Effect of attack angle on aerodynamic properties of a gap flow

Guo Huang^a, Haiming HUANG^{a,1}, Qing WANG^b Yumeng Hu^a

^{*a*}*Institute of Engineering Mechanics, Beijing Jiaotong University, Beijing,100044, China.* ^{*b*}*School of Engineering and Computing Sciences, Durham University, Durham,DH13LE,UK.*

SUMMARY

Understanding the effect of attack angle on aerodynamic properties of a gap flow in thermal protection systems of reentry vehicles is crucial for their design. A two-dimension mathematical model has been developed to explore the effect of a gap on the flow field and aerodynamic properties with different attack angles. The governing differential equations for flows at all speeds are derived, and its finite volume difference formulations are programmed in FORTRAN. The effect of attack angles on the flow field and aerodynamic surface quantities such as Mach number, velocity, temperature and heat flux is presented at the vicinity of a gap under conditions of Mach 5. The numerical results point out that a closed vortex forms at the entrance of a gap and becomes weakened with the increasing of airflow attack angle, and the maximum values of the heat loads present at the windward corner of a gap.

KEY WORDS: Aerodynamic properties, Attack angle, Finite volume methods, Gap flow

1. INTRODUTION

Thermal protection systems of high speed vehicles are generally consist of thermal insulation tiles in the form of splicing together. To avoid that these thermal insulation tiles extrude and break against each other due to thermal expansion, gaps are left between these thermal insulation tiles. The gaps can cause local aerodynamic heating. The high temperature air could flow into these gaps and damage the fuselage. Besides that, gaps could disturb the flow field and bring up severe local aerodynamic thermal effect [1]. Firstly, boundary layer of airflow separates and attaches again at the entrance of gaps, which could bring up the increase of local heat flux. Secondly, gaps may rise turbulivity, and promote the transition of the boundary layer. Thirdly, gaps

¹ Corresponding author; e-mail: huanghaiming@tsinghua.org.cn

are generally very narrow and radiative heat transfer may be blocked. Even if the heat flux in a gap is not high, the gap walls are likely to reach a high temperature.

Gap flow is a special case of cavity flow. Many flow characteristics of cavity flow have not been understood clearly owing to the flow complexity. Similarly, gap flow is extremely complicated and hard to be studied by theory analyses. Unfortunately, the narrow gap makes it hard to observe flow characteristics in the gap by experiments. The great majority of research work of gap flow is done by the USA national aeronautics and space administration (NASA) using the ground experiments from the 1970s to the 1990s. The experiments mostly contain wind tunnel experiments and arc jet tests. The study parameters include gap geometry shape, height-over-width gap ratio, boundary layer state (laminar or turbulent), etc. But these experiments also have their own limitations. For example, the empirical formula proposed by Nestlter [2] is only used to evaluate the heat flux at the bottom of a gap, which could not obtain the heat flux on other walls of a gap or flow characteristics in a gap. So far, there have been some disputes about the heat transfer mechanism in a gap. Most scholars believe that the thermal convection plays a leading role in the heat transfer mechanism of gap flows [3-5]. However, Brewer [6] pointed that the bottom of a gap were mainly heated by the thermal radiation from side walls of a gap. The experimental result by Pitts [7] showed that the thermal convection was very weak and gap walls were heated by the thermal conduction from the outer high temperature thermal insulation tiles. Not only heat transfer mechanism needs to be further studied, but also the effect of a gap width on heat flux inside a gap [8, 9]. In a word, ground experiments have limitations. Although some scholars [10, 11] presented new experimental technique to simulate the gap flow, the applicability and precision of their methods are still open to question.

By experiment the flow characteristics in a gap are hardly to be observed generally. Numerical simulation could make up for the shortage of experiments, so in recent years, a few scholars adopt numerical methods to study the gap flow. Jackson [12] studied laminar cavity flows at hypersonic speeds. The width-to- depth of the cavity in his work equals to 1, while the width-to-depth of a gap is usually less than 0.1. Many cavity flow characteristics could not be used to analyze the gap flow directly. Shen [13] analyzed the flow and heat characteristics of seal structure with a gap and a cavity under the impact of high speed airflow. He obtained the temperature distribution in a gap with no flow characteristics. However, the impact on the flow

field and aerodynamic properties due to variations in airflow attack angles has not been studied yet. In this paper, two-dimensional physical model of gap flow is established, and the effect of airflow attack angle on the flow field and the thermal environment in a gap is mainly studied by the finite volume method and the preconditioning technique for flows at all speeds. Flow field characteristics and heating environment in a gap are both analyzed.

2. MODEL

2.1 Physical model

In the actual flight of vehicles, high speed airflow impacts upon a gap on the surface of vehicles at a certain angle. For convenient description of airflow direction, define attack angle α as the angle between the airflow direction and the vehicle surface, and deflection angle ϕ as the angle between gap extension direction and airflow projection along vehicle surface. Attack angle α and deflection angle ϕ are shown in Fig. 1(a). If ϕ is not equal to 90⁰, airflow in a gap presents three-dimensional flow characteristics. While ϕ is equal to 90⁰, a three-dimensional problem could be reduced to a two-dimensional one which is shown in Fig. 1(b). In this paper, the numerical simulation has been done in the case that ϕ equals to 90⁰.



The computational domain is shown in Fig. 1(b). The width of the gap is defined as the characteristic length L_{∞} , and the width-to-depth ratio of the gap has the value of 1/12.5. The height of the computational domain is $20L_{m}$ long. The lengths of the upstream and the downstream are $40L_{\infty}$, $20L_{\infty}$, respectively. At the boundary of the external flow field, the far field boundary condition is used. The attack angle α of airflow ranges from 0^{0} to 35^{0} and takes values at regular intervals 5^{0} . The parameters of free stream are listed in Table 1. Ma_{∞} , Re and T_{∞} are Mach number, Reynolds number, temperature of free stream, respectively. At the wall, no-slip velocity and isothermal wall boundary condition with wall temperature 473.15K are adopted.

Table 1. Parameters of free steam

Ma_{∞}	Re	$T_{_{\infty}}$	α
5	5.6×10^{4}	473.15 <i>K</i>	$0^{0} \sim 35^{0}$

For conveniently describing the location of gap walls, the body-fitted curvilinear coordinate of a gap is established, which is shown in Fig. 1(c). The four corners of the gap are marked by O, A, B and C, respectively. Total length of segment O-A-B-C is L. Along the segment O-A-B-C, the distance from the point O is denoted by S. Hence, the location of a certain point on gap walls can be defined by S/L ranging from 0 to 1.

2.2. Mathematical model

2.2.1 Compressible Navier-Stokes equations

As the external flow outside a gap is at supersonic speeds, it should be solved by compressible Navier-Stokes equations. 2D integral Navier-Stokes (N-S) equations are described as

$$\int \frac{\partial \mathbf{U}}{\partial t} d\Omega + \iint_{\partial \Omega} \mathbf{F} \cdot \mathbf{n} ds - \iint_{\partial \Omega} \mathbf{F}_{v} \cdot \mathbf{n} ds = 0$$
(1)

where **U** is the vector of conservative variables, and Ω represents an area of control volume. **F** · **n** and **F**_v · **n** represent the convective flux and the viscous flux respectively. **n** is the unit normal vector of control volume faces. **F** = (**F**₁, **F**₂), **F**_v = (**F**_{1v}, **F**_{2v}), **n** = (n₁, n₂)^T. These quantities in equations (1) are given by

$$\mathbf{U} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{pmatrix}, \quad \mathbf{F}_{1} = \begin{pmatrix} \rho u \\ \rho u^{2} + P \\ \rho u v \\ u(\rho E + P) \end{pmatrix}, \quad \mathbf{F}_{2} = \begin{pmatrix} \rho v \\ \rho u v \\ \rho v^{2} + P \\ v(\rho E + P) \end{pmatrix}$$
(2a-c)
$$\mathbf{F}_{1v} = \begin{pmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ k \frac{\partial T}{\partial x} + u \tau_{xx} + v \tau_{xy} \end{pmatrix}, \quad \mathbf{F}_{2v} = \begin{pmatrix} 0 \\ \tau_{yx} \\ \tau_{yy} \\ k \frac{\partial T}{\partial y} + u \tau_{yx} + v \tau_{yy} \end{pmatrix}$$
(2d-e)

where u, v, ρ , T, P, E, $\tau_{xx}(\tau_{xy}, \tau_{yx}, \tau_{yy})$, k are x-component of velocity, y-component of velocity, density, temperature, pressure, total energy per unit mass, viscous stresses, thermal conductivity coefficient, respectively. Equations (1) are nondimisionalized as follows. $\tilde{x} = x/L_{\infty}$, $\tilde{u} = u/U_{\infty}$, $\tilde{v} = v/U_{\infty}$, $\tilde{t} = tU_{\infty}/L_{\infty}$, $\tilde{\rho} = \rho/\rho_{\infty}$, $\tilde{T} = T/T_{\infty}$, $\tilde{P} = P/(\rho_{\infty}U_{\infty}^2)$. The quantities with superscript " ∞ " represent the quantity value of free stream. The quantities with superscript "~" are the dimensionless quantities. The nondimisonalized N-S equations have the same form of equations (1). In the following text, the superscript "~" is removed from dimensionless quantities for writing conveniently. Except special statement, parameters in the formulas are dimensionless. The viscous stress τ_{xx} , τ_{yy} , τ_{yy} , τ_{yy} , and thermal conductivity coefficient k are respectively defined by

$$\tau_{xx} = \frac{\mu_L + \mu_T}{\text{Re}} \left(\frac{4}{3} \frac{\partial u}{\partial x} - \frac{2}{3} \frac{\partial v}{\partial y} \right), \quad \tau_{yy} = \frac{\mu_L + \mu_T}{\text{Re}} \left(\frac{4}{3} \frac{\partial v}{\partial y} - \frac{2}{3} \frac{\partial u}{\partial x} \right)$$
(3a-b)

$$\tau_{xy} = \tau_{yx} = \frac{\mu_L + \mu_T}{\text{Re}} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \quad k = \frac{C_P \mu_L}{\text{Pr}_L \text{Re}} + \frac{C_P \mu_T}{\text{Pr}_T \text{Re}}$$
(3c-d)

where C_P , μ_L , μ_T , Pr_L , Pr_T are specific heat coefficient at constant pressure, laminar dynamic viscosity coefficient, turbulent dynamic viscosity coefficient, laminar Prandtl number, turbulent Prandtl number, respectively. For air, $Pr_L = 0.77$, $Pr_T = 0.9$. μ_L is given by Sutherland formula defined as

$$\mu_{L} = T^{\frac{3}{2}} \frac{1 + T_{s}/T_{\infty}}{T + T_{s}/T_{\infty}}$$
(4)

where T_s is Sutherland constant. For air, $T_s = 124K$. μ_T is given by the turbulence model.

2.2.2 Preconditioning methods for flows at all speeds

In the computational fluid dynamics, the problem of high speed flow should be solved by the compressible flow equations, while the incompressible flow equations in low speed flow. However, in the problem of gap flow studied in this paper, there are both high speed flow and low speed flow. The external flow outside a gap is at supersonic speed, while velocity of flow in a gap is very low. In theory, the problem of low speed flow could be solved by compressible flow equations, but the numerical algorithms for compressible flow equations face difficulties for low Mach number conditions. This is because that the magnitude of the flow velocity becomes small in comparison with the acoustic speed in the low subsonic Mach number regime of a gap, the convective terms of the governing equations (1) become stiff, which slows down the convergence to steady state [14]. In order to solve nearly incompressible flows with numerical algorithms designed for the compressible flows, preconditioning techniques is used. As time derivative terms are multiplied by the preconditioning matrix, the eigenvalue system of primitive equations (1) can be changed, and stiff problem could be solved. There are various preconditioning matrixes, and in the present study, the preconditioning matrix proposed by Weiss et al [15] is used. The equations (1) are described to be

$$\mathbf{Q} \int \frac{\partial \mathbf{U}_{P}}{\partial t} d\Omega + \oint_{\partial \Omega} \mathbf{F} \cdot \mathbf{n} ds - \oint_{\partial \Omega} \mathbf{F}_{v} \cdot \mathbf{n} ds = 0$$
⁽⁵⁾

with $\mathbf{Q} = \partial \mathbf{U} / \partial \mathbf{U}_p$, and $\mathbf{U}_p = (p, u, v, T)^T$. Replace \mathbf{Q} in front of the time derivative by the preconditioning matrix $\mathbf{\Gamma}$, and the equations (5) is changed into

$$\Gamma \int \frac{\partial \mathbf{U}_{P}}{\partial t} d\Omega + \prod_{\partial \Omega} \mathbf{F} \cdot \mathbf{n} ds - \prod_{\partial \Omega} \mathbf{F}_{v} \cdot \mathbf{n} ds = 0$$
(6)

with

$$\Gamma = \begin{pmatrix} \theta & 0 & 0 & -\frac{\rho}{T} \\ \theta u & \rho & 0 & -\frac{\rho u}{T} \\ \theta v & 0 & \rho & -\frac{\rho v}{T} \\ \theta H - 1 & \rho u & \rho v & -\frac{\rho b^2}{2T} \end{pmatrix}$$
(7)

where H is total enthalpy. b and θ are described as

$$b^2 = u^2 + v^2$$
 (8)

$$\theta = \frac{1}{a^2 M a_r^2} + \frac{\gamma - 1}{a^2} \tag{9}$$

where *a* denotes the speed of sound, and Ma_r^2 is the preconditioning parameter which is related to the local Mach number. In order to avoid strangeness of preconditioning matrix in the vicinity of stagnation regions, Ma_r^2 should be limited. There are several approaches for limiting Ma_r^2 [16], the control method proposed by Turkel [17] is used in this study, which is described to be

$$Ma_r^2 = \min\left[\max\left(Ma^2, kMa_{\infty}^2\right), \ 1.0\right]$$
(10)

where k is an user-specified constant. It would be advantageous to select this constant to be as small as possible and k has the value of 1.0×10^{-3} in this study. Equations (6) is written as

$$\int \frac{\partial \mathbf{U}}{\partial t} d\Omega + \mathbf{Q} \mathbf{\Gamma}^{-1} \left(\iint_{\partial \Omega} \mathbf{F} \cdot \mathbf{n} ds - \iint_{\partial \Omega} \mathbf{F}_{\nu} \cdot \mathbf{n} ds \right) = 0$$
(11)

where the term $\mathbf{Q}\mathbf{\Gamma}^{-1}$ is called the conservative variable preconditioning matrix.

2.2.3 Turbulence model

In Section 2.2.1, it is mentioned that μ_T is given by the turbulence model. There are various turbulence models. In this paper, the $K-\omega$ Shear Stress Transport (SST) two-equation model of Menter is used. The SST turbulence model merges the $K-\omega$ model of Wilcox with a high Reynolds number $K-\varepsilon$ model, which combines the positive features of both models. So it is applicable in the both boundary layer and the external flow field. The dimensionless transport equations of SST turbulence model are described to be

$$\frac{\partial(\rho K)}{\partial t} + \frac{\partial(\rho u K)}{\partial x} + \frac{\partial(\rho v K)}{\partial y} = \frac{1}{\text{Re}} \left\{ \frac{\partial}{\partial x} \left[(\mu_L + \sigma_K \mu_T) \frac{\partial K}{\partial x} \right] + \frac{\partial}{\partial y} \left[(\mu_L + \sigma_K \mu_T) \frac{\partial K}{\partial y} \right] \right\} + \frac{1}{\text{Re}} P_K$$
(12)
- Re $\beta^* \rho \omega K$

$$\frac{\partial(\rho\omega)}{\partial t} + \frac{\partial(\rho\omega\omega)}{\partial x} + \frac{\partial(\rho\nu\omega)}{\partial y} = \frac{1}{\text{Re}} \left\{ \frac{\partial}{\partial x} \left[(\mu_L + \sigma_\omega\mu_T) \frac{\partial\omega}{\partial x} \right] + \frac{\partial}{\partial y} \left[(\mu_L + \sigma_\omega\mu_T) \frac{\partial\omega}{\partial y} \right] \right\} + \frac{1}{\text{Re}} P_\omega$$

$$- \text{Re} \,\beta\rho\omega^2 + \frac{1}{\text{Re}} 2(1 - f_1) \frac{\rho\sigma_{\omega^2}}{\omega} \left(\frac{\partial K}{\partial x} \frac{\partial\omega}{\partial x} + \frac{\partial K}{\partial y} \frac{\partial\omega}{\partial y} \right)$$
(13)

where *K* represents the turbulent kinetic energy, and ω denotes the rate of dissipation per unit turbulent kinetic energy. The parameters μ_T , σ_K , σ_{ω} , σ_{ω^2} , β , β^* , P_{ω} , P_K , f_1 have been given in Ref. [18], which are not listed detailly in this paper. The boundary conditions for the turbulent kinetic energy *K* and the specific dissipation ω at solid walls are

$$K = 0 \tag{14}$$

$$\omega = \frac{60\mu_L}{\rho\beta_1 d_1^2 \operatorname{Re}^2}$$
(15)

where d_1 is the distance of the first node from the wall and β_1 has the value of 0.075. The location of the grid node nearest the surface has a nontrivial effect on the accuracy of surface heat flux. For hypersonic flows, Marvin [19] recommended the stringent condition $y^+ < 0.3$, which is satisfied in this study. y^+ is the dimensionless distance from the surface and has the form of

$$y^{+} = \frac{d_{1}}{\mu} \sqrt{\rho \cdot \tau_{xy}} \tag{16}$$

The parameters of SST model for free stream are listed in Table 2.

Table 2. Parameters of SST model for free stream

K_{∞}	$\mathscr{O}_{_{\!\!\infty}}$	$\mu_{_{T^{\infty}}}$
9×10 ⁻⁹	1×10 ⁻⁶	9×10 ⁻³

3. Numerical approach

3.1 Spatial discretization

The Advection Upstream Splitting Method (AUSM) meets the goals of efficiency, accuracy, and robustness, which is often used in solving high speed flow. AUSM-family schemes have various developed forms. Liou [20] proposed a new scheme named AUSM⁺-up for the low Mach number limit, which can be used to solve flows at all speed. Considering the condition that there are both high speed flow and low speed flow in the computational domain, so AUSM⁺-up scheme is used to discretizing the convection flux $\mathbf{F} \cdot \mathbf{n}$ in the equations (11). The discretization of $\mathbf{F} \cdot \mathbf{n}$ by AUSM⁺-up scheme is defined as follows.

$$\left(\mathbf{F}\cdot\mathbf{n}\right)_{i+\frac{1}{2}} = \dot{m}_{i+\frac{1}{2}} \boldsymbol{\Phi}_{\frac{1}{2}} + \mathbf{P}_{i+\frac{1}{2}}$$
(17)

where \dot{m} is mass flux. $\mathbf{\Phi} = (1, u, v, H)^T$, $\mathbf{P} = (0, P, 0, 0)^T$. The subscript "i+1/2" denotes the interface of cells. "L", "R" represent neighboring grid cells. $\mathbf{\Phi}_{t/R}$ is defined as

$$\Phi_{\frac{1}{2}} = \begin{cases} \Phi_L, & \dot{m}_{i+\frac{1}{2}} > 0 \\ \Phi_R, & \dot{m}_{i+\frac{1}{2}} \le 0 \end{cases}$$
(18)

As AUSM⁺-up scheme only has the first order accuracy, the Monotone Upstream-centered Scheme for Conservation Laws (MUSCL) interpolation [21] is adopted in the present study to improve accuracy of AUSM⁺-up scheme to second-order accuracy. In order to prevent the generation of oscillations and spurious solutions in regions with large gradients, second-order discretization of convective flux requires the use of limiter functions. In this paper, the Van Albada limiter function based on original variables is adopted. MUSCL interpolation with Van Albada limiter function is defined to be

$$U_{i+1/2}^{L} = U_{i} + \frac{\psi}{4} \left[\left(1 - \frac{\psi}{3} \right) \Delta^{-} + \left(1 + \frac{\psi}{3} \right) \Delta^{+} \right]_{i}$$
(19a)

$$U_{i+1/2}^{R} = U_{i+1} - \frac{\psi}{4} \left[\left(1 - \frac{\psi}{3} \right) \Delta^{+} + \left(1 + \frac{\psi}{3} \right) \Delta^{-} \right]_{i+1}$$
(19b)

where U represents ρ , u, v, P, respectively. $\Delta_i^- = U_i - U_{i-1}$, $\Delta_i^+ = U_{i+1} - U_i$.

 Ψ is the Van Albada limiter function with the form of

$$\psi = \frac{2\Delta^{+} \cdot \Delta^{-} + 10^{-6}}{\left(\Delta^{+}\right)^{2} + \left(\Delta^{-}\right)^{2} + 10^{-6}}$$
(20)

The viscous flux $\mathbf{F}_{v} \cdot \mathbf{n}$ in equations (11) is discretized by the second-order central difference scheme. The discretization of convection flux in SST model is different from equations (11). The convection flux in equations (12) and (13) are discretized by first order upstream scheme. The discretization of viscous flux in equations (12) and (13) is the same with the one of equations (11), which are discretized by the second-order central difference scheme.

3.2. Temporal discretization

The implicit Lower-Upper Symmetric Gauss-Seidel (LU-SGS) scheme has features of high stability, low numerical complexity, and modest memory requirement, which are comparable to an explicit multistage. So LU-SGS scheme is implemented to discretize temporal term, which is described as follows. The equations (11) are changed into

$$\Omega \frac{\mathbf{W}^{n}}{\Delta t} + \mathbf{Q} \boldsymbol{\Gamma}^{-1} \mathbf{f}_{\partial \Omega} \left[\left(\mathbf{F}_{n} - \mathbf{F}_{\nu, n} \right)^{n+1} - \left(\mathbf{F}_{n} - \mathbf{F}_{\nu, n} \right)^{n} \right] ds = \mathbf{Q} \boldsymbol{\Gamma}^{-1} \mathbf{R}^{n}$$
(21)

with $\mathbf{W}^n = \Delta \mathbf{U}^n$, $\mathbf{F}_n = \mathbf{F}_1 n_1 + \mathbf{F}_2 n_2$, $\mathbf{F}_{\nu,n} = \mathbf{F}_{1\nu} n_1 + \mathbf{F}_{2\nu} n_2$. The superscripts "*n*" and "*n*+1" denote the time levels. Hence, \mathbf{W}^n means the flow solution at the present time *t*. Consequently, \mathbf{W}^{n+1} represents the solution at the time $t + \Delta t$. The residual term \mathbf{R}^n has the form of

$$\mathbf{R}^{n} = - \oint_{\partial \Omega} \left(\mathbf{F}_{n} - \mathbf{F}_{\nu,n} \right)^{n} ds$$
(22)

Introducing convective flux Jacobians **A** and viscous flux Jacobians \mathbf{A}_{v} , which are defined as $\mathbf{A} = \partial \mathbf{F}_{n} / \partial \mathbf{U}$, $\mathbf{A}_{v} = \partial \mathbf{F}_{v,n} / \partial \mathbf{U}$. Hence, equation (21) becomes

$$\Omega \frac{\mathbf{W}^{n}}{\Delta t} + \iint_{\partial \Omega} (\mathbf{A}' - \mathbf{A}'_{v}) \mathbf{W}^{n} ds = \mathbf{Q} \Gamma^{-1} \mathbf{R}^{n}$$
(23)

with $\mathbf{A}' = \mathbf{Q} \mathbf{\Gamma}^{-1} \mathbf{A}$, $\mathbf{A}'_{\nu} = \mathbf{Q} \mathbf{\Gamma}^{-1} \mathbf{A}_{\nu}$. In equation (23), convection flux $\mathbf{A}' \mathbf{W}^n$ is discretized by first-order upstream scheme and viscous flux $\mathbf{A}'_{\nu} \mathbf{W}^n$ by second-order central difference scheme. Hence, equation (23) becomes

$$\begin{bmatrix} \Omega \\ \Delta t + \Lambda'_{i} + \Lambda'_{j} + 2(\Lambda'_{v,i} + \Lambda'_{v,j}) \end{bmatrix} \mathbf{W}_{ij}^{n} + (\mathbf{A}')_{i+1}^{-} \mathbf{W}_{i+1}^{n} \Delta s_{i+1/2} - (\mathbf{A}')_{i-1}^{+} \mathbf{W}_{i-1}^{n} \Delta s_{i-1/2} + (\mathbf{A}')_{j+1}^{-} \mathbf{W}_{j+1}^{n} \Delta s_{j+1/2} - (\mathbf{A}')_{j-1}^{+} \mathbf{W}_{j-1}^{n} \Delta s_{j-1/2} - \Lambda'_{v,i+1} \mathbf{W}_{i+1}^{n} - \Lambda'_{v,i-1} \mathbf{W}_{i-1}^{n} - \Lambda'_{v,j+1} \mathbf{W}_{j+1}^{n} - \Lambda'_{v,j-1} \mathbf{W}_{j-1}^{n} = \mathbf{Q} \mathbf{\Gamma}^{-1} \mathbf{R}_{ij}^{n}$$
(24)

with

$$\left(\mathbf{A}'\right)^{\pm} = \frac{1}{2} \left[\mathbf{A}' \pm \frac{\Lambda'}{\Delta s} \mathbf{I}\right]$$
(25)

 Λ' is the spectral radius of A' and is defined to be

$$\Lambda' = \left(\frac{Ma_r^2 + 1}{2} |\mathbf{V} \cdot \mathbf{n}| + a'\right) \Delta s \tag{26}$$

with $\mathbf{V} = (u, v)$. *a'* presents corrected speed of sound and is given as

$$a' = \frac{1}{2} \sqrt{\left| \mathbf{V} \cdot \mathbf{n} \right|^2 \left(M a_r^2 - 1 \right) + 4 M a_r^2 \cdot a^2}$$
(27)

 Λ_{ν}' is the spectral radius of $~A_{\nu}'~$ and is defined to be

$$\Lambda_{\nu}' = \max\left[\frac{4}{3\rho}, \frac{1 + Ma_r^2(\gamma - 1)}{\rho}\right] \left(\frac{\mu_L}{\Pr} + \frac{\mu_T}{\Pr_T}\right) \frac{(\Delta s)^2}{\Omega}$$
(28)

In order to accelerate the solution of the governing equations, the local time-stepping is adopted, which is defined to be

$$\Delta t = CFL \cdot \frac{\Omega}{\left(\Lambda'_{i} + \Lambda'_{j}\right) + \left(\Lambda'_{v,i} + \Lambda'_{v,j}\right)}$$
(29)

where Courant-Friedrichs-Lewy (CFL) number has the value of 0.1.

3.3. Solving process

The solving process of equations (24) is shown as follows.

Step 1. Compute the right hand items of the equations (24) $\mathbf{Q}\mathbf{\Gamma}^{-1}\mathbf{R}_{ij}^{n}$ with the method given in the section 3.1.

Step 2. Compute $\overline{\mathbf{W}}_{ij}^{n}$.

$$\overline{\mathbf{W}}_{ij}^{n} = \frac{1}{\eta} \left[\mathbf{Q} \Gamma^{-1} \mathbf{R}_{ij}^{n} + \left(\mathbf{A}' \right)_{i-1}^{+} \mathbf{W}_{i-1}^{n} \Delta s_{i-1/2} + \left(\mathbf{A}' \right)_{j-1}^{+} \mathbf{W}_{j-1}^{n} \Delta s_{j-1/2} + \Lambda'_{\nu,i-1} \mathbf{W}_{i-1}^{n} + \Lambda'_{\nu,j-1} \mathbf{W}_{j-1}^{n} \right]$$
(30)

with $\eta = \Omega/\Delta t + \Lambda'_i + \Lambda'_j + 2(\Lambda'_{\nu,i} + \Lambda'_{\nu,j}).$ Step 3. Compute \mathbf{W}_{ij}^n .

$$\mathbf{W}_{ij}^{n} = \frac{1}{\eta} \left[\eta \overline{\mathbf{W}}_{ij}^{n} - (\mathbf{A}')_{i+1}^{-} \mathbf{W}_{i+1}^{n} \Delta s_{i+1/2} - (\mathbf{A}')_{j+1}^{-} \mathbf{W}_{j+1}^{n} \Delta s_{j+1/2} + \Lambda'_{\nu,i+1} \mathbf{W}_{i+1}^{n} + \Lambda'_{\nu,j+1} \mathbf{W}_{j+1}^{n} \right]$$
(31)

Step 4. Time marches.

$$\mathbf{U}^{n+1} = \mathbf{U}^n + \mathbf{W}^n \tag{32}$$

The solving process of equations (12) and (13) is the same with the one of

equations (24). It is supposed to be convergent that the root-mean-square values of $\mathbf{Q}\mathbf{\Gamma}^{-1}\mathbf{R}_{ij}^{n}$ are less than 10⁻⁶. The solving process is accomplished by the computer program made in FORTRAN language, and the computation process of the program is shown in Fig. 2.



Fig.2. Computation process

4. RESULTS

4.1. Characteristics of gap flow

In this section, the characteristics of gap flow are analyzed. The Mach number contour with airflow attack angle ranging from 0^0 to 35^0 is shown in Fig. 3.



Fig.3. Mach number contour

It is can be seen that at lower angle of attack, the external airflow only affects the region near the entrance of a gap. As the increasement of attack angle, the affected region expands to the inside of a gap. As a gap is very narrow, the external flow hardly rushes to the bottom of a gap. When the angle of attack is no more than 35° , airflow is at low velocity (<0.1Ma) in most regions of a gap. Near the bottom of a gap, the velocity of airflow almost equals to zero. In order to illustrate the change of characteristics of gap flow clearly, the temperature contour and streamlines with attack angle of 0° , 5° , 10° and 30° , respectively, are shown in Fig. 4. Here, the temperature is a dimensionless parameter which is nondimensionalized by the temperature value of free-stream.



Fig.4. Temperature contour and streamlines

It is can be observed that there is a closed vortex at the entrance of a gap as the angle of attack is less than 35° . As angle of attack equals to 0° , the high-temperature air flows through a gap. Then, it would separate and expand into a gap. As the width of a gap is very small, the separated gas has collided with the downstream wall of a gap, and then returns the upstream wall of a gap. As a result of the shear action of external flow, the separated gas flows downstream again. Hence, a closed vortex forms at the entrance of a gap. When the angle of attack is greater than 0° , the external gas could flow into a gap directly. As the angel of attack increases, more and more external air flows into a gap, while the vortex becomes smaller and smaller. It is also observed from the change of temperature contour. Compared with Fig. 4(a), Fig. 4(d) shows that the high-temperature zone of a gap becomes bigger as the increasement of airflow attack angle.

Known from the characteristics of gap flow, the velocity of airflow is very low at

most regions in a gap except the upper part in the case that airflow attack angle is not more than 35^{0} . Especially, gas almost keeps static at the bottom of a gap called 'dead water zone'. It is inferred that thermal conduction plays a key role in the heat transfer mechanism at the lower part in a gap, while thermal convection at the upper one.

4.2. Characteristics of thermal environment

Both the computational model with a gap and the one without a gap are solved. The value of heat flux on the gap walls is denoted with q_0 . The heat flux corresponding to point O at flat plates is denoted with q_0 . The q_0 with angle of attack ranging from 0^0 to 35^0 is shown in Fig. 5. Here, q_0 is a dimensionless quantity. As the angle of attack increases from 0^0 , airflow rushes to flat plates and is compressed more and more seriously. So aerodynamic heating becomes more and more serious, and q_0 increases with the angle of attack rising up.



Fig.5. Flat heat flux at point O

In order to represent the enhancement effect of aerodynamic heating due to a gap, the heat flux ratio q/q_0 is introduced. The q/q_0 with the attack angle of airflow having the value of 0^0 , 5^0 , 10^0 , and 30^0 , respectively, is shown in Fig. 6. In Fig. 7(*a*), the result of numerical simulation is compared with that of the supersonic wind tunnel test from the ref. [22]. It can be seen from Fig. 7(*a*) that the simulation result is consistent with the experimental one.



Fig.6. Heat flux ratio

Although the angle of attack varies, some similar laws can be got from Fig. 6. The heat flux in a gap is "U" shaped distributed. It decreases to almost zero as the increasement of depth in a gap. This is because that the external high-temperature airflow hardly rushes to the bottom of a gap. Energy attenuates gradually as the increasement of depth in a gap. The most severe aerodynamic heating occurs at the windward corner of a gap. The heat flux rises up suddenly near the windward corner. Airflow is compressed severely at the windward corner, which causes heat flux to go up. In the design of thermal protection system, the high heat flux zone at corners of a gap should be considered. It is advised that the corners of a gap could be designed into arc shapes to reduce heat flux.

The enhancement effect of aerodynamic heating under various angles of attack is different. The maximum of heat flux ratio q/q_0 with angle of attack ranging from 0^0 to 35^0 is shown in Fig. 7. It is shown that the maximum of q/q_0 goes up with the increasing of attack angles, which means that the higher attack angle of airflow causes more severe aerodynamic heating.



Fig.7. Maximum of heat flux ratio

Calorically perfect gas model is used in this paper, which is appropriate generally in the case that Mach number of free stream is no more than 5. However, in the case of higher Mach number of free stream, the high temperature air could undergo chemical reaction, dissociation and ionization. Thus calorically perfect gas model is no longer applicable, and chemistry model should be adopted.

5. CONCLUSIONS

This study applies the finite volume method and the preconditioning technique in order to investigate the flow field and aerodynamic surface quantities of supersonic gap flow. The calculations provide a detailed description of the flow field and heat environment with different attack angle. Performance results for a gap are compared to those of a flat plate without a gap. It is observed that a closed vortex forms at the entrance of a gap and becomes weakened with the increasing of attack angle of airflow. Since a gap is very narrow, external high-temperature airflow has little influence on the deep regions in a gap. The heat flux in a gap is "U" shaped distributed, and the most severe aerodynamic heating occurs at the windward corner of a gap.

ACKNOWLEDGEMENTS

The research was supported by the Fundamental Research Funds for the Central Universities (2014YJS088).

NOMENCLATURE

A convective flux Jacobians

\mathbf{A}_{v}	viscous flux Jacobians
а	speed of sound
a'	corrected speed of sound in preconditioning techniques
C_P	specific heat coefficient at constant pressure
Ε	total energy per unit mass
F	vector of connective flux
\mathbf{F}_{v}	vector of viscous flux
Н	total enthalpy
Κ	turbulent kinetic energy
Ма	Mach number
n	unit normal vector of control volume face
Р	pressure
Pr	Prandtl number
q	heat flux
q_0	heat flux of flat plate
Re	Reynold number
t	time
Δt	time step
Т	temperature
и	x-component of velocity
U	vector of conservative variables
\mathbf{U}_{P}	vector of primitive variables
v	y-component of velocity
y^+	dimensionless distance of the first node from the wall
α	angle of attack
γ	ratio of specific heat coefficient
ε	rate of turbulent energy dissipation
Λ'	spectral radius of convective flux Jacobianis
Λ'_{v}	spectral radius of viscous flux Jacobianis
μ	dynamic viscosity coefficient
ρ	density
au	viscous stresses
ϕ	angle of deflection
Ψ	Van Albada limiter function

 ω rate of dissipation per unit turbulent kinetic energy

Superscripts

n+1 new time level

Subscripts

L	laminar
Т	turbulent
i , j	index of a control volume
i + 1/2	the interface of cells
n	normal direction of control volume face
∞	in far field

REFERENCES

1. Tang GM (2000) Experimental investigation of heat transfer distributions in a deep gap. Experiments and Measurements in Fluid Mechanics 14 (4):1-6

2. Nestler DE, Saydah AR, Auxer WL (1968) Heat transfer to steps and cavities in hypersonic turbulent flow. Paper presented at the AIAA Fluid and Palasma Dynamics Conference, Los Angeles, California, 24-26 June

3. Charbonnier JM, Boerrigter H (1993) Contribution to the study of gap induced boundary layer transition in hypersonic flow. Paper presented at the AIAA/DGLR 5th International Aerospace Planes and Hypersonics Technologies Conference, Munich, Germany, 30 Nov - 3 Dec

4. Nestler D (1985) The effects of surface discontinuities on convective heat transfer in hypersonic flow. Paper presented at the AIAA 20th Thermophysics Conference, Williamsburg, VA, 19-21 June

5. Petley D, Smith D, Edwards C, Carlson A, Hamilton H (1984) Surface step induced gap heating in the Shuttle Thermal Protection System. Journal of Spacecraft and Rockets 21 (2):156-161

6. Brewer R, Saydah A, Florence D, Nestler D (1973) Thermal performance evaluation of REI panel gaps for Space Shuttle Orbiter. Journal of Spacecraft and Rockets 10 (1):23-28

7. Pitts WC, Murbach MS (1982) Flight measurements of tile gap heating on the Space Shuttle. Paper presented at AIAA/ASME 3rd Joint Thermophysics, Fluids, Plasma and Heat Transfer Conference, St. Louis, Missouri, 7-11 June

8. Norman I, Rochelle W, Kimbrough B, Ritrivi C, Ting P, Dotts R (1982) Comparison of Orbiter STS-2 development flight instrumentation data with thermal math model predictions. Paper presented at AIAA/ASME 3rd Joint Thermophysics, Fluids, Plasma and Heat Transfer Conference, St. Louis, Missouri, 7-11 June 9. Avery DE, Kerr PA, Wieting AR (1985) Experimental aerodynamic heating to simulated Shuttle tiles. Journal of Spacecraft and Rockets 22 (4):417-424

10. Gartimella S, Shollenberger K, Eibeck P, White S (1993) Flow and heat transfer in simulated re-entry vehicle tile gaps. Journal of Thermophysics and Heat Transfer 7 (4):644-650

11. Mori K, Ishibashi K, Nakamura Y, Fujii K (2012) Optical measurement of heat-flux onto 'Shaded'area in hypersonic flows (I): Preliminary measurement of heat-flux in a deep cavity. Paper presented at the 43rd AIAA Thermophyics Conference, New Orieans, Louisiana, 25-28 June

12. Jackson AP, Hillier R, Soltani S (2001) Experimental and computational study of laminar cavity flows at hypersonic speeds. Journal of Fluid Mechanics 427:329-358

13. Chun S, Xinlin X, Zhanwei C, Mingxing Y (2012) Analysis of Flow and Heat Characteristics of Seal Structure with Gap and Cavity Under the Impact of High Speed Airflow. Acta Aeronautica ET Astronautica Sinica 33 (1):34-43

14. Turkel E (1999) Preconditioning techniques in computational fluid dynamics. Annual Review of Fluid Mechanics 31 (1):385-416

15. Weiss JM, Maruszewski JP, Smith WA (1999) Implicit Solution of Preconditioned Navier-Stokes Equations Using Algebraic Multigrid. AIAA Journal 37 (1):29-36

16. Venkateswaran S, Li D, Merkle CL (2003) Influence of Stagnation Regions on Preconditioned Solutions at Low Speeds. Paper presented at 41st Aerospace Sciences Meeting and Exhibit, Reno, Nevada, 6-9 January

17. Turkel E (1987) Preconditioned methods for solving the incompressible and low speed compressible equations. Journal of Computational Physics 72 (2):277-298

18. Menter FR (1994) Two-equation eddy-viscosity turbulence models for engineering applications. AIAA Journal 32 (8):1598-1605

19. Marvin JG, Huang GP (1997) Turbulence modeling - Progress and future outlook. Paper presented at 15th International Conference on Numerical Methods in Fluid Dynamics, Monterey, CA, 24-28 June

20. Liou MS (2006) A sequel to AUSM, Part II: AUSM+-up for all speeds. Journal of Computational Physics 214 (1):137-170

21. Van LB (1979) Towards the ultimate conservative difference scheme. V. A second-order sequel to Godunov's method. Journal of Computational Physics 32 (1):101-136

22. Qian HJ, Wang Y, Tian WB (1996) Experimental Study on Heat Flux Distributions in Gaps. China Academy of Aerospace Aerodynamics, Beijing