Spin Squeezing in a Rydberg Lattice Clock

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We theoretically demonstrate a viable approach to spin squeezing in optical lattice clocks via optical dressing of one clock state to a highly excited Rydberg state, generating switchable atomic interactions. For realistic experimental parameters, these interactions are shown to generate over 10 dB of squeezing in large ensembles within a few microseconds and without degrading the subsequent clock interrogation.

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The precise measurement of frequency in atomic systems has important applications both in fundamental science, such as tests of relativity [1,2] and searches for physics beyond the standard model [3], and in technologies such as satellite navigation [4]. Frequency standards based on optical transitions have begun to surpass the performance of microwave standards, currently used to define the second [5]. In fact, comparisons between optical clocks based on single ions [6] or ensembles of neutral atoms [7,8] are now the most precise measurements ever made. Yb and Sr optical lattice clocks have reached fractional frequency instabilities of 10^{-17} in less than one hour [8,9], approaching the limit imposed by the quantum projection noise associated with measurements on independent atoms.

Squeezing, or quantum correlations between the atoms, can be used to beat this limit and improve the signal-to-noise ratio [10,11], as was proposed [12–14] and demonstrated [15,16] in the context of ion traps. Recent experiments have broken the projection noise limit on microwave clock transitions [17,18]. However, as the state-of-the-art moves towards optical standards, an outstanding challenge is to find an effective method [19–26] for generating squeezed states in lattice clocks.

In this Letter, we describe an approach to generating squeezed states of large numbers of atoms in optical lattices of various geometries by exploiting the strong interaction between highly excited atoms. The strong van der Waals interactions between Rydberg atoms provide a route to creating multipartite entangled states via the so-called dipole blockade [27], including states suitable for quantum-enhanced measurements [28–32]. Here, we consider an off resonant coupling to the Rydberg state, which introduces a switchable, long-range interaction between the atoms in the lattice clock. We show that for realistic experimental parameters, considerable squeezing can be produced within a few microseconds interaction time. The scheme requires only one additional laser, that is switched off during clock operation.

An optical lattice clock consists of an ensemble of N atoms trapped in a d-dimensional optical lattice with lattice constant $a = \lambda/2$. The clock operates on an intercombination transition between the singlet ground state ($|g\rangle$) and a long-lived excited triplet state ($|e\rangle$), as shown in Fig. 1(a). The optical lattice is tuned to a magic wavelength, λ, where

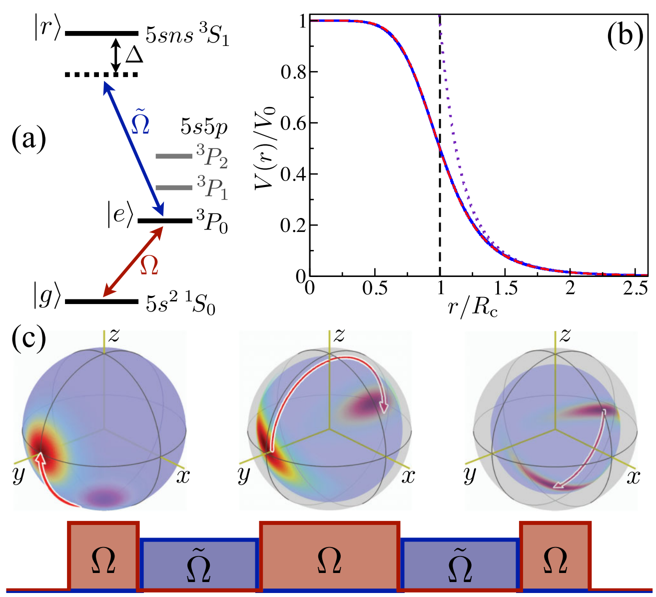

FIG. 1 (color online). (a) Energy level diagram, labeled for the specific example of strontium atoms. Transitions between the clock states $|g\rangle$ and $|e\rangle$ are laser driven with Rabi frequency Ω. A second laser off resonantly couples $|e\rangle$ to a high-lying Rydberg state $|r\rangle$ with Rabi frequency $\tilde{\Omega}$. For a large laser detuning $\Delta \gg \tilde{\Omega}$, the system reduces to effective two-level atoms with binary interactions shown by the solid curve in (b). The potential resembles the van der Waals interaction $\sim 1/r^6$ (dotted curve) at large separation r, but saturates below a critical distance R_c (vertical dashed line). For typical parameters, the interaction potential in a lattice [60] (dashed curve) is virtually identical to the continuum case (solid line), given by Eq. (3). (c) Spin-echo type squeezing protocol, consisting of linear spin rotations around the x axis and nonlinear rotations around the z axis, driven by the two laser fields. The resulting evolution of the total spin is illustrated on a generalized Bloch sphere. For clock operation, this spin-echo sequence is followed by a conventional Rabi or Ramsey scheme.

the relative Stark shifts of the two states cancel [33]. The clock transition is driven with a coupling strength Ω (see Fig. 1) for a high-precision measurement of the transition frequency through Rabi or Ramsey interrogation schemes.

An ensemble of N such two-level atoms is equivalent to a collection of effective spins described by the spin operators $\hat{\sigma}_x^{(i)} = (|g_i\rangle\langle e_i| + |e_i\rangle\langle g_i|)/2$, $\hat{\sigma}_y^{(i)} = i(|g_i\rangle\langle e_i| - |e_i\rangle\langle g_i|)/2$, and $\hat{\sigma}_z^{(i)} = (|e_i\rangle\langle e_i| - |g_i\rangle\langle g_i|)/2$. One can characterize the many-body states of this system via the total spin $\hat{\mathbf{J}} = \sum_i \hat{\boldsymbol{\sigma}}^{(i)}$ that permits convenient visualization of the system dynamics on a generalized Bloch sphere [Fig. 1(c)]. The precision of a spin measurement is fundamentally limited by the uncertainty relation $\Delta\hat{J}_{\perp,1}\Delta\hat{J}_{\perp,2} \geq |\langle\hat{\mathbf{J}}\rangle|/2$ that relates the variances $\Delta\hat{J}_{\perp,1(2)}$ along two orthogonal directions perpendicular to the mean spin $\langle\hat{\mathbf{J}}\rangle$ [34].

For uncorrelated atoms, forming an N-particle coherent spin state, the variances $\hat{J}_{\perp,1} = \hat{J}_{\perp,2} = \sqrt{|\langle\hat{\mathbf{J}}\rangle|/2}$ prescribe a minimum uncertainty circle limited by quantum projection noise in any direction. The creation of quantum correlations between the atoms permits us to reduce the uncertainty $\Delta J_{\perp,\mathrm{min}}$ along one direction at the expense of the other, leading to a squeezed uncertainty ellipse. The degree of squeezing can be quantified by the squeezing parameter

$$\xi^2 = \frac{N(\Delta J_{\perp,\mathrm{min}})^2}{\langle\vec{J}\rangle^2}, \tag{1}$$

which quantifies directly the stability gain relative to an uncorrelated coherent spin state, in the absence of additional phase fluctuations [13]. In the presence of such fluctuations, spin squeezing can still yield a significant improvement depending on the details of the noise sources and the underlying interrogation scheme [35].

Since atoms are initially uncorrelated, squeezing requires state-dependent atomic interactions [12,36]. The interaction between atoms in the clock states ($|g\rangle$ and $|e\rangle$) is very small. In contrast, the van der Waals interaction, C_6/r^6 between Rydberg atoms at a distance r takes on enormous values and, thereby, provides a natural resource for creating correlated states of neutral atoms [37–40]. For a strontium lattice clock operating at the magic wavelength of $\lambda = 813$ nm, the nearest neighbor interaction C_6/a^6 between two $|5s50s\,^3S_1\rangle$-Rydberg state atoms exceeds 10 GHz [41].

These large interaction shifts can be exploited by adding a single laser. It couples the long-lived triplet $^3P_0$ state $|e\rangle$ to a Rydberg state $|r\rangle$ via single photon excitation with a Rabi frequency $\tilde{\Omega}$ (see Fig. 1). Highly excited atomic states with principal quantum numbers $n = 50\ldots100$ have long lifetimes on the order of $\tau_\mathrm{r} \sim 100\ \mu$s. In the present situation, however, the production of large-N squeezed states requires even longer coherence times. Therefore, we consider a far off resonant coupling with detuning $\Delta \gg \tilde{\Omega}$. In this limit, the system can be described in terms of effective two-level atoms composed of the unperturbed ground state $|g\rangle$ and a new Rydberg-dressed excited clock state, $|\tilde{e}\rangle \sim |e\rangle - \varepsilon|r\rangle$. Since only a small fraction $\varepsilon = \tilde{\Omega}/(2\Delta)$ of $|r\rangle$ is admixed, the lifetime $\tau_\mathrm{r}/\varepsilon^2$ of $|\tilde{e}\rangle$ is greatly enhanced.

Most importantly, the laser coupling induces a light shift ΔE_e of the clock states $|e_i\rangle$ [28] that, due to the Rydberg-Rydberg atom interaction, is correlated with the positions $\mathbf{r}_i$ of atoms in $|e\rangle$. From perturbation theory up to fourth order in ε [42],

$$\Delta E_\mathrm{e} = \sum_i \delta_\mathrm{e}|e_i\rangle\langle e_i| + \sum_{i<j} V(|\mathbf{r}_i - \mathbf{r}_j|)|e_i e_j\rangle\langle e_i e_j|, \tag{2}$$

where $\delta_\mathrm{e} = -\Delta(1 - \sqrt{1+4\varepsilon^2})/2$ is the single atom light shift and

$$V(\mathbf{r}_i, \mathbf{r}_j) = V_{ij} = V_0 \frac{R_c^6}{|\mathbf{r}_i - \mathbf{r}_j|^6 + R_c^6}, \tag{3}$$

corresponds to an effective two-body interaction [42–49]. Figure 1(b) illustrates the characteristic shape of the potential. At large distances, independent dressing of the atoms gives rise to a potential that resembles the original interaction between the Rydberg atoms but with a reduced van der Waals coefficient $\varepsilon^4 C_6$. However, below the critical distance $R_\mathrm{c} = |(C_6/(2\hbar\Delta))|^{1/6}$ [42,44,45] simultaneous dressing of both atoms is blocked by the interactions such that the effective potential approaches a constant value $V_0 = (\tilde{\Omega}/2\Delta)^3\hbar\tilde{\Omega}$ given by the difference in the light shift of independent and fully blocked atoms [28].

Adopting the above spin notation and using this dressed-state picture, one obtains the following Ising-type Hamiltonian

$$\mathcal{H} = \hbar\Omega\hat{J}_x + \sum_{i<j}^{N} V_{ij}\hat{\sigma}_z^{(i)}\hat{\sigma}_z^{(j)} + \sum_i \delta_i\hat{\sigma}_z^{(i)}, \tag{4}$$

for the long-range interacting effective spins in a transverse field of strength $\hbar\Omega$ and an inhomogeneous longitudinal field $\delta_i = \delta_\mathrm{e} + (1/2)\sum_{j\neq i} V_{ij}$. Importantly, the transverse ($\hat{H}_x = \hbar\Omega\hat{J}_x$) and longitudinal $[\hat{H}_z = \sum_{i<j} V_{ij}\hat{\sigma}_z^{(i)}\hat{\sigma}_z^{(j)} + \sum_i \delta_i\hat{\sigma}_z^{(i)}]$ terms can be turned on and off independently via the intensities of the two laser fields, thus, providing great flexibility for the controlled creation of entangled many-body states.

To demonstrate the feasibility of spin squeezing, we consider here a spin-echo type sequence, illustrated in Fig. 1(c). Starting from all atoms in the ground state $|g\rangle$, one first applies a $\pi/2$ pulse, with the dressing laser turned off ($\tilde{\Omega} = 0$), which rotates the total spin along the x axis [see Fig. 1(c)]. Subsequent Rydberg dressing [28], i.e., application of the interaction Hamiltonian $\hat{H}_z$, for a time $\tau/2$ then leads to a twisting of the uncertainty ellipse [12] around the z axis. In order to eliminate the undesired linear spin rotation and broadening due to the inhomogeneous detunings δ_i, we subsequently apply a π pulse followed by a second dressing phase of duration $\tau/2$ and, finally, another $\pi/2$ that rotates the total spin back along the

z axis. The resulting dynamics can be solved analytically and yields for the final mean spin, $\langle \hat{J}_x \rangle = \langle \hat{J}_y \rangle = 0$,

$$\langle \hat{J}_z \rangle = -\frac{1}{2} \sum_{i=1}^{N} \prod_{k \neq i}^{N} \cos(\varphi_{ik}), \tag{5}$$

where $\varphi_{ij} = V_{ij}\tau/2\hbar$. The perpendicular spin component $\hat{J}_\perp(\theta) = \cos(\theta)\hat{J}_x + \sin(\theta)\hat{J}_y$ at an angle θ with respect to the x axis has an uncertainty

$$(\Delta J_\perp)^2 = \cos^2(\theta)\langle \hat{J}_x^2 \rangle + \sin^2(\theta)\langle \hat{J}_y^2 \rangle + \cos(\theta)\sin(\theta)\langle \hat{J}_x\hat{J}_y + \hat{J}_y\hat{J}_x \rangle, \tag{6}$$

where $\langle \hat{J}_y^2 \rangle = N/4$,

$$\langle \hat{J}_x^2 \rangle = \frac{N}{4} + \frac{1}{4} \sum_{i<j}^{N} \Big[\prod_{k \neq i,j}^{N} \cos(\varphi_{ijk}^-) - \prod_{k \neq i,j}^{N} \cos(\varphi_{ijk}^+) \Big],$$

$$\langle \hat{J}_x\hat{J}_y + \hat{J}_y\hat{J}_x \rangle = -\sum_{i<j}^{N} \sin(\varphi_{ij}) \prod_{k \neq i,j}^{N} \cos(\varphi_{ik}), \tag{7}$$

and $\varphi_{ijk}^\pm = \varphi_{ik} \pm \varphi_{jk}$.

In Fig. 2, we show the time evolution of the squeezing parameter ξ^2 obtained from Eqs. (5)–(7) upon minimizing $\Delta J_\perp$ with respect to θ. For simplicity, we assume an occupation number of one atom per site, but generalizations to higher and fluctuating lattice occupations are straightforward. For the chosen dressing parameters ($R_c/a = 3$), the squeezing parameter assumes its minimum value $\xi^2_{\rm min}$ after a short time well below $\hbar/V_0$. Higher dimensional lattices yield stronger squeezing due to a larger number of atoms $N_c \sim (R_c/a)^d$ within the soft-core radius R_c.

For small fully blockaded systems with $N < N_c$ [28], the interaction Hamiltonian reduces to $\hat{H}_z = V_0\hat{J}_z^2/2 + \delta\hat{J}_z$, such that the above pulse sequence corresponds to standard one-axis twisting by infinite-range interactions [12,28]. However, as shown in Fig. 3, the squeezing parameter $\xi^2_{\rm min}$ continues to decrease for system sizes $L = N^{1/d}a$ well beyond R_c, indicating that entanglement between distant spins extends beyond the range of the interactions. For small systems $L < R_c$ Eqs. (5)–(7) yield the familiar $\xi^2_{\rm min} \sim N^{-2/3}$ scaling [12], as indicated by the dashed line in Fig. 3. Around $L = R_c$, atoms start to explore the finite range of V_{ij} such that the length of the total spin, $\hat{J}^2$, is no longer conserved and the system is driven out of the symmetric Dicke state basis with maximum $\langle \hat{J}^2 \rangle = N(N/2+1)/2$. The corresponding decrease of $\langle \hat{J}^2 \rangle$ causes an accelerated drop of the final signal $\langle \hat{J}_z \rangle$, leading to a slight increase in $\xi^2_{\rm min}$. However, as the system size is increased further, the continual reduction of $\Delta J_{\perp,\rm min}$ compensates for the additional signal loss, such that $\xi^2_{\rm min}$ decreases well below the full-blockade limit [28] (dotted line in Fig. 3).

The $N \to \infty$ limit (dash-dotted line in Fig. 3) can be calculated efficiently since in this case $\hat{\sigma}_\alpha^{(i)} = \hat{\sigma}_\alpha^{(j)}$ ($\alpha = x, y, z$). Figure 4 shows this minimum squeezing parameter ξ^2_∞ as a function of the interaction range R_c/a. For $R_c > a$. the squeezing parameter is found to decrease as $\xi^2_\infty \sim (R_c/a)^{-0.76d} \sim N_c^{-0.76}$, i.e., showing a faster drop than small, completely blocked ensembles. For a typical value of $R_c/a = 5$, we find sizeable squeezing of $\xi^2_\infty = 0.04$ for 2D and $\xi^2_\infty = 0.005$ for 3D lattices. Equally important, the time required to obtain optimal squeezing rapidly decreases with R_c/a. For $R_c/a = 5$, optimal squeezing is reached after a total dressing time of $\tau = 0.14\hbar/V_0$ for 2D and $\tau = 0.04\hbar/V_0$ for 3D lattices, such that dressing-induced losses can be kept at a low level.

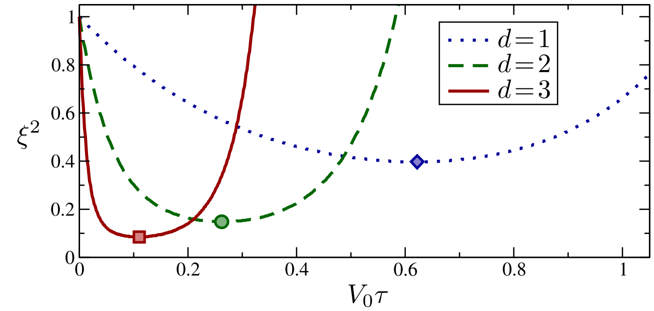

FIG. 2 (color online). Time dependence of the squeezing parameter ξ^2 in a d-dimensional optical lattice for $R_c/a = 3$. Optimal squeezing is indicated by the symbols.

Finally, we consider the specific implementation of this scheme in a Sr lattice clock, including decoherence mechanisms due to the virtual excitation of Rydberg states. Rydberg excitation of cold Sr atoms has been studied in recent experiments [50–52], and was also considered for blackbody thermometry in Sr lattice clocks [53]. The $5sns\ ^3S_1$ Rydberg series has nearly isotropic, repulsive van der Waals interactions [41], and can be accessed via single-photon excitation from the $5s5p\ ^3P_0$ level with 317 nm laser light. Tunable, solid-state lasers producing ~0.5 W of narrow-band cw light have been developed for

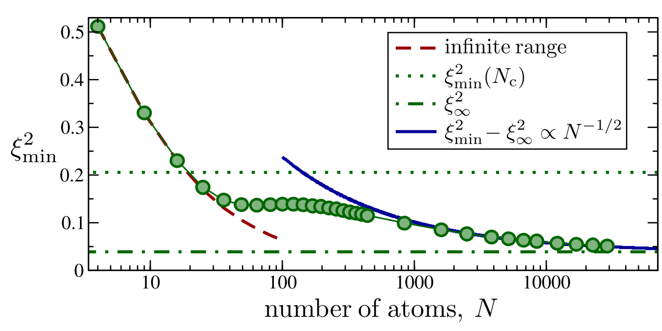

FIG. 3 (color online). Optimal squeezing parameter in a 2D lattice as a function of N for $R_c/a = 5$. The dashed curve shows the result of one-axis twisting by infinite-range interactions [12], which agrees with the exact calculations (circles) for small system sizes $N < N_c = \pi(R_c/2a)^2$ for which all atoms are blocked. However, the squeezing parameter decreases well below the corresponding value $\xi^2_{\rm min}(N_c)$ (dotted line). For large N, $\xi^2_{\rm min}$ approaches a limiting value ξ^2_∞ (dash-dotted line) as $\sim N^{-1/2}$ (thick solid curve).

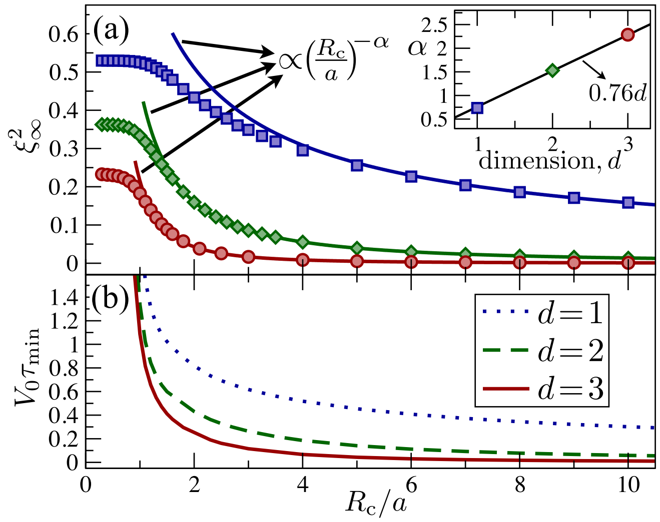

FIG. 4 (color online). (a) Minimal squeezing parameter ξ_∞^2 attainable in d-dimensional lattices as a function of R_c/a. For large R_c, ξ_∞^2 display a power-law decay with an exponent $\alpha \propto d$ (inset). Panel (b) shows the corresponding interaction time $\tau_{\min}$ to realize optimal squeezing.

similar wavelengths [54], which would permit coupling to the $|5s55s\,^3S_1\rangle$ state with a Rabi frequency of $\tilde{\Omega}/2\pi \approx 20$ MHz for a reasonable beam waist of $w = 30\ \mu$m. With $\Delta = 20\ \Omega$, these parameters yield a blockade radius of $R_c = 5a$ and $V_0 = 2$ kHz. According to Fig. 4, this results in a minimum squeezing parameter of $\xi_\infty^2 = 5\times10^{-3}$ within a dressing time of $\tau = 20\ \mu$s for a 3D lattice clock. This time scale is 3 orders of magnitude shorter than the lifetime $\tau_{\rm ryd}/\varepsilon^2 \approx 60$ ms of the Rydberg-dressed clock state. Rydberg state decay causes simple dephasing of the dressed clock transition [55,56] with a long time scale τ_{ryd}/ε^2, having no significant effect on the attainable squeezing, since $\varepsilon^2\tau/\tau_{\rm ryd} \ll 1$ for typical parameters. The finite waist of the dressing beam limits the transverse sample size for which strong squeezing can be obtained. Explicit simulations, however, show that these inhomogeneity effects are rather modest, and, e.g., increase ξ by only 15% for a cubic sample of size $\sim w/2$, corresponding to $\sim 5\times10^4$ atoms. Moreover, straightforward estimates show that typical errors in the area of the $\tilde{\Omega}$ pulses only cause frequency shifts well below additional uncertainties of lattices clocks [8,9].

Operating at the magic wavelength for the two clock states prohibits identical confinement of Rydberg atoms. In fact, the Rydberg state polarizability even changes sign [57,58] and yields an inverted lattice potential by a factor ≈ -0.7 [31]. However, due to the weak Rydberg admixture to the excited clock state, its trapping potential is modified only by a negligible fraction of $\sim 0.5\varepsilon^2 = 3\times10^{-4}$, and, hence, does not cause appreciable atomic motion. As pointed out in [59], a more significant effect may arise from motional dephasing induced by the strong van der Waals interaction between excited Rydberg atoms. In the dressing regime, these interactions, however, cause no decoherence but only modify the effective interaction potential to a negligible extend [60]. This is illustrated in Fig. 1(b) for $R_c = 5a$ and a moderate trap depth of 10 recoil energies [61]. Importantly, all of these side effects operate only within the interaction time τ, and, hence, do not affect the subsequent measurement stage during which the dressing lasers are switched off.

In conclusion, we have shown that significant spin squeezing can be obtained in existing optical lattice clocks using only a single additional laser that couples one of the clock states to a Rydberg state. Presently, the major limitation of clock stabilities stems from fluctuations of the probe laser [35], notably, the Dick effect [62], which requires to minimize the dead time of the clock cycle for optimal clock operation [8]. In this respect, an important advantage of the present squeezing protocol is that its duration is limited only by the total length of the involved Ramsey pulses, $\Omega(t)$. Hence, the signal-to-noise ratio can be improved without a significant reduction in duty cycle. Several extensions of the scheme seem promising. For example, more involved two-axis twisting schemes [12] seem possible [28] and will provide stronger squeezing. Using the present coupling scheme, simultaneous driving by the transverse ($\hat{H}_x$) and longitudinal ($\hat{H}_z$) Hamiltonians [63,64] or optimized sequences of $\hat{H}_x$ and $\hat{H}_z$ [65] could also yield enhanced squeezing. More broadly, the availability of long-lived triplet states in two-electron atoms permits nS-Rydberg-state dressing via single-photon excitation and, thus, with much higher Rabi frequency and lower decoherence than possible for two-photon dressing of alkaline atoms [42]. This opens up a promising route for the exploration of strong long-range interactions in Bose-Einstein condensates [46,47,49,66,67] and optical lattices [68] as well as long-range interacting quantum spin systems, using the setup described in this Letter.

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