

# Effects of Economic Development in China on Skill-Biased Technical Change in the US

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## Abstract

In this study, we analyze the effects of a decrease in unskilled labor in China on the direction of innovation in the US by incorporating production offshoring into a North-South model of directed technical change. We find that if offshoring is present (absent) in equilibrium, then a decrease in unskilled labor in the South would lead to skill-biased (unskill-biased) technical change in the North. This finding highlights the different implications of offshoring and conventional trade on innovation. Furthermore, we find that an increase in the Southern stock of capital reduces offshoring and also leads to skill-biased technical change. Therefore, rapid capital accumulation and a decrease in unskilled labor in China could both lead to a rising skill premium in the US. Calibrating the model to China-US data, we find that a 1% decrease in unskilled labor (1% increase in capital) in China leads to a 0.8% (0.6%) increase in the skill premium in the US under a moderate elasticity of substitution between skill-intensive and labor-intensive goods.

*JEL classification:* O14, O33, J31, F16

*Keywords:* economic growth, skill-biased technical change, offshoring

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# 1 Introduction

After three decades of economic development, China is now experiencing a rapid decrease in unskilled labor. For example, according to the Barro-Lee dataset on education attainment, the share of population (over the age of 25) in China without completion of secondary education decreased from 93.4% in 1980 to 53.7% in 2010. As a result of this dramatic decrease in unskilled labor in China, wages have been rising rapidly. For example, it is not uncommon for manufacturing plants in China to experience rising wages of 20% per year.<sup>1</sup> Given this rapidly rising wages, China is becoming a less attractive place for the offshoring of manufacturing activities. A recent article of *The Economist* documents a decreasing trend in production offshoring from developed economies to China;<sup>2</sup> for example, "[t]he Boston Consulting Group reckons that in areas such as transport, computers, fabricated metals and machinery, 10-30% of the goods that America now imports from China could be made at home by 2020". The article also argues that this decreasing trend is due to changes in the manufacturing process in developed economies such as the digitization of manufacturing;<sup>3</sup> as a result of which, "companies now want to be closer to their customers so that they can respond more quickly to changes in demand. And some products are so sophisticated that it helps to have the people who design them and the people who make them in the same place." In other words, this new manufacturing process is relatively skill-intensive.

In this study, we analyze the effects of a decrease in unskilled labor in China on the direction of innovation in the US by incorporating production offshoring into a North-South model of directed technical change. We find that if the equilibrium features offshoring, then a decrease in unskilled labor in the South would lead to skill-biased technical change in the North. In contrast, if the equilibrium does not feature offshoring, then a decrease in Southern unskilled labor would lead to unskill-biased technical change. Intuitively, when offshoring is absent in equilibrium, a reduction in the supply of unskilled labor in the South causes through international trade a price effect that improves incentives of innovation for labor-intensive goods. When offshoring is present in equilibrium, a reduction in the supply of unskilled labor in the South causes also a market size effect that improves incentives of innovation for skill-intensive goods. This finding highlights the different implications of offshoring and conventional trade on the direction of technological progress.

The above theoretical result has the following implications. When China first opened up its economy for international trade in the 1980's, there was essentially no offshoring to the economy. Together with a low level of patent protection in China at that time,<sup>4</sup> the opening of the Chinese economy implies a massive increase in the supply of unskilled labor in the world causing predominantly a price effect that improves incentives of innovation directed to the relatively scarce factor, i.e., skilled labors,<sup>5</sup> and this contributes to skill-biased technical

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<sup>1</sup>The Economist, "The End of Cheap China", March 10, 2012.

<sup>2</sup>The Economist, "The Third Industrial Revolution", April 21, 2012.

<sup>3</sup>An important technology under the digitization of manufacturing is 3D printing, "which creates a solid object by building up successive layers of material. The digital design can be tweaked with a few mouseclicks. The 3D printer can run unattended, and can make many things which are too complex for a traditional factory to handle."

<sup>4</sup>For example, the Ginarte-Park index of patent rights in China was 1.33 in 1985; see Park (2008). The Ginarte-Park index is on a scale of 0 to 5, and a larger number implies stronger patent rights.

<sup>5</sup>From 1980 to 1995, the share of population in China with at least completion of secondary education

change in developed countries. After the mid 1990's, the amount of offshoring to China started to increase rapidly. Together with an increased level of patent protection in China,<sup>6</sup> the decrease in unskilled labor in China causes mainly a market size effect that improves incentives of innovation directed to the now relatively abundant factor, i.e., skilled labors,<sup>7</sup> and this also contributes to skill-biased technical change in developed countries.

Another stylized fact of economic development in China is that capital investment as a share of gross domestic product (GDP) is about 40% and substantially higher than many developed economies. So long as the depreciation rates of capital are not substantially different across countries, China is accumulating capital at a much faster rate than developed countries. From our theoretical analysis, we find that an increase in the stock of capital in the South relative to the North would reduce offshoring. Intuitively, a larger stock of capital in China increases the wage rates of Chinese workers rendering offshoring to China less attractive. As a result, a larger stock of capital in the South also leads to skill-biased technical change in the North. Therefore, both the stylized facts of rapid capital accumulation and a decrease in unskilled labor in China could contribute to skill-biased technical change in the US.

We calibrate the model to China-US data to provide a quantitative analysis. Due to skill-biased technical change, either a decrease in unskilled labor or an increase in capital stock in the South would raise the skill premium in the North. The magnitude of the changes depends on the elasticity of substitution between skill-intensive and labor-intensive goods. We find that a 1% decrease in the supply of unskilled labor in China leads to a 0.8% (3.7%) increase in the skill premium in the US when the elasticity of substitution is 2.2 (2.4). Furthermore, a 1% increase in the capital stock in China leads to a 0.6% (2.0%) increase in the skill premium in the US when the elasticity of substitution is 2.2 (2.4).

This paper relates to studies on directed technical change, such as Acemoglu (1998, 2002, 2003), Acemoglu and Zilibotti (2001) and Gancia and Bonfiglioli (2008). These influential studies built on the literature of R&D-driven economic growth to analyze the direction of innovation.<sup>8</sup> Acemoglu (1998, 2002) analyzes skill-biased technical change and the rising skill premium in the US, whereas Acemoglu (2003), Acemoglu and Zilibotti (2001) and Gancia and Bonfiglioli (2008) analyze the implications of trade on skill-biased technical change and productivity differences across countries. However, the abovementioned studies do not consider offshoring. This paper also relates to studies on offshoring; see Grossman and Rossi-Hansberg (2008) for a recent contribution and their discussion of earlier studies. The present paper complements these two branches of literature by providing an analysis on the effects of offshoring on the direction of technological progress.

A recent study by Acemoglu *et al.* (2012) also analyzes the effects of offshoring on skill-biased technical change. In addition to some differences in modelling details, our study differs from their interesting analysis by exploring a different set of research questions. Acemoglu *et al.* (2012) analyze the effects of an offshoring-cost parameter and a patent-policy parameter

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was on average 13.7%; see the Barro-Lee dataset on educational attainment.

<sup>6</sup>The Ginarte-Park index of patent rights in China was 4.08 in 2005; see Park (2008).

<sup>7</sup>From 2000 to 2010, the share of population in China with at least completion of secondary education was on average 39.5%; see the Barro-Lee dataset.

<sup>8</sup>See Romer (1990), Segerstrom *et al.* (1990), Grossman and Helpman (1991a) and Aghion and Howitt (1992) for seminal studies in this literature and Gancia and Zilibotti (2005) for a survey.

on skill-biased technical change, whereas we analyze the effects of a decrease in unskilled labor and an increase in capital on skill-biased technical change through offshoring. Therefore, we believe that our study provides a useful complementary analysis to Acemoglu *et al.* (2012) on this uncharted area of offshoring and directed technological progress.

The rest of this study is organized as follows. Section 2 presents the model. Section 3 analyzes the effects of labor supply and capital stock in the South on the direction of innovation in the North. The final section concludes.

## 2 A North-South model of directed technical change

In this section, we consider a North-South version of the model of directed technical change based on Acemoglu (2002). The innovation process is in the form of variety expansion. When an R&D entrepreneur invents a new variety, her patents generate monopolistic profits in the Northern market and possibly also in the Southern market depending on the level of patent protection in the South. For simplicity, we assume that both countries have access to the same set of varieties of goods.<sup>9</sup> Final goods are produced using skill-intensive and labor-intensive goods, which are freely traded across countries, but capital and labors as well as the intermediate inputs that capital produces are immobile across countries. As is common in the literature, we model offshoring as "shadow migration" of workers through which the output of offshored workers in the South is combined with intermediate inputs in the North. As for the cost of offshoring, we follow Grossman and Rossi-Hansberg (2008) to assume that offshoring involves a variable cost.<sup>10</sup>

### 2.1 Households

In the North, there is a representative household with the following lifetime utility function.

$$U = \int_0^{\infty} e^{-\rho t} \ln C_t^n dt, \quad (1)$$

where  $C_t^n$  denotes consumption in the North at time  $t$ , and  $\rho > 0$  is the subjective discount rate. The household maximizes utility subject to the following asset-accumulation equation.

$$\dot{A}_t^n = r_t A_t^n + w_{h,t}^n H^n + w_{l,t}^n L^n + q_t^n K^n - C_t^n. \quad (2)$$

$A_t^n$  is the amount of financial assets in the form of patents owned by the household, and  $r_t$  is the rate of return.<sup>11</sup>  $H^n$  and  $L^n$  are respectively the inelastic supply of high-skilled and

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<sup>9</sup>See for example Grossman and Helpman (1991b), Helpman (1993) and Lai (1998) for an alternative branch of North-South models that focus on the gradual transfer of technologies from the North to the South.

<sup>10</sup>See Acemoglu *et al.* (2012) for an interesting formulation of offshoring that involves a fixed cost.

<sup>11</sup> $r_t$  is not indexed by a superscript because we assume that there is a global financial market, and our derivations are robust any distribution of financial assets across the two countries. One special case is that all financial assets are owned by the Northern household.

low-skilled labors.  $w_{h,t}^n$  and  $w_{l,t}^n$  are respectively the wage rates of high-skilled and low-skilled labors.  $K^n$  is the inelastic supply of capital,<sup>12</sup> and  $q_t^n$  is the rental price of capital. From standard dynamic optimization, the familiar Euler equation is

$$\frac{\dot{C}_t^n}{C_t^n} = r_t - \rho. \quad (3)$$

As for the South, there are analogous conditions. Finally, we assume that the North is more skill-abundant than the South (i.e.,  $H^n/L^n > H^s/L^s$ ) and that the North is also more capital-abundant than the South (i.e.,  $K^n/L^n > K^s/L^s$ ).<sup>13</sup>

## 2.2 Final goods

The production of final goods is perfectly competitive; therefore, it does not matter where production takes place. Final goods are produced with the following CES aggregator.

$$Y_t = \left[ \gamma (Y_{l,t}^n + Y_{l,t}^s)^{(\varepsilon-1)/\varepsilon} + (1-\gamma) (Y_{h,t}^n + Y_{h,t}^s)^{(\varepsilon-1)/\varepsilon} \right]^{\varepsilon/(\varepsilon-1)}, \quad (4)$$

where  $Y_{l,t}^n$  and  $Y_{l,t}^s$  are respectively labor-intensive goods produced in the North and in the South, and  $Y_{h,t}^n$  and  $Y_{h,t}^s$  are respectively skill-intensive goods produced in the North and in the South.  $\varepsilon > 1$  is the elasticity of substitution between the two types of goods,<sup>14</sup> and  $\gamma$  determines their relative importance.  $\{Y_{l,t}^n, Y_{l,t}^s, Y_{h,t}^n, Y_{h,t}^s\}$  are freely traded across countries subject to international prices  $\{P_{l,t}, P_{h,t}\}$ . The standard price index of final goods is

$$1 = [\gamma^\varepsilon (P_{l,t})^{1-\varepsilon} + (1-\gamma)^\varepsilon (P_{h,t})^{1-\varepsilon}]^{1/(1-\varepsilon)}, \quad (5)$$

where we have set the price of final goods (numeraire) to one. The resource constraint on final goods is

$$Y_t = R_t + C_t^n + C_t^s, \quad (6)$$

where  $R_t$  is the global amount of final goods devoted to R&D.

## 2.3 Labor-intensive goods

In the South, the production function of labor-intensive goods is

$$Y_{l,t}^s = \frac{(l_t^s)^\beta}{1-\beta} \left( \int_0^{N_{l,t}} [x_{l,t}^s(i)]^{1-\beta} di \right) (N_{l,t})^{1-\beta}, \quad (7)$$

where  $\beta > (\varepsilon - 2)/(\varepsilon - 1)$  determines the elasticity of substitution between intermediate inputs.  $l_t^s$  is the amount of Southern unskilled labor employed in the production of  $Y_{l,t}^s$ .

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<sup>12</sup>We differ from Acemoglu (2002) by assuming that intermediate goods are produced using capital instead of final goods. This modification allows us to analyze the effects of changes in the supply of capital. For simplicity, we focus on an inelastic supply of capital; see the conclusion for a discussion of this assumption.

<sup>13</sup>See for example, Bai *et al.* (2006) for a discussion on the relatively low capital-labor ratio in China.

<sup>14</sup>See Acemoglu (2003) for a discussion of evidence for  $\varepsilon > 1$ .

In addition to using labor, the production of  $Y_{l,t}^s$  requires differentiated intermediate inputs  $x_{l,t}^n(i)$  for  $i \in [0, N_{l,t}]$ , where  $N_{l,t}$  is the number of differentiated inputs for labor-intensive goods that have been invented as of time  $t$ . The term  $(N_{l,t})^{1-\beta}$  captures an externality effect of  $N_{l,t}$  on the production of  $Y_{l,t}^s$  in order to ensure a balanced growth path along which  $N_{l,t}$  and  $Y_{l,t}^s$  grow at the same rate.<sup>15</sup>

In the North, the production function of labor-intensive goods is given by

$$Y_{l,t}^n = \frac{(l_t^n + \delta \mathbf{l}_t^s)^\beta}{1 - \beta} \left( \int_0^{N_{l,t}} [x_{l,t}^n(i)]^{1-\beta} di \right) (N_{l,t})^{1-\beta}, \quad (8)$$

where  $\mathbf{l}_t^s$  is the amount of Southern unskilled labor employed by Northern firms to produce  $Y_{l,t}^n$  capturing the offshoring of production. Following Grossman and Rossi-Hansberg (2008), we use a parameter  $\delta \in (0, 1)$  to capture the variable cost of offshoring. A higher cost of offshoring is reflected by a smaller value of  $\delta$ . If  $\delta = 0$ , then offshoring of labor-intensive goods would be absent,<sup>16</sup> and the model is left with conventional trade in  $\{Y_{l,t}^n, Y_{l,t}^s, Y_{h,t}^n, Y_{h,t}^s\}$ . We refer to a larger  $\delta$  as a higher degree of offshoring. As a result of offshoring, the resource constraint for Southern unskilled labor is  $l_t^s + \mathbf{l}_t^s = L^s$ , whereas the resource constraint for Northern unskilled labor is  $l_t^n = L^n$ .

## 2.4 Skill-intensive goods

In the South, the production function of skill-intensive goods is given by

$$Y_{h,t}^s = \frac{(h_t^s)^\beta}{1 - \beta} \left( \int_0^{N_{h,t}} [x_{h,t}^s(j)]^{1-\beta} dj \right) (N_{h,t})^{1-\beta}. \quad (9)$$

$h_t^s$  is the amount of Southern skilled labor employed in the production of  $Y_{h,t}^s$ . In addition to using labor, the production of  $Y_{h,t}^s$  requires differentiated intermediate inputs  $x_{h,t}^n(j)$  for  $j \in [0, N_{h,t}]$ , where  $N_{h,t}$  is the number of differentiated inputs for skill-intensive goods that have been invented as of time  $t$ . The term  $(N_{h,t})^{1-\beta}$  captures an externality effect of  $N_{h,t}$  on the production of  $Y_{h,t}^s$  in order to ensure a balanced growth path along which  $N_{h,t}$  and  $Y_{h,t}^s$  grow at the same rate.

In the North, the production function of labor-intensive goods is given by

$$Y_{h,t}^n = \frac{(h_t^n)^\beta}{1 - \beta} \left( \int_0^{N_{h,t}} [x_{h,t}^n(j)]^{1-\beta} dj \right) (N_{h,t})^{1-\beta}, \quad (10)$$

where we have ruled out offshoring of skill-intensive goods.<sup>17</sup> Due to the absence of offshoring for skill-intensive goods, the resource constraint for Southern skilled labor is  $h_t^s = H^s$ , whereas the resource constraint for Northern skilled labor is  $h_t^n = H^n$ .

<sup>15</sup>In Acemoglu (2002), this externality is not needed because  $x_{l,t}^n(i)$  is produced from final goods, whereas  $x_{l,t}^n(i)$  is produced from a fixed supply of capital in the present study.

<sup>16</sup>In fact, we find that if  $\delta$  is below a threshold value, then offshoring would be absent in equilibrium.

<sup>17</sup>We have found that if and only if a knife-edge condition holds such that the costs of offshoring for labor-intensive and skill-intensive goods are the same (i.e.,  $\delta_h = \delta_l = \delta > 0$ ), then the model would feature offshoring in both sectors. Given that our focus is on the offshoring of labor-intensive goods, we consider the case of  $0 \leq \delta_h < \delta_l = \delta$  under which the equilibrium features zero offshoring of skill-intensive goods and is identical to the case of  $\delta_h = 0$ .

## 2.5 Intermediate inputs

For notational convenience, we suppress the index  $i \in [0, N_{l,t}]$  for the intermediate inputs of labor-intensive goods and the index  $j \in [0, N_{h,t}]$  for the intermediate inputs of skill-intensive goods. In the North, the production function of each differentiated intermediate input is

$$x_{z,t}^n = k_{z,t}^n, \quad (11)$$

where  $z \in \{h, l\}$ . In other words, one unit of capital produces one unit of intermediate input. Given the capital-rental price  $q_t^n$  in the North, the monopolistic producer of each differentiated intermediate input charges a profit-maximizing markup  $\eta^n$  over  $q_t^n$  such that

$$p_{z,t}^n = \eta^n q_t^n, \quad (12)$$

where  $z \in \{h, l\}$  and  $\eta^n = 1/(1 - \beta) > 1$ . Therefore, the amount of profit captured by each intermediate input in the North is

$$\pi_{z,t}^n = (1 - 1/\eta^n) p_{z,t}^n x_{z,t}^n = \beta p_{z,t}^n x_{z,t}^n, \quad (13)$$

where  $z \in \{h, l\}$ . Due to symmetry, the resource constraint on capital in the North is  $N_{l,t} x_{l,t}^n + N_{h,t} x_{h,t}^n = K_{l,t}^n + K_{h,t}^n = K^n$ .

In the South, the production function of each differentiated intermediate input is

$$x_{z,t}^s = k_{z,t}^s, \quad (14)$$

where  $z \in \{h, l\}$ . Given the capital-rental price  $q_t^s$  in the South, the monopolistic producer of each differentiated intermediate input charges a markup  $\eta^s$  over  $q_t^s$  such that

$$p_{z,t}^s = \eta^s q_t^s, \quad (15)$$

where  $z \in \{h, l\}$ . Here we follow Goh and Olivier (2002) to model incomplete patent protection that constrains the markup in the South;<sup>18</sup> specifically, we assume that  $\eta^s = 1/(1 - \phi) \leq \eta^n$  where  $\phi \in [0, \beta]$ . Intuitively, the presence of potential imitation due to incomplete patent protection forces the monopolistic producers to lower their markup in the South. If  $\phi = \beta$  ( $\phi = 0$ ), then patent protection is complete (zero) in the South. The amount of profit captured by each intermediate input in the South is

$$\pi_{z,t}^s = (1 - 1/\eta^s) p_{z,t}^s x_{z,t}^s = \phi p_{z,t}^s x_{z,t}^s, \quad (16)$$

where  $z \in \{h, l\}$ . The resource constraint on capital in the South is  $N_{l,t} x_{l,t}^s + N_{h,t} x_{h,t}^s = K_{l,t}^s + K_{h,t}^s = K^s$ .

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<sup>18</sup>See also Li (2001), Chu (2011) and Iwaisako and Futagami (2013).

## 2.6 R&D

There is a continuum of entrepreneurs investing in R&D, and the invention of a new variety of skill-intensive or labor-intensive inputs requires  $\mu$  units of final goods. If  $\mu$  is the same across the two countries, then the location of R&D is indeterminate, and our derivations are robust to any geographical distribution of R&D. If  $\mu$  is smaller in the North than in the South, then innovation takes place only in the North as in for example, Acemoglu and Zilibotti (2001) and Gancia and Bonfiglioli (2008).<sup>19</sup> When an entrepreneur invents a new variety, she obtains patents in both the North and the South.<sup>20</sup> The innovation process is

$$\dot{N}_{z,t} = R_{z,t}/\mu, \quad (17)$$

where  $z \in \{h, l\}$ . Suppose we denote  $V_{z,t}$  as the value of an invention. Free entry ensures that

$$(V_{z,t} - \mu)\dot{N}_{z,t} = 0, \quad (18)$$

where  $z \in \{h, l\}$ . The familiar Bellman equation is

$$r_t = \frac{\pi_{z,t}^n + \pi_{z,t}^s + \dot{V}_{z,t}}{V_{z,t}}, \quad (19)$$

where  $z \in \{h, l\}$ . Intuitively, the Bellman equation equates the interest rate to the asset return per unit of asset, where the asset return is the sum of monopolistic profits  $\pi_{z,t}^n + \pi_{z,t}^s$  and any potential capital gain  $\dot{V}_{z,t}$ .

## 2.7 Decentralized equilibrium

The equilibrium is a time path of prices  $\{r_t, w_{l,t}^n, w_{l,t}^s, w_{h,t}^n, w_{h,t}^s, q_t^n, q_t^s, P_{l,t}, P_{h,t}, p_{l,t}^n(i), p_{l,t}^s(i), p_{h,t}^n(j), p_{h,t}^s(j)\}$  and a time path of allocations  $\{R_{l,t}, R_{h,t}, C_t^n, C_t^s, Y_t, Y_{l,t}^n, Y_{l,t}^s, Y_{h,t}^n, Y_{h,t}^s, x_{l,t}^n(i), x_{l,t}^s(i), x_{h,t}^n(j), x_{h,t}^s(j), l_t^n, l_t^s, l_t^s, h_t^n, h_t^s\}$ . Also, at each instance of time, the followings hold:

- Households maximize utility taking  $\{r_t, w_{l,t}^n, w_{h,t}^n, q_t^n, w_{l,t}^s, w_{h,t}^s, q_t^s\}$  as given;
- Competitive final-goods firms produce  $\{Y_t\}$  to maximize profit taking prices  $\{P_{l,t}, P_{h,t}\}$  as given;
- Competitive labor-intensive goods firms in the two countries produce  $\{Y_{l,t}^n, Y_{l,t}^s\}$  to maximize profit taking the international price  $\{P_{l,t}\}$  as given;
- Competitive skill-intensive goods firms in the two countries produce  $\{Y_{h,t}^n, Y_{h,t}^s\}$  to maximize profit taking the international price  $\{P_{h,t}\}$  as given;
- Monopolistic intermediate-goods firms in the labor-intensive sector produce  $\{x_{l,t}^n(i), x_{l,t}^s(i)\}$  and choose  $\{p_{l,t}^n(i), p_{l,t}^s(i)\}$  to maximize profit taking prices  $\{q_t^n, q_t^s\}$  as given;

<sup>19</sup>See Acemoglu and Zilibotti (2001) for a discussion of evidence that 90% of global R&D is performed in OECD countries and 35% in the US.

<sup>20</sup>It is useful to note that given the global financial market, patents that are based on a variety invented in the North (South) are not necessarily solely owned by Northern (Southern) households.



- Monopolistic intermediate-goods firms in the skill-intensive sector produce  $\{x_{h,t}^n(j), x_{h,t}^s(j)\}$  and choose  $\{p_{h,t}^n(j), p_{h,t}^s(j)\}$  to maximize profit taking prices  $\{q_t^n, q_t^s\}$  as given;
- R&D firms choose  $\{R_{l,t}, R_{h,t}\}$  to maximize profit taking  $\{V_{h,t}, V_{l,t}\}$  as given;
- The market-clearing condition for unskilled labor in the two countries holds such that  $l_t^n = L^n$  and  $l_t^s + \mathbf{l}_t^s = L^s$ ;
- The market-clearing condition for skilled labor in the two countries holds such that  $h_t^n = H^n$  and  $h_t^s = H^s$ ;
- The market-clearing condition for capital in the two countries holds such that  $N_{l,t}x_{l,t}^n + N_{h,t}x_{h,t}^n = K^n$  and  $N_{l,t}x_{l,t}^s + N_{h,t}x_{h,t}^s = K^s$ ;
- The market-clearing condition for final goods holds such that  $Y_t = R_{l,t} + R_{h,t} + C_t^n + C_t^s$ .

## 2.8 Balanced growth equilibrium

In this subsection, we discuss the balanced growth equilibrium of the model. The model features a unique steady-state value of  $N_{h,t}/N_{l,t}$ . If the initial value of  $N_{h,t}/N_{l,t}$  is above (below) this steady-state value, then the equilibrium initially features R&D in labor-intensive (skill-intensive) goods only until the economy reaches the balanced growth path along which  $N_{h,t}$  and  $N_{l,t}$  grow at the same rate. On the balanced growth path, the equilibrium features a positive amount of offshoring if and only if  $\delta$  is sufficiently large. We summarize these results in Proposition 1.

**Proposition 1** *The dynamics of  $N_{h,t}/N_{l,t}$  is characterized by global stability such that the economy converges to a unique and stable balanced growth path along which  $N_{h,t}$  and  $N_{l,t}$  grow at the same rate. If and only if  $\delta > [(K^s/L^s)/(K^n/L^n)]^{1-\beta}$ , then the equilibrium would feature a positive amount of offshoring (i.e.,  $\mathbf{l}^s > 0$ ).*

**Proof.** See Appendix A. ■

The threshold value of  $\delta$  above which the equilibrium features offshoring is given by  $[(K^s/L^s)/(K^n/L^n)]^{1-\beta} < 1$ . Intuitively, in the presence of offshoring, the wage rate of unskilled labor in the South must be a fraction  $\delta$  of that in the North. However, if the capita-labor ratio in the South is sufficiently high relative to the North, then it would be impossible for the South to have such a low relative wage in equilibrium.

### 3 How the South affects innovation in the North

In this section, we analyze the effects of a reduction in the supply of Southern unskilled labor  $L^s$  and an increase in Southern capital stock  $K^s$  on the direction of Northern innovation. In Section 3.1, we analyze a special case of zero patent protection in the South (i.e.,  $\phi = 0$ ). In Section 3.2, we analyze another special case of complete patent protection in the South (i.e.,  $\phi = \beta$ ). In Section 3.3, we analyze the general case of incomplete patent protection in the South (i.e.,  $0 < \phi < \beta$ ).

#### 3.1 Zero patent protection in the South

Here we sketch out the results in the main text and relegate the detailed derivations to Appendix A. We focus on the balanced growth path and omit the time subscript for convenience. From (8) and (10), one can derive the following conditional demand functions for  $x_l^n(i)$  and  $x_h^n(j)$ .

$$x_l^n(i) = \left[ \frac{P_l(N_l)^{1-\beta}}{p_l^n(i)} \right]^{1/\beta} (l^n + \delta \mathbf{l}^s), \quad (20)$$

$$x_h^n(j) = \left[ \frac{P_h(N_h)^{1-\beta}}{p_h^n(j)} \right]^{1/\beta} h^n. \quad (21)$$

Under the special case of zero patent protection (i.e.,  $\phi = 0$ ) in the South, the steady-state version of (19) simplifies to

$$\frac{V_h}{V_l} = \frac{\pi_h^n}{\pi_l^n} = \frac{p_h^n x_h^n}{p_l^n x_l^n}, \quad (22)$$

where  $p_h^n = p_l^n = \eta^n q^n$ . Substituting (20) and (21) into (22) yields

$$\frac{V_h}{V_l} = \left( \frac{N_h}{N_l} \right)^{(1-\beta)/\beta} \underbrace{\left( \frac{P_h}{P_l} \right)^{1/\beta}}_{\text{price effect}} \underbrace{\frac{H^n}{L^n + \delta \mathbf{l}^s}}_{\text{market size effect}}. \quad (23)$$

The above expression is similar to the one in Acemoglu (2002) except for the terms  $\delta \mathbf{l}^s$  and  $(N_h/N_l)^{(1-\beta)/\beta}$ , which captures the externality effect. A decrease in Southern unskilled labor  $L^s$  reduces the offshoring  $\mathbf{l}^s$  of labor-intensive goods. Therefore, we obtain the following intuition from (23). When offshoring is absent (i.e.,  $\mathbf{l}^s = 0$ ), a reduction in the supply of Southern unskilled labor  $L^s$  leads to only a negative price effect by decreasing  $P_h/P_l$ . As a result,  $V_h/V_l$  decreases causing innovation to be directed towards labor-intensive goods. However, when offshoring is present (i.e.,  $\mathbf{l}^s > 0$ ), a reduction in the supply of Southern unskilled labor  $L^s$  leads to also a positive market size effect by increasing  $H^n/(L^n + \delta \mathbf{l}^s)$ . As a result,  $V_h/V_l$  increases causing innovation to be directed towards skill-intensive goods, and this gives rise to skill-biased technical change (i.e.,  $N_h/N_l$  increases). We summarize this main result in Proposition 2.

**Proposition 2** *If the equilibrium features a positive amount of offshoring, then a decrease (an increase) in the supply of Southern unskilled labor  $L^s$  would lead to skill-biased (unskill-biased) technical change. If the equilibrium does not feature offshoring, then a decrease (an increase) in the supply of Southern unskilled labor  $L^s$  would lead to unskill-biased (skill-biased) technical change.*

**Proof.** See Appendix A. ■

The intuition of this result can be explained as follows. Without offshoring, any change in the supply of unskilled labor in the South causes only a price effect on the value of inventions. In this case, the market size effect is absent due to zero (or more generally, weak) patent protection in the South, so that production in the South generates zero monopolistic profit (or more generally, a small amount of profit). With offshoring, some Southern workers are hired to work with Northern intermediate inputs that are protected by complete patent protection. As a result, a change in the supply of Southern unskilled labor causes through offshoring an additional market size effect on the value of inventions, and this result is consistent with the finding in Acemoglu (2003) under complete Southern patent protection *without* offshoring. In other words, Southern patent protection and offshoring serve as two substitutable channels through which the supply of unskilled labor in the South causes a market size effect on the value of inventions in the North.

The result in Proposition 2 has the following implications. First, the opening of the Chinese economy for international trade in the 1980's implies a massive increase in the supply of unskilled labor and causes skilled-biased technical change because there was very little offshoring to China at that time. Second, the substantial amount of offshoring to China in the present implies that a decrease of unskilled labor in China would also lead to skill-biased technical change.

From (7) and (8), one can derive the following conditional demand functions for  $l^s$  and  $l^n$ .

$$w_l^s = \frac{\beta P_l N_l}{1 - \beta} \left( \frac{K_l^s}{l^s} \right)^{1-\beta}, \quad (24)$$

$$w_l^n = \frac{\beta P_l N_l}{1 - \beta} \left( \frac{K_l^n}{l^n + \delta \mathbf{l}^s} \right)^{1-\beta}, \quad (25)$$

where we have applied symmetry on  $x_l^s(i) = x_l^s = K_l^s/N_l$  and  $x_l^n(i) = x_l^n = K_l^n/N_l$ . A larger  $K^s$  leads to an increase in  $K_l^s$ ; as a result,  $w_l^s$  increases holding other variables constant. Given that the equality  $w_l^s = \delta w_l^n$  must hold when offshoring is present (i.e.,  $\mathbf{l}^s > 0$ ), we have

$$\left( \frac{K_l^s}{L^s - \mathbf{l}^s} \right)^{1-\beta} = \delta \left( \frac{K_l^n}{L^n + \delta \mathbf{l}^s} \right)^{1-\beta}, \quad (26)$$

where we have used  $l^s + \mathbf{l}^s = L^s$  and  $l^n = L^n$ . Therefore, an increase in  $K_l^s$  reduces  $\mathbf{l}^s$ ; intuitively, a larger  $K_l^s$  increases the wage rate of Southern unskilled labor rendering offshoring less attractive. This reduction in  $\mathbf{l}^s$  triggers a market size effect as shown in (23). As a result, a larger capital stock in the South also leads to skill-biased technical change (i.e.,  $N_h/N_l$  increases). We summarize this result in Proposition 3.

**Proposition 3** *When the equilibrium features offshoring, an increase in Southern capital stock  $K^s$  leads to skill-biased technical change. When the equilibrium does not feature offshoring, an increase in Southern capital stock  $K^s$  also leads to skill-biased technical change.*

**Proof.** See Appendix A. ■

Although the comparative statics of  $N_h/N_l$  with respect to  $K^s$  are the same regardless of whether or not the equilibrium features offshoring, the intuition behind the two scenarios is quite different. In the absence of offshoring, the effect of  $K^s$  on  $N_h/N_l$  operates through the price effect. Suppose there is a zero supply of high-skill labor  $H^s$  in the South. Then, a larger capital stock  $K^s$  expands only the production of labor-intensive goods  $Y_l^s$ , which leads to a positive price effect by increasing  $P_h/P_l$  and consequently skill-biased technical change. A similar intuition also applies to the more general case of  $H^s/L^s < H^n/L^n$ , which we have assumed throughout the analysis.

### 3.2 Complete patent protection in the South

In this subsection, we consider complete patent protection in the South (i.e.,  $\phi = \beta$ ), and the rest of the model is the same as in the previous subsection with offshoring in labor-intensive goods. In this case, the steady-state ratio of  $N_h/N_l$  can be expressed as<sup>21</sup>

$$\frac{N_h}{N_l} = \left[ \left( \frac{1-\gamma}{\gamma} \right)^\varepsilon \left( \frac{H^n + \delta H^s}{L^n + \delta L^s} \right)^{\beta(\varepsilon-1)} \right]^{\frac{1}{1-(1-\beta)(\varepsilon-1)}}, \quad (27)$$

where  $\varepsilon > 1$  and  $1 - (1-\beta)(\varepsilon-1) > 0$  because  $\beta > (\varepsilon-2)/(\varepsilon-1)$ . Equation (27) shows that under offshoring, a decrease in  $L^s$  leads to an increase in  $N_h/N_l$  as before; however,  $N_h/N_l$  is independent of  $K^s$  in this case. Intuitively, although a larger  $K^s$  reduces offshoring  $\mathbf{l}^s$ , any decrease in  $\mathbf{l}^s$  is offset by an equal increase in unskilled labor  $l^s$  devoted to production in the South. Because of complete Southern patent protection, the market size effect of unskilled labor depends on  $L^s$  regardless of its distribution in  $\mathbf{l}^s$  and  $l^s$ . Therefore, despite its effect on offshoring  $\mathbf{l}^s$ , a larger Southern capital stock  $K^s$  no longer leads to skill-biased technical change under complete patent protection. We summarize these results in Proposition 4.

**Proposition 4** *Under complete patent protection in the South, a decrease (an increase) in the supply of Southern unskilled labor  $L^s$  leads to skill-biased (unskill-biased) technical change. However, changes in Southern capital stock  $K^s$  have no effect on  $N_h/N_l$ .*

**Proof.** See (27). ■

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<sup>21</sup>Equation (27) can be derived by setting  $\phi = \beta$  in (A11) of Appendix A.

### 3.3 Incomplete patent protection in the South

In the previous subsections, we show that whenever offshoring is present, a decrease in unskilled labor leads to skill-biased technical change regardless of whether Southern patent protection is zero or complete. Therefore, one can conjecture that the same result also applies to the general case of incomplete Southern patent protection; however, we are also interested in quantitative implications. Therefore, in this subsection, we calibrate the model for the general case of incomplete Southern patent protection in order to provide an illustrative numerical investigation on the effects of changes in unskilled labor and capital in China on the direction of innovation in the US. The model features the following set of parameters  $\{\varepsilon, \delta, \gamma, \rho, \beta, \phi, L^s, H^s, L^n, H^n, K^s, K^n\}$ .<sup>22</sup> We either consider standard values of these parameters or calibrate them using empirical moments in China and the US.

For the discount rate, we set  $\rho$  to a standard value of 0.03. For the parameter on labor share, we set  $\beta$  to the lower value of 0.4 in China because it also implies a more realistic markup  $\eta^n = 1/(1 - \beta) = 1.67$ . According to the Ginarte-Park index of patent rights, the level of patent protection in China from 1995 to 2005 is on average 63.5% of that in the US, so we set  $\eta^s - 1 = 0.635(\eta^n - 1)$ , which implies  $\phi = 0.30$ . We normalize  $L^s$  to unity and compute  $H^s$  using data on the share of population in China with at least some tertiary education (i.e.,  $H^s/(H^s + L^s)$ ), which is on average 4.6% from 1995 to 2010 according to the Barro-Lee dataset on education attainment). Similarly, we compute  $L^n$  and  $H^n$  using data on the share of population in the US with at least some tertiary education (i.e.,  $H^n/(H^n + L^n)$ ), which is on average 51% from 1995 to 2010 according to the Barro-Lee dataset) and the relative population size between China and the US (i.e.,  $(H^s + L^s)/(H^n + L^n)$ ), which is on average 4.44 from 1995 to 2009 according to the Penn World Table). We normalize  $K^n$  to unity and compute  $K^s$  using data on the relative GDP between China and the US (i.e.,  $(P_h Y_h^s + P_l Y_l^s)/(P_h Y_h^n + P_l Y_l^n)$ ), which is on average 0.47 from 1995 to 2009 according to the Penn World Table).<sup>23</sup> For the remaining parameters  $\{\varepsilon, \delta, \gamma\}$ , we consider a range of values of  $\varepsilon \in \{2.0, 2.2, 2.4\}$ . For each value of  $\varepsilon$ , we calibrate the values of  $\{\delta, \gamma\}$  using the following moments. For the offshoring parameter, we calibrate  $\delta$  using the value of processing trade surplus in China as a share of GDP (i.e.,  $w_l^s \mathbf{1}^s/(P_h Y_h^s + P_l Y_l^s)$ ), which is on average 4.7% from 1995 to 2008).<sup>24</sup> Finally, we calibrate the value of  $\gamma$  using the college premium in the US (i.e.,  $w_h^n/w_l^n$ , which is on average about 1.7 from 1995 to recent time). Table 1 reports the calibrated parameter values.<sup>25</sup>

**Table 1: Calibrated parameter values**

$\varepsilon$	$\delta$	$\gamma$	$\rho$	$\beta$	$\phi$	$L^s$	$H^s$	$L^n$	$H^n$	$K^s$	$K^n$
2.0	0.16	0.49	0.03	0.4	0.3	1	0.05	0.12	0.12	0.14	1
2.2	0.16	0.48	0.03	0.4	0.3	1	0.05	0.12	0.12	0.14	1
2.4	0.16	0.48	0.03	0.4	0.3	1	0.05	0.12	0.12	0.14	1

<sup>22</sup>It can be shown that the calibration and simulation of the interested variables are independent of  $\mu$ .

<sup>23</sup>There are two versions of data on China in the Penn World Table, and we compute our values using both versions and taking an average of the two values.

<sup>24</sup>Data on the value of processing trade surplus in China is obtained from Xing (2012). Data on China's GDP is obtained from United Nations: National Account Main Aggregates Database.

<sup>25</sup>We provide the equilibrium expressions for calibration in an unpublished appendix (see Appendix B).

We consider two policy experiments. First, we reduce the supply of unskilled labor in the South and examine its effect on  $N_h/N_l$  and  $w_h^n/w_l^n$ .<sup>26</sup> Second, we increase the capital stock in the South and examine its effect on  $N_h/N_l$  and  $w_h^n/w_l^n$ . Table 2 reports the results.<sup>27</sup> Due to skill-biased technical change, either a decrease in  $L^s$  or an increase in  $K^s$  would raise the skill premium in the North. The magnitude of the changes is sensitive to the value of  $\varepsilon$  (i.e., the elasticity of substitution between skill-intensive and labor-intensive goods) as is well known in the literature. Suppose we consider a moderate value of  $\varepsilon = 2.2$  as our benchmark. Then, we find that a 1% decrease in the supply of unskilled labor  $L^s$  in China would lead to a 0.8% increase in the skill premium in the US, whereas a 1% increase in the capital stock  $K^s$  in China could lead to a 0.6% increase in the skill premium in the US. If we consider a larger value of  $\varepsilon = 2.4$ , then a 1% decrease in unskilled labor (1% increase in capital) in China would raise the skill premium in the US by as much as 3.7% (2.0%).

**Table 2a: 1% decrease in  $L^s$**

$\varepsilon$	2.0	2.2	2.4
$\Delta N_h/N_l$	0.7%	1.4%	4.6%
$\Delta w_h^n/w_l^n$	0.1%	0.8%	3.7%

**Table 2b: 1% increase in  $K^s$**

$\varepsilon$	2.0	2.2	2.4
$\Delta N_h/N_l$	0.5%	0.8%	2.3%
$\Delta w_h^n/w_l^n$	0.3%	0.6%	2.0%

To have a better understanding of the effects of  $L^s$  and  $K^s$  on the skill premium  $w_h^n/w_l^n$ , we derive<sup>28</sup>

$$\frac{w_h^n}{w_l^n} = \left[ \left( \frac{1-\gamma}{\gamma} \right)^{\varepsilon/(\varepsilon-1)} \left( \frac{H^n + \delta H^s}{L^n + \delta L^s} \right)^{-1/(\varepsilon-1)} \left( \frac{H^n + \frac{\phi}{\beta} \delta H^s}{L^n + \frac{\phi}{\beta} \delta L^s + \delta \mathbf{I}^s(\beta - \phi)/\beta} \right) \right]^{\zeta}, \quad (28)$$

where  $\zeta \equiv (\varepsilon - 1) / [1 - (1 - \beta)(\varepsilon - 1)] > 0$  because  $\varepsilon > 1$  and  $\beta > (\varepsilon - 2)/(\varepsilon - 1)$ . Suppose we consider the special case of complete Southern patent protection (i.e.,  $\phi = \beta$ ). Then, (28) simplifies to

$$\frac{w_h^n}{w_l^n} = \left[ \left( \frac{1-\gamma}{\gamma} \right)^{\varepsilon/(\varepsilon-1)} \left( \frac{H^n + \delta H^s}{L^n + \delta L^s} \right)^{(\varepsilon-2)/(\varepsilon-1)} \right]^{\zeta}. \quad (29)$$

Under complete Southern patent protection, a decrease in  $L^s$  raises the skill premium  $w_h^n/w_l^n$  if and only if  $\varepsilon$  is greater than a threshold value of 2. Under incomplete Southern patent protection (i.e.,  $\phi < \beta$ ), our numerical results indicate that this threshold value of  $\varepsilon$  can be slightly below 2. Another interesting implication from (29) is that under complete Southern patent protection,  $w_h^n/w_l^n$  is independent of  $K^s$ . In other words, an increase in  $K^s$  raises the skill premium if and only if  $\phi < \beta$ , under which  $K^s$  affects  $w_h^n/w_l^n$  through offshoring  $\mathbf{I}^s$ .

<sup>26</sup>It is useful to note that  $w_h^n/w_l^n = w_h^s/w_l^s$  in this model.

<sup>27</sup>The results in Table 2 are expressed as percent changes in  $N_h/N_l$  and  $w_h^n/w_l^n$ .

<sup>28</sup>We provide the derivations in an unpublished appendix (see Appendix B).

## 4 Conclusion

In this study, we have analyzed how economic development in China could affect skill-biased technical change in the US. In our analysis, we have assumed that the supply of skilled/unskilled labors and the capital stock are exogenous. In reality, they are all endogenous variables. In the case of China, their changes are mainly driven by economic development. As the economy develops, the share of skilled labor in the work force and the stock of physical capital increase. As a result, the smaller supply of unskilled labor and the larger supply of physical capital reinforce each other in triggering skill-biased technical change through offshoring. Furthermore, if the reduction in the supply of unskilled labor also increases the skill premium in both the US and China as in our simulation results,<sup>29</sup> then there would be more incentives for skill acquisition in both countries increasing the supply of skilled labor and triggering further skill-biased technical change. Therefore, we believe that our results are robust to the endogenous accumulation of physical and human capital. However, allowing for these additional features would significantly complicate our analysis, so that we leave these interesting extensions to future research.

## References

- [1] Acemoglu, D., 1998. Why do new technologies complement skills? Directed technical change and wage inequality. *Quarterly Journal of Economics*, 113, 1055-1089.
- [2] Acemoglu, D., 2002. Directed technical change. *Review of Economic Studies*, 69, 781-809.
- [3] Acemoglu, D., 2003. Patterns of skill premia. *Review of Economic Studies*, 70, 199-230.
- [4] Acemoglu, D., Gancia, G., and Zilibotti, F., 2012. Offshoring and directed technical change. manuscript.
- [5] Acemoglu, D., and Zilibotti, F., 2001. Productivity differences. *Quarterly Journal of Economics*, 116, 563-606.
- [6] Aghion, P., and Howitt, P., 1992. A model of growth through creative destruction. *Econometrica*, 60, 323-351.
- [7] Bai, C., Hsieh, C., and Qian, Y., 2006. The return to capital in China. *Brookings Papers on Economic Activity*, 37, 61-102.
- [8] Chu, A., 2011. The welfare cost of one-size-fits-all patent protection. *Journal of Economic Dynamics and Control*, 35, 876-890.
- [9] Gancia, G., and Bonfiglioli, A., 2008. North-South trade and directed technical change. *Journal of International Economics*, 76, 276-295.

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<sup>29</sup>In our model, the skill premiums in the North and the South are the same; i.e.,  $w_h^n/w_l^n = w_h^s/w_l^s$ .

- [10] Gancia, G., and Zilibotti, F., 2005. Horizontal innovation in the theory of growth and development. In: Aghion, P., Durlauf, S., (Eds.), *Handbook of Economic Growth*, 1A, 111-170. North-Holland, Amsterdam.
- [11] Goh, A.-T., and Olivier, J., 2002. Optimal patent protection in a two-sector economy. *International Economic Review*, 43, 1191–1214.
- [12] Grossman, G., and Helpman, E., 1991a. Quality ladders in the theory of growth. *Review of Economic Studies*, 58, 43-61.
- [13] Grossman, G. and Helpman, E., 1991b. Endogenous product cycles. *Economic Journal*, 101, 1214–1229.
- [14] Grossman, G., and Rossi-Hansberg, E., 2008. Trading tasks: a simple theory of offshoring. *American Economic Review*, 98, 1978-1997.
- [15] Helpman, E., 1993. Innovation, imitation, and intellectual property rights. *Econometrica*, 61, 1247–1280.
- [16] Iwaisako, T., and Futagami, K., 2013. Patent protection, capital accumulation, and economic growth. *Economic Theory*, forthcoming.
- [17] Lai, E., 1998. International intellectual property rights protection and the rate of product innovation. *Journal of Development Economics*, 55, 133-153.
- [18] Li, C.-W., 2001. On the policy implications of endogenous technological progress. *Economic Journal*, 111, C164-C179.
- [19] Park, W., 2008. International patent protection: 1960-2005. *Research Policy*, 37, 761-766.
- [20] Romer, P., 1990. Endogenous technological change. *Journal of Political Economy*, 98, S71-S102.
- [21] Segerstrom, P., Anant, T.C.A. and Dinopoulos, E., 1990. A Schumpeterian model of the product life cycle. *American Economic Review*, 80, 1077-91.
- [22] Xing, Y., 2012. Processing trade, exchange rates and China’s bilateral trade balances. *Journal of Asian Economics*, 23, 540-547.



## Appendix A

In this appendix, we provide proofs of the propositions. Before we proceed to the proofs, it would be helpful to first present the following preliminary derivations. The prices of intermediate inputs do not depend on  $z \in \{l, h\}$ , so that  $p_{l,t}^n = p_{h,t}^n = \eta^n q_t^n = p_t^n$  and  $p_{l,t}^s = p_{h,t}^s = \eta^s q_t^s = p_t^s$ . The conditional demand functions for labors are

$$w_{l,t}^s = \frac{\beta P_{l,t}}{1-\beta} (l_t^s)^{\beta-1} (x_{l,t}^s)^{1-\beta} (N_{l,t})^{2-\beta}, \quad (\text{A1-a})$$

$$w_{l,t}^n = \frac{\beta P_{l,t}}{1-\beta} (l_t^n + \delta \mathbf{l}_t^s)^{\beta-1} (x_{l,t}^n)^{1-\beta} (N_{l,t})^{2-\beta}, \quad (\text{A1-b})$$

$$w_{h,t}^s = \frac{\beta P_{h,t}}{1-\beta} (h_t^s)^{\beta-1} (x_{h,t}^s)^{1-\beta} (N_{h,t})^{2-\beta}, \quad (\text{A1-c})$$

$$w_{h,t}^n = \frac{\beta P_{h,t}}{1-\beta} (h_t^n)^{\beta-1} (x_{h,t}^n)^{1-\beta} (N_{h,t})^{2-\beta}. \quad (\text{A1-d})$$

The conditional demand functions for intermediate inputs are

$$x_{l,t}^s = (P_{l,t})^{\frac{1}{\beta}} (p_t^s)^{-\frac{1}{\beta}} (l_t^s) (N_{l,t})^{\frac{1-\beta}{\beta}}, \quad (\text{A1-e})$$

$$x_{l,t}^n = (P_{l,t})^{\frac{1}{\beta}} (p_t^n)^{-\frac{1}{\beta}} (l_t^n + \delta \mathbf{l}_t^s) (N_{l,t})^{\frac{1-\beta}{\beta}}, \quad (\text{A1-f})$$

$$x_{h,t}^s = (P_{h,t})^{\frac{1}{\beta}} (p_t^s)^{-\frac{1}{\beta}} (h_t^s) (N_{h,t})^{\frac{1-\beta}{\beta}}, \quad (\text{A1-g})$$

$$x_{h,t}^n = (P_{h,t})^{\frac{1}{\beta}} (p_t^n)^{-\frac{1}{\beta}} (h_t^n) (N_{h,t})^{\frac{1-\beta}{\beta}}. \quad (\text{A1-h})$$

When offshoring takes place in equilibrium (i.e.,  $\mathbf{l}_t^s > 0$ ), the marginal productivity of domestic unskilled labor must be proportional to the marginal productivity of foreign unskilled labor subject to the offshoring cost  $\delta$ ; therefore, we have  $\delta w_{l,t}^n = w_{l,t}^s$ . Using this condition along with the above first-order conditions, we obtain

$$\frac{p_t^n}{p_t^s} = \delta^{\frac{\beta}{1-\beta}}. \quad (\text{A2})$$

Because the final-goods sector is perfectly competitive, profit maximization implies

$$\frac{P_{h,t}}{P_{l,t}} = \frac{1-\gamma}{\gamma} \left( \frac{Y_{h,t}^n + Y_{h,t}^s}{Y_{l,t}^n + Y_{l,t}^s} \right)^{-\frac{1}{\varepsilon}}. \quad (\text{A3})$$

The production functions (7)-(10) can be re-expressed as

$$Y_{l,t}^s = \frac{l_t^s}{1-\beta} (P_{l,t})^{\frac{1-\beta}{\beta}} (N_{l,t})^{\frac{1}{\beta}} (p_t^s)^{-\frac{1-\beta}{\beta}}, \quad (\text{A4-a})$$

$$Y_{l,t}^n = \frac{l_t^n + \delta \mathbf{l}_t^s}{1-\beta} (P_{l,t})^{\frac{1-\beta}{\beta}} (N_{l,t})^{\frac{1}{\beta}} (p_t^n)^{-\frac{1-\beta}{\beta}}, \quad (\text{A4-b})$$

$$Y_{h,t}^s = \frac{h_t^s}{1-\beta} (P_{h,t})^{\frac{1-\beta}{\beta}} (N_{h,t})^{\frac{1}{\beta}} (p_t^s)^{-\frac{1-\beta}{\beta}}, \quad (\text{A4-c})$$

$$Y_{h,t}^n = \frac{h_t^n}{1-\beta} (P_{h,t})^{\frac{1-\beta}{\beta}} (N_{h,t})^{\frac{1}{\beta}} (p_t^n)^{-\frac{1-\beta}{\beta}}. \quad (\text{A4-d})$$

Taking into account (A4) together with the labor-market-clearing conditions, (A2) and (A3) imply

$$\frac{P_{h,t}}{P_{l,t}} = \left( \frac{1-\gamma}{\gamma} \right)^{\frac{\beta\varepsilon}{1+\beta(\varepsilon-1)}} \left( \frac{N_{h,t}}{N_{l,t}} \right)^{-\frac{1}{1+\beta(\varepsilon-1)}} \left( \frac{H^n + \delta H^s}{L^n + \delta L^s} \right)^{-\frac{\beta}{1+\beta(\varepsilon-1)}}, \quad (\text{A5})$$

which serves as the first condition that we will use to solve for the steady-state equilibrium values of  $\{N_{h,t}/N_{l,t}, P_{h,t}/P_{l,t}, \mathbf{l}_t^s\}$ . The other two conditions can be derived as follows.

The R&D conditions imply that  $V_{z,t} = \mu$  and thus  $\dot{V}_{z,t} = 0$  when  $\dot{N}_{z,t} > 0$  for  $z \in \{l, h\}$ . Using (19), we obtain

$$r_t = \frac{\pi_{z,t}^n + \pi_{z,t}^s}{\mu}. \quad (\text{A6})$$

The equilibrium bias is  $V_{h,t}/V_{l,t} = (\pi_{h,t}^n + \pi_{h,t}^s)/(\pi_{l,t}^n + \pi_{l,t}^s) = 1$ . Also using (13), (16), (A1) and (A2), we derive

$$\frac{P_{h,t}}{P_{l,t}} = \left( \frac{N_{h,t}}{N_{l,t}} \right)^{-(1-\beta)} \left( \frac{\frac{\phi}{\beta} \delta H^s + H^n}{\frac{\phi}{\beta} \delta (L^s - \mathbf{l}_t^s) + L^n + \delta \mathbf{l}_t^s} \right)^{-\beta}. \quad (\text{A7})$$

Finally, the capital-market conditions give rise to<sup>30</sup>

$$\frac{P_{h,t}}{P_{l,t}} = \left( \frac{N_{h,t}}{N_{l,t}} \right)^{-1} \left( \frac{\left( \delta^{1/(1-\beta)} \frac{K^n}{K^s} + \delta \right) \mathbf{l}_t^s + L^n - \delta^{1/(1-\beta)} L^s \frac{K^n}{K^s}}{\delta^{1/(1-\beta)} H^s \frac{K^n}{K^s} - H^n} \right)^{\beta}, \quad (\text{A8})$$

noting (A1) and (A2). The steady-state equilibrium values of  $\{N_{h,t}/N_{l,t}, P_{h,t}/P_{l,t}, \mathbf{l}_t^s\}$  are determined by (A5), (A7) and (A8) along with the resource constraint  $\mathbf{l}_t^s \in [0, L^s]$ .

**Proof of Proposition 1 .** Using (A7), one can show that if the following inequality holds,

$$\frac{P_{h,t}}{P_{l,t}} > \left( \frac{N_{h,t}}{N_{l,t}} \right)^{-(1-\beta)} \left( \frac{\frac{\phi}{\beta} \delta H^s + H^n}{\frac{\phi}{\beta} \delta (L^s - \mathbf{l}_t^s) + L^n + \delta \mathbf{l}_t^s} \right)^{-\beta}, \quad (\text{A9})$$

then  $V_{h,t} = (\pi_{h,t}^n + \pi_{h,t}^s)/r_t = \mu$  and  $V_{l,t} < \mu$ , which imply that  $\dot{N}_{h,t} > 0$  and  $\dot{N}_{l,t} = 0$ . Combined with (A5), this inequality can be rewritten as

$$\frac{N_{h,t}}{N_{l,t}} < \left( \frac{1-\gamma}{\gamma} \right)^{\frac{\varepsilon}{1-(1-\beta)(\varepsilon-1)}} \left( \frac{H^n + \delta H^s}{L^n + \delta L^s} \right)^{-\frac{1}{1-(1-\beta)(\varepsilon-1)}} \left( \frac{\frac{\phi}{\beta} \delta H^s + H^n}{\frac{\phi}{\beta} \delta (L^s - \mathbf{l}_t^s) + L^n + \delta \mathbf{l}_t^s} \right)^{\frac{1+\beta(\varepsilon-1)}{1-(1-\beta)(\varepsilon-1)}}, \quad (\text{A10})$$

where  $\mathbf{l}^s \in [0, L^s]$  is given by its steady-state equilibrium value. Thus, following Acemoglu and Zilibotti (2001), we have shown that there is only one type of innovation off the steady

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<sup>30</sup>To derive (A8), we use

$$\frac{K^s}{K^n} = \frac{x_{h,t}^s N_{h,t} + x_{l,t}^s N_{l,t}}{x_{h,t}^n N_{h,t} + x_{l,t}^n N_{l,t}} = \frac{N_{l,t} (P_{l,t})^{\frac{1}{\beta}} (p_t^s)^{-\frac{1}{\beta}} (\mathbf{l}_t^s) (N_{l,t})^{\frac{1-\beta}{\beta}} + N_{h,t} (P_{h,t})^{\frac{1}{\beta}} (p_t^s)^{-\frac{1}{\beta}} (H^s) (N_{h,t})^{\frac{1-\beta}{\beta}}}{N_{l,t} (P_{l,t})^{\frac{1}{\beta}} (p_t^n)^{-\frac{1}{\beta}} (L^n + \delta \mathbf{l}_t^s) (N_{l,t})^{\frac{1-\beta}{\beta}} + N_{h,t} (P_{h,t})^{\frac{1}{\beta}} (p_t^n)^{-\frac{1}{\beta}} (H^n) (N_{h,t})^{\frac{1-\beta}{\beta}}}.$$

state, and the economy monotonically reaches the balanced growth path in finite time. On the balanced growth path,  $N_{h,t}$  and  $N_{l,t}$  grow at the same rate. The same proof can be applied to an economy starting from  $N_{h,t}/N_{l,t}$  larger than the right-hand side of (A10).

In the rest of this proof, we consider the existence and uniqueness of the equilibrium. Using (A5), (A7) and (A8), we derive the following two conditions that can be used to solve for the steady-state equilibrium values of  $\{N_h/N_l, \mathbf{l}^s\}$ .

$$\begin{aligned} \frac{N_h}{N_l} &= \left( \frac{1-\gamma}{\gamma} \right)^{\frac{\varepsilon}{1-(1-\beta)(\varepsilon-1)}} \left( \frac{H^n + \delta H^s}{L^n + \delta L^s} \right)^{-\frac{1}{1-(1-\beta)(\varepsilon-1)}} \left( \frac{\frac{\phi}{\beta} \delta H^s + H^n}{\frac{\phi}{\beta} \delta (L^s - \mathbf{l}^s) + L^n + \delta \mathbf{l}^s} \right)^{\frac{1+\beta(\varepsilon-1)}{1-(1-\beta)(\varepsilon-1)}} \\ &\equiv F(\mathbf{l}^s), \end{aligned} \quad (\text{A11})$$

$$\begin{aligned} \frac{N_h}{N_l} &= \left( \frac{\gamma}{1-\gamma} \right)^{\frac{\varepsilon}{\varepsilon-1}} \left( \frac{H^n + \delta H^s}{L^n + \delta L^s} \right)^{\frac{1}{\varepsilon-1}} \left( \frac{\left( \delta^{\frac{1}{1-\beta}} \frac{K^n}{K^s} + \delta \right) \mathbf{l}^s + L^n - \delta^{\frac{1}{1-\beta}} \frac{K^n}{K^s} L^s}{\delta^{\frac{1}{1-\beta}} \frac{K^n}{K^s} H^s - H^n} \right)^{\frac{1+\beta(\varepsilon-1)}{(\varepsilon-1)}} \\ &\equiv G(\mathbf{l}^s). \end{aligned} \quad (\text{A12})$$

$F(\mathbf{l}^s)$  is (weakly) decreasing in  $\mathbf{l}^s$  because  $\phi \leq \beta$ . As for  $G(\mathbf{l}^s)$ , it depends on the value of  $\delta$ ; specifically, there are three parameter spaces to consider: (a)  $\delta > [(H^n/H^s)(K^s/K^n)]^{1-\beta}$ , (b)  $[(L^n/L^s)(K^s/K^n)]^{1-\beta} < \delta < [(H^n/H^s)(K^s/K^n)]^{1-\beta}$ ,<sup>31</sup> and (c)  $\delta \leq [(L^n/L^s)(K^s/K^n)]^{1-\beta}$ . Recall that  $[(H^n/H^s)(K^s/K^n)]^{1-\beta} > [(L^n/L^s)(K^s/K^n)]^{1-\beta}$  because  $H^n/L^n > H^s/L^s$ .

Case (a): If  $\delta > [(H^n/H^s)(K^s/K^n)]^{1-\beta}$ , then  $G(\mathbf{l}^s)$  is strictly increasing in  $\mathbf{l}^s$  guaranteeing the uniqueness of the equilibrium (if it exists). To establish its existence, we need to ensure that  $F(\mathbf{l}^s)$  and  $G(\mathbf{l}^s)$  cross within  $\mathbf{l}^s \in [0, L^s]$ . First,  $F(0) > G(0)$  because  $F(0) > 0$  and  $G(0) < 0$  as a result of  $L^n - \delta^{\frac{1}{1-\beta}} \frac{K^n}{K^s} L^s < 0$ . Second,  $F(L^s) < G(L^s)$  would also hold if and only if  $\gamma$  is sufficiently large.

Case (b): If  $[(L^n/L^s)(K^s/K^n)]^{1-\beta} < \delta < [(H^n/H^s)(K^s/K^n)]^{1-\beta}$ , then  $G(\mathbf{l}^s)$  would be decreasing in  $\mathbf{l}^s$ . Furthermore,  $G(\mathbf{l}^s)$  would be positive if and only if  $\mathbf{l}^s < \left( \frac{\delta^{1/1-\beta} K^n/K^s - L^n/L^s}{\delta^{1/1-\beta} K^n/K^s + \delta} \right) L^s$ . As  $\mathbf{l}^s \rightarrow \left( \frac{\delta^{1/1-\beta} K^n/K^s - L^n/L^s}{\delta^{1/1-\beta} K^n/K^s + \delta} \right) L^s$ ,  $G(\mathbf{l}^s) = 0 < F(\mathbf{l}^s)$ . Finally,  $G(0) > F(0)$  would also hold if and only if  $\gamma$  is sufficiently large; in this case, it can be shown that  $G(\mathbf{l}^s)$  crosses  $F(\mathbf{l}^s)$  exactly once from above.<sup>32</sup>

Case (c): If  $\delta \leq [(L^n/L^s)(K^s/K^n)]^{1-\beta}$ , then  $\delta < [(H^n/H^s)(K^s/K^n)]^{1-\beta}$  implying that  $\delta^{\frac{1}{1-\beta}} \frac{K^n}{K^s} H^s - H^n < 0$  in  $G(\mathbf{l}^s)$ . In this case,  $G(\mathbf{l}^s)$  must be nonpositive for  $\mathbf{l}^s \in [0, L^s]$  because  $L^n - \delta^{\frac{1}{1-\beta}} \frac{K^n}{K^s} L^s \geq 0$ ; therefore, an offshoring equilibrium does not exist. ■

**Proof of Propositions 2 and 3.** In the following proofs, we consider the special case of zero patent protection in the South. Setting  $\phi = 0$  in (A11), we obtain

$$F(\mathbf{l}^s) = \left( \frac{1-\gamma}{\gamma} \right)^{\frac{\varepsilon}{1-(1-\beta)(\varepsilon-1)}} \left( \frac{H^n + \delta H^s}{L^n + \delta L^s} \right)^{-\frac{1}{1-(1-\beta)(\varepsilon-1)}} \left( \frac{H^n}{L^n + \delta \mathbf{l}^s} \right)^{\frac{1+\beta(\varepsilon-1)}{1-(1-\beta)(\varepsilon-1)}}, \quad (\text{A11-a})$$

<sup>31</sup>It can be shown that if  $\delta = [(H^n/H^s)(K^s/K^n)]^{1-\beta}$ , then  $\mathbf{l}^s = \left( \frac{\delta^{1/1-\beta} K^n/K^s - L^n/L^s}{\delta^{1/1-\beta} K^n/K^s + \delta} \right) L^s$  instead of being determined by (A12).

<sup>32</sup>On the other hand, if  $G(0) < F(0)$ , then the model may feature multiple equilibria, which we rule out by imposing a sufficiently large  $\gamma$  to ensure that  $G(0) > F(0)$  holds.

and  $G(\mathbf{I}^s)$  is the same as in (A12). The unique steady-state equilibrium values of  $\{N_h/N_l, \mathbf{I}^s\}$  are implicitly determined by solving these two equations. We need to consider the two parameter spaces under which offshoring exists: (a)  $\delta > [(H^n/H^s)(K^s/K^n)]^{1-\beta}$ , and (b)  $[(L^n/L^s)(K^s/K^n)]^{1-\beta} < \delta < [(H^n/H^s)(K^s/K^n)]^{1-\beta}$ .

Case (a): If  $\delta > [(H^n/H^s)(K^s/K^n)]^{1-\beta}$ , then  $G(\mathbf{I}^s)$  is increasing in  $\mathbf{I}^s$ . In this case, an increase in  $K^s$  shifts up  $G(\mathbf{I}^s)$  and gives rise to a larger equilibrium value of  $N_h/N_l$ .

Case (b): If  $[(L^n/L^s)(K^s/K^n)]^{1-\beta} < \delta < [(H^n/H^s)(K^s/K^n)]^{1-\beta}$ , then  $G(\mathbf{I}^s)$  is decreasing in  $\mathbf{I}^s$  and crossing  $F(\mathbf{I}^s)$  exactly once from above given a sufficiently large  $\gamma$ . In this case, an increase in  $K^s$  shifts down  $G(\mathbf{I}^s)$  and also gives rise to a larger equilibrium value of  $N_h/N_l$ . We summarize these results in the following Lemma.

**Lemma 1:** If  $\delta > [(L^n/L^s)(K^s/K^n)]^{1-\beta}$ , then an increase in  $K^s$  would lead to an increase in  $N_h/N_l$ .

Using (A11-a) and (A12), we derive the following condition that implicitly determines  $N_h/N_l$ .

$$\begin{aligned} & \frac{H^n}{H^s} \left( \delta^{\frac{1}{1-\beta}} \frac{K^n}{K^s} + \delta \right) \left( \frac{N_h}{N_l} \right)^{-\frac{1-(1-\beta)(\varepsilon-1)}{1+\beta(\varepsilon-1)}} \\ = & \delta \left( \delta^{\frac{1}{1-\beta}} \frac{K^n}{K^s} - \frac{H^n}{H^s} \right) \left( \frac{N_h}{N_l} \right)^{\frac{\varepsilon-1}{1+\beta(\varepsilon-1)}} + \delta^{\frac{1}{1-\beta}} \frac{K^n}{K^s} \left( \frac{\gamma}{1-\gamma} \right)^{\frac{\varepsilon}{1+\beta(\varepsilon-1)}} \left( \frac{H^n}{H^s} + \delta \right)^{\frac{1}{1+\beta(\varepsilon-1)}} (L^n + \delta L^s)^{\frac{\beta(\varepsilon-1)}{1+\beta(\varepsilon-1)}}. \end{aligned} \quad (\text{A13})$$

Once again, we need to consider the two parameter spaces under which offshoring exists: (a)  $\delta > [(H^n/H^s)(K^s/K^n)]^{1-\beta}$ , and (b)  $[(L^n/L^s)(K^s/K^n)]^{1-\beta} < \delta < [(H^n/H^s)(K^s/K^n)]^{1-\beta}$ .

Case (a): If  $\delta > [(H^n/H^s)(K^s/K^n)]^{1-\beta}$ , then the right-hand side of (A13) is increasing in  $N_h/N_l$ , whereas the left-hand side of (A13) is always decreasing in  $N_h/N_l$ . In this case, a decrease in  $L^s$  shifts down the right-hand side and gives rise to a larger equilibrium value of  $N_h/N_l$ .

Case (b): If  $[(L^n/L^s)(K^s/K^n)]^{1-\beta} < \delta < [(H^n/H^s)(K^s/K^n)]^{1-\beta}$ , then the right-hand side of (A13) is also decreasing in  $N_h/N_l$  and crosses the left-hand side exactly once from below. In this case, a decrease in  $L^s$  shifts down the right-hand side and also gives rise to a larger equilibrium value of  $N_h/N_l$ . We summarize these results in the following Lemma.

**Lemma 2:** If  $\delta > [(L^n/L^s)(K^s/K^n)]^{1-\beta}$ , then a decrease in  $L^s$  would lead to an increase in  $N_h/N_l$ .

**Zero-offshoring equilibrium:** Now we consider the case of  $\delta \leq [(L^n/L^s)(K^s/K^n)]^{1-\beta}$ , under which offshoring does not take place in equilibrium (i.e.,  $\mathbf{I}^s = 0$ ). In this case, we derive three equilibrium conditions,

$$\frac{P_h}{P_l} = \left( \frac{1-\gamma}{\gamma} \right)^{\frac{\beta\varepsilon}{1+\beta(\varepsilon-1)}} \left( \frac{N_h}{N_l} \right)^{-\frac{1}{1+\beta(\varepsilon-1)}} \left( \frac{(H^s)(p^s)^{-(1-\beta)/\beta} + (H^n)(p^n)^{-(1-\beta)/\beta}}{(L^s)(p^s)^{-(1-\beta)/\beta} + (L^n)(p^n)^{-(1-\beta)/\beta}} \right)^{-\frac{\beta}{1+\beta(\varepsilon-1)}}, \quad (\text{A14})$$

$$\frac{P_h}{P_l} = \left( \frac{H^n}{L^n} \right)^{-\beta} \left( \frac{N_h}{N_l} \right)^{-(1-\beta)}, \quad (\text{A15})$$

$$\left( \frac{p^n}{p^s} \right)^{1/\beta} = \frac{K^s}{K^n} \frac{(L^n) + (P_h/P_l)^{1/\beta} (H^n)(N_h/N_l)^{1/\beta}}{(L^s) + (P_h/P_l)^{1/\beta} (H^s)(N_h/N_l)^{1/\beta}}, \quad (\text{A16})$$

which correspond to (A5), (A7) and (A8), respectively. Substituting (A15) and (A16) into (A14), we obtain

$$\frac{N_h}{N_l} = \left( \frac{1-\gamma}{\gamma} \right)^{\frac{\varepsilon}{1-(1-\beta)(\varepsilon-1)}} \left( \frac{H^n}{L^n} \right)^{\frac{1+\beta(\varepsilon-1)}{1-(1-\beta)(\varepsilon-1)}} \left( \frac{L^s \left( \frac{K^s}{K^n} \left( 1 + \frac{N_h}{N_l} \right) \right)^{(1-\beta)} + L^n \left( \frac{L^s}{L^n} + \frac{H^s}{H^n} \frac{N_h}{N_l} \right)^{1-\beta}}{H^s \left( \frac{K^s}{K^n} \left( 1 + \frac{N_h}{N_l} \right) \right)^{(1-\beta)} + H^n \left( \frac{L^s}{L^n} + \frac{H^s}{H^n} \frac{N_h}{N_l} \right)^{1-\beta}} \right)^{\frac{1}{1-(1-\beta)(\varepsilon-1)}}. \quad (\text{A17})$$

Because  $H^n/L^n > H^s/L^s$ , the right-hand side is monotonically increasing and concave in  $N_h/N_l$ , which ensures the unique existence of a steady-state equilibrium. One can show that the right-hand side is increasing in  $L^s$  and  $K^s$ , so we can prove the following lemma by means of a usual graphical analysis.

**Lemma 3:** If  $\delta \leq [(L^n/L^s)(K^s/K^n)]^{1-\beta}$ , then there would be no outsourcing in equilibrium (i.e.,  $\mathbf{l}^s = 0$ ); in this case, an increase in  $L^s$  or  $K^s$  leads to an increase in  $N_h/N_l$ .

Finally, note that Lemma 2 and Lemma 3 prove Proposition 2, whereas Lemma 1 and Lemma 3 prove Proposition 3. ■

## Appendix B: Not for publication

In this appendix, we provide the equilibrium expressions for calibrating the model: (a) offshoring as a share of GDP  $w_l^s \mathbf{l}^s / (P_l Y_l^s + P_h Y_h^s)$ , (b) the relative GDP  $(P_l Y_l^s + P_h Y_h^s) / (P_h Y_h^n + P_l Y_l^n)$ , and (c) the skill premium  $w_h^n / w_l^n$ . Note that (5) implies

$$P_l = \left( \gamma^\varepsilon + (1 - \gamma)^\varepsilon \left( \frac{P_h}{P_l} \right)^{1-\varepsilon} \right)^{\frac{1}{\varepsilon-1}}, \quad (\text{B1-a})$$

$$P_h = \left( \gamma^\varepsilon \left( \frac{P_h}{P_l} \right)^{-(1-\varepsilon)} + (1 - \gamma)^\varepsilon \right)^{\frac{1}{\varepsilon-1}}. \quad (\text{B1-b})$$

Then, using the capital-market condition for  $s$  and (A1), we obtain

$$p^s = (K^s)^{-\beta} \left( (P_l)^{\frac{1}{\beta}} (l^s) (N_l)^{\frac{1}{\beta}} + (P_h)^{\frac{1}{\beta}} (H^s) (N_h)^{\frac{1}{\beta}} \right)^\beta. \quad (\text{B2})$$

As for  $P_l Y_l^n + P_h Y_h^n$ , we use (A4) to obtain

$$P_h Y_h^n + P_l Y_l^n = \frac{\delta^{\frac{\beta}{1-\beta}} K^n}{1 - \beta} (P_l) (N_l) \left( \frac{H^s L^n - H^n L^s + (H^n + \delta H^s) \mathbf{l}^s}{\delta^{\frac{1}{1-\beta}} K^n H^s - K^s H^n} \right)^\beta, \quad (\text{B3})$$

noting (A2) and (A8). Using (A1), we obtain

$$w_l^s = \frac{\beta (P_l) (N_l)}{1 - \beta} \left( \frac{P_l N_l}{p^s} \right)^{\frac{1-\beta}{\beta}}. \quad (\text{B4})$$

Using (A4), (A8) and (B2), we obtain

$$P_l Y_l^s + P_h Y_h^s = \frac{K^s}{1 - \beta} (P_l) (N_l) \left( \frac{H^s L^n - H^n L^s + (H^n + \delta H^s) \mathbf{l}^s}{\delta^{\frac{1}{1-\beta}} K^n H^s - K^s H^n} \right)^\beta. \quad (\text{B5})$$

Using (B2), (B4) and (B5), we obtain

$$\frac{w_l^s \mathbf{l}^s}{P_l Y_l^s + P_h Y_h^s} = \frac{\beta \left( \delta^{\frac{1}{1-\beta}} K^n H^s - K^s H^n \right) \mathbf{l}^s}{K^s (H^s L^n - H^n L^s + (H^n + \delta H^s) \mathbf{l}^s)}. \quad (\text{B6})$$

Using (B3) and (B5), we obtain

$$\frac{P_l Y_l^s + P_h Y_h^s}{P_h Y_h^n + P_l Y_l^n} = \frac{K^s}{\delta^{\frac{\beta}{1-\beta}} K^n}. \quad (\text{B7})$$

Finally, using (A1), we obtain

$$\frac{w_h^n}{w_l^n} = \left( \frac{P_h}{P_l} \frac{N_h}{N_l} \right)^{\frac{1}{\beta}}. \quad (\text{B8})$$

By (A5) and (A7),

$$\left( \frac{P_h}{P_l} \frac{N_h}{N_l} \right)^{\frac{1}{\beta}} = \left( \left( \frac{1 - \gamma}{\gamma} \right)^{\varepsilon/(\varepsilon-1)} \left( \frac{H^n + \delta H^s}{L^n + \delta L^s} \right)^{-1/(\varepsilon-1)} \left( \frac{H^n + \frac{\phi}{\beta} \delta H^s}{L^n + \frac{\phi}{\beta} \delta L^s + \delta \mathbf{l}^s (\beta - \phi)/\beta} \right) \right)^{\frac{\varepsilon-1}{1-(1-\beta)(\varepsilon-1)}}. \quad (\text{B9})$$

Then, (B8) and (B9) imply (28).