in this model has been analysed extensively [11–13], mainly in canonical quantisation. It also arises naturally in models of tensionless strings interacting on contact: in [14] and [15], building on [16], it was shown that the expectation value of the super-Wilson loop associated with a closed curve for an Abelian gauge theory is generated by the spinning string action integrated over world-sheets spanning the curve. Generalising this to the non-Abelian theory requires a way of extending the path-ordering associated with the boundary into the spanning world-sheet, this can be achieved by the introduction of additional world-sheet fields with boundary values $\tilde{\varphi}$, φ. A full discussion of this string model is unnecessary here, save for the requirement that the Lie algebra generators be traceless. We rewrite (9) as

$$\int_0^\infty dT \int \mathscr{D}(w,\psi)\,\mathcal{N}\,e^{-\frac{1}{2}\int_0^{2\pi} dt\left(\frac{\dot{w}^2}{T}+\psi\cdot\dot{\psi}\right)}\,\Omega\,Z[\mathcal{A}],$$

where

$$Z[\mathcal{A}] = \int \mathscr{D}(\tilde{\varphi},\varphi)\,e^{-\frac{1}{2}\int_0^{2\pi} dt\,\tilde{\varphi}\cdot\mathcal{D}\varphi}, \quad \mathcal{D} = \frac{d}{dt} + \mathcal{A},$$

$$\text{and}\quad \Omega = \int_0^{2\pi} dt\,\psi\cdot\dot{w}\,\psi\cdot\delta A\,\frac{\delta}{\delta\mathcal{A}}. \tag{10}$$

There is a world-line supersymmetry underlying this model. As previously observed the action for w and ψ results from gauge-fixing the locally supersymmetric action (7). Similarly the new term $\int dt\,\tilde{\varphi}\cdot\mathcal{D}\varphi$ results from the reparametrisation invariant

$$\int dt\Big(\tilde{\varphi}\left\{\frac{d}{dt} + \dot{w}\cdot A - e\psi\cdot\partial A\cdot\psi\right\}\varphi + e\left(\tilde{z}\,\psi\cdot A\,\varphi + \tilde{\varphi}\,\psi\cdot A\,z + \tilde{z}z\right)\Big) \tag{11}$$

on integrating out $\tilde{z}$ and z. This is invariant under the supersymmetry (8) when it is extended to act on the anti-commuting $\tilde{\varphi}$, φ

and their commuting superpartners $\tilde{z}$ and z

$$\delta\varphi = \epsilon z, \quad \delta\tilde{\varphi} = \epsilon\tilde{z}, \quad \delta z = -\epsilon\left(\frac{\dot{\varphi}}{e} + \frac{\chi z}{2e}\right),$$
$$\delta\tilde{z} = -\epsilon\left(\frac{\dot{\tilde{\varphi}}}{e} + \frac{\chi\tilde{z}}{2e}\right). \tag{12}$$

To apply this to the Standard Model we use $\tilde{\varphi}$ and φ with five components partitioned into sets of three and two to accommodate Lie algebra generators of $SU(3)\times SU(2)\times U(1)$

$$(T^R) = i\left(\lambda^b/2, \quad \sigma^a/2, \quad 1_2/2 - 1_3/3\right), \tag{13}$$

where λ and σ are the 3×3 Gell-Mann and 2×2 Pauli matrices, and the form of the $U(1)$-generator is dictated by the tracelessness condition inherited from the underlying string model. Eq. (13) coincides with the embedding into $SU(5)$ of the Georgi–Glashow model [17], however we make no further assumptions about the particle representations to use as these will emerge naturally from the model. Separate couplings are associated with each of the three algebras, but these have been implicitly absorbed into the gauge-field. Integrating over $\tilde{\varphi}$ and φ gives $Z[\mathcal{A}] = \mathrm{Det}\,(i\mathcal{D})$ which we compute by solving the eigenvalue problem (with anti-periodic boundary conditions, as for ψ)

$$i\mathcal{D}\,v(t) = \mu\,v(t), \quad v(2\pi) = -v(0) \tag{14}$$

which has the solution $v(t) = g(t)\,v(0)\exp(-i\mu t)$ and $\mu = n + 1/2 + \log(\rho)/(2\pi i)$ where $v(0)$ is required to be an eigenvector of the matrix $g(2\pi)$ with eigenvalue ρ, and n is an integer. Now

$$\prod_{n=-\infty}^{\infty}\left(1 + \frac{\log\rho}{2\pi i(n+1/2)}\right) = \frac{\sqrt{\rho} + 1/\sqrt{\rho}}{2}, \tag{15}$$

giving

$$\mathrm{Det}\,(i\mathcal{D})_{\mathrm{ap}} \propto \det\left(\sqrt{g(2\pi)} + 1/\sqrt{g(2\pi)}\right). \tag{16}$$

$g(2\pi)$ is block-diagonal with a 3×3 piece

$$\exp\left(\int_0^{2\pi}\frac{i}{3}\mathcal{A}^{U(1)}dt\right)\mathcal{P}\exp\left(-\int_0^{2\pi}\frac{i}{2}\mathcal{A}^{SU(3)}\cdot\lambda dt\right) \tag{17}$$

which we denote by $e^{-i\theta/3}g_{SU(3)}$, with $\theta = -\int_0^{2\pi}\mathcal{A}^{U(1)}dt$, and a 2×2 piece

$$\exp\left(-\int_0^{2\pi}\frac{i}{2}\mathcal{A}^{U(1)}dt\right)\mathcal{P}\exp\left(-\int_0^{2\pi}\frac{i}{2}\mathcal{A}^{SU(2)}\cdot\sigma dt\right) \tag{18}$$

which we denote by $e^{i\theta/2}g_{SU(2)}$. The 3×3 piece has eigenvalues $e^{i\xi_a}$ with $(\xi_a) = (-\theta/3+a, -\theta/3+b-a, -\theta/3-b)$, whilst the 2×2 piece has eigenvalues $e^{i\zeta_a}$ with $(\zeta_a) = (\theta/2+c, \theta/2-c)$ so (16) is

$$\prod_1^2\left(e^{i\zeta_a/2} + e^{-i\zeta_a/2}\right)\prod_1^3\left(e^{i\xi_a/2} + e^{-i\xi_a/2}\right)$$
$$= \left(e^{i\theta/2} + e^{ic} + e^{-ic} + e^{-i\theta/2}\right)$$
$$\times\left(e^{i\theta/2} + e^{i\theta/6}\left(e^{ia} + e^{i(b-a)} + e^{-ib}\right)\right.$$
$$\left. + e^{-i\theta/6}\left(e^{-ia} + e^{i(a-b)} + e^{ib}\right) + e^{i\theta/2}\right)$$

enabling the determinant to be expressed as a sum of products of traces

$$\mathrm{Det}\,(i\mathcal{D})_{\mathrm{ap}} \propto \left(e^{i\theta/2} + \mathrm{tr}\,g_{SU(2)} + e^{-i\theta/2}\right)$$
$$\times\left(e^{i\theta/2} + e^{i\theta/6}\,\mathrm{tr}\,g_{SU(3)} + e^{-i\theta/6}\,\mathrm{tr}\,g^{\dagger}_{SU(3)} + e^{-i\theta/2}\right). \tag{19}$$

The eigenvalues become $\mu = n + \log(\rho)/(2\pi i)$ if we impose periodic boundary conditions on the $\tilde{\varphi}$ and φ, and

$$\left(\prod_{n=-\infty}^{-1}\left(1 + \frac{\log\rho}{2\pi in}\right)\right)\log\rho\left(\prod_{n=1}^{\infty}\left(1 + \frac{\log\rho}{2\pi in}\right)\right) = \sqrt{\rho} - 1/\sqrt{\rho},$$

giving

$$\mathrm{Det}\,(i\mathcal{D})_{\mathrm{p}} \propto \left(e^{i\theta/2} - \mathrm{tr}\,g_{SU(2)} + e^{-i\theta/2}\right)$$
$$\times\left(-e^{i\theta/2} + e^{i\theta/6}\,\mathrm{tr}\,g_{SU(3)} - e^{-i\theta/6}\,\mathrm{tr}\,g^{\dagger}_{SU(3)} + e^{-i\theta/2}\right) \tag{20}$$

and if at the same time we impose periodic boundary conditions on ψ the effect is to generate a γ_5 insertion. Adding together the result of computing (10) with anti-periodic boundary conditions on all fermions to that of imposing periodic boundary conditions results in the sum of (19) and (20) multiplied by γ_5 provided we *choose* the values of the normalisations of the functional integrals, $\mathcal{N}$, in the two cases appropriately. Note that we have simultaneously imposed the same boundary conditions on all the anticommuting variables ψ, φ and $\tilde{\varphi}$. This is necessary to preserve the underlying world-line supersymmetry, because the periodicity of the bosonic variables requires that ϵ in (8) is periodic or anti-periodic when ψ is, and so (12) implies that φ and $\tilde{\varphi}$ are too.

Multiplying out the terms results in the projection operators P_L and P_R multiplied by various exponentials of gauge fields:

$$e^{i\theta}\,P_L + e^{i\theta/2}\,\mathrm{tr}\,g_{SU(2)}\,P_R + P_L + e^{i2\theta/3}\,\mathrm{tr}\,g_{SU(3)}\,P_R$$
$$+ e^{i\theta/6}\,\mathrm{tr}\,g_{SU(3)}\,\mathrm{tr}\,g_{SU(2)}\,P_L + e^{-i\theta/3}\,\mathrm{tr}\,g_{SU(3)}\,P_R$$
$$+ e^{i\theta/3}\,\mathrm{tr}\,g^{\dagger}_{SU(3)}\,P_L + e^{-i\theta/6}\,\mathrm{tr}\,g^{\dagger}_{SU(3)}\,\mathrm{tr}\,g_{SU(2)}\,P_R$$
$$+ e^{-i2\theta/3}\,\mathrm{tr}\,g^{\dagger}_{SU(3)}\,P_L + P_R$$
$$+ e^{-i\theta/2}\,\mathrm{tr}\,g_{SU(2)}\,P_L + e^{-i\theta}\,P_R. \tag{21}$$

From which we can read off the representations and chiralities of the fermions. Each term has a piece $e^{iY\theta}$ where Y is the $U(1)$-charge. $\mathrm{tr}\,g_{SU(2)}$ represents the coupling of an $SU(2)$-doublet, $\mathrm{tr}\,g_{SU(3)}$ the coupling in the fundamental representation of $SU(3)$ and $\mathrm{tr}\,g^{\dagger}_{SU(3)}$ its complex conjugate. This set of twelve terms corresponds to the $U(1)$-charges, $SU(2)$, $SU(3)$ representations and chirality assignments of the fermions and their anti-particles in the Standard Model, augmented by a sterile neutrino (useful in modelling neutrino masses): $(\bar{l}_R, \bar{E}_L, \bar{\nu}_R, U_R, Q_L, D_R, \bar{D}_R, \bar{Q}_L, \bar{U}_R, \nu_R, E_L, l_R)$, respectively.

4. Concluding remarks

We have described a simple generalisation of the world-line approach to chiral fermions that automatically produces the sum over the $SU(3)\times SU(2)\times U(1)$ representations and chiralities that occur in a single generation of the Standard Model augmented by a sterile right-handed neutrino. In the language of field theory the one-loop effective action in a background gauge-field is

$$\log\int\mathscr{D}(E_L, l_R, Q_L, U_R, D_R)\exp\Big(-\int d^4x\,(E_L^{\dagger}\bar{\sigma}\cdot D\,E_L$$
$$+ l_R^{\dagger}\sigma\cdot D\,l_R + \nu_R^{\dagger}\sigma\cdot D\,\nu_R + Q_L^{\dagger}\bar{\sigma}\cdot D\,Q_L$$
$$+ U_R^{\dagger}\sigma\cdot D\,U_R + D_R^{\dagger}\sigma\cdot D\,D_R)\Big),$$

where the gauge-covariant derivative, D, depends on the representation of the field it acts on. By contrast the world-line expression for the variation of this effective action under a change of gauge-field is just

$$\int dT\, \mathscr{D}(w, \psi, \tilde{\varphi}, \varphi)\, \Omega \exp\left(-\frac{1}{2}\int dt \left(\frac{\dot{w}^2}{T} + \psi \cdot \dot{\psi} + \tilde{\varphi} \cdot \mathcal{D}\varphi\right)\right)$$

summed over anti-periodic and periodic boundary conditions on the anti-commuting variables. ($\mathcal{D}$, $\mathcal{A}$ and Ω are defined in (10) and (6).) The information about representations and chiralities is generated by the functional determinant resulting from integrating out the additional anti-commuting world-line variables, $\tilde{\varphi}$ and φ. This model arises naturally in the context of tensionless strings with contact interactions where it becomes necessary to extend the notion of path-ordering along a closed curve into the body of a world-sheet spanning the curve.

We made a number of choices. We chose the gauge group to be that of the Standard Model, with the generators of $SU(3)$ and $SU(2)$ as in (13). The $U(1)$ generator was required to be traceless due to the underlying string model, but we chose its overall normalisation in (13). We also added the result of applying periodic and anti-periodic boundary conditions on all the fermions. This is like a GSO projection, as might be anticipated given the connection with an underlying spinning string theory.

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