Gravity and the Stability of the Higgs Vacuum

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We discuss the effect of gravitational interactions on the lifetime of the Higgs vacuum where generic quantum gravity corrections are taken into account. Using a "thin-wall" approximation, we provide a proof of principle that small black holes can act as seeds for vacuum decay, spontaneously nucleating a new Higgs phase centered on the black hole with a lifetime measured in millions of Planck times rather than billions of years. The corresponding parameter space constraints are, however, extremely stringent; therefore, we also present numerical evidence suggesting that with thick walls, the parameter space may open up. Implications for collider black holes are discussed.

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With the discovery of the Higgs boson at the LHC [1,2] and the measurement of its mass, it seems we live in interesting times: The running of the coupling of the Higgs boson quite possibly means that our vacuum is only metastable, and the true Higgs vacuum in fact lies at large expectation values of the Higgs and negative vacuum energy. Although this metastability at first might seem alarming, in order for the vacuum to decay, it must tunnel through a sizable energy barrier. The probability for this typically has an exponential factor

$$\Gamma \sim A e^{-B/\hbar}, \tag{1}$$

where B is the action of a solution to the Euclidean field equations, "the bounce," which interpolates between the metastable (false) and true vacua; the prefactor A is determined from fluctuations around the bounce. Since the action B is usually large (we will set $\hbar$ to 1 for the rest of this Letter) the probability of vacuum decay is very low. For the decay of a false vacuum, the process was understood and the probability computed in a series of papers by Coleman, Callan, and De Luccia [3–5]. This "gold standard" calculation is now used ubiquitously to estimate decay rates and the half-life of a false vacuum state in field theory; for the Higgs vacuum it predicts a lifetime well in excess of the age of the Universe.

The Coleman *et al.* picture of vacuum decay is, however, very idealized, in that an exactly homogeneous and isotropic false vacuum decays into a very nearly symmetric configuration: a completely spherical bubble of true vacuum which expands outwards with uniform acceleration. In everyday physics, however, first-order phase transitions are far from clean; they often proceed not via some perfect nucleation process, but rather by impurities acting as sites for the condensation of a new phase. Recently in [6] we investigated the impact of gravitational impurities, in the guise of black holes, on the usual Coleman–De Luccia (CDL) picture (see also [7,8] for early investigations of this issue) and found a significant enhancement of vacuum decay in a wide range of cases—the intuition of an impurity seeding nucleation of a true vacuum bubble was entirely borne out by the analysis.

To recall Coleman's original intuition: The nucleation of a bubble costs energy because a wall with energy and tension is formed as a barrier between the false and true vacua; to counter that, energy is gained by the volume of space inside the bubble having lower energy by virtue of having transitioned to the true vacuum. A bubble of just the right size then optimizes this energy payoff and, once formed, it grows. The picture with a gravitational inhomogeneity is similar: The bubble forms around the (Euclidean) black hole, but because of the distortion of space the payoff between volume inside a bubble and its surface area is changed; bubbles form at a smaller radius and, hence, the "cost" of forming them is lower. The instanton has a smaller action and the decay process can be significantly enhanced. Alternatively, in terms of the original energy argument of Coleman and De Luccia, the addition of a seed black hole that is eliminated or reduced by the bubble can change the energy balance dramatically.

Can this process affect the lifetime of the Higgs vacuum? We will show that it can, although only if small black holes nucleate the decay. Such black holes could result from gradual evaporation of primordial black holes formed in the early Universe [9]; alternatively, if there are "large" extra dimensions [10,11] responsible for producing a hierarchically large Planck scale in our universe, then small black holes can be produced at the LHC [12]. Depending on the tension of the bubble wall, which is directly related to parameters in the Higgs potential, the enhancement of vacuum decay can be large.

To briefly review the Higgs potential, note that at large values of the Higgs field, we can pick any component ϕ and approximate the potential using an effective coupling constant λ_{eff},

$$V(\phi) = \frac{1}{4}\lambda_{\mathrm{eff}}(\phi)\phi^4. \tag{2}$$

The effective coupling is obtained by combining the running of λ under the renormalization group with the low-energy particle physics parameters. Two-loop calculations of the running coupling [13], including contributions from all of the standard model fields, yield a high-energy approximation

$$\lambda_{\mathrm{eff}} \approx \lambda_* + b\left(\ln\frac{\phi}{\phi_*}\right)^2, \tag{3}$$

where the fit to the results of [13] give parameter ranges $-0.01 \lesssim \lambda_* \lesssim 0$, $0.1M_p \lesssim \phi_* \lesssim M_p$, and $b \sim 10^{-4}$. These parameters ranges are mostly due to experimental uncertainties in the Higgs boson and top quark masses; however, with the currently measured values it seems that λ_{eff} near the Planck scale is small with a preference towards negative values.

Of course, this discussion assumes no impact from new physics between the TeV scale and the Planck scale. At the very least, quantum gravity effects will have to be taken into account. On dimensional grounds, we might expect some modifications to the potential of the following form [14]:

$$V(\phi) = \lambda_{\mathrm{eff}}(\phi)\frac{\phi^4}{4} + (\delta\lambda)_{\mathrm{bsm}}\frac{\phi^4}{4} + \frac{\lambda_6}{6}\frac{\phi^6}{M_p^2} + \frac{\lambda_8}{8}\frac{\phi^8}{M_p^4} + \cdots, \tag{4}$$

where $(\delta\lambda)_{\mathrm{bsm}}$ includes possible corrections to the running coupling from physics beyond the standard model, and the polynomial terms arise with new physics identified with the Planck scale. If the coefficients λ_6, etc., are similar in magnitude, then the small size of λ_{eff} at the Planck scale means that the interesting physics occurs where the potential is determined predominantly by λ_{eff} and λ_6. In Fig. 1 we illustrate the effect of these corrections on the standard model potential with $\lambda_* = -0.01$.

Quantum tunneling in a corrected potential such as this has been looked at by Branchina *et al.* [15], who take $\lambda_* \sim -0.1$, where the potential barrier occurs at $\phi \ll M_p$, and a negative $\lambda_6 = -2$. They argue the existence of a greatly enhanced tunneling rate; however, their discussion entirely neglected gravitational backreaction of the instanton on the geometry.

In order to explore the impact of a gravitational impurity, we will extend the method of Coleman and De Luccia [5] to include a black hole. We find that within the CDL "thin-wall" description, the tunneling amplitude can be significantly enhanced by a small black hole, albeit within a small region of parameter space. This provides a "proof of principle" and motivates a numerical analysis of Higgs instantons, which confirms the presence of strongly enhanced decay in the presence of black holes.

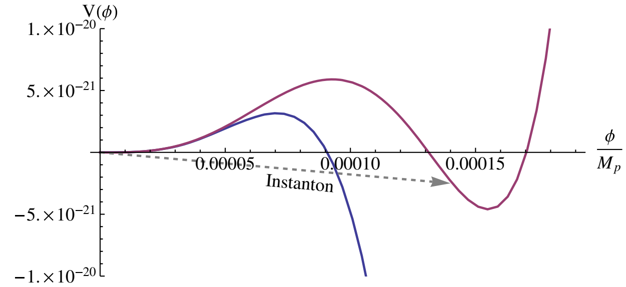

FIG. 1 (color online). An illustration of the impact of the quantum gravity correction (in red), here shown for $\lambda_* = -0.01$, $\phi_* = 2M_p$, and $\lambda_6 = 63$ K. The path of the bounce is sketched.

The equations of motion for a wall bounding two different regions of spacetime with different cosmological constants and black-hole masses can be expressed in the form $\dot{R}^2 + V(R) = 0$, where R is the bubble radius (as a function of Euclidean time) and $V(R)$ is an effective potential involving the wall tension. For the decay of the Higgs vacuum, we assume the standard model has $\Lambda_+ = 0$, and write the true vacuum cosmological constant as $\Lambda_- = -3/\ell^2$; then, the potential V depends on ℓ, the black-hole masses $M_\pm$, and the surface tension of the bubble wall. (See [16] for explicit forms of this potential.)

To recapitulate the results of [6], the action of a general instanton with a black hole was found to be

$$B = \frac{\mathcal{A}_h^+}{4G} - \frac{\mathcal{A}_h^-}{4G} + \frac{1}{4G}\oint d\lambda\{(2R - 6GM_+)\dot{\tau}_+ - (2R - 6GM_-)\dot{\tau}_-\}, \tag{5}$$

where $R(\tau_\pm)$ is the solution for the bubble wall and $\mathcal{A}_h^\pm$ are the black-hole horizon areas corresponding to $M_\pm$. This result includes a careful treatment of the conical deficits that can arise in the Euclidean section when the periodicity of the bubble solution is not the same as that of the black hole; although the specifics of computing actions in vacuum and anti–de Sitter (AdS) space vary from that of de Sitter space, the essence of the calculation remains the same as the presentation in [6], and the result, Eq. (5), identical in form.

This bounce action feeds directly into the exponent in (1), and following Callan and Coleman [4], we estimate the prefactor by taking a factor of $(B/2\pi)^{1/2}$ for the single time-translational zero mode of the instanton, but we use the light crossing time of the black hole, $(GM_+)^{-1}$, as a rough estimate of the remaining determinant of fluctuations, giving

$$\Gamma_D \approx \left(\frac{B}{2\pi G M_+}\right)^{1/2} e^{-B}. \quad (6)$$

Typically, the CDL action is of order $\mathcal{O}(10^{3\text{–}6})$ for the Higgs potentials, leading to a huge exponential suppression of the decay rate, and to the conclusion that gravitational tunneling is irrelevant.

However, the effect of a black hole, Eq. (5), on the tunneling action can be very significant for low-tension bubble walls and small-mass black holes. As the seed black-hole mass M_+ is switched on, the instanton action drops rapidly, and the bubble initially nucleates by removing the black hole. However, as the seed black-hole mass continues to increase, a critical mass M_C is reached at which the potential $V(R)$ has a single point at which $V = V' = 0$, and there exists a static bubble-wall solution. In this case, an unstable static bubble nucleates, which will either recollapse or expand with roughly equal probability. As the seed black-hole mass increases further, the nucleated bubble now has a black-hole remnant in the bubble interior, with the action now rising with increasing seed mass. The quantitative values of this critical mass, and the maximal suppression of the bounce action at M_C, depend on the wall tension parameter σ and the true vacuum energy; however, unless the combination $\sigma\ell$ is Planck scale, this suppression is several orders of magnitude at M_C, thus changing the exponential factor in Eq. (6) from an irrelevant 10^6 to a potentially extremely relevant $10^{0\text{–}2}$.

Whether or not this enhancement is relevant depends on its magnitude relative to other physical decay processes, specifically, black-hole evaporation. The key indicator is, therefore, the branching ratio of the static tunneling decay rate to the Hawking evaporation rate, $\Gamma_H \approx 3.6 \times 10^{-4}(G^2 M_+^3)^{-1}$ [17],

$$\Gamma_D/\Gamma_H \approx 44(M_+^2/M_p^2)B^{1/2}e^{-B}. \quad (7)$$

For our thin-wall instantons, there is indeed a range of M_+ (small, though still above the Planck mass) for which we have very strong enhancement of bubble tunneling.

The main wrinkle in this argument is that the condition for the thin-wall approximation requires that the energy at the potential minimum is smaller than the potential barrier height; scanning through parameter space, we find that requiring a thin wall is very constraining: The range of λ_6 for which this occurs is very small, and it occurs for large values of the parameter $\lambda_6 \gtrsim 10^3\text{–}10^5$, depending on λ_*. On the other hand, computing the branching ratio, Eq. (7), for these models shows that tunneling does indeed dominate. Thus, while our pseudoanalytic discussion is limited in the sense of parameter space, it has provided a proof of principle that black holes could potentially seed vacuum decay.

In order to decide whether this effect is restricted to a niche of parameter space or is potentially relevant, a full exploration of instantons outside of the thin-wall approximation is necessary. Motivated by our thin-wall results, in which the enhanced tunneling takes place with the static instanton (as $M_+ > M_C$, which is typically less than the Planck mass), we have made a preliminary numerical investigation of static instantons, taking $\lambda_* = -0.01$ and $b = 10^{-4}$ as representative values for the Higgs potential.

Static bounce solutions to the Einstein-scalar equations with rotational symmetry on a black-hole AdS background can be found using a spherically symmetric metric ansatz,

$$ds^2 = f(r)e^{2\delta(r)}d\tau^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (8)$$

where

$$f = 1 - \frac{2G\mu(r)}{r}. \quad (9)$$

The solutions are obtained using a shooting technique, varying the value of the scalar field at the black-hole horizon and aiming for $\phi \to 0$ as $r \to 0$. In Ref. [6], it was shown that the action is given by the area terms in Eq. (5), as in the thin-wall case. The resulting values of the action for a selection of Higgs models is shown in Fig. 2. Note that the semiclassical bubble nucleation argument only applies when the action $B > 1$.

Computing the branching ratio now with these "thick-wall" solutions gives Fig. 3. Although black holes produced in the early Universe start out with relatively high masses, their temperature is nonetheless above that of the microwave background, and they evaporate down into the range plotted in Fig. 3. At this point, the mass hits a range in which vacuum decay is more probable; i.e., the tunneling half-life becomes smaller than the (instantaneous) Hawking

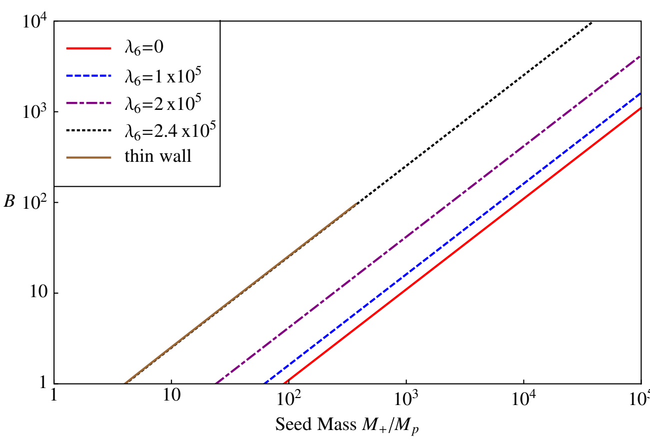

FIG. 2 (color online). The action for a bounce solution. Each plot corresponds to a different value of λ_6 in the Higgs potential (4), with $\lambda_* = -0.01$ and $b = 1.0 \times 10^{-4}$. The largest value of λ_6 is within the range of the thin-wall approximation, and the thin-wall result is shown for comparison.

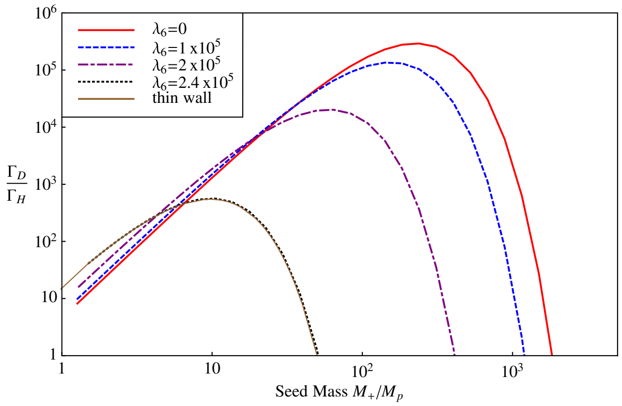

FIG. 3 (color online). The branching ratio of the false vacuum nucleation rate to the Hawking evaporation rate as a function of the seed mass for a selection of Higgs models. Each plot corresponds a different value of λ_6 in Eq. (4), with $\lambda_* = -0.01$.

lifetime of the black hole. Note that this range is well above the Planck mass, where we have some confidence in the validity of the vacuum decay calculation. Given that this evaporation time scale is $\sim 10^{-28}$ s for a $10^5 M_p$-mass black hole, it is clear that once a primordial black hole nears the end of its life cycle, it *will* seed vacuum decay in these models. Hence, with these Higgs potentials, the presence of any primordial black holes will eventually trigger a catastrophic phase transition from our standard model vacuum, thus ruling out potentials with parameters in these ranges.

Because our results show that it is precisely for small black holes that the risk of seeded tunneling is greatest, a natural question is what happens with collider black holes. These can be produced if the fundamental (higher-dimensional) Planck scale is near the TeV scale [12]. These black holes have features inherited from their higher-dimensional nature, and while there are no known exact solutions, evaporation rates have been computed assuming a higher-dimensional Myers-Perry solution [18], with emission cross sections appropriate to a brane-world scenario [19].

Black-hole-seeded tunneling is now a more involved process, as it should involve a bubble forming around the higher-dimensional black hole triggered by the Higgs field transitioning on the brane, with the bubble then expanding out to fill the extra dimensions before finally becoming effectively four-dimensional and seeding true decay of our Universe. While this process is beyond the reach of the analytic approximations we have used here, we can estimate the effect by modeling the instanton with a higher-dimensional counterpart of the solutions described above. In this case, the form of the potential $V(R)$ for the bubble motion is modified, but is of a remarkably similar form, essentially replacing $R \to R^{n+1}$, where n is the number of extra dimensions. Assuming the static bubble, we can then calculate the horizon radius and area: The action will be the difference in seed and remnant black-hole horizon areas. It turns out this calculation is relatively insensitive to the number of extra dimensions (the horizon areas $\mathcal{A} \propto M^{(n+2)/(n+1)}$), whereas the evaporation rate of black holes is enhanced, in part because of the increased Hawking temperature, $T \propto M^{-1/(n+1)}$, and in part because of grey-body factors. The branching ratio tends to be suppressed with extra dimensions, making collider black holes less risky for vacuum decay; however, black holes produced by particle collisions could still cause vacuum decay in certain regions of parameter space. Fortunately, we have some reassurance about the safety of the LHC from the fact that cosmic ray collisions have occurred at energies higher than those reached at the collider [20].

To sum up, we have shown that the Coleman–De Luccia result for the lifetime of our Universe in Higgs potentials with metastability seems crucially dependent on the absence of inhomogeneities: The presence of primordial black holes can dramatically reduce the barrier to vacuum decay, and seed nucleation to a universe with a very different "standard model." Such a conclusion of course depends on the existence of said small black holes—by no means a certainty—and a detailed numerical study of parameter space. However, these results are suggestive that the issue of metastability of our Universe may not be as simple as initially thought.

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