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Selected Topics in Heavy Flavour Physics

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Abstract

We review the status of flavour physics in spring 2014. The numerous accurate new measurements of flavour experiments have enabled us to test our theoretical understanding of flavour processes with an unprecedented precision. At first sight the dominant amount of measurements seems to be standard model like. Having a closer look one finds, however, that in most of the observables there is still some considerable space for new effects. In addition many discrepancies are still not settled yet. For further investigations and definite conclusions an improvement of the theoretical precision as well as the experimental one is mandatory.

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1 Introduction

The standard model of particle physics [1, 2, 3] is finally complete. The Higgs particle that was predicted in 1964 [4, 5, 6] was found in 2012 at LHC by the ATLAS and CMS Collaborations [7, 8]. Knowing the value of the mass of the Higgs particle, for the first time a complete electro-weak precision fit could be performed without having any unmeasured standard model parameters. This was done in [9] and slightly later in [10] and a very good overall consistency has been found.
Despite these and numerous other successes, the standard model leaves many open questions. Far reaching ones like the quest for the quantisation of gravity, or an understanding of dark energy. We also do not know the origin of dark matter and we might want to answer simple sounding questions, like, why are there three generations of matter in nature. Another very profound question is, where does matter, i.e. an excess of matter over anti-matter in the universe come from. Sakharov has shown already in 1967 [11] that a matter-anti-matter asymmetry can be created dynamically if the fundamental laws of nature have the following basic properties:

- Baryon number is violated.
- There was a phase out-of thermal equilibrium in the early universe.
- C and CP are violated.

Focusing on the requirement of CP-violation one finds that this effect is included in the standard model in the quark mixing matrix[1], the Cabibbo-Kobayashi-Maskawa (CKM) matrix [12, 13]. The CKM matrix describes the coupling of the weak charged gauge bosons to quarks. It allows also non-diagonal couplings of the charged currents, i.e., the u-quark does not only couple to the d-quark via a charged W boson, but it also couples to the s-quark and the b-quark. The entries of the CKM-matrix give the respective coupling strengths

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} . \tag{1.1}$$

The couplings of, e.g. the up- and down-quark to the charged we current is given by:

$$\text{Coupling} \ \propto \frac{g_2}{2\sqrt{2}}\gamma_\mu(1-\gamma_5)V_{ud} \, . \tag{1.2}$$

The CKM matrix is by construction unitary and it can be parameterised by four variables, three real angles and one complex phase. The latter one describes CP-violation. The so-called standard parameterisation reads

$$V_{CKM3} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} , \tag{1.3}$$

with

$$s_{ij} := \sin(\theta_{ij}) \quad \text{and} \quad c_{ij} := \cos(\theta_{ij}) \, . \tag{1.4}$$

[1] We will not discuss here the possibility of having CP violation also in lepton mixing or in the strong sector.

The three angles are denoted by θ_{12}, θ_{23} and θ_{13} and the complex phase describing CP-violation is δ_{13}. This parameterisation is typically used for numerical calculations. There is also a very transparent parameterisation, the so-called Wolfenstein parameterisation [14]. This parameterisation uses the experimentally found hierarchy $V_{ud} \approx 1 \approx V_{cs}$ and $V_{us} \approx 0.225 \equiv \lambda$ to perform a Taylor expansion in λ. Here one also has 3 real parameters λ, A and ρ and one complex coupling denoted by η.

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} . \tag{1.5}$$

In this form the hierarchies can be read of very nicely. The most recent numerical values read (status March 2014 from the CKMfitter page [15] - similar results are obtained by UTfit [16])

$$\lambda = 0.22457^{+0.00185}_{-0.00014} , \tag{1.6}$$

$$A = 0.823^{+0.012}_{-0.033} , \tag{1.7}$$

$$\bar{\rho} \equiv \left(1 - \frac{\lambda^2}{2}\right)\rho = 0.1289^{+0.0176}_{-0.0094} , \tag{1.8}$$

$$\bar{\eta} \equiv \left(1 - \frac{\lambda^2}{2}\right)\eta = 0.348^{+0.012}_{-0.012} . \tag{1.9}$$

The investigation of the CKM parameters goes hand in hand with the determination of the so-called unitarity triangle. By construction we have

$$V_{CKM} V_{CKM}^\dagger = 1 . \tag{1.10}$$

In the case of three generations this gives us nine conditions. Three combinations of CKM elements, whose sum is equal to one and six combinations whose sum is equal to zero, in particular

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 . \tag{1.11}$$

Using the Wolfenstein parameterisation we get for this sum

$$A\lambda^3 \left[(\overline{\rho} + i\overline{\eta}) - 1 + (1 - (\overline{\rho} + i\overline{\eta}))\right] = 0 . \tag{1.12}$$

Since A and λ are already quite well known one concentrates on the determination of ρ and η. The above sum of three complex numbers can be represented graphically as a triangle, the so-called unitarity triangle, see Fig. 1. The determination of the unitarity triangle is in particular interesting since a non-vanishing η describes CP-violation in the standard model. In principle the following strategy is used (for a review see, e.g. [17]):
Compare the experimental value of some flavour observable with the corresponding theory expression, where ρ and η (or the angles α, β and γ) are left as free parameters and plot the constraint on these two parameters in the complex $\rho - \eta$ plane. Four prominent examples of constraints are:

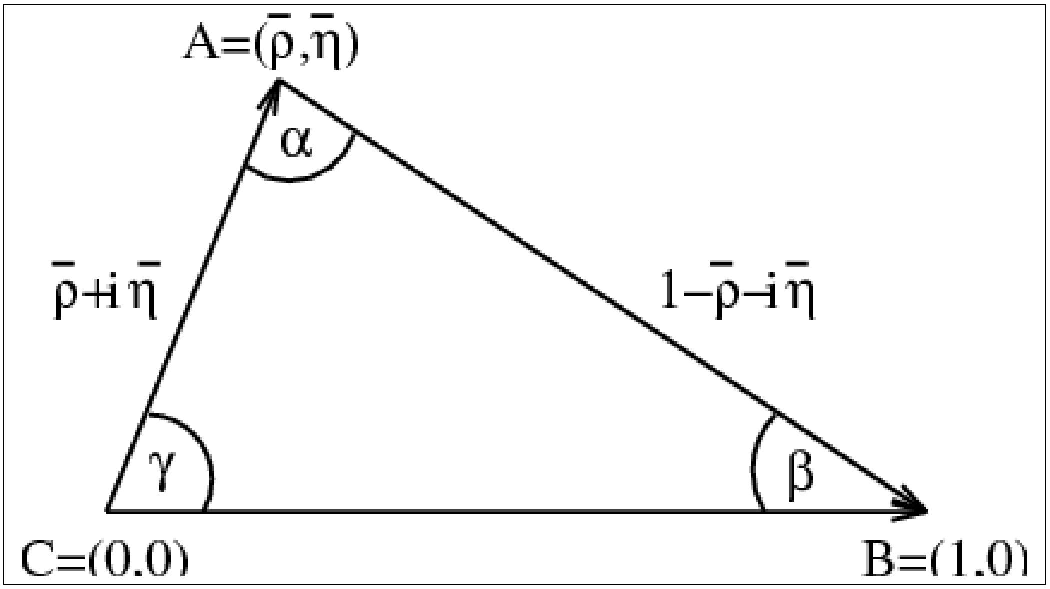

Figure 1: The unitarity triangle.

- The amplitude of a beauty-quark decaying into an up-quark is proportional to V_{ub}. Therefore the branching fraction of B-mesons decaying semi leptonically into mesons that contain the up-quark from the beauty decay is proportional to $|V_{ub}|^2$:

$$B(B \to X_u e\overline{\nu}) = \tilde{a}_{\text{theory}} \cdot |V_{ub}|^2 = a_{\text{theory}} \cdot \left(\rho^2 + \eta^2\right) ,$$

$$\Rightarrow \rho^2 + \eta^2 = \frac{B^{\text{Exp.}}(B \to X_u e\overline{\nu})}{a_{\text{theory}}} , \tag{1.13}$$

  where a contains the result of the theoretical calculation. By comparing experiment and theory for this decay and leaving ρ and η as free parameters we get a constraint in the $\rho - \eta$-plane in the form of a circle around $(0, 0)$ with the radius $B^{\text{Exp.}}(B \to X_u e\overline{\nu})/a_{\text{theory}}$.

- Investigating the system of neutral B-mesons one finds that the physical eigenstates are a mixture of the flavour eigenstates. This effect is due the so-called box diagrams shown in Fig. 2, that enable transitions between a $\bar{B}_d = (b\bar{d})$-meson and a $B_d = (\bar{b}d)$-meson. As a result of this mixing the two physical eigenstates have different masses, the difference of the two masses is denoted by ΔM_{B_d}. Theoretically one finds $\Delta M_{B_d} \propto |V_{td}|^2 \propto (\rho - 1)^2 + \eta^2$. Comparing experiment and theory we obtain a circle around $(1, 0)$.

- Comparing theory and experiment for the CP-violation effect in the neutral K-system, denoted by the quantity ϵ_K, we get an hyperbola in the $\rho - \eta$-plane.

- Bigi, Carter and Sanda have shown that the angle β can be extracted directly, with almost no theoretical uncertainty from the following CP-asymmetry in exclusive B-

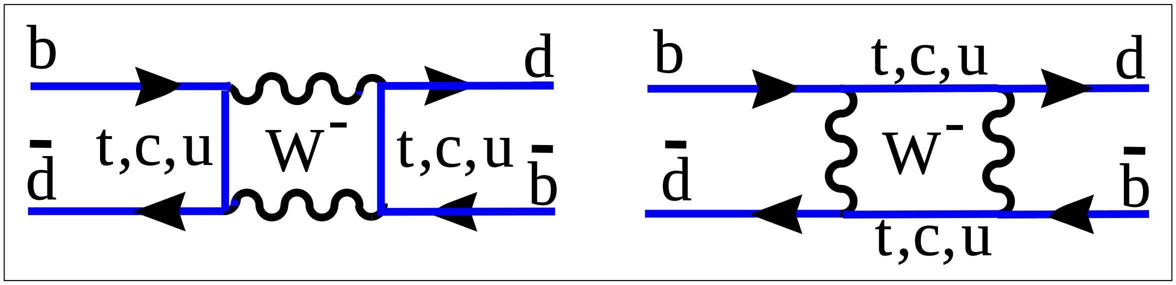

Figure 2: Box diagrams contributing to the mixing of B_d-mesons. The Feynman diagram on the left can only contribute to M_{12}^q, because the W-bosons are always off-shell, this is also the case for the top-quark contribution of the diagram on the right. Contributions to Γ_{12}^q can only arise from the up- and charm-quark on the right.

decays [18, 19].

$$a_{CP} := \frac{\Gamma(B \to J/\Psi + K_S) - \Gamma(\bar{B} \to J/\Psi + K_S)}{\Gamma(B \to J/\Psi + K_S) + \Gamma(\bar{B} \to J/\Psi + K_S)} \propto \sin 2\beta \tag{1.14}$$

Because of its theoretical cleanness this decay mode is called the gold-plated mode.

The overlap of all these regions gives finally the values for ρ and η. In Fig. 3 all the above discussed quantities are included schematically. The constraint from the semi leptonic decay is shown in green, the constraint from B-mixing is shown in blue and the hyperbolic constraint from ϵ_K is displayed in pink. Later on we present a figure with the latest experimental numbers. Experimentally CP-violation was already found in 1964 [20] as a tiny effect in the

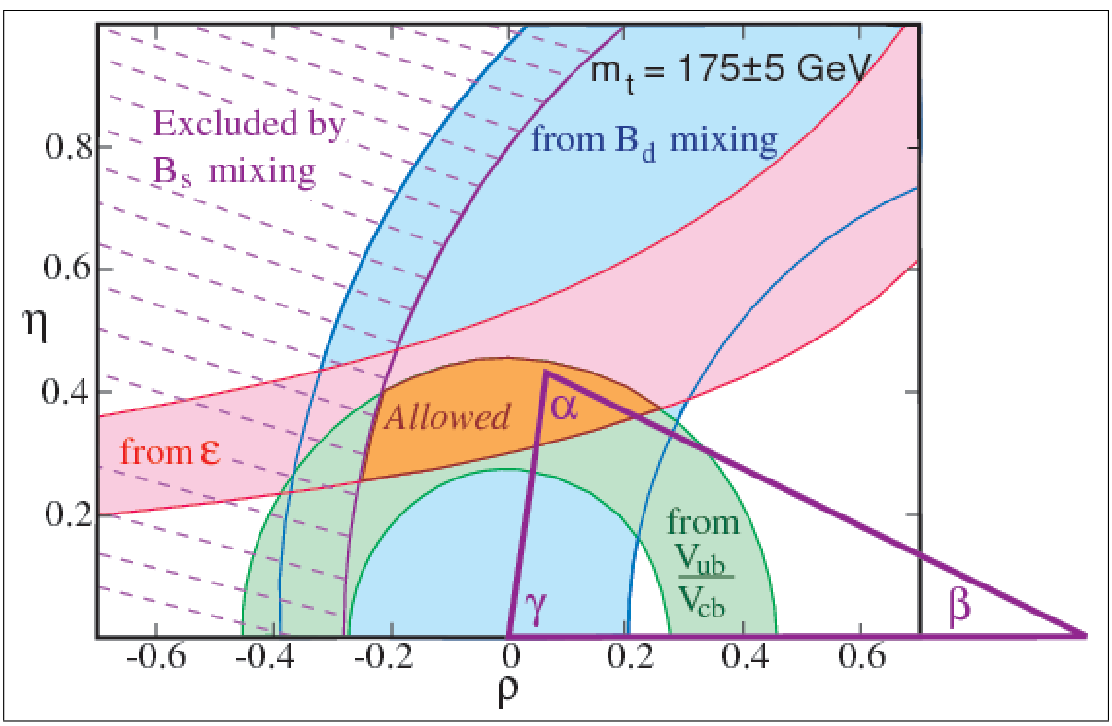

Figure 3: Bounds on the unitarity triangle from V_{ub}, CP-violation in the Kaon-system ϵ_K and the mass difference of neutral B-mesons, ΔM_d and ΔM_s.

decay of Kaons, denoted above by ϵ_K. Larger effects were predicted already in 1981 [18, 19] in the decay of B-mesons, in particular in the decay $B_d \to J/\psi K_S$. Indirect CP violation in the B_d-system, i.e., a non-vanishing value of the angle β, was then found subsequently in 2001 by BaBar [21] and Belle [22]. Besides the B_d-system, where direct CP-violation was established in 2006 at Belle [23] in the decay $B_d \to \pi^+\pi^-$ and in 2007 at BaBar [24] in the decays $B_d \to \pi^+\pi^-, K^+\pi^-$, direct CP violation has also been found by the LHCb Collaboration in 2012 in the decay $B^+ \to DK^+$[25] and in 2013 in the decay $B_s \to K^-\pi^+$ [26]. Indirect CP violation in the B_s sector, which is predicted to be very small in the standard model (see, e.g. the review [27]), has not yet been detected, despite intense searches, e.g. [28, 29]. In 2011 there were also some indications at LHCb that there might be direct CP violation in the charm sector [30], which were, however, not confirmed by more recent studies in 2013 [31], see also the discussion in [32].
One of the reasons for the interest in flavour physics stems from the fact that meson decays are currently the only processes in nature, where CP violation has been detected. By studying these decays in detail, one hopes to deepen our understanding of the origin of CP violation. Moreover flavour physics enables indirect searches for new physics, where very precise measurements are compared with very precise standard model calculations. Significant deviations might then point towards beyond standard model contributions. Here processes that are strongly suppressed in the standard model are particularly well suited. Examples are the box-diagrams, shown in Fig. 2 and so-called penguin diagrams depicted in Fig. 4, that enable flavour changing neutral currents (FCNC) at the loop-level. In the standard model there is e.g., no tree-level transition of a b-quark into a s-quark. Penguin diagrams were invented in 1975 by Shifman, Vainshtein and Zakharov [33] and baptised by John Ellis in 1977 [34]. Such a programme is complementary to the direct searches at, e.g. the general purpose

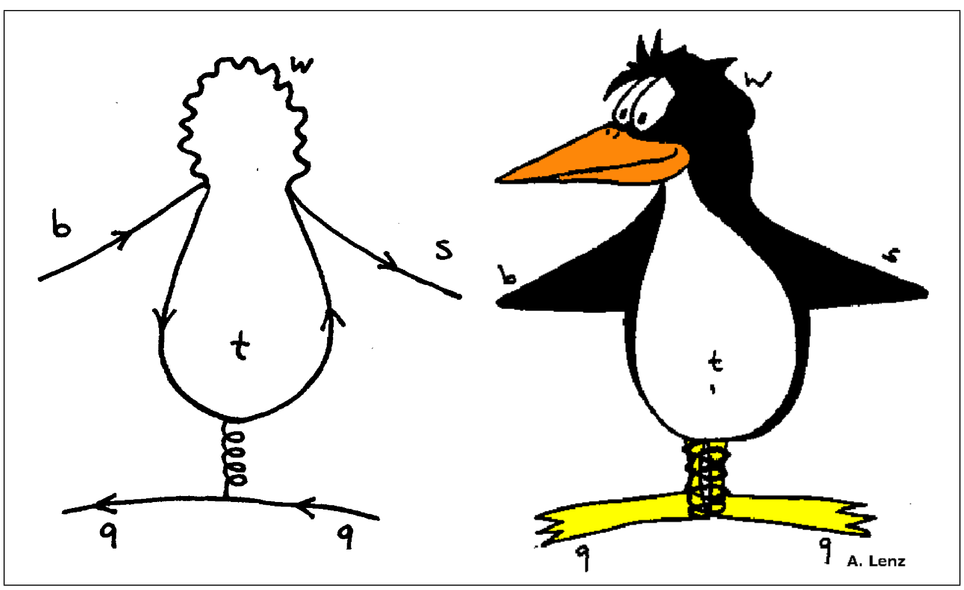

Figure 4: Penguin diagrams enable a transition of e.g., a b-quark into a strange-quark, which is in the standard model forbidden at tree-level.

detectors ATLAS and CMS, where one hopes to detect decay products of directly produced new particles. As long as no direct evidence for new physics is found, indirect searches might provide the first hints for new effects at a higher energy scale than directly accessible and as soon as direct evidence for new particles is found, the precision study of flavour effects will be helpful in determining the new flavour couplings. Indirect searches rely of course heavily on our control of the corresponding hadronic uncertainties in flavour transitions. For the case of b-hadrons two facts turn out to be very useful in that respect. First, the strong coupling at the scale $\mu = m_b$ is relatively small $\alpha_s(m_b) \approx 0.2$ and second, there exists an expansion of decay rates in terms of the inverse heavy quark mass, the heavy quark expansion (HQE) [35], which allows precise predictions. Several non-trivial cross-checks for these tools to handle QCD effects will be discussed below. An interesting question is of course, to what extent the HQE methods can be applied in the charm sector, where the expansion parameters $\alpha_s(m_c)$ and $1/m_c$ are considerably larger. A final motivation for flavour physics studies are precise determinations of many standard model parameters, like the values of the CKM parameters or also some quark masses.

After the big success of the B-factories with the detectors BaBar and Belle, see, e.g. [36] and the results from TeVatron, see, e.g. [37], the field is currently dominated by the LHCb experiment (see, e.g. [38] for some earlier results), but there are also some important contributions from ATLAS, see, e.g. [39], and CMS, see, e.g. [40], as will be discussed below.

In Section 2 we will study inclusive quantities like lifetimes, but also the mixing system as well as individual inclusive branching ratios. Most of the corresponding theory predictions rely on the heavy quark expansion. In Section 3 we switch to exclusive quantities, starting from leptonic decays, over semi-leptonic decays to non-leptonic ones. In Section 4 we discuss some consequences for searches for new physics models and in Section 5 we conclude.

2 Inclusive decays

We start our discussion with inclusive decays. Such decays are characterised by the fact that we do not specify the hadronic final state, simplifying thus the non-perturbative physics considerably. The prime example of an inclusive quantity are lifetimes of b- and c-hadrons, as well as observables related to the mixing of neutral mesons. Finally we discuss also individual semi- and non-leptonic inclusive decay modes.

2.1 Lifetimes

Lifetimes are among the most fundamental properties of a particle. We compare here recent measurements for the lifetime of D-mesons, B-mesons and b-baryons with the latest theory predictions.

2.1.1 Theory

Total decay rates can be written according to the heavy quark expansion - see [35] for the first systematic expansion and [41] for a review of the extensive literature - as

$$\Gamma = \Gamma_0 + \frac{\Lambda^2}{m_q^2}\Gamma_2 + \frac{\Lambda^3}{m_q^3}\Gamma_3 + \frac{\Lambda^4}{m_q^4}\Gamma_4 + \ldots \; . \tag{2.15}$$

If the mass m_q of the decaying quark is heavy and the hadronic scale Λ is not very large, then the expansion in Eq.(2.15) is expected to converge quickly. In particular because there are no corrections of order Λ/m_q. Each of the coefficient Γ_i for $i \geq 2$ consists of perturbatively calculable Wilson-coefficients and of non-perturbative matrix-elements, that have to be determined, e.g. with lattice calculations or QCD sum rules. For more details we refer the interested reader to the review [41].

2.1.2 Charmed mesons

For charmed mesons one finds experimentally a huge spread in the lifetime ratios [42]

$$\frac{\tau(D^+)}{\tau(D^0)}^{\mathrm{Exp.}} = 2.536 \pm 0.017 \,, \qquad \frac{\tau(D_s^+)}{\tau(D^0)}^{\mathrm{Exp.}} = 1.219 \pm 0.017 \,. \tag{2.16}$$

Besides the fact that $1/m_c$ does not look like a good expansion parameter, the values in Eq.(2.16) indicate huge corrections in Eq.(2.15), if not a complete breakdown of the expansion. Nevertheless, studies within the HQE were performed, see, e.g. [41] for the history of these efforts. In [43] an investigation of the D-meson lifetimes including α_s-corrections to Γ_3 and the LO-corrections to Γ_4 obtained

$$\frac{\tau(D^+)}{\tau(D^0)}^{\mathrm{HQE}} = 2.2 \pm 0.4^{+0.3}_{-0.7} \,, \qquad \frac{\tau(D_s^+)}{\tau(D^0)}^{\mathrm{HQE}} = 1.19 \pm 0.12 \pm 0.04 \,. \tag{2.17}$$

The first error stems from the uncertainties in the non-perturbative matrix elements of the arising four-quark operators. For these matrix elements some assumptions had to be made in [43], since there is no first principle calculation available. Such an endeavour would be very desirable. The second error in Eq.(2.17) stems from the renormalisation scheme dependence, which could be reduced by a NNLO-QCD calculation. Contrary to the naive expectation the HQE seems to be capable of describing the huge lifetime ratios in the D-meson system, but for more profound statements, lattice values for the arising non-perturbative matrix elements are mandatory.

2.1.3 B-mesons

For B-mesons the measured lifetime ratios are very close to one [44]

$$\frac{\tau(B^+)}{\tau(B_d)}^{\mathrm{Exp.}} = 1.079 \pm 0.007 \,, \qquad \frac{\tau(B_s)}{\tau(B_d)}^{\mathrm{Exp.}} = 0.998 \pm 0.009 \,. \tag{2.18}$$

More recent experimental numbers, that are not yet included in the HFAG average can be found in [45]. Because of the larger value of m_b one expects now a better convergence of Eq.(2.15). Unfortunately it turns out, see, e.g. the detailed discussion in [41], that pronounced cancellations are occuring in the theory predictions of these ratios, that lead to a strong sensitivity on the bag parameters and the most recent determination of these parameters stems already from 2001 [46]. Relying on these old non-perturbative values and including the NLO-QCD corrections from [47, 48] one gets [41]

$$\frac{\tau(B^+)}{\tau(B_d)}^{\mathrm{HQE}} = 1.04^{+0.05}_{-0.01} \pm 0.02 \pm 0.01 \,, \qquad \frac{\tau(B_s)}{\tau(B_d)}^{\mathrm{HQE}} = 1.001 \pm 0.002 \,. \tag{2.19}$$

The ratio of the neutral mesons is in perfect agreement with data, while the prediction for $\tau(B^+)/\tau(B_d)$ is slightly smaller than the measurement quoted in Eq.(2.18), but for more far-reaching statements precise bag parameters are urgently needed. Since the lifetime ratio $\tau(B_s)/\tau(B_d)$ is affected by very pronounced numerical cancellations (see, e.g. [41] for a detailed description) in the standard model, this quantity can also be used as an important bound on hidden B-decay channels due to new physics, see, e.g. the recent investigation in [50].

2.1.4 b-baryons

The Λ_b lifetime suffered from a longstanding discrepancy between experiment and theory that was finally settled experimentally. HFAG gave in 2003 an average of

$$\frac{\tau(\Lambda_d)}{\tau(B_d)}^{\mathrm{HFAG\,2003}} = 0.80 \pm 0.05 \; . \tag{2.20}$$

Older numbers resulted in even smaller ratios. The value in Eq.(2.20) was in disagreement with early estimates based on the HQE, see, e.g. [51] (see [41] for a more detailed history of prediction of the Λ_b lifetime)

$$\frac{\tau(\Lambda_d)}{\tau(B_d)}^{\mathrm{HQE\,1986}} \approx 0.96 \; . \tag{2.21}$$

Again the theory prediction depends strongly on the value of the non-perturbative matrix elements and in this case we have only an exploratory lattice study from 1999 [52], which yielded quite large numerical values for the bag parameters, leading to a larger deviation of the ratio from one. Taking these numbers and also looking for some additional effects that might reduce the ratio, one could arrive at values as low as [53]

$$\frac{\tau(\Lambda_d)}{\tau(B_d)}^{\mathrm{HQE\,2004}} = 0.86 \pm 0.05 \; . \tag{2.22}$$

In recent years there were a lot of new measurements from CDF [54] and D0 [55] at TeVatron and also from CMS [56], ATLAS [57] and of course LHCb [45, 58] that found considerably higher values for the Λ_b-lifetime. The current HFAG [44] average reads

$$\frac{\tau(\Lambda_d)}{\tau(B_d)}^{\mathrm{HFAG\,2013}} = 0.941 \pm 0.016 \; . \tag{2.23}$$

In [41] the Λ_b lifetime was re-investigated, using spectroscopic information for matrix elements (following [59]) and the NLO-QCD result from [48], as well as the $1/m_b$ corrections from [53], with the result

$$\frac{\tau(\Lambda_d)}{\tau(B_d)}^{\mathrm{HQE\,2014}} = 0.935 \pm 0.054 \; . \tag{2.24}$$

The final number depends, however, crucially on the precise value of the bag parameter, where we are lacking a first principle calculation, see the discussion in [41].

2.1.5 Lifetime upshot

The above comparison between experiment and theory shows that the HQE seems to work well for lifetimes of heavy hadrons, even in the case of D-mesons. For more precise statements new lattice investigations are urgently needed. Further examples like the B_c meson lifetime and the Ξ_b lifetime are discussed in the review [41].

2.2 Mixing Quantities

The phenomenon of particle-antiparticle mixing is a macroscopic quantum effect. It arises due to so-called box diagrams shown in Fig. 2, which enable a transition of a neutral meson state, defined by its quark flavour content into its anti-particle. This effect shows that the flavour eigenstates of the neutral mesons, e.g., $B_d = (\bar{b}d)$ and $\bar{B}_d = (b\bar{d})$, do not coincide with the mass eigenstates, which we denote by B_H and B_L, where H stands for heavy and L for light. In the Kaon system the notation K_S and K_L is used, where S stands for short-lived and L for long-lived. Performing a change of basis, one finds a mass difference ΔM_d and a decay rate difference $\Delta\Gamma_d$ of the mass eigenstates.

$$\Delta M_d := M_H - M_L \ , \tag{2.25}$$

$$\Delta\Gamma_d := \Gamma_L - \Gamma_H \ , \tag{2.26}$$

where M_H denotes the mass of the heavy eigenstate et cetera.

2.2.1 A very brief history of mixing

Mixing is by now well established in several systems of neutral mesons:

1956 K^0-system: Mixing in the neutral K-system was theoretically developed in 1955 by Gell-Mann and Pais [60]. Based on that framework the quantum mechanical phenomenon of regeneration was predicted in the same year by Pais and Piccioni [61]. Experimentally this phenomenon was confirmed in 1960 [62]. A huge lifetime difference between the two neutral K-mesons (K_S and K_L) was established already in 1956 [63].

1986 B_d-system: Mixing in the B_d-system was found in 1986 by UA1 at CERN [64] (UA1 attributed the result however to B_s mixing) and in 1987 by ARGUS at DESY [65]. The large result for the mass difference ΔM_d can be seen as the first clear hint for an (at that time) unexpected large value of the top quark mass [66] [2]. For the decay rate difference - which is expected to have a small value in the standard model [49, 50] - currently only upper bounds are available from BaBar [68], Belle [69], D0 [70] and LHCb [45]. Here further experimental studies are very welcome, because in this quantity there is still room for some sizable new physics effects [50].

2006/12 B_s-system: The large mass difference in the B_s-system was established in 2006 by the CDF Collaboration at TeVatron [71]. In 2012 the LHCb Collaboration measured for the first time a non-vanishing value of the decay rate difference in the B_s-system [72].

2007 D^0-system: Here we had several experimental evidences (BaBar, Belle, Cleo, CDF, E791, E831 FOCUS, LHCb) for values of $\Delta\Gamma/\Gamma$ and $\Delta M/\Gamma$ at the per cent level, but the first single measurement with a statistical significance of more than five standard deviations was done only in 2012 by the LHCb collaboration [73].

[2] To avoid a very large value of the top quark mass, also different new physics scenarios were investigated, in particular a scenario with a heavy fourth generation of fermions and a top quark mass of the order of 50 GeV, see, e.g. [67].

2.2.2 Theory

Mass differences and decay rate differences of neutral B-mesons can be expressed to a very high accuracy as (see, e.g. [27] for the explicit form of the tiny corrections)

$$\Delta M_q = 2|M_{12}^q| , \tag{2.27}$$

$$\Delta \Gamma_q = 2|\Gamma_{12}^q| \cos(\phi_q) , \tag{2.28}$$

with M_{12}^q being the dispersive part of the box diagrams, see Fig. 2, and Γ_{12}^q being the absorptive part. The mixing phase reads $\phi_q = \arg\left(-M_{12}^q/\Gamma_{12}^q\right)$. The dispersive part M_{12}^q is sensitive to off-shell intermediate states; in the case of the neutral B mesons, the by far largest contribution stems from the virtual top quark in the loop. This part is also very sensitive to hypothetical heavy new physics particles in the loop. For the B-meson system one gets after integrating out the heavy W-boson and the top-quark

$$M_{12}^q = \frac{G_F^2}{12\pi^2} (V_{tq}^* V_{tb})^2 M_W^2 S_0(x_t) B_{B_q} f_{B_q}^2 M_{B_q} \hat{\eta}_B . \tag{2.29}$$

Let us sketch the origin of this structure. G_F denotes the Fermi-constant containing the weak coupling and the mass of the W-boson. $(V_{tq}^* V_{tb})^2$ is the CKM-structure arising in the dominant contribution to the diagrams in Fig. 2. The evaluation of the 1-loop box-diagram gives the so-called Inami-Lim function $S_0(x_t)$ [74] with $x_t = m_t^2/M_W^2$, which describes the dependence on the top-quark mass. Perturbative QCD corrections to the box-diagrams are denoted by $\hat{\eta}_B$ [75, 76], they turned out to be ample. The arising non-perturbative matrix element of the four quark operator Q

$$Q = \bar{q}_\alpha \gamma_\mu (1-\gamma_5) b_\alpha \cdot \bar{q}_\beta \gamma^\mu (1-\gamma_5) b_\beta , \tag{2.30}$$

where α and β denote colour indices, is for historical reasons parameterised in terms of a bag parameter B_{B_q} and a decay constant f_{B_q}:

$$\langle \bar{B}_q | Q | B_q \rangle = \frac{8}{3} B_{B_q} f_{B_q}^2 M_{B_q}^2 . \tag{2.31}$$

The bag parameter and the decay constant have to be determined with non-perturbative methods like lattice QCD or QCD sum rules.
Γ_{12}^q is sensitive to on-shell intermediate states; thus only the up- and charm-quark on the r.h.s. of Fig. 2 can contribute. After integrating out the heavy W-bosons one performs a second operator-product expansion (OPE), the HQE, yielding a similar form as in Eq.(2.15)

$$\Gamma_{12} = \frac{\Lambda^3}{m_b^3}\left(\Gamma_3^{(0)} + \frac{\alpha_s(\mu)}{4\pi}\Gamma_3^{(1)} + \ldots\right) + \frac{\Lambda^4}{m_b^4}\left(\Gamma_4^{(0)} + \ldots\right) + \ldots . \tag{2.32}$$

Again Γ_i consists of perturbative Wilson coefficients and non-perturbative matrix elements. Because of the arising CKM matrix elements both M_{12}^q and Γ_{12}^q can be complex.

2.2.3 Charm mixing

The mixing observables in the charm system are typically denoted as x and y

$$x = \frac{\Delta M}{\Gamma} , \qquad y = \frac{\Delta \Gamma}{2\Gamma} . \tag{2.33}$$

In contrast to B-mixing, where we have the very heavy top-quark, as well as the charm- and the up-quark as virtual loop particles, charm mixing proceeds via internal d-,s- and b-quarks. The lower masses of the internal and external particles lead to the fact that the methods used for the determination of the mixing observables in the B-system are much less justified for describing D-oscillations.
Because of the promising result of the investigations of D-meson lifetimes one might try nevertheless to use the HQE for a determination of Γ_{12}, as it was done in [77]. In that case, however, a second problem arises. The leading term in the HQE, Γ_3 suffers from an almost perfect GIM [78] cancellation. Thus the idea came up quite some time ago [79, 80] that D-mixing is described by higher orders in the HQE, i.e. Γ_6 and Γ_9, where the GIM cancellation is much less pronounced, see [81]. Until now no satisfactionary calculation of these higher order effects was performed. The conclusion of [81] was that standard contributions to x and y of up to 1% are not excluded, while [77] concluded that they are probably smaller. It was also shown in [77] that the enhancement effect suggested in [79, 80, 81] could also lead to CP violating effects in mixing of the order of several per mille. But here clearly more work has to be done. Because of these drawbacks it was also tried to use an exclusive approach in order to describe charm mixing [82, 83], leading to a similar conclusion: x and y might have values of about 1% in the standard model.
Experimentally D-mixing is now well settled. HFAG [44] quotes as averages

$$x^{\text{Exp.}} = \left(0.39^{+0.16}_{-0.17}\right)\% \,, \qquad y^{\text{Exp.}} = \left(0.67^{+0.07}_{-0.08}\right)\% \,, \tag{2.34}$$

while CP violation in D-mixing is still quite weakly constrained by experiment [84], which will, however, change in future, see, e.g. [85].
Despite the drawbacks related to our insufficient understanding of the standard model contribution, the D-mixing system is, however very well suited to look for new physics effects, because the contribution of heavy new particles, can be calculated more reliably, see, e.g. [86, 87].

2.2.4 B-mixing

In B-mixing the theory is under much better control and we predict for the mass differences [49]

$$\Delta M_d^{\text{Theory}} = (0.543 \pm 0.091)\ \text{ps}^{-1} \,, \qquad \Delta M_s^{\text{Theory}} = (17.3 \pm 2.6\,)\,\text{ps}^{-1} \,. \tag{2.35}$$

The large theory uncertainty is dominated by the values of the hadronic matrix elements. We have used the most recent result from FLAG [88] for $f_{B_q} B^2$, which is simply the result from [89]. Similar values were obtained in [90] and slightly higher ones in [91]. These predictions can be compared with the most recent experimental averages from HFAG [44][3]

$$\Delta M_d^{\text{Exp.}} = (0.510 \pm 0.004)\ \text{ps}^{-1} \,, \qquad \Delta M_s^{\text{Exp.}} = (17.69 \pm 0.08)\ \text{ps}^{-1} \,. \tag{2.36}$$

The central values agree perfectly with the standard model predictions, but due to the large theory uncertainties there is still some room for new physics effects.
The calculation of the decay rate difference relies on the HQE, which was questioned in

[3]The most precise measurements for ΔM_d [92] and ΔM_s [93] are currently obtained from the LHCb collaboration.

particular for the case of $\Delta\Gamma_s$, which is governed by the quark level decay $b \to c\bar{c}s$. In that case the energy release is substantially limited compared to a b-decay into mass-less final states and thus the expansion parameter of the HQE naively seems to be large. An explicit calculation including NLO-QCD corrections [94, 95, 96, 97] and subleading HQE corrections [98, 99] gives [50, 49]

$$\Delta\Gamma_d^{\rm HQE} = (0.0029 \pm 0.0007)\ {\rm ps}^{-1}\,, \qquad \Delta\Gamma_s^{\rm HQE} = (0.087 \pm 0.021)\ {\rm ps}^{-1}\,. \tag{2.37}$$

$\Delta\Gamma_s$ was measured for the first time in 2012 by the LHCb Collaboration [72]. The current average from HFAG [44] reads

$$\Delta\Gamma_s^{\rm Exp.} = (0.081 \pm 0.011)\ {\rm ps}^{-1}\,, \tag{2.38}$$

it includes the measurements from LHCb [29], ATLAS [100], CDF [101] and D0 [102]. Experiment and theory agree perfectly for $\Delta\Gamma_s$, excluding thus huge violations of quark hadron duality. The experimental uncertainty will be reduced in future, while the larger theory uncertainty is dominated from unknown matrix elements of dimension seven operators, see [97, 49]. Here a first lattice investigation or a continuation of the QCD sum rule study in [103, 104] would be very welcome.
$\Delta\Gamma_d$ has not been measured yet. The HFAG average [44] includes measurements from BaBar [68] and Belle [69], but there were also two investigations from D0 [70] and LHCb [45]. LHCb compared the difference in the effective lifetimes of $B_d \to J/\psi K^*$ and $B_d \to J/\psi K_S$, while D0 found that $\Delta\Gamma_d$ can give a sizable contribution [105] to the dimuon asymmetry [106, 107, 108, 70]. The different values read

$$\frac{\Delta\Gamma_d}{\Gamma_d}^{\rm HFAG} = (1.5 \pm 1.8)\,\%\,, \tag{2.39}$$

$$\frac{\Delta\Gamma_d}{\Gamma_d}^{\rm D0} = (0.50 \pm 1.38)\,\%\,, \tag{2.40}$$

$$\frac{\Delta\Gamma_d}{\Gamma_d}^{\rm LHCb} = (-4.4 \pm 2.7)\,\%\,. \tag{2.41}$$

All these bounds are compatible with the small standard model prediction [49]

$$\frac{\Delta\Gamma_d}{\Gamma_d}^{\rm HQE} = (0.42 \pm 0.08)\,\%\,, \tag{2.42}$$

but they also leave a lot of space for beyond standard model effects. It is interesting to note that the long-standing problem of the dimuon asymmetry [106, 107, 108, 70] could be solved by a large value of the $\Delta\Gamma_d$, i.e., $\Delta\Gamma_d = (6.3 \pm 1.6) \cdot \Delta\Gamma_d^{\rm SM}$. In a model-independent study [50] it was shown that large enhancements of $\Delta\Gamma_d$ do not violate any other experimental bounds, which is in contrast to the situation with $\Delta\Gamma_s$, where no enhancement being considerably larger than the hadronic uncertainties is possible, see, e.g. [109]. Therefore it would be very eligible to have more precise experimental bounds on this quantity.
There is also a third class of observables in the mixing systems, related to CP violation, the so-called flavour-specific or semi-leptonic asymmetries. They are defined as

$$a_{sl}^q = \frac{\Gamma(\bar{B}_q(t) \to f) - \Gamma(B_q(t) \to \bar{f})}{\Gamma(\bar{B}_q(t) \to f) + \Gamma(B_q(t) \to \bar{f})} = \left|\frac{\Gamma_{12}^q}{M_{12}^q}\right| \sin\phi_q\ , \tag{2.43}$$

where f denotes a flavour-specific final state - semi-leptonic states are a special case of flavour specific ones. In the standard model these asymmetries are tiny [49]

$$a_{sl}^{d,\mathrm{HQE}} = (-4.1 \pm 0.6) \cdot 10^{-4} \;, \qquad a_{sl}^{s,\mathrm{HQE}} = (+1.9 \pm 0.3) \cdot 10^{-5} \;. \tag{2.44}$$

The first measurements of the dimuon asymmetry [106, 107, 108] pointed towards a large enhancement of the semi-leptonic asymmetries. At that time the measured asymmetry A_{CP} was interpreted as having only contributions from CP violation in mixing:

$$A_{CP} \propto A_{sl}^b = C_d a_{sl}^d + C_s a_{sl}^s \;. \tag{2.45}$$

The first measurement of A_{CP} [106, 107] was a factor of 42 larger[4] than the standard model prediction in [97]. A successive measurement [108] gave a slightly smaller value, but the statistical significance of the deviation increased to 3.9 standard deviations. The findings from the D0 Collaboration can be tested by individual measurements of the semi-leptonic asymmetries, which have been performed for the B_d-system by D0 [111] and BaBar [112] and for the B_s-system by D0 [113] and LHCb [114].

$$a_{sl}^{d,\mathrm{D0}} = (+0.68 \pm 0.45 \pm 0.14)\,\% \;, \;\; a_{sl}^{s,\mathrm{D0}} = (-1.12 \pm 0.74 \pm 0.17)\,\% \;, \tag{2.46}$$

$$a_{sl}^{d,\mathrm{BaBar}} = \left(+0.06 \pm 0.17^{+0.38}_{-0.23}\right)\% \;, \;\; a_{sl}^{s,\mathrm{LHCb}} = (-0.06 \pm 0.50 \pm 0.36)\,\% \;. \tag{2.47}$$

These numbers are consistent with the standard model predictions, but because of the still sizable uncertainties they also do not exclude the large enhancement of the dimuon asymmetry. Last year Borissov and Hoeneisen [105] identified a new source contributing to the measured value of A_{CP}, leading to

$$A_{CP} \propto A_{sl}^b + C_{\Gamma_d} \frac{\Delta\Gamma_d}{\Gamma_d} + C_{\Gamma_s} \frac{\Delta\Gamma_s}{\Gamma_s} \;. \tag{2.48}$$

The contribution due to $\Delta\Gamma_s$ turns out to be negligible, but even the tiny standard model value of $\Delta\Gamma_d^{\mathrm{SM}}$ gives a sizable share. Investigating different regions for the muon impact parameter separately, it is possible to extract individual values for a_{sl}^d, a_{sl}^s and $\Delta\Gamma_d$ from the D0 measurements [70]:

$$a_{sl}^{d,\mathrm{D0}} = (-0.62 \pm 0.43)\% \;, \quad a_{sl}^{s,\mathrm{D0}} = (-0.82 \pm 0.99)\% \;, \quad \frac{\Delta\Gamma_d}{\Gamma_d}^{\mathrm{D0}} = (0.50 \pm 1.38)\% \,. \tag{2.49}$$

This result differs from the combined standard model expectation for the three observables by $3.0\,\sigma$. If one instead assumes that the semi-leptonic asymmetries a_{sl}^d and a_{sl}^s are given by their standard model values, then the decay rate difference $\Delta\Gamma_d$ measured by [70] using (2.48) is

$$\frac{\Delta\Gamma_d}{\Gamma_d}^{\mathrm{D0}} = (2.63 \pm 0.66)\% \;, \tag{2.50}$$

which differs by $3.3\,\sigma$ from the SM prediction.

[4]See e.g., [110] for the profound implications of this enhancement factor.

2.2.5 Mixing upshot

The mixing observables ΔM_d, ΔM_s and $\Delta\Gamma_s$ in the B-system agree well with the standard model predictions. Because of some sizable hadronic uncertainties there is still plenty of room for new physics effects. In the case of the semi-leptonic asymmetries the current experimental values are compatible with the tiny standard model expectations, but the uncertainties of the measurements are still one (a_{sl}^d) to two orders (a_{sl}^s) of magnitude larger than the central values of the standard model predictions. The longstanding discrepancy in the dimuon asymmetry might point towards some new physics effects in a_{sl}^d, a_{sl}^s and $\Delta\Gamma_d$, a possibility that is currently not excluded by any other experimental constraint.
In the charm system similar statements cannot be made because of the largely unknown size of the standard model contribution. Here an improvement in our theoretical understanding is very desirable. As long as this does not happen, it is not excluded that new physics was already occuring in D-mixing and we simply could not identify it.

2.3 Inclusive decays

Inclusive quark decays rely on the same theoretical footing as the lifetimes, the HQE, which seems to be well tested now. These decays are experimentally difficult to study, but they might enable searches for hidden decay channels of heavy hadrons, see, e.g. [115].

2.3.1 Theory

NLO-QCD corrections turned out to be crucial for the inclusive b-quark decays, see, e.g. [116]. They were determined for $b \to cl^-\bar{\nu}$ already in 1983 [117], for $b \to c\bar{u}d$ in 1994 [118], for $b \to c\bar{c}s$ in 1995 [119], for $b \to$ no charm in 1997 [120] and for $b \to sg$ in 2000 [121, 122]. Since there were several misprints in [119] - leading to IR divergent expressions -, the corresponding calculation was redone in [115] and the numerical result was updated by using modern input parameters.[5] NNLO corrections for semi-leptonic decays have been calculated in [123, 124, 125, 126, 127, 128] and some first investigations for non-leptonic decays were done in [129]. A complete NNLO-QCD study of the non-leptonic decays seems to be doable now, also because the $\Delta B = 1$-Wilson coefficients are known at NNLO precision [130].

2.3.2 Semi-leptonic and radiative decays

Inclusive semi-leptonic decays can be used for the determination of the CKM elements V_{cb} and V_{ub}, see the PDG [42] article *Semi-leptonic B meson decays and the determination of V_{cb} and V_{ub}*. The current values for these CKM elements read [42]

$$V_{cb}^{\text{Inclusive}} = (42.4 \pm 0.9) \cdot 10^{-3} , \qquad V_{ub}^{\text{Inclusive}} = (4.41 \pm 0.15^{+0.15}_{-0.17}) \cdot 10^{-3} . \tag{2.51}$$

These values are larger than the values obtained by investigating exclusive decays like $\bar{B} \to D^* l \bar{\nu}_l$ or $\bar{B} \to \pi l \bar{\nu}_l$ where one gets [42]

$$V_{cb}^{\text{Exclusive}} = (39.5 \pm 0.8) \cdot 10^{-3} , \qquad V_{ub}^{\text{Exclusive}} = (3.23 \pm 0.31) \cdot 10^{-3} . \tag{2.52}$$

[5] The authors of [119] left particle physics and it was not possible to obtain the correct analytic expressions. The numerical results presented in [119] were, however, correct.

Currently it is not clear what the origin of this longstanding discrepancy is, see, e.g. [42] for the discussion of experimental issues and problems related to estimating the hadronic uncertainties, but also for some ideas how new physics could be responsible for the shift. A more recent experimental investigation at Belle [131] in 2013 yielded the result

$$V_{ub}^{\text{Exclusive}} = (3.52 \pm 0.29) \cdot 10^{-3} \,. \tag{2.53}$$

A further related observable is the semi-leptonic branching ratio. Its standard model value reads [115]

$$Br_{sl}^{\text{HQE}} = \frac{\Gamma(b \to c e^- \bar{\nu}_e)^{\text{HQE}}}{\Gamma_{tot}^{\text{HQE}}} = (11.6 \pm 0.8)\,\% \,, \tag{2.54}$$

which can be compared to the following experimental values

$$Br_{sl}(B_d)^{\text{Exp.}} = (10.33 \pm 0.28)\,\% \,, \tag{2.55}$$

$$Br_{sl}(B^+)^{\text{Exp.}} = (10.99 \pm 0.28)\,\% \,, \tag{2.56}$$

$$Br_{sl}(B_s)^{\text{Exp.}} = (10.61 \pm 0.89)\,\% \,, \tag{2.57}$$

where the first two values are taken from the PDG [42] and the value for B_s is from [132]. These numbers agree well with the theory prediction, which will be probably affected notably by the inclusion of NNLO-QCD effects.
Finally we would like to mention the penguin induced decay $b \to s\gamma$ (similar to the diagram shown in Fig. 4), that is quite well measured [44]

$$Br(b \to s\gamma)^{\text{HFAG 2013}} = (3.43 \pm 0.21 \pm 0.07) \cdot 10^{-4} \tag{2.58}$$

and agrees well with the standard model prediction of [133]

$$Br(b \to s\gamma)^{\text{Theory}} = (3.15 \pm 0.23) \cdot 10^{-4} \,. \tag{2.59}$$

This decay gives serious constraints on different extensions of the standard model, like Two-Higgs-Doublet models or Supersymmetry.

2.3.3 Non-leptonic decays

Non-leptonic inclusive decays are not well studied experimentally, they might, however, be interesting for searching for new effects in a model and even decay channel independent way, see, e.g. [115]. The updated theory predictions read for $b \to c$ transitions

$$Br(b \to c\bar{u}d)^{\rm HQE} = 0.446 \pm 0.014 \,, \tag{2.60}$$

$$Br(b \to c\bar{c}s)^{\rm HQE} = 0.232 \pm 0.007 \,, \tag{2.61}$$

$$Br(b \to ce\bar{\nu}_e)^{\rm HQE} = 0.116 \pm 0.008 \,, \tag{2.62}$$

$$Br(b \to c\mu\bar{\nu}_\mu)^{\rm HQE} = 0.116 \pm 0.008 \,, \tag{2.63}$$

$$Br(b \to c\tau\bar{\nu}_\tau)^{\rm HQE} = 0.027 \pm 0.001 \,, \tag{2.64}$$

$$Br(b \to c\bar{u}s)^{\rm HQE} = 0.024 \pm 0.001 \,, \tag{2.65}$$

$$Br(b \to c\bar{c}d)^{\rm HQE} = 0.0126 \pm 0.0005 \,, \tag{2.66}$$

$$Br(b \to u\bar{u}d)^{\rm HQE} = 0.0063 \pm 0.0018 \tag{2.67}$$

and for subleading $b \to u$-transitions or penguins (see Fig. 4)

$$Br(b \to sg)^{\rm HQE} = 0.0050 \pm 0.0009 \,, \tag{2.68}$$

$$Br(b \to u\bar{c}s)^{\rm HQE} = 0.0043 \pm 0.0012 \,, \tag{2.69}$$

$$Br(b \to u\bar{u}s)^{\rm HQE} = 0.0024 \pm 0.0012 \,, \tag{2.70}$$

$$Br(b \to d\bar{d}s)^{\rm HQE} = 0.0022 \pm 0.0011 \,, \tag{2.71}$$

$$Br(b \to s\bar{s}s)^{\rm HQE} = 0.0018 \pm 0.0009 \,, \tag{2.72}$$

$$Br(b \to ue\bar{\nu}_e)^{\rm HQE} = 0.0017 \pm 0.0005 \,, \tag{2.73}$$

$$Br(b \to u\mu\bar{\nu}_\mu)^{\rm HQE} = 0.0017 \pm 0.0005 \,, \tag{2.74}$$

$$Br(b \to u\tau\bar{\nu}_\tau)^{\rm HQE} = 0.0006 \pm 0.0002 \,, \tag{2.75}$$

$$Br(b \to dg)^{\rm HQE} = 0.00024 \pm 0.00010 \,, \tag{2.76}$$

$$Br(b \to u\bar{c}d)^{\rm HQE} = 0.00023 \pm 0.00007 \,, \tag{2.77}$$

$$Br(b \to s\bar{s}d)^{\rm HQE} = 0.00009 \pm 0.00006 \,, \tag{2.78}$$

$$Br(b \to d\bar{d}d)^{\rm HQE} = 0.00008 \pm 0.00005 \,. \tag{2.79}$$

2.3.4 Inclusive upshot

The theory of inclusive decays is theoretically quite solid. There is, however, the longstanding discrepancy in the extraction of the CKM elements V_{ub} and V_{cb}, which has to be settled by further experimental and theoretical investigations. Non-leptonic inclusive decays might provide a complementary testing ground for beyond standard model effects. Here any experimental investigation would be very welcome. On the theory side the extension to NNLO-QCD seems to be worthwhile and doable.

3 Exclusive Decays

We present here certain exclusive decays, that seem to be very promising in searching for new physics effects or determining standard model parameters. We start with leptonic decays, that have the simplest hadronic structure, because they only depend on a decay constant. Next we discuss semi-leptonic decays that depend on form factors and finally we briefly discuss non-leptonic decays, where some additional assumptions, like QCD factorisation [134, 135, 136], have to be made in order to describe them theoretically.

3.1 Leptonic decays

The decay $B \to \tau\nu$ proceeds in the standard model via an annihilation into a W-boson. If there exists, e.g. an extended Higgs sector, the W-boson could simply be replaced by a charged Higgs-boson. For quite some time the experimental value of the corresponding branching ratio was about three standard deviations above the theory prediction [15] (see also [16] for similar results) of

$$Br(B^+ \to \tau^+ \nu_\tau)^{\rm SM} = (0.739^{+0.091}_{-0.071}) \cdot 10^{-4} \, . \tag{3.80}$$

However, a new measurement at Belle [137] found, using a hadronic tagging method

$$Br(B^+ \to \tau^+ \nu_\tau)^{\rm Belle} = (0.72^{+0.29}_{-0.27}) \cdot 10^{-4} \, , \tag{3.81}$$

which is now perfectly consistent with the standard model expectation. An independent confirmation of this result would be very helpful. The current world average reads [44]

$$Br(B^+ \to \tau^+ \nu_\tau)^{\rm HFAG\,2013} = (1.14 \pm 0.22) \cdot 10^{-4} \, , \tag{3.82}$$

which is still larger than the theory prediction. A detailed discussion of B-meson decays into final states with a τ-lepton can be found in [138].
The decay $B_s \to \mu^+\mu^-$ proceeds in the standard model on the loop-level, either via penguins or via a box diagram. It is thus also perfectly suited to search for new physics effects. Very recently the theory prediction was updated [139] including NNLO-QCD corrections to obtain

$$\bar{Br}(B_s \to \mu^+\mu^-)^{\rm SM} = (3.65 \pm 0.23) \cdot 10^{-9} \, , \tag{3.83}$$

$$\bar{Br}(B_d \to \mu^+\mu^-)^{\rm SM} = (1.06 \pm 0.09) \cdot 10^{-10} \, . \tag{3.84}$$

$\bar{Br}$ denotes the average time-integrated branching ratio that includes effects of a finite value of $\Delta\Gamma_q$ [140]. The current experimental numbers read [141]

$$\bar{Br}(B_s \to \mu^+\mu^-)^{\rm Exp.} = (2.9 \pm 0.7) \cdot 10^{-9} \, , \tag{3.85}$$

$$\bar{Br}(B_d \to \mu^+\mu^-)^{\rm Exp.} = \left(3.6^{+1.6}_{-1.4}\right) \cdot 10^{-10} \, , \tag{3.86}$$

which are averages from the CMS value [142] and the LHCb value [143]. Standard model and experiment agree for the measured B_s-decay, but there is still room for substantial deviations,

due to the large experimental uncertainties. The current bound on the B_d-decay is higher than the standard model expectation, but here we have to wait for future more precise measurements to see, whether there are some first hints of new physics in these decays or not. Already the current experimental precision gives some interesting constraints on 2HDM models or SUSY-models, in particular in the large $\tan\beta$-region.
In [139] also the theory predictions of interesting and experimentally almost unexplored decays like $B_q \to \tau^+\tau^-$ were updated.

$$\bar{B}r(B_s \to \tau^+\tau^-)^{\mathrm{SM}} = (7.73 \pm 0.49) \cdot 10^{-7} , \tag{3.87}$$

$$\bar{B}r(B_d \to \tau^+\tau^-)^{\mathrm{SM}} = (2.22 \pm 0.19) \cdot 10^{-8} . \tag{3.88}$$

These decays are very helpful for new physics searches, see, e.g. [50, 109] and they are currently quite unconstrained. For $B_s \to \tau^+\tau^-$ no direct bound exists at all and for $B_d \to \tau^+\tau^-$ there is a weak bound from BaBar [144] (at 90% C.L.)

$$\bar{B}r(B_d \to \tau^+\tau^-)^{\mathrm{BaBar}} < 4.1 \cdot 10^{-3} . \tag{3.89}$$

3.2 Semi-leptonic decays

Exclusive, semi-leptonic B-meson decays are crucial for the determination of V_{cb} and V_{ub}, which was already discussed in the Section 2.3.2. For this purpose one investigates decays with electrons or muons in the final states. Having instead τ-leptons, e.g. in $B \to \bar{D}^{(*)}\tau^+\nu_\tau$[6], one finds some deviations between experiment and theory. Usually the ratios

$$R(D^{(*)}) = \frac{\Gamma(B \to \bar{D}^{(*)}\tau^+\nu_\tau)}{\Gamma(B \to \bar{D}^{(*)}l^+\nu_l)} \tag{3.90}$$

are investigated, with l denoting e or μ. The theory prediction reads [145]

$$R(D)^{\mathrm{SM}} = 0.296 \pm 0.016 , \tag{3.91}$$

$$R(D^*)^{\mathrm{SM}} = 0.252 \pm 0.003 . \tag{3.92}$$

BaBar measured in 2012 [146, 147] the following values

$$R(D)^{\mathrm{BaBar}} = 0.440 \pm 0.058 \pm 0.042 , \tag{3.93}$$

$$R(D^*)^{\mathrm{BaBar}} = 0.332 \pm 0.024 \pm 0.018 , \tag{3.94}$$

which differ sizably from the standard model expectation. Unfortunately there exists no recent number from Belle. Updating the 2010 values from [148] the analysis in [147] finds

$$R(D)^{\mathrm{Belle}} = 0.34 \pm 0.10 \pm 0.06 , \tag{3.95}$$

$$R(D^*)^{\mathrm{Belle}} = 0.43 \pm 0.06 \pm 0.06 , \tag{3.96}$$

[6]Here we consider both $B_d \to D^{-(*)}\tau^+\nu_\tau$ and $B^+ \to \bar{D}^{0(*)}\tau^+\nu_\tau$ and our final results are averages of the two possibilities. For semi-leptonic decays the $B \to \bar{D}^*$-transition is roughly two times as common as the $B \to \bar{D}$ one.

Here more data will be necessary to clarify this situation. A detailed discussion of the experimental situation can be found in [138]. On the theory side, there was an ab-initio lattice calculation in [149], which obtained

$$R(D)^{\mathrm{Lattice}} = 0.316 \pm 0.12 \pm 0.07\,. \tag{3.97}$$

A similar result $R(D) = 0.31 \pm 0.02$ was obtained in [150], being still lower than the experimental number.
There is a second class of semi-leptonic decays that triggered a lot of interest: $B \to K^{(*)}\mu^+\mu^-$. In contrast to $B \to \bar{D}^{(*)}\tau^+\nu_\tau$, which is a tree-level decay in the standard model, the decay $B \to K^{(*)}\mu^+\mu^-$ is triggered by a $b \to s\mu^+\mu^-$-penguin or box diagram, as the decay $B_s \to \mu^+\mu^-$.
Using $1fb^{-1}$ of data, LHCb measured in 2005 [151] a 4.4 σ deviation of the isospin asymmetry A_I, defined as

$$A_I = \frac{Br(B_d \to K^0\mu^+\mu^-) - \frac{\tau(B_d)}{\tau(B^+)}Br(B^+ \to K^+\mu^+\mu^-)}{Br(B_d \to K^0\mu^+\mu^-) + \frac{\tau(B_d)}{\tau(B^+)}Br(B^+ \to K^+\mu^+\mu^-)}\,, \tag{3.98}$$

from the tiny standard model prediction [152, 153, 154]. In 2014 this measurement was updated [155] with the full data set of $3fb^{-1}$ and the deviation disappeared (it was reduced to 1.5 standard deviations). On the other hand, the measured branching fractions of the four $B \to K^{(*)}\mu^+\mu^-$ decays [155, 156] and the decay $B_s \to \phi\mu^+\mu^-$ [157] have all lower values than the standard model expectations [158, 159].
The same large data set was used in [160] to perform an angular analysis of charged and neutral $B \to K\mu^+\mu^-$ decays and no deviation from the small standard model expectations [161, 162, 163, 164, 165] was found.
The angular analysis of the decay $B \to K^{0*}\mu^+\mu^-$ was published in 2013 [166] with a data set of $1fb^{-1}$. This decay can be expressed in terms of the eight form factor like parameters F_L, S_{3-9} [167]. In [168] it was suggested to use instead the parameters $P'_j = S_j/\sqrt{F_L(1-F_L)}$ for $j = 4, 5, 6, 8$, because some hadronic contributions cancel. LHCb measured [166] the four parameters P'_j in six different q^2-bins and from the 24 measurements 23 agreed with the standard model, while the 24th one, related to P'_5, deviated by 3.7 σ. This discrepancy triggered a lot of theoretical interest, see, e.g. [169, 170, 171, 172, 173, 174, 175, 176, 177]. Future investigations of the hadronic uncertainties as well as the results of the $3fb^{-1}$ data set will give further clues.

3.3 Non-leptonic decays

Hadronic $B \to DK$-decays can be used to extract the CKM angle γ directly, see, e.g., the GLW-method [178, 179], the ADS-method [180, 182] or the GGSZ-method [181] . These methods provide a clean consistency check of the CKM picture with decays that proceed only via tree-level and are thus expected to be less sensitive to new physics effects. Currently values from LHCb [183], BaBar[184] and Belle [185] are available

$$\gamma^{\mathrm{LHCb}} = \left(72.0^{+14.7}_{-15.6}\right)^\circ\,, \tag{3.99}$$

$$\gamma^{\mathrm{BaBar}} = \left(69^{+17}_{-16}\right)^\circ\,, \tag{3.100}$$

$$\gamma^{\mathrm{Belle}} = \left(68^{+15}_{-14}\right)^\circ\,, \tag{3.101}$$

which can be compared with the CKM-fit result [15, 16]

$$\gamma^{\text{CKMfitter}} = \left(69.7^{+1.3}_{-2.8}\right)^{\circ} \ . \tag{3.102}$$

These numbers agree, but the precision of the direct determination is not yet comparable to the indirect one. Here it will be very interesting to see what happens, when the experimental precision is improving.
The angle β can be obtained by studying the decay $B_d \to J/\psi K_S$. In contrast to the previous case, which was dominated by tree-level contributions, the dependence on the CKM-angle β arises from the interference between mixing and decay and is thus related to a loop process. The values for β from LHCb [186], BaBar [187] and Belle [188] read

$$\sin 2\beta^{\text{LHCb}} = 0.73 \pm 0.07 \pm 0.04 \ , \tag{3.103}$$

$$\sin 2\beta^{\text{BaBar}} = 0.687 \pm 0.028 \pm 0.012 \ , \tag{3.104}$$

$$\sin 2\beta^{\text{Belle}} = 0.667 \pm 0.023 \pm 0.012 \ , \tag{3.105}$$

which can again be compared with the CKM-fit result [15, 16]

$$\sin 2\beta^{\text{CKMfitter}} = 0.775^{+0.020}_{-0.049} \ . \tag{3.106}$$

This deviation caused some discussion in the literature, see, e.g., [189] and references therein. It might be related to new physics in B_d-mixing and/or to the extraction of V_{ub}.
The related decays $B_s \to J/\psi K^+K^-, J/\psi\pi^+\pi^-, ...$ can be used to extract the mixing phase β_s in the B_s-system, which is predicted to be very small in the standard model [15, 16].

$$\beta_s^{\text{CKMfitter}} = 0.01821^{+0.00081}_{-0.00079} \ . \tag{3.107}$$

Using $1 fb^{-1}$ of data LHCb found [29]

$$\beta_s^{\text{LHCb}} = 0.01 \pm 0.07 \pm 0.01 \ . \tag{3.108}$$

Both numbers are consistent, but the experimental uncertainty is still considerably larger than the theoretical one. The phase β_s should not be mixed up, with the mixing phase ϕ_s defined below Eq.(2.28), see, e.g. the "note added" in [190, 191].
Finally we would like to briefly discuss direct CP violation in hadronic D-meson decays - see [32] and references therein for a more detailed discussion. ΔA_{CP} is defined as the difference of the CP asymmetries of a neutral D-meson decaying into KK and $\pi\pi$ final states.

$$\Delta A_{CP} := A_{CP}(D^0 \to K^+K^-) - A_{CP}(D^0 \to \pi^+\pi^-) \ . \tag{3.109}$$

The first measurements in 2011 [30, 192, 193] gave a combined value of

$$\Delta A_{CP} = -0.678 \pm 0.147\% \, . \tag{3.110}$$

Such a large value was quite unexpected in the standard model. LHCb performed, however, subsequent measurements where the significance went down [194] and also some which resulted in a different sign [31]. Taking these new numbers into account the new combination turns out to be [44]

$$\Delta A_{CP} = -0.329 \pm 0.121\% \, . \tag{3.111}$$

The statistical significance for CP violation in hadronic D decays went now down considerably, but the central value is still larger than to be expected naively in the standard model. Here clearly further experimental input is needed to settle this issue.

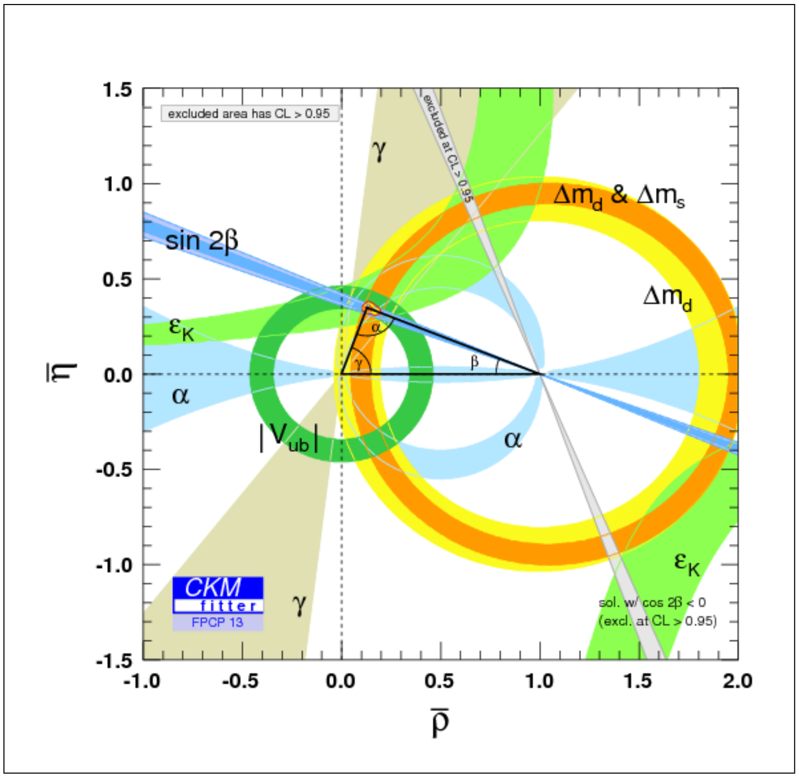

Figure 5: The current status of the CKM fit taken from [15], similar results can be obtained from [16].

4 Consequences for new physics models

A general test of the consistency of the CKM picture is provided by the usual fit of the unitarity triangle, see, e.g., [15] and [16]. Here observables like V_{ub}, ΔM_d, ΔM_s, $\sin 2\beta$ and CP-violation in the Kaon system, ϵ_K, are included. As can be seen from Fig. 5 the currently available amount of flavour data is very well compatible with the CKM paradigm. Nevertheless, this does not exclude the possibility of having sizable new physics contributions in the flavour sector, which will be investigated below.

4.1 Model independent search for new physics

There are different ways of performing model independent searches for new physics effects. Mixing seems to be a promising place to search for beyond standard model effects, because it is a loop effect. In [189] and [195] new physics effects in mixing were estimated under the assumption of having only considerable effects in mixing, in M_{12}, while the tree-level decay amplitudes are dominated by standard model contributions, i.e. the relation between the *true* values of M_{12} and Γ_{12} and their standard model counterparts $M_{12}^{\rm SM}$ and $\Gamma_{12}^{\rm SM}$ reads

$$M_{12}^q = M_{12}^{q,\rm SM} \Delta_q \, , \tag{4.112}$$

$$\Gamma_{12}^q = \Gamma_{12}^{q\rm SM} \, , \tag{4.113}$$

where Δ is an arbitrary complex number, encoding the new physics contribution. This assumption corresponds also to neglecting new penguin contribution in the decays $B_d \to$

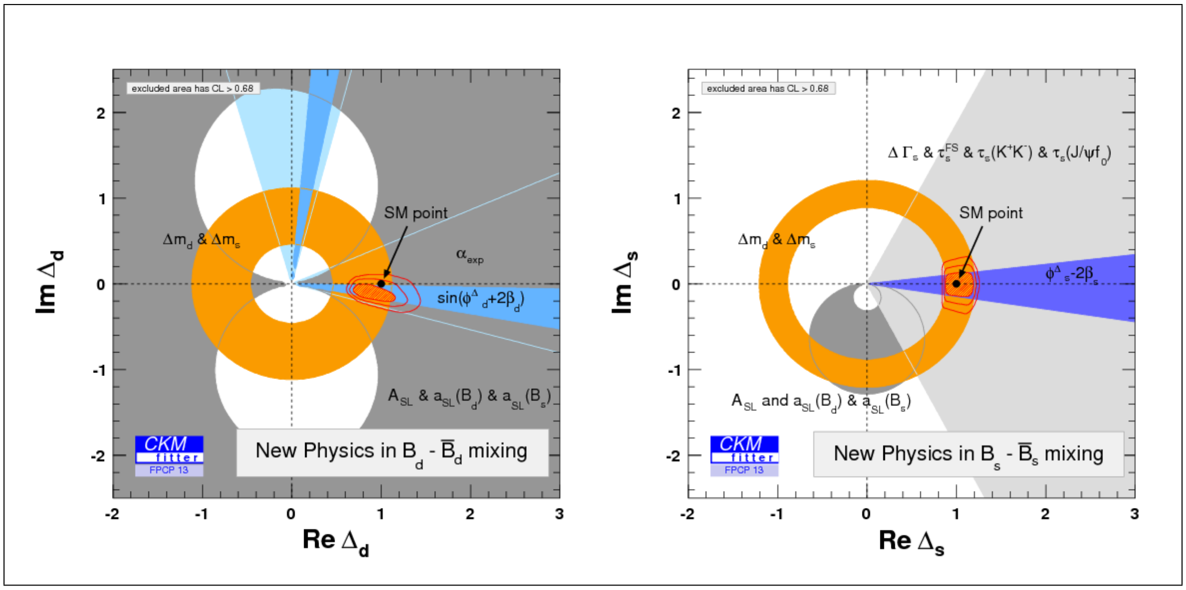

Figure 6: Allowed space for new physics effects in B_d- and B_s-mixing. The figures are taken from [15], they are an update of [195].

$J/\psi K_S$ and $B_s \to J/\psi\phi$ and thus the values for β and β_s give information on the mixing phase $\phi_q = \arg(-M_{12}^q/\Gamma_{12}^q) = \phi_q^{\rm SM} + \phi_q^\Delta$. Combing all available data till 2013 for the B_d-system one gets the bounds shown on the left hand side of Fig. 6, while the bounds on the B_s-system are displayed on the right hand side of Fig. 6. In the B_d-system the fit prefers a negative value of the phase of Δ_d, but the deviation from the standard model is less than 2 σ. On the other hand values of ϕ_d^Δ of about $-10°$, which would be a quite large new physics contributions, are clearly not ruled out. In the B_s-system the fit prefers the standard model value, but it leaves also space for considerable deviations. In that case up to $\pm 20°$ are not ruled out yet.
Similar studies were performed for the Wilson coefficients C_7, C_9 and C_{10} of $b \to s$-penguins transitions. In [169] P_5' could be explained by a negative shift in C_9, while C_7 stays standard model like, see e.g., Fig. 7. Similar, but sometimes less pronounced results, i.e., better

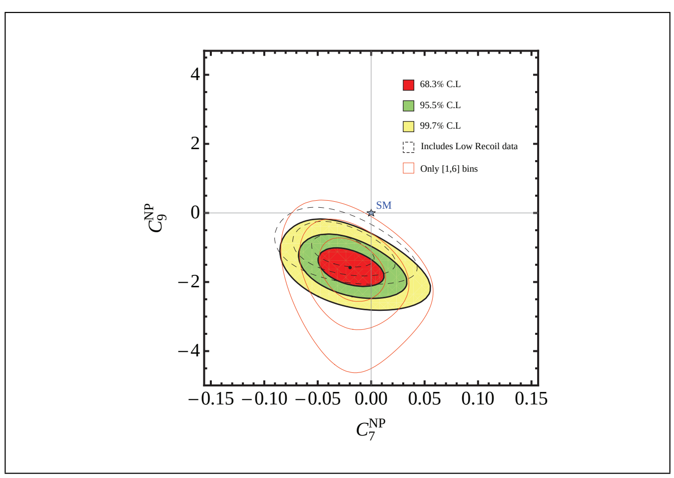

Figure 7: Constraints on the new physics contributions to the Wilson coefficients C_7 and C_9 from penguin induced $b \to s$ transitions. The figure is taken from [169].

consistent with the standard model, were found, e.g. in [170, 172, 175, 176].
A complementary study was performed in [50]. Here the space of new physics effects in the Wilson coefficients C_1 and C_2 of tree-level decays like $b \to c\bar{u}d$, $b \to c\bar{c}d$, $b \to u\bar{c}d$ and $b \to u\bar{u}d$ was investigated and it was found that notable deviations from the standard model expectations are still possible, see Fig. 8 for the bounds on C_1 and C_2 for the case of $b \to u\bar{u}d$-transitions.

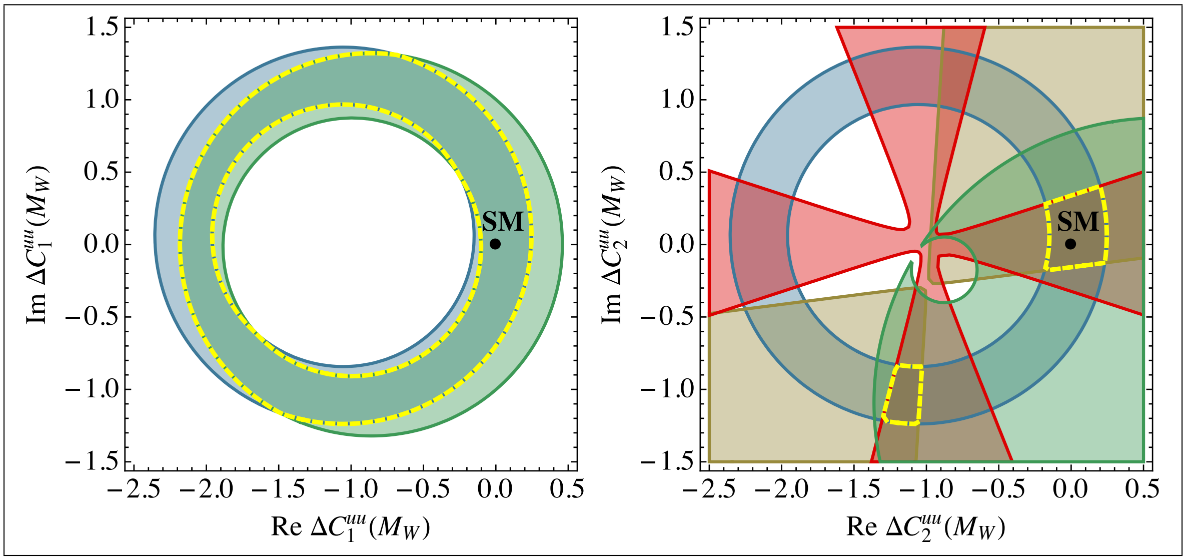

Figure 8: Allowed space for new physics effects in the Wilson coefficients C_1 and C_2 of the current-current operators for the case of $b \to u\bar{u}d$-transitions. The figures are taken from [50].

4.2 Decay channel independent search for new physics

An even more general BSM search strategy might be provided by the study of inclusive non-leptonic decays in the spirit of the missing charm puzzle [202]. This was advocated again in [115]. Comparing experiment and theory for partially summed branching ratios, like $Br(b \to \text{no charm})$ one could get information on all invisible decay modes, e.g., $B \to \tau\tau$ or even more fancy possibilities like an invisible decay into light dark matter particles. The latest experimental studies in that direction, a determination of n_c - the average number of charm quarks per b-decay - date back to 2006 [203]. Non-leptonic inclusive decays might also gain some insight into CP violation, see, e.g. [204].

4.3 Model dependent search for new physics

There exists an enormous amount of literature about model dependent searches for new physics effects in flavour observables. A corresponding discussion is beyond the scope of this review.

5 Conclusion

We have discussed a selected choice of topics in flavour physics in order to give an idea of the current status of the field. For different choices of topics see, e.g. the reviews [196, 197, 198, 199, 200, 201]. Splitting up the current investigations in three areas: testing the theoretical tools, determining standard model parameters and search for new physics we come to the following conclusions:

1. Testing of our theoretical tools: Applying the HQE for lifetimes of b- and c-hadrons as well as for $\Delta\Gamma_s$ gives very promising results and there is no huge space for violations of duality anymore. Old discrepancies like the Λ_b-lifetime have finally been settled experimentally, unknown quantities like $\Delta\Gamma_s$ have been measured for the first time in perfect agreement with theory and it looks like even D-meson lifetimes might be described by the HQE. Since it is clear now that the HQE works, the new question is, how precise is the HQE. To answer that, lattice results for many of the arising observables are urgently needed.
   There are, however, also some areas where it is not clear yet, how to describe them in theory. For D-mixing it might be worthwhile to push the HQE to its limits and determine dimension-nine and dimension-twelve contributions. For more complicated problem like ΔA_{CP} it is almost unclear, how to proceed, although there are some interesting ideas related to the lattice, see e.g. [205]

2. Determining standard model parameters: in this field a huge progress has been made, as can be seen, e.g. in Fig. 5 or in the current precision of the CKM-element V_{cb}. Among many other observables, future measurements of γ will provide a clean cross-check of the CKM-picture.

3. Search for new physics: here we have three results: a) most observables are standard model like, b) there is nevertheless still a lot of space for effects beyond the standard model and c) there is still a notable number of remaining discrepancies.

   (a) Most observables are standard model like:
   This should actually not be a source of disappointment, since it is an amazing success of our theory. Complicated loop observables like ΔM_q, $\Delta\Gamma_s$ or $b \to s\gamma$ are described very well by the standard model. Even very rare decays like $B_s \to \mu^+\mu^-$ have been predicted many years before their discovery (e.g., Buras quoted 1998 [206] in his Les Houches Lectures a value of $(3.4 \pm 1.2) \cdot 10^{-9}$). This is a real impressive success of the standard model.

   (b) Despite being standard model-like, there is still a lot of room for new physics in observables like $B_s \to \mu^+\mu^-$, Br_{sl}, ΔM_q, $\Delta\Gamma_d$, a_{sl}^q, β_s and β. Here more precise measurements as well as more precise theoretical studies will either shrink further the space for new physics effects or find the first convincing hints.

   (c) There are still remaining discrepancies at the 3 σ-level, e.g., V_{ub}, the dimuon asymmetry, $Br(B^+ \to \tau^+\nu_\tau)$, $R(D^{(*)})$, $Br(B_d \to \mu^+\mu^-)$, $Br(B \to K^{(*)}\mu^+\mu^-)$, $Br(B_s \to \phi\mu^+\mu^-)$, P_5', ΔA_{CP}, β,... Here again more precise measurements as well as more precise theoretical studies will shed light on the origin of these discrepancies.

For future searches we also would like to stress some lesser known, but nevertheless promising observables like $B \to \tau^+\tau^-$, $\Delta\Gamma_d$ and inclusive non-leptonic decays.

So we are heading towards exciting, but strenuous times in flavour physics.

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