When Does a Way of Working Become a Methodology?

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Running Head: A Way of Working

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ABSTRACT

We discuss the question of how to determine what are valid methodologies in mathematics education and the problems associated with developing new ways of working into acceptable research methods. We develop the idea of a two-stage validation process and look at the consequences for communicating between what we term methodological neighbours, an alternative diffusion model that explores the relationship between teachers and researchers and the role of the PhD within the validation process. We then describe our own way of working and suggest how it is beginning to evolve into a methodology.

WHEN DOES A WAY OF WORKING BECOME A METHODOLOGY?

Day (1995) recalls "as a teacher in school I never knowingly read a piece of research and ... was certainly sceptical of its relevance." In his transition from being a teacher to a researcher in a university, and in the transition he has seen in his colleagues, he notes the tensions between "continuing their development as good teachers ... and learning about becoming a researcher". This paper is drawn from our attempts to ease those tensions.

Like many (if not all) in mathematics education, we are interested in finding out how people learn mathematics and how people can become better teachers of it. When we first began working on these interests together, the outcome of such work was primarily meant to inform our own teaching, with the thought that it might also be of interest to others.

Such an approach to our initial work did not involve us in discussing methodologies (though we worked in a particular way), in literature reviews (though we read what other people had to say and our reading informed our discussions) or in data collection (though we gathered and used evidence in a way that we might now see as systematic). Our work together was not motivated by a wish (or requirement) to do *research* since what we did probably did not conform to any image of mathematics education research we then had. Recently, however, a question has been raised for us: can the work we do, the development of theoretical perspectives on learning by

- asking questions of our own learning,
- sharing our experiences and

- reflecting (from the learner's viewpoint) on the incidents that stand out in our teaching

properly be called research? Is the way of working we developed to provide our own theoretical perspective a *methodology*?

Inevitably, this question about our own work has led us to reflect on whether, and how, our work can be sited amongst the work of others in the mathematics education field, and from that to the more fundamental question of what research is in that field. The purpose of this paper is to explore this fundamental question, discuss how we might begin to situate our work within the mathematics education field and begin to examine whether our way of working has the properties we expect of a methodology.

RESEARCH IN MATHEMATICS EDUCATION

The question of what constitutes research in mathematics education is hardly new. Begle (1969) was asking the question at the first ICME. Traditionally, perhaps, answers to this question come in the form of *lists* of methodologies that have been applied successfully to problems in the teaching and learning of mathematics. Such an approach works well as an aid to classification: matching methodologies with the situations in which they may be appropriate to use. However, in many ways it begs the question of how new methodologies come into being and, more importantly, come to be accepted as valid for given situations.

For those working within a traditional methodology taken from such a list the issue is not an important one: they can ask questions and provide answers in a way that is provably acceptable to a wide community. However, we suggest that any given list of

acceptable methodologies will effectively silence answers to some questions in the field and an approach to validating methodologies by *criteria* rather than listing may be more useful.

Our own way of working, which is explored in more depth later, consists of an amalgam of three parts, which we call introspection, co-spection and 'as if from inside'. We combine the examination of our own learning, with a sharing of the stories we construct from that examination to provide two (often very different) accounts and use them as a starting point to consider what internal explanations may account for the incidents we see amongst learners. Because this way of working has developed from our spontaneous discussion with each other and is personal to us, it does not fit neatly within a list of existing methodologies. Even within the broader (and more useful) classification of research into 'traditions' that Bishop (1992) delineates, we can only begin to see how our work fits within a larger framework.

Indeed, the notion of a framework — any fixed skeleton of research traditions upon which one must hang the particular research paradigm used if work is to be described as 'research' at all — has resonances with a word coined by Daly (1973): "methodolatry". In Belenky et al. (1986) this word is used to suggest that by having only a fixed number of acceptable forms of research, other ways of working and questions that cannot adequately be answered by existing methodologies are 'rendered invisible'. Moreover, Belenky et al. note that when a group "insists upon a particular methodology in the research it supports, it outlaws questions that cannot be answered in that fashion".

METHODOLATRY

Rather than merely highlighting for us the problems of addressing methodological issues by listing what is acceptable, the word methodolatry, and Belenky's interpretation of its meaning, led us to see an ebb and flow between question and answer in research. In one direction, the question one is interested in answering strongly influences the form which the answers take; in the other, if one is interested in having answers in a particular form then that strongly influences what questions may be accessible.

We might adapt Gilligan's notion of a *voice* (Gilligan, 1982) as an analogy for methodology to highlight these directions: on the one hand, the voice with which one speaks influences what one can say while, on the other, if one has something particular to say, one may need a particular choice of voice in which to say it. Such an analogy can be extended in both directions: 'what are we speaking about' and 'to whom are we speaking'.

This notion allows us to extend this ebb and flow of question and answer in research into two aspects which we call the *domain* and *range* of the research: 'what (or who) is the subject of our research?' and 'who might use the results of our work?'. By adding another influential question to this 'tidal system' of research — 'what are the results used for?' — we developed what we saw as five important questions for us to answer which could be addressed in either direction, indicated in Figure 1.

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The five questions we developed from our discussions on the implications of the notion of methodolatry match closely those we found in Sierpinska et. al. (1993):

> What is the specific object of study in mathematics education?
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> What are the aims of research in mathematics education?
>
> What are the specific research questions or problematiques of research in mathematics education?
>
> What are the results of research in mathematics education?
>
> What criteria should be used to evaluate the results of research in mathematics education?

However, we think there are some levels on which the two sets of questions differ. While Sierpinska's are ones that need to be asked by mathematics education as a community, of its work as a whole, our questions need to be asked by individual researchers or groups about their own work. We also suggest that there is an implicit direction to Sierpinska's questions: the objects of study need to come *before* the aims of research. Our figure 1, however, implies that the questions can be answered in either direction.

For example, if we want our answers to be used to influence, say, government policy, the form the answers take must be one which a government can access. This decision determines what questions can legitimately be asked, which to some extent determines what domain the research is in. In the most obvious of cases, where a researcher is commissioned by an agency to conduct some research, the commissioning body has the right to expect answers in a form which it can use and the questions which are asked are those of the agency and not the researcher. For

example, as part of a major review of the state of mathematics teaching and learning in the UK (Cockcroft, 1982), work was commissioned to inform the government committee about the role of mathematics in employment (Fitzgerald, 1983) in which the focus was directed by the committee's needs.

Alternatively, if we are interested in a particular research domain, this determines the kinds of useful questions that can be asked within it. Similarly, these types of questions determine the form of the answers and the form of the answers determines who has access to using them and what use they may be put to. Teachers who notice some phenomenon of interest in their classroom may be tempted to explore it initially for themselves and, as they ask questions of their situation, may begin to generate answers for themselves. This may be the starting point for a long exploration in which, eventually the form of the answers is seen as accessible and of value to others (as was the case in Markovits and Hershkowitz, 1997 who, it appears, began from a noticing in a classroom which led to a long investigation of children's estimation abilities). Of course, in these cases the existence of the initial phase of asking questions for oneself is often hidden in the final publication.

METHODOLOGICAL AND META-METHODOLOGICAL VALIDATION

Having decided upon this set of questions and discovered their community-wide analogies developed by Sierpinska, we began to see that we might be able to answer our original question about the transition from a way of working to a methodology. The five questions we list encourage us to make explicit what we are doing and why we are doing it. We call these type of questions *meta-methodological.* They are not

questions within what Bishop refers to as a research tradition, they are questions *about* such traditions.

By providing a set of questions which make explicit what a particular methodology hopes to achieve and how it intends to achieve it, rather than listing existing acceptable research methods, we can provide a route by which new ones can be validated.

It is in the role of validation that we find one of the most important consequences of this re-examination of methodological status. One of the most concise definitions of research is that of Stenhouse (1984) who defines it as "systematic enquiry made public". Within this definition, it appears to us that validation of something as research is made by someone (or some community of people) who decides that the enquiry has indeed been systematic on the basis of what has been made public. By splitting the issue into two: what we might term *methodological validation* and *meta-methodological validation*, we are suggesting that such a decision needs to be made on two levels and by two communities.

If someone makes public work they claim has been done within a particular, already accepted methodology, then a community needs to validate the application of the methodology as appropriate for the question. Bishop highlights the importance of this with reference to the work of Krutetskii:

> Several researchers were quite surprised when reading Krutetskii's (1976) work to meet first a vitriolic attack on another's use of factor analysis and then to see Krutetskii himself using factor analysis. The point, of course, is

> not that this particular method is right or wrong, but that it was considered by Krutetskii to be inappropriate for the first problem and appropriate for the second.

Gravemeijer (1994) talks about this form of validation as 'internal', it "concerns the correctness of the findings within the actual situation". (In contrast, Gravemeijer uses 'external validity' to mean something concerned with "the bearing of the results on other situations" which is only part of what we mean by meta-methodological validity.)

Meta-methodological validation, however, is not about whether a particular instance of the use of a methodology is appropriate or not, but about whether the way of working that someone posits as a methodology does indeed work in the way intended: that is, if someone puts forward a way of working and addresses how this way of working answers the meta-methodological questions, then a community needs to confirm that the questions have been appropriately considered and answered. Mason (1994) put forward a new form of enquiry – noticing – in a plenary session of the Psychology of Mathematics Education conference. In doing so he gave access to his ideas about this new way of thinking to as wide a range of mathematics educators as possible.

It seems clear then that the validating communities here are necessarily different. In the first form of validation, the best possible people to determine whether a particular use of a methodology is appropriate are those who themselves use that methodology: it is surely better that Krutetskii, who uses factor analysis himself, should be one of the people who determine whether others' use of factor analysis is appropriate rather

than someone who has no grounding in the use of such statistical methods criticising it for this reason.

However, determining whether or not some new way of working is valid as a methodology obviously cannot be done solely by those working within it. Instead, we suggest that it is for the mathematics education community at large to ensure that the meta-methodological questions have been sensibly answered. Thus the wider community ensures that the domain the methodology has been developed to address, will give rise to the kinds of questions the methodology can ask and will give answers in a form accessible to those to whom the researcher wishes to speak. Our image of these two types of validation are embodied in figure 2.

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METHODOLOGICAL NEIGHBOURHOODS

One consequence of this two-part validation is that we can begin to see the different roles taken by the mathematics education community in relation to the work of an individual, or a particular research group. On the larger scale the whole of the community is available to make a meta-methodological validation of the individual's way of working. Such a validation, however, only needs to be done once. Indeed, it only needs to be done at all if the researcher wishes to develop a new 'voice' to speak about some new aspect or to speak to a new audience, rather than adopting a traditional voice or one which has already been validated for someone else's work.

The methodological validation, however, needs to be done repeatedly: each new piece of work being validated by those who work with that methodology. We call the closer community that does this form of validation the *methodological neighbourhood*: the group of researchers who have the same (or similar) methodologies.

It is important to note that we consider the term methodological neighbourhood to be just as nebulous — but useful — a notion as that of a neighbourhood in general. We may talk of our street as our neighbourhood, or our town or our region of the country. The notion of neighbourhood may have many different levels and methodologically we may talk about our neighbourhood as only those who use exactly the same methods as us, or those who use similar ones or those, perhaps, who are in the same broad research tradition that Bishop defines. Moreover, it may be important to note that we can be members of many different neighbourhoods at once - just as one may consider the people in one's street as neighbours and the people in the same sports club as neighbours in a different sense, one may belong to a number of different methodological neighbourhoods of groups of researchers working in areas and in ways that fit with one's own different ways of working.

The concept of the methodological neighbourhood also allows us to reassess Gravemeijer's (1994) discussion of the research-development-diffusion (RDD) model. The RDD model is one in which there is a clear direction to the flow of communication: from the researcher, to the curriculum developer to the teacher. This can lead to a separation of research from implementation to the extent that, as Gravemeijer notes from Fullan's (1982) work, "a teacher could use the materials and

alter some teaching behaviors without coming to grips with the conceptions or beliefs underlying the change".

Our view of the role of the two complementary communities of 'local' methodological neighbours and the 'global' mathematics education community (shown in figure 3) allows us to see communication flowing in different directions and removes the hierarchy in the RDD model that sees researchers at one end and teachers at the other. Researchers in one methodological neighbourhood may pick up ideas from their neighbours and may reinterpret them, or translate them, for their neighbours in another methodological neighbourhood. Indeed, two of our five meta-methodological questions require us to address the issue of who uses the results of our research and what they use them for: answers to these may well include other researchers who may use our results to stimulate, direct or challenge their own thinking.

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Repeated application of this process of translation between neighbourhoods leads to a diffusion of results around the community — a single community made up of many neighbourhoods and which includes within it both teachers and researchers. Some neighbourhoods may contain both teachers and researchers together, allowing them to communicate directly; others may contain only researchers (the results likely to be inaccessible to teachers or not immediately of value to them) but where the results may be diffused through translations into teacher-accessible neighbourhoods.

Such a model allows us to reconsider Saul's (1995) view:

> mathematics education research is rarely helpful to the teacher. I search the standard journals in vain for insights into my classroom or my curricula. They are few and far between. Those ideas I do glean could be communicated more convincingly and more efficiently if they were not couched as the result of a scientific investigation

This view highlights the importance of language in mathematics education. As we have indicated, particular methodologies may be seen as akin to having a particular voice. In order to talk to our methodological neighbours - which is vital for the methodological validation process - we need to speak in the language of the methodology. Such languages may not be accessible to those outside the methodology. For example, some of those who work in the field of semiotics in mathematics education have described the problems they have in finding those who can listen critically to their work and this is certainly an area of mathematics education in which many find the methodological language difficult or inaccessible. Saul's description of the vain search for classroom insights in research journals speaks clearly of the problems of inter-neighbourhood communication.

The alternative to the RDD model we pose, then, shows that the diffusion between neighbourhoods is vital for all the community: for example, while many may feel that the work in semiotics is in a language foreign to them, closer methodological neighbours can pick up useful ideas from it and develop them in their own work which *is* in an accessible language. Similarly, people who work in neighbourhoods closer to teachers pick up ideas from research journals in languages they can

assimilate into their own research, thus making those concepts accessible to teachers. Similarly, some (like Lerman, 1989) who might see their work as centred in the philsophy of mathematics education, can write for people outside their neighbourhood and help diffuse the ideas across the community as a whole.

Again, one of the important features of this model is that the process can work in either direction: those in the neighbourhood of teachers (who may be teachers themselves, or researchers who have that neighbourhood as one of their methodological areas) can pick up ideas that teachers express in their writing about their classrooms, translate them for those who are further away from teachers but who may absorb them into their thinking. Indeed we see a neat symmetry in this: a teacher may begin from incidents in their classroom and move towards the research literature to explore possible reasons for the incidents and a traditional researcher may begin from their knowledge of the literature, seeing issues raised by theories, and move to teaching contexts to explore and examine those issues in more depth.

THE ROLE OF THE PHD IN MATHEMATICS EDUCATION

The two-community model of validation and communication also highlights what we feel may be a significant issue in mathematics education - the role of the PhD. In an early presentation of some of the ideas within this paper (Duffin and Simpson, 1996), we found very different responses from two classes of people. Young PhD researchers, who may only have come into mathematics education research very recently, described a model of their research that fitted only one direction on our meta-methodological diagram (figure 1) - they were obliged to adopt a traditional methodology, which restricted the choice of question they could ask. Indeed, they

seemed unaware of the possibility of an alternative approach. However, some researchers who had worked in mathematics education for longer, but who had only recently begun work towards, or gained, a PhD had a different view of how their working could be mapped on to the diagram. They felt that they had developed new methodologies to allow them to ask the questions they wished to answer, but found that such an approach made gaining acceptance for their work much more difficult.

In terms of our model, these two different reactions make sense: on the one hand, those working within a traditional methodology have only the methodological validation to do: a validation process that involves only the methodological neighbours (represented by the examiners of the thesis). Those who wish to use new methodologies, however, must undergo a two stage validation process, with different audiences for both stages: a meta-methodological stage and a methodological stage. At the first stage, the validation process must be performed at a community-wide level, for which a PhD thesis is singularly badly placed because it is usually accessed only by methodological neighbours who, for a new methodology, may be rare.

This implies, we suggest, that those who want to use new methodologies, or wish to ask questions inaccessible to old methodologies, need to be in a position to publish to a wider community *before* they begin working to answers to their research questions — that is, *before* a PhD.

A number of such researchers have begun to emerge in the domain of mathematics education. One of these started from early experience as a secondary teacher of mathematics before moving towards research. In an earlier publication (Jaworski,

1991) she was tentatively trying out alternative approaches to the teaching process which later (Jaworski, 1994) emerged as a fully integrated research method from a constructivist perspective.

MAKING OUR WAY OF WORKING EXPLICIT

While the consequences of our model are an important aid in helping us understand the relationships between communities in mathematics education, our aim at the start of our discussions was to site our work amongst that of others and to see how our way of working might be developing into a methodology. We have suggested that this development takes place by making explicit our answers to the meta-methodological questions, such as those we raise in figure 1. We need to ensure that the relationship between the domain, questions, answers and range of the work is clear to the wider community. Before we can do that, however, we have to be explicit about exactly what our way of working is.

We began working together when we discovered a shared interest in a single piece of work done by an eight year old girl who had recorded her attempts to work out the value of the squares of the first 20 natural numbers (Duffin and Simpson, 1991). It was our initial realisation that we had viewed the piece of work from two totally different perspectives that generated the stimulus for continuing to discuss such incidents.

One of the features of these early discussions was that we found we were constantly referring back to our own learning as well as calling upon our current work in our

respective spheres. We became aware of the importance of our own learning to what we brought to the discussion. We also became aware of what was important to us in the sharing: the way in which this drew attention to our differing perceptions and, through talking and sharing, enlarged and changed the perceptions of both of us.

It was only at a later date that our attention was drawn to the contrasts between the approaches of Piaget (seeing learning as rooted in the individual) and Vygotsky (seeing learning as rooted in interaction between individuals) as we listened to colleagues more experienced in research than ourselves. For us, both our individual ways of viewing what we experienced, and the important changes and developments arising spontaneously from our interaction, became cornerstones in our own development.

More recently, as our thoughts turned to ideas about connecting our work with that of others, we began to feel the need to be explicit about these features of our way of working. We identified three essential characteristics:

- Introspection
- Co-spection
- 'As if from inside'

By introspection we mean constantly seeking to discern our individual perceptions of experiences, both past and present, and our reactions to them. We suggest that looking at ourselves from the inside gives us an access to the mental processes of a learner that we cannot have in studying other people. For example, in initially examining the work of the eight year old girl, one of us viewed it from the perspective of a mathematician, concentrating on the mathematics within the exercise

and looking to generalise the processes used by the girl. The other came from the perspective of a mathematics educator, observing the way in which the processes she used evolved from the iconic to the symbolic, refining and minimising the recording.

We use the term ‘co-spection’ to mean the sharing of our own personal reactions to experience, with the deliberate intention of using samenesses and differences to further both our individual and our shared perceptions. For example, in sharing our views of the work the eight year old girl had produced we found that our appreciation of the work was enhanced and, in consequence, decided to continue discussing any significant classroom episodes we encountered during the course of our work. We also became aware of what was important to us in the sharing: the way in which this drew attention to our different perceptions and, through the talking, enlarged and changed the perceptions of both of us.

By ‘as if from inside’ we mean that we try to approach the observation of the actions of others, usually in some kind of learning context, from a viewpoint which tries to take into account what individual learners’ own perception of their experiences might be. For example, in exploring further, and in coming to write about the creativity involved in the eight year old girl's method for working out square numbers, we began to postulate the ways in which she may have been thinking that might have caused her to develop her method.

We recognise that some of the language we use in describing our way of working stems from that of Mason and from adapting our perception of his ideas to it. It is he who points out that the sole use of introspection as a research tool in psychology was

strongly challenged by workers such as Watson (1913) and that the excesses of treating personal, inward looking accounts as unchallengeable partially led to the development of 'objective' behaviourism (Mason, 1994). Mason's work and his introduction of the terms extra-, intra- and interspection were influential for us, both in concept and in the choice of vocabulary we made to describe one aspect of our way of working.

We accept that there are problems with introspection as a research method. Both Mason and we are trying to solve these problems but we appear to be doing it in different ways. Mason does it by abandoning the term introspection in favour of extra-, intra- and inter-spection, while we do it through adding the notions of co-spection and 'as if from inside' to validate, challenge and extend the findings of introspection.

For us it is the synthesis of all *three* characteristics — examining our individual responses to experiences, sharing them closely with the other who tends initially to respond differently to experiences, and using the similarities and differences of the internal processes we get from these to consider our observation of others 'as if from inside' — which constitutes our way of working. The sharing through co-spection and the examination of any theory we develop through the 'as if from inside' process provide us with a challenge to our individual, internal accounts which introspection alone cannot do.

The phrase 'as if from inside' comes from reflecting on Mason (1987) when he classifies researchers as follows:

> We are all trying to model or describe the inner world of experience. Some of us proceed by contemplating and studying other people, or by studying ourselves as if from the outside; others proceed by contemplating and studying ourselves from inside.

We find a partial symmetry here: relating the two dimensions of studying ourselves/others and studying from the inside/outside. But while Mason relates the self to both inside and outside study, he only mentions the notion of studying others from outside. It seems to us that our attempts to look at others by considering what internal influences there might have been on their actions brings in a fourth concept of 'studying others as if from inside', which appears to complete the symmetry.

This idea entered our work at an early stage before we had encountered the literature that gave rise to its vocabulary. It arose when we were developing our theory of learning, *natural*, *conflicting* and *alien* (Duffin and Simpson, 1993). Initially the theory was based only on the two concepts of natural and conflicting but, in examining an incident about a seven year old boy, we became conscious that a clear contradiction, which for us would have caused a conflict, did not perturb him in the slightest; he merely ignored it.

The incident related to a subtraction the boy had performed, recorded in vertical form and using the well known `bug' of always taking the smaller digit from the larger one in subtraction. In consequence he arrived at an incorrect answer and wrote beside it `but the real answer is...' alongside the correct solution. We immediately recognised that, for us, this would have constituted a conflict causing us to look again at what we had done but for the boy this did not appear to be a cause of concern. Our exploration

of this incident led us to think about what kind of mental constructs this boy had had to cause him to write as he did. In this sense we were attempting to examine the boy's work `as if from the inside'. It also led us to consider what, in the mental processes of a learner, might lead to this kind of response, resulting in the third concept of our theory: alien. It also led us to realise that an essential feature of our work was that we were trying to observe learning incidents that came our way as if from the viewpoint of the learner involved: as if from the inside.

OUR WAY OF WORKING AND THE FIVE QUESTIONS

Our aim, in beginning our work on these methodological and meta-methodological issues, was only to examine how our work could fit amongst those of others and to see if the ways in which we were asking and answering the questions of interest to us could have a methodological status. The act of deriving our meta-methodological questions and the notions of the methodological neighbourhood with the consequences they have for an alternative diffusion model for research, has done much more than we expected, but still leaves our main aim unaddressed.

We believe that we have provided a framework in which we can see the ways in which methodologies can come into being – through meta-methodological validation: by addressing the issues of the domain and range of the research which we have come to refer to as our five meta-methodological questions. Our own search for that validation, by making explicit what we do and how it addresses these question, is now only beginning. That beginning, however, highlights many of the aspects of this work by demonstrating the abstract notions in a concrete (and for us, personal) example.

Our meta-methodological theory suggests that there can be an ebb-and-flow between the domain, questions, answers, range and use within both a single piece of research and, as in our case, a longer term research theme. Our earliest work, in which we tried to describe and explain our different perceptions of the eight year old girl's work, was the result of a personally motivated question: how was it possible for two people to see the same incident in such different ways?

So, in this case, our domain was determined for us by this question — it is clearly a question about our individual perceptions. The question also influenced the form of the answers we required: they were essentially answers for ourselves. Hence also the domain (us) was determined and the use was also set down: to provide us with a better framework for understanding each other and for appreciating the possibility of different interpretations of incidents.

The act of beginning to answer the question the pupil's work raised for us led us to issues about creativity (Duffin and Simpson, 1991). Differentiating between what we termed 'personal', 'local' and 'global' creativity raised issues which, in discussion, others found of interest. Thus we were encouraged to make our answers accessible to other researchers. The emphasis changed to a focus on the range for our research, that then led us to need to reformulate the answers. This influence extends back to the kinds of subsequent questions we now ask, which have a particular form of answer and proposed 'user'. It is clear, then, that the flow has reversed.

This ebb-and-flow makes the act of addressing the meta-methodological questions considerably harder. If you wish to write down five clear answers to the questions, which one do you start with? Any choice of a particular question to start with will force a direction on the meta-methodological validation that may well not be present in one's research. Moreover, in trying to answer any one question, we could find no way to avoid bringing in aspects of the other questions (Duffin and Simpson, 1996).

Our exploration of the five questions is only in its early stages so attempts to be more explicit about them may be premature. What we are currently aware of is that it is difficult to treat the questions as distinct and separate and it may be that, after further work, we will want to change them in some way.

Moreover, in trying to answer these questions, we have become aware of an almost threatening aspect to them when they are separated: perhaps the research process (particularly as it grows) becomes so entangled that the individual branches of the meta-methodological validation process cannot be discerned without destroying the unique form that a researcher's (or a research group's) work takes.

Our effort, and the effort of others who wish to ask questions which may be rendered invisible by existing methodologies, might therefore be better devoted to expressing clearly what they are doing and why they do it, using the meta-methodological questions as a background. It may be that this will enable us to find a fit with Stenhouse's description of "systematic enquiry made public."

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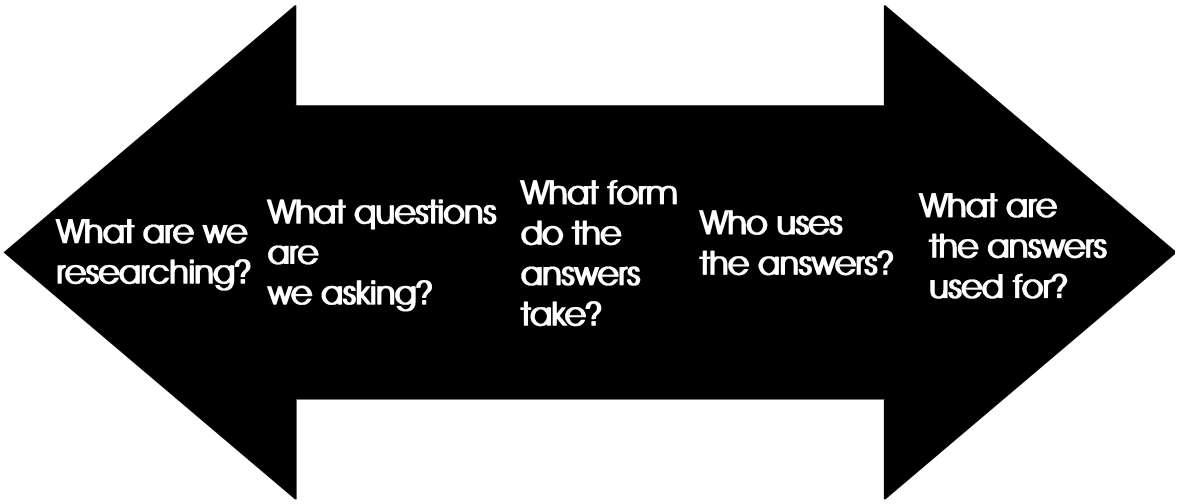

Figure 1. Questions and Answers in Research

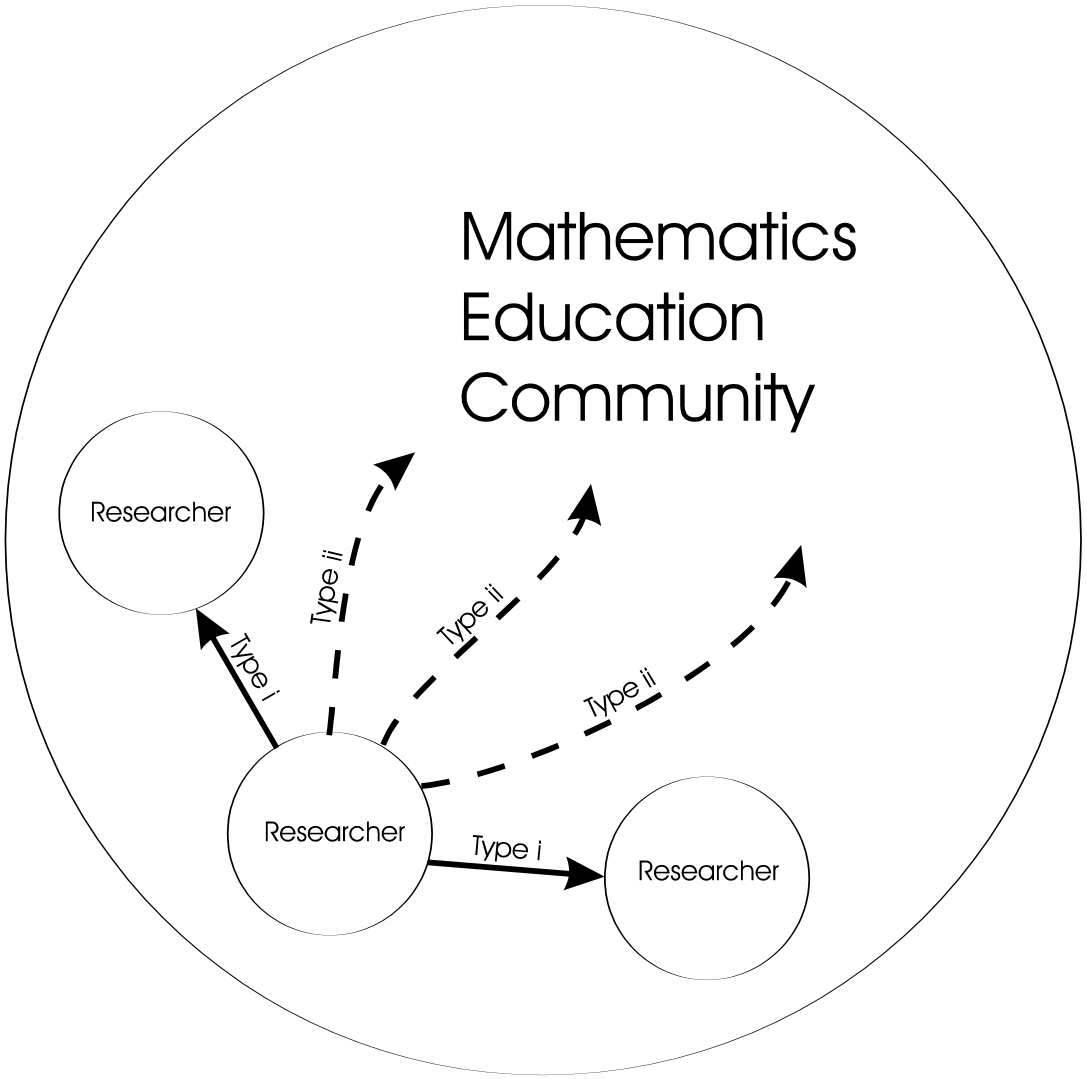

Figure 2. Two forms of validation

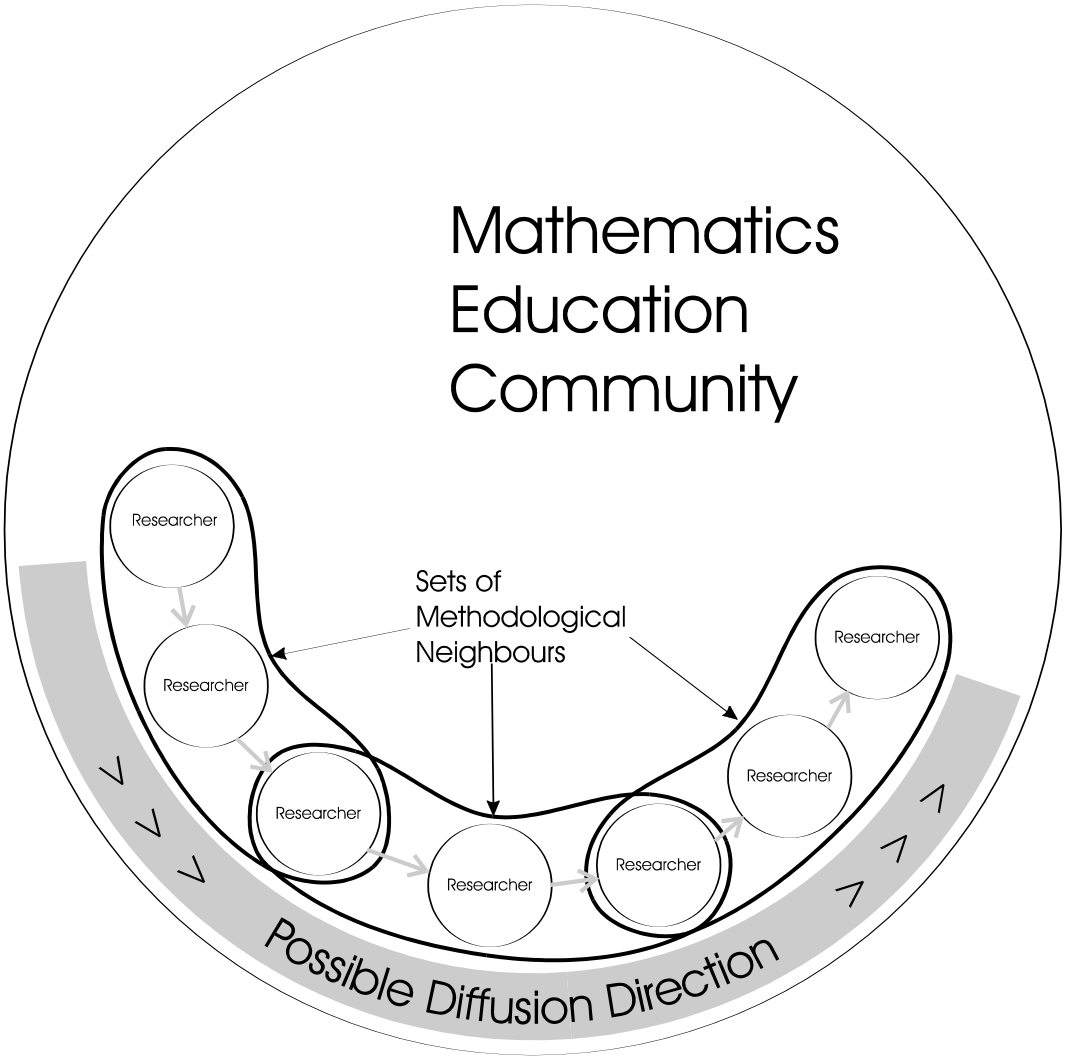

Figure 3. Research diffusion