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# A Financial Engineering Approach to Identify Stock Market Bubble

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### Abstract

In this paper we adopt an engineering method based on Al-Anaswah and Wilfling, state space model with Markov-switching, to capture the speculative bubbles of stock markets in China and US. We present the VAR log linear asset pricing model in state space model with Markov-switching, so that we can capture the unobservable speculative bubbles. Based on the dataset from Stock markets in China and US, we find empirically that the engineering technique we choose detect the stock markets bubbles effectively, and that the switching probabilities between the surviving and collapsing regimes. In-the-sample and out-of-sample forecasting further support our empirical evidence.

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Keywords:Bubble;VAR-loglinear asset pricing; State space model with Markov-switching; financial engineering

# 1. Introduction

After several decades of research, financial economists remain unsatisfied about how to measure and identify stock market bubble, especially how to capture and forecast the regime-switching behaviour of speculative bubbles, which is also essential to authorities and investors. Due to the fact that stock bubbles are unobservable, this question has puzzled people for a long time. What is more, since China's stock market is more complicated, could the bubbles be explained in the same way like other countries such as US? What are the similarities and the differences of the stock market bubbles between China and US?

In this paper, we express the VAR-loglinear asset pricing model into state space form in order to capture the unobservable bubble process by using Kalman filter. On top of that we add a two-state Markov-switching to analyse the regime features and the switching probabilities of bubbles. In other words, we adopt a state space model with Markov-switching to identify the stock market bubbles both in China and US.

We obtain an encouraging overall result that the econometric framework we adopt is able to detect bubble periods not only in US, but also in China. This result indicates that we are able to identify the surviving regime and the collapsing regime as well as the switching probabilities in our samples, by employing state space model with Markov-switching. In both countries, the stock market bubble collapses periodically after some asymmetric duration of one regime. However, the duration of the collapsing regime is longer in China than it is in US. Besides that, our in-the-sample and out-of-sample forecasting further support our empirical evidence.

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The contributions of this study are twofold. First, based on the dividend multiple measure of fundamentals, we succeed in overcoming the shortage of the data in China's stock market and in finding statistically significant regime-switching characteristics of the stock-price bubble processes in China as well as in US. Second, we find substantial similarities and differences of the stock market bubbles between China and US, consequently enriching the current empirical research of the stock market bubbles.

The rest of this paper is organised as follows. In section 2 we present the literature review of related research. In section 3 we derive the state space model with Markov-switching, and show how switching regression model can capture the characteristics of stock market bubbles. Section 4 presents the data, the methodology used to construct fundamental values, the main tests, and our estimate outcome, while in section 5 we examine the in-the-sample and out-of-sample forecasting ability. Section 6 concludes.

#### 2. Literature review

It has been a long time for the economists to be concerned about the econometric detection of stock market bubbles. Initially, these studies focused on the indirect identification of speculative bubbles in stock market [2-5].However, indirect tests of stock market bubbles are often criticized, not only since speculative bubbles always display explosive growth but also because effects of bubble in stock prices could not be distinguished easily from the effects of unobservable market fundamentals, see [6-7]. For this reason, so-called direct bubble tests have been developed, which directly specify and examine the presence of a particular form of speculative bubble [7-9].West [8] sees bubble as an alternative regime definitely different from the stationary autoregressive process, and takes a statistically significant difference between two alternative estimators of one certain parameter as an indication of a speculative bubble.

Hall, Psaradakis and Sola [10] in the first time introduce Markov-switching (Hamilton [11, 12]) technique into ADF tests, in order to distinguish discrete regimes (surviving regime and collapsing regime) in time series data of bubbles. Although this technique is criticized for the size distortion by Van Norden and Vigfusson [13], the methodology of Markov-switching has been used by more and more people due to the fact that bubbles often switch between two or more regimes [14,15].

As for the China's stock market bubbles, [16-18] employ three-regime model in their empirical research respectively. Fan, Lu and Wang [19] also use a three-regime MS-VECM model to study the relationship between China and US. However, based on the indirect test method, Meng, Zhou and Wang [20] obtain a two-regime behaviour formula that the price bubble fulfils and argue that the third regime is insignificant in China's stock market. Apart from that, Zhao and Zeng [7] find that the monthly return process can be obviously divided into two states, namely, bubbles survival state and bubbles collapse state. Li [21] describes an expansion and collapsing process of speculative bubble in China's stock market by suing a regime switch model to study the non-linear relationship of return and turnover. Based on the literatures above and the flexibility of computing, we choose a two-regime specification in this paper.

Due to the fact that asset bubbles are unobservable, state space model is frequently applied in research of asset bubbles. It is well known that, based on state space model, Kalman filter is a powerful econometric tool in estimating the unobservable variable. Wu [9] estimates the deviation of stock prices from the present-value model and construct the time series of S&P500 using the state space model and Kalman filter. In China, Hang, Liu and Cao [22] adopt this methodology in their research of estate bubble of Shanghai, Tang, Cai and Xie [23] use the dynamic distributed lag model and the state space model to analyses the stock market wealth. Wang [24] applies the state space model and the general Kalman filter method to the real estate market of the main cities in China.

Al-Anaswah and Wilfling [1] treat the bubble as an unobservable variable as in Wu [9] but extend the framework by allowing the bubble to switch between alternative regimes. Through this they succeeded in separating two distinct periods in the bubble process from each other. Technically speaking, we implement Markov-switching in our unobserved-components framework by adopting the methodology from Kim and Nelson [25] who show how to use state space models that are subject to regime-switching. Hitherto, this econometric technique has mainly been used for the detection of turning points in business-cycle research (see [26-28]) and its application to the stock market bubbles in China, compared to the US, constitutes the innovation of this paper.

#### 3. State space model with Markov-switching

## 3.1. Theoretical model

Like Al-Anaswah and Wilfling [1], we also define the bubble as the deviation of the stock price from the fundamental value, and the fundamental value can be calculated from Campbell and Shiller [29, 30]:

As suggested by Wu [9], we assure that we can approximate the dividend process  $\{d_t\}$  by an ARIMA (k, 1, 0):

$$\Delta \mathbf{d}_{t} = \mu + \sum_{j=1}^{k} \varphi_{j} \Delta d_{t-j} + \delta_{t}$$
<sup>(1)</sup>

with  $d_t$  denoting the log real dividend paid at date t, and  $\delta_t \sim N(\theta, \sigma_{\delta}^2)$  denoting a Gaussian white-noise error term in which we can estimate the autoregressive order k from the data.

Following Diba and Grossman [2], Zhou and Yang [31], we assume  $B_t$  representing the rational speculative bubble component and the bubble process  $\{B_t\}$  satisfying the homogeneous difference equation:

$$B_{t} = \left(\frac{1}{\psi}\right)B_{t-1} + \eta_{t} \tag{2}$$

with  $\eta_t \sim N(0, \sigma_{\eta}^2)$ , uncorrelated with the dividend innovation  $\delta_t$  in equation (1). The bubble grows when  $\frac{1}{\psi} > 1$  and collapses when  $\frac{1}{\psi} < 1$ .

As for the fundamental value, it can be obtained from Campbell and Shiller [29, 30]. Briefly, consider the following rational-expectations model of stock-price determination:  $q = \kappa + mE_t(p_{t+1}) + (1 - \psi)d_t - p_t$ , where q is the required log gross return rate;  $E_t$  is the mathematical expectation operator conditional on all information available at date t;  $p_t$  is the log real stock price at date t; and  $\kappa$ , mare parameters of linearization.

Imposing the transversality condition  $\lim_{i\to\infty} m^i E_t(p_{t+i}) = 0$ , we can obtain the particular solution to the

difference equation, which is the intrinsic value:  $p_t^f = \frac{\kappa - q}{1 - m} + (1 - m) \sum_{i=0}^{\infty} m^i E_t(d_{t+i})$ . Obviously, the general

solution is:  $p_t = p_t^f + B_t$ . Sorting these equations, we can easily get:

$$\Delta p_{t} = \Delta p_{t}^{f} + \Delta B_{t} = (1 - m) \sum_{i=0}^{\infty} m^{i} [E_{t} (d_{t+i}) - E_{t-i} (d_{t+i-1})] + \Delta B_{t}$$
(3)

This equation cannot be estimated directly because the bubble component is unobservable. Therefore, we need to express the equations in state space form so that the Kalman filter technique can be used to tackle this problem.

#### 3.2. State space model

Suppose  $\beta_t$  is a  $(n \times 1)$  vector of unobservable variables, namely state variables, and  $\mathbf{g}_t$ ,  $\mathbf{z}_t$  are  $(g \times 1)$  and  $(f \times 1)$  observable variables, namely input and output variables, respectively. Then the state space model consists of a state equation and an output equation, written as follows:

$$\boldsymbol{\beta}_t = \mathbf{F} \boldsymbol{\beta}_{t-1} + \boldsymbol{\xi}_t \,, \tag{4}$$

$$\mathbf{z}_t = \mathbf{H}\boldsymbol{\beta}_t + \mathbf{D}\mathbf{g}_t + \boldsymbol{\zeta}_t, \tag{5}$$

Where **F**, **H** and **D** are constant real matrices;  $\xi_t$  and  $\zeta_t$  are  $(n \times 1)$  and  $(g \times 1)$  vectors of disturbances, which are uncorrelated with each other and  $E(\xi_t) = 0$ ,  $E(\zeta_t) = 0$ ,  $E(\zeta_t, \zeta_t') = \Omega$ ,  $E(\zeta_t, \zeta_t') = \mathbb{R}$ .

In order to use Kalman filter to estimate the unobservable bubble, the theoretical model can be written as:  $\boldsymbol{\beta}_{t} = (B_{t}, B_{t-1})', \ \mathbf{z}_{t} = (\Delta d_{t}, \Delta p_{t})', \ \mathbf{g}_{t} = (1, \Delta d_{t}, \Delta d_{t-1}, \Delta d_{t-2}, ..., \Delta d_{t-h})',$ 

$$\boldsymbol{\xi}_{t} = (\boldsymbol{\eta}_{t}, 0)', \boldsymbol{\zeta}_{t} = (\boldsymbol{\delta}_{t}, 0)'$$
$$\mathbf{F} = \begin{pmatrix} 1/\psi & 0\\ 1 & 0 \end{pmatrix}, \quad \mathbf{H} = \begin{pmatrix} 0 & 0\\ 1 & -1 \end{pmatrix}$$
(6)

$$\mathbf{D} = \begin{pmatrix} \mu & 0 & \phi_1 & \phi_2 & \dots & \phi_{h-1} & \phi_h \\ 0 & (1-m) & (1-m)m & (1-m)m^2 & \dots & (1-m)m^{h-1} & (1-m)m^h \end{pmatrix}$$

$$\mathbf{\Omega} = \begin{pmatrix} \sigma_{\eta}^2 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} \sigma_{\delta}^2 & 0 \\ 0 & 0 \end{pmatrix}$$
(7)

#### 3.3. State space model with Markov-switching

In order to do identification, we allow the bubble to switch between alternative regimes, namely one in which bubble survives and one in which it collapses. In what follows, we call them regime 1 and 2  $(S_t = 1, 2)$  for surviving regime and collapsing regime respectively. More details can be found in Kim and Nelson [25].

Introducing a first-order Markov-process with transition probabilities  $p_{ij} = \Pr[S_t = j/S_{t-1} = i]$  and a transition-probability matrix  $\mathbf{\Pi} = \begin{pmatrix} p_{11} & 1-p_{22} \\ 1-p_{11} & p_{22} \end{pmatrix}$ , then the state space model should be rewritten as:

$$\boldsymbol{\beta}_t = \mathbf{F}_{S_t} \boldsymbol{\beta}_{t-1} + \boldsymbol{\xi}_t, \tag{8}$$

$$\mathbf{z}_{t} = \mathbf{H}_{S} \mathbf{\beta}_{t} + \mathbf{D}_{S} \mathbf{g}_{t} + \boldsymbol{\zeta}_{t}, \tag{9}$$

$$\begin{pmatrix} \boldsymbol{\xi}_t \\ \boldsymbol{\zeta}_t \end{pmatrix} \sim N \begin{pmatrix} \boldsymbol{0}, \begin{pmatrix} \boldsymbol{\Omega}_{S_t} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}_{S_t} \end{pmatrix} \end{pmatrix}$$
(10)

Considering the related algorithm and approximation approach in Kim and Nelson [25], then the general Kalman filter can be expressed, after denoting  $S_{t-1} = i, S_t = j$ , as follows:

$$\boldsymbol{\beta}_{\boldsymbol{\mu} - 1}^{(i,j)} = \mathbf{F}_{j} \boldsymbol{\beta}_{i-\boldsymbol{\mu} - 1}^{i}, \tag{11}$$

$$\mathbf{P}_{t|t-1}^{(i,j)} = \mathbf{F}_j \mathbf{P}_{t-1|t-1}^i \mathbf{F}_j' + \mathbf{\Omega}_j, \tag{12}$$

$$\boldsymbol{\zeta}_{t_{l^{t-1}}}^{(i,j)} = \mathbf{z}_t - \mathbf{H}_j \boldsymbol{\beta}_{t_{l^{t-1}}}^{(i,j)} - \mathbf{D}_j \mathbf{g}_t, \tag{13}$$

$$\mathbf{f}_{l|t-1}^{(i,j)} = \mathbf{H}_j \mathbf{P}_{l|t-1}^{(i,j)} \mathbf{H}_j' + \mathbf{R}_j, \tag{14}$$

$$\boldsymbol{\beta}_{t|t}^{(i,j)} = \boldsymbol{\beta}_{t|t-1}^{(i,j)} + \mathbf{P}_{t|t-1}^{(i,j)} \mathbf{H}_{j}' \left[ \mathbf{f}_{t|t-1}^{(i,j)} \right]^{-1} \boldsymbol{\zeta}_{t|t-1}^{(i,j)},$$
(15)

$$\mathbf{P}_{t|t}^{(i,j)} = \left[\mathbf{I} - \mathbf{P}_{t|t-1}^{(i,j)}\mathbf{H}_{j}'\left[\mathbf{f}_{t|t-1}^{(i,j)}\right]^{-1}\mathbf{H}_{j}\right]\mathbf{P}_{t|t-1}^{(i,j)},\tag{16}$$

$$\boldsymbol{\beta}_{t/t}^{j} = \frac{\sum_{i=1}^{2} \Pr\left[S_{t-1} = i, S_{t} = j \mid \Psi_{t}\right] \boldsymbol{\beta}_{t/t}^{(i,j)}}{\Pr\left[S_{t} = j \mid \Psi_{t}\right]}$$
(17)

$$\mathbf{P}_{t/t}^{j} = \frac{\sum_{i=1}^{2} \Pr[S_{t-1} = i, S_{t} = j \mid \Psi_{t}] \{\mathbf{P}_{t/t}^{(i,j)} + (\mathbf{\beta}_{t/t}^{j} - \mathbf{\beta}_{t/t}^{(i,j)}) (\mathbf{\beta}_{t/t}^{j} - \mathbf{\beta}_{t/t}^{(i,j)})'\}}{\Pr[S_{t} = j \mid \Psi_{t}]}$$
(18)

where 
$$\boldsymbol{\beta}_{t,j}^{(i,j)} = \mathbb{E}\left[\boldsymbol{\beta}_{t} \mid \boldsymbol{\Psi}_{t-1}, S_{t} = j, S_{t-1} = i\right], \ \mathbf{P}_{t-1}^{(i,j)} = \mathbb{E}\left[(\boldsymbol{\beta}_{t} - \boldsymbol{\beta}_{t|t-1})(\boldsymbol{\beta}_{t} - \boldsymbol{\beta}_{t|t-1})' \mid \boldsymbol{\Psi}_{t-1}, S_{t} = j, S_{t-1} = i\right], \ \boldsymbol{\Psi}_{t-1} \ \text{denoting}$$

the vector of observations available as of date *t*-1,  $\boldsymbol{\beta}_{t-1|t-1}^{i}$  is an inference on  $\boldsymbol{\beta}_{t-1}$  based on information up to time *t*-1 given  $S_{t-1} = i$ ;  $\boldsymbol{\beta}_{t|t-1}^{(i,j)}$  is an inference on  $\boldsymbol{\beta}_{t}$  based on information up to time *t*-1 given  $S_{t} = j, S_{t-1} = i$ ;  $\boldsymbol{\zeta}_{t|t-1}^{(i,j)}$  is the conditional forecast error of  $\mathbf{z}_{t}$  based on information up to time *t*-1 given  $S_{t} = j, S_{t-1} = i$ ;  $\boldsymbol{\zeta}_{t|t-1}^{(i,j)}$  is the conditional forecast error  $\boldsymbol{\zeta}_{t}$  based on information up to time *t*-1 given  $S_{t} = j, S_{t-1} = i$ ;  $\mathbf{f}_{t|t-1}^{(i,j)}$  is the conditional variance of the forecast error  $\boldsymbol{\zeta}_{t|t-1}^{(i,j)}$ .

In order to reduce the complexity of our models, we only allow the autoregressive coefficient  $\frac{1}{\psi}$  to switch between the two regimes, leaving all other parameters as non-switching between the Markov-regimes. At last we can get  $\frac{1}{\psi_1}$  and  $\frac{1}{\psi_2}$  from matrices  $\mathbf{F}_{S_t}$ . If  $\frac{1}{\psi_1} \neq \frac{1}{\psi_2} \neq 0$  significantly, we can say we succeed in discriminating the surviving and collapsing phases from the bubble process.

# 4. Empirical results

## 4.1. Data and fundamental values

The data of China we use to identify bubbles are 222 monthly observations on the SSE (A Shares in Shanghai Stock Exchange) Composite index for the period July 1991- December 2009, which is taken form Wind Data Base and CEIC China. Because of the well-known problem of the dividend data in China, we use a three-step method based on Dividend Multiple Model of Van-Norden and Schaller [32] to cope with it. Firstly, we simply calculate the sum of yearly dividend of every share as the yearly dividend of the SSE, which could be divided by 12 to get monthly data. In addition, we transform it into real variables. Finally, we take the latest 12 month moving average to get enough variation.

As for the data of US, we take 1668 observations from Shiller [33], on the S&P 500 for the period January 1871 – December 2009. The monthly dividend and price series are transformed into real variables using the monthly US Consumer Price – All Items Seasonally Adjusted Index reported by Shiller [33].

## 4.2. Unit root test and parameter specification

The results of ADF unit root tests are listed in table 1, from which it is evident that the log dividends in both two countries follow the standard unit root process. The results of PPERRON test are similar but not listed here for brevity.

Table 1. ADF unit root test

Variables Horizontal test First order difference test

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	Statistic	1% cm	5% cm	10%cm Statistic	1% cm	5% cm	10%cm
Log dividend, China	-0.900	-3.470	-2.882	-2.572 -6.952***	-3.470	-2.882	-2.572
Log dividend, USA	-1.062	-3.430	-2.860	-2.570 -5.038***	-3.430	-2.860	-2.570
Notes: Cm denotes criti	ool volue *	* ** ***	Denote	statistical significan	ca at 5%	1% 01	% lavale

Notes: Cm denotes critical value. \*, \*\*, \*\*\* Denote statistical significance at 5%, 1%, 0.1% levels.

According to the autocorrelation and partial correlation tests, we choose the autoregressive specification of order 1. The results of AR (1) are listed in table 2, from which it is apparent that  $\mu = 0, k = 0$  in equation (1).

Table 2. AR (1) model of real dividend

Data of China	lnd (real log dividend)	Data of US	dlnd(differenced real log dividend)
Lag.lnd	0.994***	Lag.dlnd	0.999****
T statistic	(149.21)	T statistic	(1271.32)
_constant	0.00365	_constant	0.00295
T statistic	(0.75)	T statistic	(1.55)
Ν	221	Ν	1667

Notes: \*, \*\*, \*\*\* Denote statistical significance at 5%, 1%, 0.1% levels.

As for parameter h, from the perspective of accuracy, obviously, it is the bigger, the better. On the other hand, the value of this parameter is limited by the sample size and may cause loss of information as well as computational burden. We have examined this parameter using 1, 2, 3 and found significant results already when we use h=3.

And we have done specification test of Markov-switching model against its linear counterpart. Because of the computational burden and infeasibility of the LRT test in Hansen [34] and Garcia [35], we choose the method in Ang and Bekaert [36]. The results are listed in table 3.

Table 3. S	Specification	test of Markov-	switching	property

Sample	Log likelihood of	Log likelihood of	Likelihood ratio	
	Markov-switching model	linear model	test statistic	
China,1991-2009	-1193.5926	-1203.5367	19.8882***	
US, 1871-2009	-4604.6888	-4654.2353	99.0930****	

Notes: \*, \*\*, \*\*\* Denote statistical significance at 5%, 1%, 0.1% levels.

## 4.3. Main results

The main results of the state space model with Markov-switching we used are listed in table 4. There are similar characteristics in both two samples:

First of all, it is clear that the bubble parameters  $\frac{1}{\psi_1}$  and  $\frac{1}{\psi_2}$  are significantly different from each other, and

 $0 < \frac{1}{\psi_1} < 1$ ,  $\frac{1}{\psi_2} > 1$ . It indicates that periodically collapsing stock bubbles expand in surviving regime,

whereas contract in collapsing regime. Overall, there are regime switching phenomena in bubble processes, and we succeed in discriminating surviving and collapsing phases from the bubble process, as well as detecting the turning points and identifying bubbles.

Moreover, in both samples  $\eta_i$  and  $\delta_t$  are also no zero significantly. It means that the stock price and dividend process show some stochastic characteristics, corresponding to the reality. It reveals the volatility in stock index and noise in stock market.

In addition, transition probabilities  $p_{11}$  and  $p_{12}$  are statistically significant, and the values of them are also in expected range. It can be interpreted that once the bubble in some regime, it is expected that there is some length of duration in this regime. From the values of them, we can say each regime is highly persistent.

Finally, the expected duration of each regime can be calculated by  $D(S_i)=1/(1-p_{ii})$ , according to Kim and Nelson [25]. By comparison of transition probabilities, it can be easily found that the collapsing regime is more persistent that the surviving regime.

	$p_{11}$	$p_{22}$	$\psi_1$	$\delta_{\mathrm{t}}$	$\eta_t$	т	$\frac{1}{\psi_2}$	Log likelihood
China, 1991-2009	0.9867	0.9029	0.5043	6.6513	11.3695	0.9919	1.0221	-1193.5926
Standard error	0.0081	0.0251	0.0461	0.3200	0.2862	0.0369	0.0164	
T statistic	122.3311	35.9162	10.9309	20.7850	39.7252	26.8542	62.4583	
US, 1871-2009	0.9845	0.6150	0.6580	1.4643	3.8067	0.3526	1.0015	-4590.9054
Standard error	0.0043	0.0810	0.0198	0.0254	0.0714	0.1055	0.0020	
T statistic	228.9367	7.5932	33.1551	57.6179	53.2786	3.3427	494.6577	
T statistic US, 1871-2009 Standard error T statistic	0.0081 122.3311 0.9845 0.0043 228.9367	0.0251 35.9162 0.6150 0.0810 7.5932	0.0461 10.9309 0.6580 0.0198 33.1551	0.3200 20.7850 1.4643 0.0254 57.6179	0.2862 39.7252 3.8067 0.0714 53.2786	0.0369 26.8542 0.3526 0.1055 3.3427	0.0164 62.4583 1.0015 0.0020 494.6577	-4590.9054

Table 4. Main results

On the other hand, there are also some differences between the China's stock market and US stock market:

Firstly, both the dividend innovation  $\delta_t$  and the bubble innovation  $\eta_t$  in China's stock market are bigger

than those in US stock market. It appears that there is more noise and bigger volatility in China's stock market. What's more, the expected duration of each regime in China's stock market is longer than those in US stock market. It illustrates that the stock bubbles in China are bigger that those in US.

At last, the value of log likelihood demonstrates the degree of fitting of the model we used. It seems that the state space model with Markov switching suits the stock market in US better. One possible explanation is that there are more non-marketing characters in China's stock market, including short sale constraints, up and down limits, etc.

## 5. Predictive analysis

Now turn to predictions. The filtered probabilities provide information about the regime in which the bubble is most likely to be at every point in the sample. They are therefore very useful for dating the various switches. The smoothed probabilities use the information up to the end of the sample and are more accurate since they are based on more information. We use the filtered probabilities and smoothed probabilities to be the signals of in-sample and out-of-sample forecasting, respectively.

Figs 1 - 3 show the real price process, the smoothed surviving-regime probabilities along with filtered surviving-regime probabilities for China's Data ranging from 1991 to 2009. Evidently, we have captured bubbles in the periods of 1992July-1993July, 1994Oct, 1999July-Dec, 2008May-Dec, by employing the state space model with Markov-switching. Not only smoothed probabilities, but also filtered probabilities shoot up dramatically in these corresponding times. And the two probabilities rise up to some extent in 1997May-June, 2006June-July, 2001Aug-2005Apr, coinciding with several smaller bubbles in these periods. The results are in accordance of previous research, such as Meng, Zhou and Wang [20], Zhao and Zeng [7], etc.



Fig. 1 real price process, China, 1991-2009



Likewise, Figs 4 -6 display the real price process, the smoothed surviving-regime probabilities along with filter surviving-regime probabilities for the entire US Data set ranging from 1871 to 2009. When the Great Depression, World War II, the Black Monday in 1987 and the recent financial crisis are taken into account, it is understandable that both smoothed probabilities and filtered probabilities soar in 1877Aug-1877Nov, 1893July, 1907Feb, 1930Oct-1940Feb, 1943Aug-Nov, 1946Sep-Nov, 1962June, 1970May, 1974July-Oct, 1987Oct-Nov, 2008Oct-Dec.



0

0.2



0.3

0.2

Fig.6 filtered probabilities, USA, 1871-2009

# 6. Conclusions

This paper expresses the VAR-loglinear asset pricing model into state space form to capture the unobservable bubble process using Kalman filter. On top of that we add a two-state Markov-switching to analyse the regime features and the switching probabilities of bubbles. In other words, we adopt a state space model with Markov-switching to identify the stock market bubbles both in China and US.

We obtain the encouraging overall result that the engineering technique we adopt is able to detect bubble periods not only in US, but also in China. The results indicate that we are able to identify the surviving regime and the collapsing regime as well as the switching probabilities in our samples, by employing the state space model with Markov-switching. In both countries, the stock market bubble collapses periodically after some asymmetry duration of one regime. However, the duration of the collapsing regime is longer in China than in US. Besides that, our inthe-sample and out-of-sample forecasting further support our empirical evidence.

The contributions of this study are twofold. First, based on the dividend multiple measure of fundamentals, we succeed in overcoming the shortage of the data in China's stock market and in finding statistically significant regime-switching characteristics of the stock-price bubble processes in China as well as in US. Second, we find substantial similarities and differences of the stock market bubbles between China and US, which, of course, enriches the existing empirical research of the stock market bubbles.

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