DISCUSSION

THE SUM RULE HAS NOT BEEN TESTED*

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The debate between Glymour ([2]) and Fine ([1]) hinges in part on a comparison of the width of the incoming wave packet in momentum space with the angles intercepted by the detectors in the Cross-Ramsey experiment. As Fine argues, it follows from the quantum formalism that the initial dispersion will be conserved in Compton scattering, and he allows that the Sum Rule is constrained by the statistical results of quantum mechanics. The Sum Rule may fail, but it will not fail in any way that violates quantum mechanics. Thus most of the time the individual momentum value does not change more than the width of the initial wave packet: usually the value of $Q_{\gamma e}$ at t must equal the value of Q_{γ} at t' plus the value of Q_e at $t' \pm \Delta p/2$, where Δp represents the dispersion in momentum for the initial gamma ray. Thus in order to refute Fine's thesis, the detectors must discriminate finely enough to pick out changeovers of individual values within Δp.

Standard treatments of experiments like Cross-Ramsey usually assume that the gamma beam is in an eigenstate of momentum. If this were so, conservation would follow immediately from standard quantum theory, there could be no switchover of values on Fine's analysis, and Compton-Simon or Cross-Ramsey experiments would be irrelevant for testing Fine's thesis. But this assumption is clearly an oversimplification. At most the initial beam could be in a "near" eigenstate. So the question arises, is it "near enough" to make Cross-Ramsey irrelevant? Glymour says at the end of his paper that he sees no reason to think so. I see three reasons. In order for a Cross-Ramsey experiment to test the Sum Rule, the detectors must respond to angle spreads *smaller* than the dispersion in the initial beam. But (1) this is not the case in Cross-Ramsey; (2) it is never the case in scattering experiments; and (3) if it were the case in Cross-Ramsey, it is unclear whether the detectors would be detecting *momentum* at all.

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(0) Before looking at the Cross-Ramsey experiment, there is one general difficulty with the Glymour program that should be noted. To tell whether or not the final value of the sum is within Δp of the initial momentum value, the initial momentum of the individual photon must be determined within an interval δp much smaller than Δp. Attempts to do so, however, would normally be taken to reprepare the photon in a new state with dispersion $(\Delta p)' = \delta p$. This merely resets the problem. Given the new wave function, it follows from quantum mechanics that most probably the initial momentum of the photon equals the sum of the final momenta, plus or minus $(\Delta p)'/2$, and to test the Sum Rule, the individual values must be determined more precisely than $(\Delta p)'$. Thus in order to test the Sum Rule, it must be possible to measure the photon without repreparing it. This is thought by many to be impossible. But since it is a matter of philosophical controversy, it is worth considering other problems with the experimental arrangement. The chief problem is that the Cross-Ramsey detectors are too coarse-grained—by orders of magnitude—to test the Sum Rule.

(1) Cross and Ramsey are concerned with both energy and momentum conservation. For simplicity, I shall concentrate entirely on momentum and assume that the incoming beam is represented by a wave packet with a narrow spread around E_0. If the photons are to have approximately the same energy, yet there is to be a dispersion among their momenta, this must be due to a spread in direction. (See figure 1.) Not all are moving straight down the $\vec{z}$ axis. Let $(\Delta p)^2 = \langle(\vec{p} - \vec{p}_0)^2\rangle = 4p_0^2 \sin^2 \phi/2$ where $\vec{p}_0$ is the mean momentum.

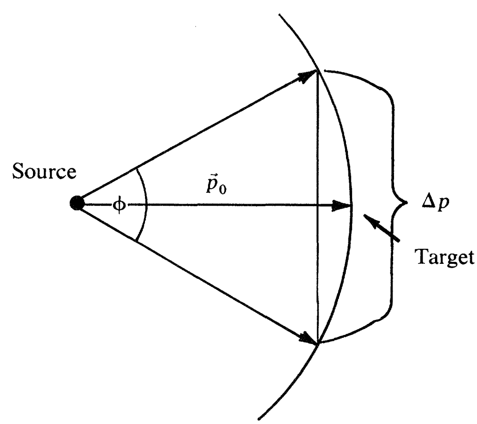

Figure 1

Hence $E_0^2 = |p_0|^2/2m$. How large, then, must Δp be to test the Sum Rule with the Cross-Ramsey apparatus?

Since they have a finite extension in space, the Cross-Ramsey detectors will pick up an angular spread in momentum. From the diagram in the Cross-Ramsey paper, the electron detector is approximately 6 *cm* from the target. It is 13 × 17 *mm* in area. It turns out that we need be only interested in orders of magnitude, so I shall take it to be a circle 6 *mm* in radius, rather than a rectangle. The gamma detector is about the same, so the total angular spread is about 2/10, which corresponds to angles of about 10°. If Cross-Ramsey is to test the Sum Rule, the angular spread in the initial gamma beam must be much greater than that in the detectors. But to the contrary. The beam passes through 28 *cm* of lead before exiting through a hole 1.5 *mm* in radius. Thus the angular spread for photons cannot be much greater than 1/200.

(2) It is no accident that the detectors do not discriminate within the angular spread of the scattered beams. Scattering experiments are deliberately designed that way, both for experimental and for theoretical reasons. If the original momentum spread is too great, what starts out as a particle may not continue to act like one. The wave packet will smear over space and classical limits will no longer be applicable. Goldberger and Watson ([3]) stress this in their encyclopedic work on scattering theory. Throughout the entire book they consider only near momentum eigenstates for the initial beam, because ". . . under the conditions that permit macroscopic observations, the spreading of the quantum mechanical wave packet is ordinarily quite negligible. In fact the experimental techniques for measuring cross sections assume this to be true, since classical mechanics is conventionally used in calculating orbits in accelerators, through focussing magnets, and so on" ([3]).

Moreover, it is an essential assumption in the general theory of scattering processes that the detector not discriminate among values in the packet. This is crucial to guarantee that the scattering cross section does not depend on the shape of the incoming wave.[1] Thus it would, at best, be a messy business to analyze a scattering experiment which would test the Sum Rule; for one could not rely on standard scattering theory in calculating corrections, as do Cross and Ramsey and the authors they cite.

(3) An experiment to test the Sum Rule must, in contrast to Cross and Ramsey, use a broad initial beam and extremely fine detectors. Besides practical questions, an interesting philosophical problem

[1] See, for example, [4], pages 193–197.

confronts any such experiment. The problem arises because momentum direction can only be measured by taking pairs of position readings. A particle initially localized in a region around the origin is detected a short time later near R; to within θ, the momentum is supposed, therefore, to be directed along $\vec{R}$. (See figure 2.) Classically, for a

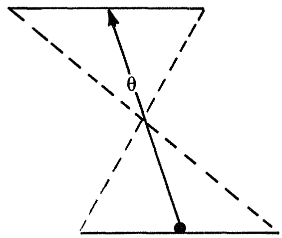

Figure 2

free particle this assumption is trivially valid. Quantum mechanically it is not. A necessary condition on its truth is that the probability for the particle to be found at R must equal (or be proportional to) the probability that the momentum be directed along $\vec{R}$. These two probabilities, however, are formally calculated in different ways. The probability of detection at $\vec{R}$ at t for a particle in state Ψ is given by

(1) $$\text{Prob}\,(r = \vec{R}, t = t) = \Psi^*(\vec{R}, t)\Psi(\vec{R}, t)$$

whereas the probability of having momentum $(m/t)\vec{R}$ is given by $|\langle\Psi|e^{i\vec{K}\cdot\vec{r}}\rangle|^2$ for $\vec{K} = \vec{P}/\hbar = m\vec{R}/\hbar t$, or,

(2) $$\text{Prob}\,(\vec{p} = m\vec{R}/t) = \left| \int_{-\infty}^{+\infty} \Psi(\vec{r},0)\, exp\,(im\vec{R}\cdot\vec{r}/\hbar t)\, d^3r \right|^2$$

In a normal scattering experiment, where Δp is small, the electron which arrives at the detector can be represented fairly well as a minimum uncertainty wave packet, *i.e.* a packet in which $\Delta r \Delta p = \hbar/2$. In this case, it is trivial to show that the two probabilities are proportional. The position density for a minimum uncertainty wave packet is given by the Gaussian distribution[2]

(3) $$[2\pi((\Delta r)^2 + (\Delta p)^2 t^2/m^2)]^{-3/2}\, exp\,(-(\vec{r} - \vec{v}_0 t)^2/2\,((\Delta r)^2 + (\Delta p)^2 t^2/m^2).$$

[2]Cf. [4], pages 14–16.

For times during which $(\Delta p)^2 t^2/m^2$ is small compared to (Δr),^{2} the packet acts like a classical particle with velocity $\vec{p}_0/m$; but as $(\Delta p)^2 t^2/m^2$ gets large, the dispersion in momentum produces a spread in the spatial uncertainty. It is then sensible to ask if the density at a point is proportional to the probability that the momentum is directed toward that point.

In order to separate the density due to relocation of the whole packet from that due to the momentum dispersion, it is best to choose a coordinate system in which the particle is at rest, *i.e.* one in which $\vec{p}_0 = 0$. Then $A(\vec{K}) = \langle \Psi | e^{i\vec{K}\cdot\vec{r}} \rangle$ is

(4) $$A(\vec{K}) = (2(\Delta r)^2/\pi)^{3/4}\, exp\,(-K^2(\Delta r)^2)$$

and

(5) $$|\, A(m\vec{R}/\hbar t)\,|^2 = (2(\Delta r)^2/\pi)^{3/2}\, exp\,(-2m^2 R^2 (\Delta r)^2/\hbar^2 t^2)$$

The spatial density outside the original packet will not be appreciable unless $(\Delta p)^2 t^2/m^2 \gg (\Delta r)$.^{2} In this case we may set (Δr) approximately equal to 0 in the coefficient of (3). Evaluating at $\vec{R}$ and using $(\Delta r)^2(\Delta p)^2 = \hbar^2/4$, (3) becomes

(6) $$\text{Prob}\,(r = \vec{R},\, t = t) = (2\pi\hbar^2 t^2/4m^2(\Delta r)^2)^{-3/2}$$
$$exp\,(-2m^2 R^2 (\Delta r)^2/\hbar^2 t^2)$$

Thus for fixed t the two probabilities are proportional. If we choose $m^2\hbar^2/t^2 \approx 1$, (which is easily seen to be consistent with the earlier restriction on $(\Delta p)\,t/m$) they are identical.

The demonstration of (6) is exceedingly simple because both the distribution in space and the distribution in momentum are Gaussian. But if the initial wave is not a minimum uncertainty packet, this will no longer be true, and a more general proof must be offered.

This is just the situation which confronts Glymour. Even with high energy beams and the best detectors available, the direction of an electron can only be determined within a few microradians. To test the Sum Rule, the momentum dispersion, Δp, must be at least of this order of magnitude. So the electron would have to be localized within $\Delta r \approx \hbar \times 10^7 \approx 10^{-27}\, m$ in order to have a minimum uncertainty wave packet. But this is absurd; the classical radius of the electron is only of the order of $10^{-15}\, m$. So the scattered beam cannot be a minimum uncertainty packet if the Sum Rule is to be tested by a Compton scattering experiment. But if it is not, the issue of whether the Sum Rule is in principle testable is not settled. For an argument must still be given for identifying the probability of detection with the momentum probability.

REFERENCES

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[4] Rodberg, L. S. and Theler, R. M. *Introduction to the Quantum Theory of Scattering*. New York and London: Academic Press, 1967.