PHYSICAL REVIEW D **89,** 125018 (2014)

Novel Higgs potentials from gauge mediation of exact scale breaking

Steven Abel and Alberto Mariotti

Institute for Particle Physics Phenomenology, Durham University,
South Road, Durham DH1 3LE, United Kingdom

(Received 10 January 2014; revised manuscript received 9 May 2014; published 24 June 2014)

We present a gauge mediation principle for beyond standard model theories where *exact* UV scale invariance is broken in a hidden sector. The relevant configurations are those in which the standard model and a hidden sector emanate from a scale invariant pair of UV theories that communicate only via gauge interactions. We compute the radiatively induced Higgs potential which contains logarithmic mass-squared terms that lead to unusual Higgs self-couplings. Its other couplings are unchanged.

DOI: 10.1103/PhysRevD.89.125018 PACS numbers: 12.60.Fr, 11.30.Qc, 12.60.Cn, 12.60.Jv

I. INTRODUCTION AND OVERVIEW

The existence in nature of mass scales that are much smaller than the Planck scale suggests an additional symmetry at work in the standard model (SM). Lately the old idea that the relative lightness of the Higgs boson has something to do with scale invariance [1–4] has been gaining some ground [5–41].

It has to be said that scale invariance is not an obvious candidate for a symmetry to protect the Higgs boson, because the standard model has scale anomalies. In the absence of a Higgs mass the theory is only "classically scale invariant" (which is to say that it isn't), and anomalous symmetries only buy a loop of protection. Of course scaling anomalies are precisely the starting point of the Coleman-Weinberg mechanism [2,3], but whether or not classical scale invariance can have a well-defined meaning in a UV complete theory is still a matter for debate [42].

The notion of scale invariance is clearly on firmer footing if the SM ultimately emanates from a theory that is *exactly* scale invariant in the ultraviolet (UV). This is a fixed point of the renormalization group (RG) which means that the theory stops running at very high energies. The couplings and anomalous dimensions will change as the theory flows towards the infrared (IR), but the fact that it flows out of a UV fixed point renders all UV divergences harmless. This idea was pioneered in the context of asymptotic safety [43] (for a recent review, see [44]), and may have relevance in other scenarios including a number that address the hierarchy problem (for example [45–49]). General aspects of this idea were recently discussed in [30].

Unfortunately, finding calculable predictions in such theories is generally difficult because it is not possible to follow the renormalization group flow analytically from the UV fixed point all the way to the IR. However, here we will present one situation that *can* give perturbatively calculable results, and we will calculate the resulting Higgs potential which deviates substantially from the SM one. We suppose first that at the UV fixed point there are additional degrees of freedom that communicate to the SM only through its gauge couplings. Second, we assume that they dominate the RG flow away from the fixed point, while the SM components and gauge couplings remain almost stationary.

This type of situation is referred to as a "saddle point," and the previous statement amounts to saying that the UV fixed point is "attractive" in the SM directions and "repulsive" in the directions of the additional degrees of freedom. As we shall see, such systems can easily be constructed by sewing together two theories, an SM augmented so that it has an attractive (IR-stable) fixed point, and another theory that has a repulsive (UV-stable) fixed point. The latter we will refer to as the "hidden sector."

One expects that the dominant RG flow in the hidden sector can lead to spontaneous breaking of scale invariance. In other words it causes some scalar to acquire a vacuum expectation value (VEV), f_c. This breaking may arise from radiative terms, or it may arise in the context of some scale invariant UV complete theory, for example Nambu–Jona-Lasinio models of strongly coupled fermionic systems [45,50]. As there are only gauge couplings between these degrees of freedom and the SM, the spontaneous breaking of scale breaking leaks through to the SM in a controlled way. In particular it yields calculable and novel results for the Higgs potential.

This configuration, depicted in Fig. 1, is tractable because it is modular, in the sense that gauge mediation of supersymmetry breaking (GMSB) is modular (for a review, see [52]), and therefore we refer to the framework as gauge mediated exact scale breaking (GMESB). Its principle can be stated as follows.

The SM resides in an extended conformal theory in the UV. It couples only via gauge interactions to a sector that initiates flow away from the fixed point and spontaneously breaks scale invariance at a scale, f_c.

Readers familiar with spontaneously broken scale invariance will recognize the field χ getting the VEV $\langle\chi\rangle = f_c$ as its Goldstone mode, namely the dilaton. Ultimately, since the Higgs boson itself gets a (much smaller) VEV, the *actual* dilaton would be composed mainly of χ with a small admixture of Higgs bosons. Therefore, the problems that

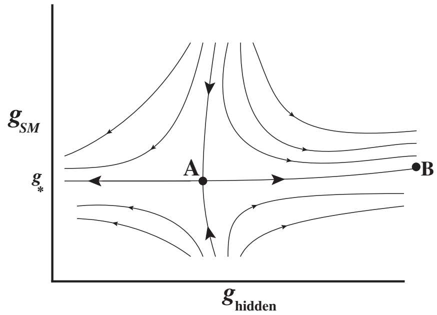

FIG. 1. Schematic depiction of the UV fixed-point structure required for GMESB. The SM couplings $g_{\rm SM}$ and the anomalous dimension are attracted to the fixed point at A, while the additional (hidden) sector to which it couples only via gauge interactions initiates a flow to point B where it spontaneously breaks scale invariance.

plague the usual Higgs-as-dilaton idea [8,51] do not apply. In particular, it would have all the same couplings to the other particles of the SM—although as we shall see its self-couplings are generally different.

The obvious example of such a system would be if the SM gauge couplings ran to zero at a Gaussian UV fixed point. In this case the Yukawa couplings and Higgs self interactions would also run to zero, and the SM would just become a free field theory in the UV. In this case, UV divergent contributions to the Higgs mass would be tamed by the mediation itself flowing to zero in the UV. (This is in essence the approach discussed in [53] and evoked in the discussion of [19].) This case would be difficult to treat analytically.

The case of interest here is rather easier to analyze: it has a UV fixed point that is interacting, with nonzero couplings and anomalous dimensions. When $\langle\chi\rangle \to 0$, scale invariance is broken only by the Higgs VEV which in that limit becomes the true dilaton of the theory. All the couplings in the SM remain at their UV fixed-point values, g_*, and therefore the Higgs mass itself is zero to all orders. Once $\langle\chi\rangle = f_c$ is turned on and becomes dominant (so that the Higgs boson is no longer the true dilaton) we shall find, by expanding about $\langle\chi\rangle = 0$, that the effective low energy theory has relevant operators proportional to f_c. (The leading contributions to the Higgs mass come from would-be logarithmic UV divergences that are tamed by small nonzero anomalous dimensions at the UV fixed point.)

One might suppose that the phenomenology of such a theory would be Coleman Weinberg–like in the IR, but it is not. In accord with our proverb about anomalous symmetries, one finds all kinds of classically dimensionful operators in the IR, of the order of f_c suppressed by loops. This illustrates the general fact that classically scale invariant theories and truly scale invariant theories are not generally speaking close cousins. Crucially, if there is ever a return to exact scale invariance in the UV, the scale at which that happens will be the one that governs the relevant operators in the effective IR theory.

In particular, we will show that the effective IR theory has a potential containing a logarithmic Higgs mass-squared term:

$$V = \frac{\lambda}{4}\phi^4 + \frac{1}{4}\phi^2\left(-m_h^2 + (m_h^2 - 2\langle\phi\rangle^2\lambda)\log\left[\frac{\phi^2}{\langle\phi\rangle^2}\right]\right),$$
$$m_h^2 \sim \text{Loop-factors} \times f_c^2. \tag{1}$$

Note that there will be the usual logarithmic terms for the quartic coupling as well, but their effect on electroweak symmetry breaking is negligible. The coupling λ is arbitrary as it can be present at the UV fixed point, whereas we will derive m_h^2 and $\langle\phi\rangle$ from radiative corrections.

Expanding the potential (1) about the minimum as $\phi = \langle\phi\rangle + h$, the Higgs self-couplings are given by

$$V = \frac{1}{4}(\langle\phi\rangle^4\lambda - m_h^2\langle\phi\rangle^2) + \frac{m_h^2 h^2}{2} + \left(\frac{m_h^2}{6\langle\phi\rangle} + \frac{2\langle\phi\rangle\lambda}{3}\right)h^3$$
$$+ \left(\frac{\lambda}{3} - \frac{m_h^2}{24\langle\phi\rangle^2}\right)h^4 + \mathcal{O}(h^5). \tag{2}$$

The SM potential and couplings are recovered in the limit that one chooses the SM value of $\lambda = \frac{m_h^2}{2\langle\phi\rangle^2}$. An interesting alternative however is $\lambda = \frac{m_h^2}{\langle\phi\rangle^2}$, which gives an electroweak breaking minimum that is degenerate with the symmetric minimum at the origin. One can of course arrange a long lived metastable minimum as well. Finally, if λ is negligible, the potential takes the "running-Higgs-mass-squared" form, comparable to that of the simplest (e.g. UDD) F- and D-flat directions in the minimal supersymmetric standard model:

$$V = \frac{1}{4}m_h^2\phi^2\left(\log\frac{\phi^2}{\langle\phi\rangle^2} - 1\right). \tag{3}$$

Roughly speaking the appearance of the logarithmic potential can be understood as follows. Any field that changes the β function couples through the scale anomaly to the corresponding gauge bosons. Therefore, the Yang-Mills terms in the effective Lagrangian pick up contributions of the form

$$\mathcal{L} \supset \beta(\phi,\chi)FF, \tag{4}$$

where the ϕ dependence is dominated by the familiar top quark and Higgs contributions to the beta functions. The χ dependence arises from the running to the fixed point around the scale $\langle\chi\rangle = f_c$. In some scenarios χ could be a nondynamical auxilliary field (i.e. a spurion) encoding for example fermion masses. It will not be crucial for our discussion if χ is a dynamical field or not.

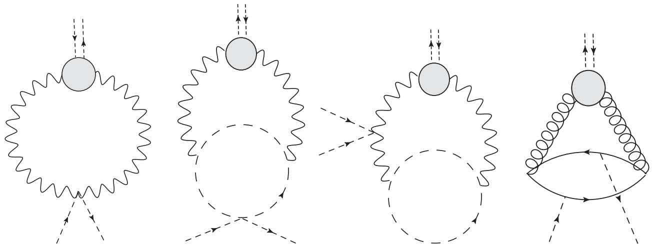

FIG. 2. Leading radiative contributions to the Higgs mass-squared coupling. The blob represents the scale anomaly of the $SU(2)_W$ and $SU(3)_c$ gauge groups, respectively.

At the leading order the contributions to the gauge propagator can be split into the visible and hidden parts [54], $C \supset C_{\rm vis}(\phi) + C_{\rm hid}(\chi)$. One should recall that in nonsupersymmetric systems the one-loop gauge propagator is not proportional to the β function (as the latter also gets vertex contributions) so the analysis is unfortunately more complicated than simply putting the dressed propagators into the perturbative expansion. Nevertheless, higher order terms begin with the cross-term $C \supset C_{\rm vis}C_{\rm hid}$. Diagrams that do not involve C at all have no dependence on ϕ or f_c and therefore cancel at the fixed point [55]. The vanishing of the β functions at the UV fixed point guarantees that terms in the effective potential derived from $C_{\rm vis}(p)$ can contribute only to finite quartic terms for the visible sector Higgs boson. Finally the interesting new terms arise from the cross-terms (effectively the second, third and fourth diagrams of Fig. 2), and from two-loop diagrams like the first diagram of Fig. 2. It turns out that the two-loop and three-loop top diagram have no logarithmic terms; they are effectively computable portal terms. The logarithmic piece in the potential derives from two diagrams involving the Higgs loop. The coefficient of all these terms is relatively small so that the potential naturally has a minimum at $\langle\phi\rangle \ll f_c$. Note that, although the minimum is generated radiatively, the obvious advantage over the visible sector Coleman-Weinberg mechanism is that $\langle\phi\rangle$ and f_c are essentially independent parameters, and so one avoids the problematic loop-suppressed Higgs mass.

In the next section we first derive the effective potential by carefully analyzing the contributions from the diagrams of Fig. 1. In the section that follows we discuss the global configuration, presenting a simple perturbative example of a QCD theory with a UV fixed point. Finally, we discuss the phenomenological aspects, including the expected scale of new physics, which turns out to be naturally of the order of 1–10^3 TeV.

II. THE CALCULATION

As described in the introduction, the setup we consider is a theory that can be split into a visible (SM) sector and a hidden sector, by sending the gauge coupling to zero. The hidden sector consists of a part that is responsible for spontaneously breaking scale invariance at a scale, f_c, and a messenger sector containing fields that are charged under the SM gauge groups, to which it directly couples. The gauge group of the SM appears as a global symmetry to the hidden sector dynamics that causes breaking of scale invariance. At a conformal fixed point (where $f_c = 0$), any nongauge (e.g. portal-like) coupling connecting the visible and hidden sectors is vanishing.

As we mentioned, such a configuration can be built by considering the properties of the visible + messenger theory (i.e. by turning off the hidden sector gauge groups) and hidden + messenger (i.e. by turning off the SM gauge groups) independently. The latter should have a UV stable fixed point while the visible + messenger sector should be a scale invariant version of the SM, which possesses a nontrivial IR stable fixed point. (Thus, when all gauge couplings are turned on one expects to find the saddle-point behavior described above with the hidden sector initiating the flow away from the fixed point in the IR.) Any portal-like coupling connecting the visible and hidden sectors is vanishing at the UV fixed point [56].

In the simplest case, the SM can be made conformal by adding extra states that couple only via gauge interactions to the SM (and not via Yukawa couplings). If these states couple to the hidden sector as well then they act as messengers. This is of course trivial to arrange (with direct couplings to the SM forbidden with symmetries) for the non-Abelian factors of the SM gauge group. For example, vector-like pairs of $SU(3)$ triplets and $SU(2)$ doublets suffice. Doing the same for the hypercharge coupling would require some kind of unification, so we shall not attempt to build a complete model here.

Despite that, we shall in the next section present a toy configuration of an $SU(3)$ model with an IR fixed point coupled via bifundamental fermions (messengers) to a "hidden" $SU(N)$ model with a repulsive UV fixed point that has all the desired properties. In particular, this minimal example shows that one does not necessarily expect to find additional states at mass scales much below f_c.

On the other hand, more elaborate implementations could include additional states in the visible sector to make it conformal, for example colored scalars coupling to the Higgs boson through quartic interactions. Those states should not acquire a mass of the order of f_c (i.e. they should not couple to the hidden sector directly as well), otherwise scale breaking would be directly mediated to the Higgs boson. They would be expected to acquire masses proportional to f_c but loop suppressed, much like the Higgs boson itself. This is of course also analogous to GMSB, the new extra states being similar to gauginos and sfermions which acquire masses comparable to the Higgs boson through gauge mediation. Here, we shall focus on the simplest case and assume such states are absent.

Let us now proceed with the computation of the Higgs potential by the spontaneous breaking of scale invariance in the hidden sector. We allow the Higgs to have a tree level quartic potential with the coupling λ, consistent with the requirements of conformal symmetry. As alluded to in the introduction our goal is to obtain calculable expressions, so we assume that the SM gauge couplings are always perturbative. This makes it possible to evaluate the effective potential for the Higgs boson by considering loops of SM gauge fields, with insertions of current correlators. From now on we neglect the contributions mediated by $U(1)_Y$.

The standard model gauge bosons couple perturbatively both to the currents of the standard model matter $J^\mu_{\rm vis}$ and to the currents representing the hidden sector $J^\mu_{\rm hid}$ as

$$\mathcal{L} \supset g A^a_\mu (J^{\mu a}_{\rm hid} + J^{\mu a}_{\rm vis}), \tag{5}$$

where here A^a_μ can represent the W bosons or the gluons for the $SU(2)$ and $SU(3)$ case, respectively, with a running over the generators of the gauge group. Henceforth, we work with the generic case of an $SU(N)$ gauge group. We will specialize to $SU(2)$ and $SU(3)$ at the end of the computation.

The fact that the two currents are not directly coupled but communicate only through gauge interactions can be taken as the formal definition of gauge mediation. We parametrize the two point functions of the currents with dimensionless functions [57], C:

$$\langle J^{\mu a}_{\rm vis} J^{\nu b}_{\rm vis}\rangle = -(p^2\eta_{\mu\nu} - p_\mu p_\nu) C^{ab}_{\rm vis}(p^2, \phi^2) \tag{6}$$

$$\langle J^{\mu a}_{\rm hid} J^{\nu b}_{\rm hid}\rangle = -(p^2\eta_{\mu\nu} - p_\mu p_\nu) C^{ab}_{\rm hid}(p^2, f_c^2). \tag{7}$$

The visible sector two point function depends on the Higgs field VEV, here denoted by ϕ, while the hidden sector one depends on the characteristic scale f_c. For simplicity, we take the two point functions to be diagonal in the non-Abelian indices. We also assume that these functions depend only quadratically on ϕ and f_c.

In this parametrization, the current two point functions correct the propagator of the gauge boson producing an effective potential induced by a loop of the gauge boson:

$$V_{\rm eff} = \frac{3}{2}{\rm Tr}\int \frac{d^4p}{(2\pi)^4}\log\left(1 + \frac{m_V^2}{p^2} + g^2 C_{\rm vis} + g^2 C_{\rm hid}\right), \tag{8}$$

where we denoted by m_V the gauge boson mass, relevant for the electroweak case. The trace is over the gauge group indices; for $SU(N)$ gauge groups and the diagonal C functions and mass this gives an overall $N^2 - 1$ factor.

In evaluating the possible quantum corrections, we will of course be focussing on the terms that are proportional to $f_c^2\phi^2$, which will determine the quadratic part of the potential for the Higgs. There are other contributions, resulting for example from the discontinuities in the anomalous dimension induced by the Higgs VEV, which can generate only quartic terms for the Higgs boson. These terms are negligible in determining the minimum.

Our gauge mediation principle implies that in the limit $f_c \to 0$ the effective potential mixing the two sectors should vanish. We can enforce this, and at the same time select only the contributions of interest, by computing the difference between the effective potential (8) with $f_c \neq 0$ and the effective potential with $f_c = 0$:

$$\delta_{f_c} V_{\rm eff} = V_{\rm eff}(f_c^2) - V_{\rm eff}(f_c^2 = 0). \tag{9}$$

Note that in each term there are divergences that do not involve the scale f_c. These correspond to wave function renormalization and simply set the anomalous dimensions at the conformal fixed point and cancel in Eq. (9). Of course the advantage of the GMESB approach is that we do not need to consider them.

Expanding the effective potential in the gauge coupling, we obtain at $\mathcal{O}(g^4)$

$$\delta_{f_c} V_{\rm eff} = \frac{3(N^2-1)}{2}\int \frac{d^4p}{(2\pi)^4}\frac{p^2}{p^2+m_V^2}(g^2\delta_{f_c}C_{\rm hid} - \frac{g^4}{2}(2C_{\rm vis}\delta_{f_c}C_{\rm hid} + (\delta_{f_c}C_{\rm hid})^2)). \tag{10}$$

The quadratic terms for the Higgs field ϕ appear here in the massive propagator for the gauge boson or inside the $C_{\rm vis}$ function. Of the former contributions, we need keep only the leading order ones, i.e. those at $\mathcal{O}(g^2)$ inside the parenthesis. We arrive at the final generic expression for the Higgs potential induced by an $SU(N)$ gauge group:

$$\delta_{f_c} V_{\rm eff} = \frac{3(N^2-1)}{2}\int \frac{d^4p}{(2\pi)^4}\left(g^2\frac{p^2}{p^2+m_V^2}\delta_{f_c}C_{\rm hid} - g^4 C_{\rm vis}(\phi^2)\delta_{f_c}C_{\rm hid}\right). \tag{11}$$

Note that the first term in this expression gives rise to a Higgs mass proportional to f_c once we expand the propagator. As we shall now see, the second term can give rise to both mass terms and logarithms, due to the dependence of $C_{\rm vis}$ on the Higgs field.

The form of the function $C_{\rm vis}(p^2, \phi^2)$ is completely determined by the visible sector and changes if there are extra states (massless at the tree level) that couple to the Higgs. As mentioned above, in this paper we will explicitly consider only the simplest case where such states are absent. The function $C_{\rm hid}$ is an unknown function provided by the hidden sector. The hidden sectors can be different for the $SU(2)$ or $SU(3)$ gauge groups, having in principle different matter content and also dynamical scales. We distinguish between them with an extra index, e.g. $\delta_{f_{c(2)}} C^{(2)}_{\rm hid}$ for the hidden sector of $SU(2)$.

The main contribution to the integral of (11) comes from the large momentum region, with $p^2 \gg \phi^2$. Hence, in order to find an approximate expression for $\delta_{f_c} V_{\rm eff}$ that already encodes all the relevant features, we can use the one-loop large momentum expansion of the $C_{\rm vis}$, namely [58]

$$8\pi^2 C^{(2)} = \log\left(\frac{\mu^2}{p^2}\right) b'^{(2)} + \frac{m_H^2}{4p^2}\left(1 + \log\frac{m_H^2}{p^2}\right) - 6\frac{m_t^2}{p^2} + \frac{m_W^2}{4p^2}\left(51 - 13\log\frac{m_W^2}{p^2}\right) + \mathcal{O}(1/p^4)$$
$$8\pi^2 C^{(3)} = \log\left(\frac{\mu^2}{p^2}\right) b'^{(3)} - 6\frac{m_t^2}{p^2} + \mathcal{O}(1/p^4) \tag{12}$$

for $SU(2)$ and $SU(3)$ gauge bosons, respectively. Here, $b'^{(2)}$ and $b'^{(3)}$ are the wave function renormalization coefficients of the $SU(2)$ and $SU(3)$ gauge bosons. The dependence on the Higgs doublet is encoded in the masses as $m_H^2 = 4\lambda HH^\dagger$, $m_W^2 = \frac{g_2^2}{2} HH^\dagger$ and $m_t^2 = \lambda_t^2 HH^\dagger$.

Inserting these expressions into (11), expanding the massive propagator, and keeping only the terms quadratic in the Higgs field, we get for the $SU(2)$ induced effective potential

$$\delta_{f_c} V_{\rm eff}^{(2)} = \frac{9 g_2^4 \mathcal{A}_2}{16\pi^2} f_{c(2)}^2 HH^\dagger \left(4\pi^2 - 6\lambda_t^2 + \lambda\left(1 + \frac{\mathcal{B}_2}{\mathcal{A}_2}\right) + \frac{51}{8} g_2^2 \left(1 - \frac{13}{51}\frac{\mathcal{B}_2}{\mathcal{A}_2}\right) + \lambda \log\frac{4\lambda HH^\dagger}{f_{c(2)}^2} - \frac{13}{8} g_2^2 \log\frac{g_2^2 HH^\dagger}{2 f_{c(2)}^2}\right), \tag{13}$$

where we denote

$$\mathcal{A}_2 = -\frac{1}{f_{c(2)}^2}\int \frac{d^4 p}{(2\pi)^4} \frac{\delta_{f_{c(2)}} C_{\rm hid}^{(2)}}{p^2}$$
$$\mathcal{B}_2 = -\frac{1}{f_{c(2)}^2}\int \frac{d^4 p}{(2\pi)^4} \frac{\delta_{f_{c(2)}} C_{\rm hid}^{(2)}}{p^2} \log\frac{f_{c(2)}^2}{p^2}. \tag{14}$$

Note that $C_{\rm hid}$ are at least one loop, hence, $\mathcal{A}_2$ and $\mathcal{B}_2$ are effectively at least at two loop, and they are dimensionless. An analogous computation for $SU(3)$ leads to

$$\delta_{f_c} V_{\rm eff}^{(3)} = \frac{24 g_3^4 \mathcal{A}_3}{16\pi^2} f_{c(3)}^2 HH^\dagger (-6\lambda_t^2), \tag{15}$$

with $\mathcal{A}_3$ defined as in (14) but with $f_{c(3)}^2$ and $C_{\rm hid}^{(3)}$.

The total effective potential is then the sum of (13) and (15). If there is not a huge hierarchy between $\mathcal{A}_2$ and $\mathcal{B}_2$, one can neglect the terms of the order of $g_2^4\lambda$ and g_2^6 in (13) compared to λ_t^2 (one can check that including these terms does not affect the discussion). On the other hand, we must retain the terms of this order multiplying the logarithm of $HH^\dagger$ which can become large at the minimum.

The complete potential then simplifies to

$$V_{\rm eff} = \lambda (HH^\dagger)^2 + \frac{9 g_2^4 \mathcal{A}_2}{16\pi^2} f_{c(2)}^2 HH^\dagger \times \left(4\pi^2 - \lambda_t^2 \left(6 + 16 \frac{f_{c(3)}^2 \mathcal{A}_3}{f_{c(2)}^2 \mathcal{A}_2} \frac{g_3^4}{g_2^4}\right) + \left(\lambda - \frac{13}{8} g_2^2\right) \log\frac{HH^\dagger}{f_c^2}\right). \tag{16}$$

This is the generic effective potential for the Higgs in models of GMESB. It contains two arbitrary scales, $f_{c(2)}$ and $f_{c(3)}$, and two dimensionless quantities, $\mathcal{A}_2$ and $\mathcal{A}_3$, which are entirely specified by the hidden sectors to which the gauge bosons of $SU(2)$ and $SU(3)$ couple. Even in strongly coupled cases they are given simply by the two point functions of the hidden sector currents that break scale invariance.

The potential of Eq. (16) can be specified in terms of one dimensionful and one dimensionless combination of parameters, and it can also be simplified in terms of physical scales. Indeed, we can rewrite the potential by parametrizing the Higgs field as $H = e^{i\xi.\tau}\begin{pmatrix} 0 \\ \phi/\sqrt{2}\end{pmatrix}$. Imposing the minimization condition at $\phi = \langle\phi\rangle$, and trading the unknown parameters $\mathcal{A}_2 f_{c(2)}^2$ for the second derivatives of the potential around $\phi = \langle\phi\rangle$, we arrive at the generic GMESB Higgs potential:

$$V = \frac{\lambda}{4}\phi^4 + \frac{1}{4}\phi^2\left(-m_h^2 + (m_h^2 - 2\langle\phi\rangle^2\lambda)\log\left[\frac{\phi^2}{\langle\phi\rangle^2}\right]\right). \tag{17}$$

The original parameters are related to λ and m_h as

$$Y = 13 g_2^2 - 8\lambda; \qquad X = \frac{2\langle\phi\rangle^2\lambda}{m_h^2} - 1$$
$$\mathcal{A}_2 = \frac{64\pi^2}{9 g_2^4}\frac{X}{Y}\frac{m_h^2}{f_{c(2)}^2}$$
$$\mathcal{A}_3 = \frac{\pi^2}{18 g_3^4 \lambda_t^2}\left(1 + 16\frac{X}{Y}(2\pi^2 - 3\lambda_t^2) - \frac{X}{\pi^2}\log\frac{\langle\phi\rangle^2}{2 f_{c(2)}^2}\right)\frac{m_h^2}{f_{c(3)}^2}.$$

Since $\mathcal{A}_2$ is generically two-loop suppressed, Eq. (18) implies that, in the absence of any fine-tuning, the scale of new physics is larger than the electroweak scale by a factor of two to three loops. Hence, we have a generic prediction that

$$f_c \sim 10^3\text{–}10^6\ \text{GeV}. \tag{18}$$

It is interesting to note that the value of λ gives direct information about the hidden sector. Taking into account that $\frac{m_h^2}{v^2} \simeq \frac{1}{4}$, Eq. (18) implies that if $\lambda < \frac{m_h^2}{2v^2} \simeq \frac{1}{8}$ then the factor $\mathcal{A}_2$ should be negative in order to have electroweak symmetry breaking. If $\frac{m_h^2}{2v^2} < \lambda < \frac{13}{8} g_2^2$ then it should be positive, and if $\lambda > \frac{13}{8} g_2^2$ it should again be negative. An interesting observation is that the SM value, i.e. $\lambda = \frac{m_h^2}{2\langle\phi\rangle^2}$, is realized for $\mathcal{A}_2 = 0$ and $\mathcal{A}_3 = \frac{\pi^2 m_h^2}{18 f_{c(3)}^2 g_3^4 \lambda_t^2}$; in other words the canonical SM potential results when conformal invariance in the hidden sector is communicated dominantly by states charged under $SU(3)$.

In the next subsection we discuss the minimal implementation of GMESB, which has a positive $\mathcal{A}_2$ as a natural outcome. This leads to correct electroweak symmetry breaking when the quartic coupling lies in the window $\frac{m_h^2}{2v^2} < \lambda < \frac{13}{8} g_2^2$. However, it is clear from the above that the generic prediction of GMESB is that the Higgs potential takes the form in Eq. (17) regardless of the details of the hidden sector, and that the tree level Higgs quartic coupling at the fixed point, λ, encapsulates the entire difference between this and the standard model.

In Sec. IV we will discuss the phenomenology of the potential (17) in the different regimes.

We conclude this discussion with a remark concerning the effective potential (17). The logarithmic/quadratic terms in the effective potential deserve comment as they may be surprising to readers familiar with perturbative computations and dilaton effective actions. They result from the fact that UV scale invariance means that the leading contributions to the quadratic term must come from higher order radiative corrections that mix UV (where f_c resides) and IR (where ϕ resides) contributions [59].

We can in fact verify that the method of computation above gives the correct answer in a much simpler (two-loop) example than the one studied in this paper, namely a theory with two scalars coupled with a quartic coupling, where one scalar has a large mass, f_c. The mass f_c is playing the role of the order parameter for the spontaneous breaking of scale invariance, corresponding to the VEV of the would-be dilaton. The effective potential at the two-loop order can be computed directly as in [60], and one finds terms going as $\lambda\lambda_{12}\phi^2 f_c^2 \log\phi^2$, where ϕ is the tree level massless field, λ_{12} is the quartic coupling between the two scalars, and λ is the quartic coupling for ϕ. The other dimensionful argument of the log is typically the renormalization scale. One can now realize scale invariance nonlinearly by using the prescription of Ref. [61], in which one replaces the renormalization scale with the dilaton. This results in a term going like $\log\phi^2/f_c^2$. The same terms are obtained if one uses the approach we adopted above for the derivation of Eq. (16). Thus, we argue that our results would be consistent with those derived by performing the entire three-loop computation of the effective action in the theory under study.

A. Minimal GMESB

In this subsection we introduce a minimal model of GMESB in which the hidden sector is constrained by certain assumptions. We will see how the constraint of UV scale invariance allows one to extract quantitative results for C_{hid}, and hence determine the parameters $\mathcal{A}_{2,3}$ governing the Higgs potential completely.

The minimal model is defined as follows. First, we will assume that the visible sector contains only the SM. Second, we assume that the hidden sector contains at the fixed-point bosonic and fermionic states with unknown anomalous dimensions, induced by the hidden sector dynamics (which could be strongly coupled), and we will take the anomalous dimensions of multiplets for each group to be degenerate, γ_F for the fermions and γ_B for the bosons, with e.g. $\dim(\phi) = 1 + \gamma_B$. Note that we will take there to be two independent hidden sectors, one coupled to $SU(3)$ and one to $SU(2)$, each determined by their content of bosonic and fermionic matter. (One could in principle allow states that are charged under $SU(3)$ and $SU(2)$ simultaneously.) We will also assume that all the hidden sector states have a universal mass [62], f_c. It will turn out that for consistency γ_F and γ_B must be negative.

Whatever the hidden sector configuration, once the matter content is known, the function C_{hid} can in principle be expanded perturbatively in the SM gauge couplings. We will here consider only the one-loop contribution to C. In order to do this, let us first recall the canonical weakly coupled expression for an $SU(N)$ gauge theory with a generic matter content of massive scalars and fermions, together with its asymptotics (derived in Appendix A). Schematically, it can be written as

$$C^1 = \frac{1}{8\pi^2}\left(\sum_\phi C(r_\phi) C_B^1(m_\phi) + \sum_\psi C(r_\psi) C_F^1(m_\psi) + \sum_G C(\mathrm{adj}) C_G^1\right), \quad (19)$$

where $C_B^1(m_\phi), C_F^1(m_\psi), C_G^1$ denote the one-loop contribution to the gauge boson propagator from complex bosons, Weyl fermions and gauge bosons [63] running in the loop, respectively, and (at the risk of confusion) $C(r)$ represents the Casimir index of the representation r. The asymptotic expressions are at a large momentum (with respect to the characteristic mass scale of the field),

$$C_B^1(m_{\phi})) = \frac{1}{3}\log\left(\frac{\mu^2}{p^2}\right) + 2\frac{m_\phi^2}{p^2}\left(1 + \log\left(\frac{m_\phi^2}{p^2}\right)\right) + O(1/p^4),$$

$$C_F^1(m_\psi) = \frac{2}{3}\log\left(\frac{\mu^2}{p^2}\right) - 4\frac{m_\psi^2}{p^2} + O(1/p^4), \quad (20)$$

and at a low momentum,

$$C^1_B(m_{\phi)}) = -\frac{1}{3}\log\left(\frac{m_\phi^2}{\mu^2}\right) + O(p^2)$$
$$C^1_F(m_\psi) = -\frac{2}{3}\log\left(\frac{m_\psi^2}{\mu^2}\right) + O(p^2), \qquad (21)$$

where we denote with m_ϕ and m_ψ the mass of the scalar and of the fermion, respectively.

More generally, the anomalous dimensions of the hidden sector matter determine the structure of the $C_{\rm hid}$ function and one can argue (as in e.g. [30]) that the generic expression is obtained by simply modifying the canonical one, such that all dimensionful quantities scale with appropriate exponents (equivalently that propagators scale as $1/(p^2 - m^2)^{1-\gamma_B}$). Under these hypotheses, the large and small momentum expansions for $C_{\rm hid}$ are respectively

$$C^1_{\rm hid} \to \frac{\delta^{ab}}{8\pi^2}\left[\log\frac{\mu^2}{p^2}\left(\sum_\phi(1-\gamma_B)\frac{C(r_\phi)}{3} + \sum_\psi(1-\gamma_F)\frac{2C(r_\psi)}{3}\right) + 2\frac{f_c^2}{p^2}\left(\frac{f_c^{-2\gamma_B}}{p^{-2\gamma_B}}\sum_\phi C(r_\phi) - 2\frac{f_c^{-2\gamma_F}}{p^{-2\gamma_F}}\sum_\psi C(r_\psi)\right)\right.$$
$$\left. + 2\frac{f_c^{2(1-\gamma_B)}}{p^{2(1-\gamma_B)}}(1-\gamma_B)\log\frac{f_c^2}{p^2}\sum_\phi C(r_\phi)\right] + \mathcal{O}(1/p^4), \qquad (22)$$

$$C^1_{\rm hid} \to -\frac{\delta^{ab}}{8\pi^2}\log\frac{f_c^2}{\mu^2}\left(\sum_\phi(1-\gamma_B)\frac{C(r_\phi)}{3} + \sum_\psi(1-\gamma_F)\frac{2C(r_\psi)}{3}\right) + \mathcal{O}(p^2). \qquad (23)$$

The terms proportional to $\log(\mu)$ can be recognized as the modification to the beta function of the gauge coupling coming from fields with large anomalous dimensions; we denote the contribution to the beta function coefficient by

$$b'_{\rm HS} \equiv \left(\sum_\phi(1-\gamma_B)\frac{C(r_\phi)}{3} + \sum_\psi(1-\gamma_F)\frac{2C(r_\psi)}{3}\right). \qquad (24)$$

The assumption that conformal invariance is spontaneously broken only by f_c imposes some constraints on the asymptotics of the C functions. In particular, the β function for the gauge boson should be zero at a large momentum. The β function is a combination of the wave function renormalization for the gauge boson, encapsulated by the asymptotic logarithm of $C_{\rm tot} = C_{\rm vis} + C_{\rm hid}$, the vertex renormalization and the wave function renormalization of matter. In order for the β function of the gauge coupling to vanish in the UV, the asymptotic behavior of $C_{\rm tot}$ should be such that [64,65]

$$b'_{\rm HS} + b'_{\rm SM} \simeq 2N_c. \qquad (25)$$

This should be valid both for the $SU(2)$ and the $SU(3)$ gauge groups. We write an approximate equality since the two-loop contributions could be relevant in determining the fixed point; however, even in that case the above relation holds approximately.

We can use the above asymptotics to compute the integrals (14) determining the effective potential by splitting the domain of integration into the regions $p^2 \ll f_c^2$ and $p^2 \gg f_c^2$. We use a shorthand notation for the effective number of fermions and bosons in the hidden sector:

$$n_B = \sum_\phi C(r_\phi) \qquad n_F = \sum_\psi C(r_\psi). \qquad (26)$$

The anomalous dimensions of the hidden sector fields should be negative, in order for the integrals to be convergent. Including for completeness the integral $\mathcal{B}_2$, we obtain

$$\mathcal{A}_a = \frac{1}{(16\pi^2)^2}\left(2(b'_{\rm HS})_{(a)} + \frac{4n_B^{(a)}}{(\gamma_B^{(a)})^2} - \frac{8n_F^{(a)}}{\gamma_F^{(a)}}\right)$$
$$\mathcal{B}_2 = \frac{1}{(16\pi^2)^2}\left(4(b'_{\rm HS})_{(2)} + \frac{4n_B^{(2)}(2-\gamma_B^{(2)})}{(\gamma_B^{(2)})^3} - \frac{8n_F^{(2)}}{(\gamma_F^{(2)})^2}\right), \qquad (27)$$

where $a = 2, 3$. We note that, unless $\frac{1}{\gamma} \gg 1$, there is not a large hierarchy between $\mathcal{A}_2$ and $\mathcal{B}_2$, so we can consistently neglect the terms multiplying $\mathcal{B}_2$ as anticipated. The complete potential is then as in (16) with $\mathcal{A}_2$ and $\mathcal{A}_3$ given by Eq. (27). As expected, the quantities $\mathcal{A}_a$ and $\mathcal{B}_2$ are two-loop suppressed. Moreover, the relation (25) implies that $(b'_{\rm HS})_{(a)}$ are positive, both for $SU(2)$ and $SU(3)$ gauge groups (they are respectively $(b'_{\rm HS})_{(2)} \simeq \frac{19}{6}$ and $(b'_{\rm HS})_{(3)} \simeq 7$). Since the anomalous dimensions are negative, the factors $\mathcal{A}_a$ are positive definite. This is a peculiarity of the minimal model and is not a generic property of GMESB models. However, it implies that in the minimal model we cannot realize scenarios with purely logarithmic electroweak symmetry breaking and a negligible quartic coupling (see the discussion at the end of the previous section).

In minimal GMESB, the expected scale of new physics and the precise value of the quartic coupling clearly depend sensitively on the field content of the hidden sector and on the anomalous dimensions, since they determine the integrals (14).

Let us consider a simple example to demonstrate. Suppose that the $SU(2)$ and the $SU(3)$ hidden sectors contain four and eight pairs of Weyl fermions in the fundamental and antifundamental representation of the gauge group, respectively, so $n_{F(2)} = 4$ and $n_{F(3)} = 8$. Both hidden sectors do not contain bosons for simplicity, i.e. $n_{B(2)} = n_{B(3)} = 0$. Then we can consider two cases for the anomalous dimensions of the fermions in the two hidden sectors: the first has $\gamma_F^{(2)} = -0.034$ and $\gamma_F^{(3)} = -0.26$, the second $\gamma_F^{(2)} = -0.023$ and $\gamma_F^{(3)} = -0.20$. Note that the fermionic matter content of each hidden sector is such that Eq. (25) is roughly satisfied. By imposing Eqs. (18) and demanding the correct $\langle\phi\rangle$ and m_h^2, the resulting values for the quartic coupling and the scale f_c are

$$\gamma_F^{(2)} = -0.034, \quad \gamma_F^{(3)} = -0.26 \Rightarrow \lambda = 0.15, \quad f_c = 2.5\,\mathrm{TeV}$$
$$\gamma_F^{(2)} = -0.023, \quad \gamma_F^{(3)} = -0.20 \Rightarrow \lambda = 0.5, \quad f_c = 15\,\mathrm{TeV}.$$

In the first case, since λ is close to the limiting SM value of $\frac{1}{8}$, there has to be a partial cancellation in the factor $X = \frac{2\langle\phi\rangle^2\lambda}{m_h^2} - 1$ in Eq. (18), leading to a scale of new physics f_c that is at the lower end of the expected window in Eq. (18). In the second case, the value of λ is larger than the SM one, there is no cancellation in Eq. (18), and the resulting scale of new physics is correspondingly larger.

Clearly since $\mathcal{A}_2$ is positive definite in minimal GMESB, Eq. (18) tells us that λ will always lie between the SM value and $\frac{13}{8} g_2^2$ in this case, as demonstrated in the above examples. The lower limit yields a potential for the Higgs essentially identical to the SM one, and new physics at a relative accessible scale. The upper limit yields a Higgs potential with different self-couplings from the SM potential, but the scale of new physics is pushed beyond the reach of the LHC. It is interesting that either limit offers the prospect of experimental detection. In Sec. IV we will describe the phenomenological properties of the Higgs potential in the minimal case as well as the other nonminimal cases described in Sec. II.

III. THE GLOBAL PICTURE: A TOY MODEL

We wish to briefly discuss the global picture in more detail. In particular, an important issue that we address is the nature of the fixed points. It is useful to have a specific model in mind for this question. Here, we shall consider a pair of coupled QCD theories, one standing for the visible sector and one for the hidden.

First consider a single $SU(N)$ QCD with F flavors of (Dirac) quarks, a singlet scalar playing the role of the SM Higgs, and a large degenerate Yukawa coupling, y, for $F' \leq F$ of the pairs. These F' pairs represent the visible sector matter fields whereas the $F - F'$ quarks that do not couple to the Higgs boson are messengers; in a more realistic model one would among other things probably wish to introduce Yukawa hierarchies. The Lagrangian can be written as

$$\mathcal{L} = -\frac{1}{4g^2} FF + i\bar{\psi}\gamma.D\psi + \frac{1}{2}(\partial h)^2 - (y\bar{t}th + \mathrm{h.c.}) - \lambda h^4, \tag{28}$$

where the F' quarks with degenerate Yukawa coupling are labeled t. Let $a_g = g^2/(16\pi^2)$, $a_y = y^2/(16\pi^2)$, and $a_\lambda = \lambda/(16\pi^2)$. Then we obtain to the first order in the Yukawa and quartic coupling, and two-loop order in the gauge coupling, the following beta functions:

$$\begin{aligned}
\beta_{\alpha_g} &= -2\alpha_g^2 N\left[\frac{11}{3} - \frac{2}{3}\frac{F}{N} + \alpha_y\frac{F'}{N}\right] \\
&\quad - 2a_g^3 N^2\left[\frac{34}{3} - \frac{F}{N}\left(\frac{13}{3} - \frac{1}{N^2}\right)\right] \\
\beta_{\alpha_y} &= 2\alpha_y\left[4F'\alpha_y - 3\frac{N^2-1}{N}\alpha_g\right] \\
\beta_{\alpha_\lambda} &= 2[10\alpha_\lambda^2 + 2NF'\alpha_\lambda\alpha_y - NF'\alpha_y^2].
\end{aligned} \tag{29}$$

The canonical fixed point is the Caswell-Banks-Zaks (CBZ) fixed point [66,67], when the number of flavors is chosen so as to make the one-loop contribution to $\beta_{\alpha_g} \approx 0$. That is, writing $\beta_{\alpha_g} = -\frac{2}{3}\alpha_g^2 N[b_0 + b_1\alpha_g + b_y\alpha_y]$ we require $b_0 \approx 0$. In order to remain perturbative while still having the two-loop contribution compensate for the one-loop contribution, we then require $F = \frac{11N}{2}(1-\epsilon)$, with $b_0 = \epsilon \sim 1/N$. The usual CBZ fixed point is an asymptotically free theory ($b_0 > 0$) with a perturbative IR fixed point which requires $1 \gg \epsilon > 0$. Perturbativity is still possible because, since $b_1 \sim N$, then $b_1\alpha_g \sim b_0$ implies that the 't Hooft coupling $4\pi N\alpha_g \sim \epsilon \sim 1/N$ can still be much less than one. This type of fixed point is a natural place to consider putting our conformally extended SM. As long as it is IR stable, the theory does not stray from the fixed point until it is disturbed by f_c sized deformations. As we shall see the latter are naturally f_c sized mass terms for the $F - F'$ (messenger) quarks that do not couple to the ϕ in Yukawa couplings.

In addition we would like the SM to remain perturbative so that the fixed-point values of the gauge couplings are not dramatically different from their usual SM values.

To the leading order in $1/N$, one can find a nontrivial fixed point at

$$4\pi N\{\alpha_{g*}, \alpha_{y*}, \alpha_\kappa\} = 4\pi\frac{11\epsilon}{50}\left\{\frac{4}{3}, \frac{N}{F'}, \frac{N}{2F'}\right\}. \tag{30}$$

Hence, we require $\epsilon > 0$, but clearly the couplings can be arbitrarily small at the fixed point provided that $F' \gtrsim N$.

Defining $\gamma_i = \frac{\alpha_i}{\alpha_i^*} - 1$, and diagonalizing the linearized renormalization group equations (RGEs) in (29) about the fixed points, the β functions have the eigenvalues

$$\beta_{\gamma_i} = 4\frac{11}{25}\epsilon\left\{\frac{11}{9}\epsilon, \frac{N}{2}, 1\right\}, \tag{31}$$

and, since $\epsilon > 0$, all eigenvalues are positive, indicating an IR-stable fixed point as required. The small eigenvalue corresponds mainly to the gauge coupling for which the fixed point is only weakly attractive (so-called quasifixed behavior).

What about the hidden sector theory? We need to ensure that this theory flows away from its fixed point, so that it is able to generate the spontaneous breaking of the scale invariance f_c through some dynamical mechanism, possibly confinement with a mass-gap. This requires a fixed point that is unstable in the IR. It is possible to show (using an analysis similar to the one in [68]) that one cannot have a perturbative UV fixed point in which all directions are UV stable. Essentially the issue is that, as can be seen from (29), when $b_0 \approx 0$ (for perturbativity) then $b_1 < 0$, whereas a UV-stable fixed point requires $b_1 > 0$. In fact this is true in general for any additional states that one adds. However, (unlike [68] which was concerned with solving Landau pole problems) here we only require that *some* directions are unstable in the IR in order to start a flow.

Consider therefore $SU(N)$ QCD with F'' flavors of (Dirac) quarks, f, with $SU(F'')_L \times SU(F'')_R$ flavor symmetry, and a Yukawa coupling Y to a flavor bifundamental scalar, Φ. The Lagrangian can be written as

$$\mathcal{L} = -\frac{1}{4g^2}FF + i\bar{f}\gamma.Df + \frac{1}{2}(\partial\Phi)^2 - (Y\bar{f}f\Phi + \text{h.c.}) - \lambda[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \kappa\mathrm{Tr}[(\Phi^\dagger\Phi)^2]. \tag{32}$$

Note that similar systems were analyzed in [69]. To the one-loop order in the Yukawa and quartic coupling, and two-loop order in the gauge coupling, the β functions are

$$\begin{aligned}\beta_{\alpha_g} &= -2\alpha_g^2 N\left[\frac{11}{3} - \frac{2}{3}\frac{F''}{N} + \alpha_y\frac{F''^2}{N}\right] \\ &\quad - 2a_g^3N^2\left[\frac{34}{3} - \frac{F''}{N}\left(\frac{13}{3} - \frac{1}{N^2}\right)\right] \\ \beta_{\alpha_Y} &= 2\alpha_g\left[(F'' + N)\alpha_Y - 3\frac{N^2-1}{N}\alpha_g\right] \\ \beta_{\alpha_\lambda} &= 2[4N\alpha_\lambda^2 + 2N\alpha_\lambda\alpha_Y - F''\alpha_Y^2].\end{aligned} \tag{33}$$

To this order, κ runs independently and has a fixed point at $\kappa = 0$. Again to retain perturbativity we choose $F'' = \frac{11}{2}N(1-\epsilon)$, with $b_0 = \epsilon \sim 1/N$. To the leading order in $1/N$ and ϵ, there is a fixed point at

$$4\pi N\{\alpha_{g*}, \alpha_{y*}, \alpha_{\kappa*}\} = -4\pi\frac{22}{19}\epsilon\left\{1, \frac{13}{6}, \frac{\sqrt{23}-1}{4}\right\}. \tag{34}$$

Therefore, now we require $\epsilon < 0$ for this to be physical. Since $b_0 < 0$ as well, one suspects that the fixed point is a saddle point, and indeed it is. Diagonalizing the RGEs about the fixed points, the β functions have the eigenvalues

$$\beta_{\gamma_i} = 13\frac{22}{19}|\epsilon|\left\{-\frac{11}{9}|\epsilon|, 4\frac{\sqrt{23}}{13}, 1\right\}. \tag{35}$$

The fixed point is weakly repulsive (as required) in the direction that is mainly comprised of the gauge coupling.

Now we can consider a global configuration in which two such theories are coupled. For our SM theory, we focus on $SU(3)_c$. (Obviously in a full model there would have to be some degree of unification in order to include the full SM gauge groups including the hypercharge.) There are 6 $SU(3)_c$ flavors in the SM, and to be near a CBZ fixed point we require $F \lesssim 33/2$. Indeed, from Eq. (30) it is natural to choose $\frac{88\pi}{225}\epsilon \sim \alpha_s$ or $\epsilon \approx 0.1$, so that the fixed point value of the $SU(3)_c$ coupling is not far from its weak-scale value. We can choose $F = 15$ (and hence $\epsilon = 1/11$), and therefore need to add nine pairs of messenger quarks with $F - F' = 9$. By (30) the couplings at the fixed points are given by

$$4\pi\{\alpha_{g*}, \alpha_{y*}, \alpha_{\kappa*}\} = \frac{2\pi}{25}\left\{\frac{4}{3}, \frac{1}{5}, \frac{1}{10}\right\}, \tag{36}$$

so the theory is just as perturbative as the SM there. Note that the Yukawa couplings are $y \approx 0.8$ and the quartic coupling is $\lambda \approx 0.3$. These are reasonable values for what is after all just a toy model.

Meanwhile, the nine pairs of quarks have an $SU(9)_L \times SU(9)_R$ flavor symmetry. We gauge the diagonal anomaly-free part to be our hidden sector gauge group. Thus, the hidden sector already sees three pairs of messenger quarks (since the bifundamentals are charged under $SU(3)_c$), and as we saw above we need to make this up to $F'' = \frac{11}{2}N(1-\epsilon)$ where $N = 9$, with $\epsilon < 0$. Here, the solution can be virtually as weakly coupled as desired, since when $F'' = 50$ one has $\epsilon = 1/99$. Since GMESB works best with large anomalous dimensions in the hidden sector, relatively strong coupling is perhaps more desirable however. This may also be required for the dynamics that lead to the spontaneously breaking of scale invariance. Although it is not obligatory, the minimal and perturbative choice is that the field Φ is the dilaton that gets a VEV, f_c. We should stress however that the hidden sector gauge group may become strongly coupled in the IR in which case the mediation may be better described by a low energy theory of bound states. This would of course greatly complicate

TABLE I. A toy model for GMESB. The $SU(3)_c$ theory sits at an IR fixed point while the $SU(9)_{\rm hid} = [SU(9)_L \times SU(9)_R]_{\rm diag}$ theory flows from a repulsive UV fixed point. The F'' pairs of the $SU(9)_{\rm hid}$ fermions $f \equiv \{\psi_{\rm mess}, \psi_{\rm hid}\}$ and $\bar{f} \equiv \{\bar{\psi}_{\rm mess}, \bar{\psi}_{\rm hid}\}$ include the bifundamental messenger fields.

	$SU(3)_c$	$SU(9)_L$	$SU(9)_R$
$6 \times \psi_{\rm SM}$	$\square$	1	1
$6 \times \bar{\psi}_{\rm SM}$	$\overline{\square}$	1	1
ϕ	1	1	1
$\psi_{\rm mess}$	$\square$	$\overline{\square}$	1
$\bar{\psi}_{\rm mess}$	$\overline{\square}$	1	$\square$
$(F''-3)\times\psi_{\rm hid}$	1	$\overline{\square}$	1
$(F''-3)\times\bar{\psi}_{\rm hid}$	1	1	$\square$
Φ	1	$\square$	$\overline{\square}$

the explicit computation of the parameters $\mathcal{A}_{2,3}$. The particle content is summarized in Table I.

IV. SUMMARY AND PHENOMENOLOGICAL DISCUSSION

Let us recap what we have found. We assumed that the standard model emanates from a UV scale invariant theory that also contains a messenger sector of fields charged under the SM gauge groups but without couplings to the Higgs boson. If the messenger fields gain masses of the order of f_c from spontaneous scale breaking in the rest of the hidden sector, then mass terms and novel logarithmic couplings are induced radiatively in the Higgs potential. For this to be phenomenologically viable, the scale of new physics should be of the order of 1–10^3 TeV, at which scale one would expect a large number of new states charged under the standard model gauge group to appear. Nevertheless, there are interesting footprints in the standard model, in particular the Higgs self-couplings.

Indeed, the effective potential of the SM Higgs boson in the IR theory was found to be

$$V = \frac{\lambda}{4}\phi^4 + \frac{1}{4}\phi^2\left(-m_h^2 + (m_h^2 - 2\langle\phi\rangle^2\lambda)\log\left[\frac{\phi^2}{\langle\phi\rangle^2}\right]\right). \tag{37}$$

The phenomenology depends entirely on how close the quartic coupling λ is to its SM value, $\lambda = m_h^2/2\langle\phi\rangle^2$. This is in turn related via Eq. (18) to the shift in the vacuum polarization of the gauge bosons communicated by the messenger fields,

$$\mathcal{A}_a = \frac{1}{f_{c(a)}^2}\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2}(C_{\rm hid}^{(a)}(p^2,0) - C_{\rm hid}^{(a)}(p^2,f_c^2)), \tag{38}$$

where a labels the gauge group. We stress that these quantities, and hence λ, are calculable in many specific models. We computed it for the minimal case of a

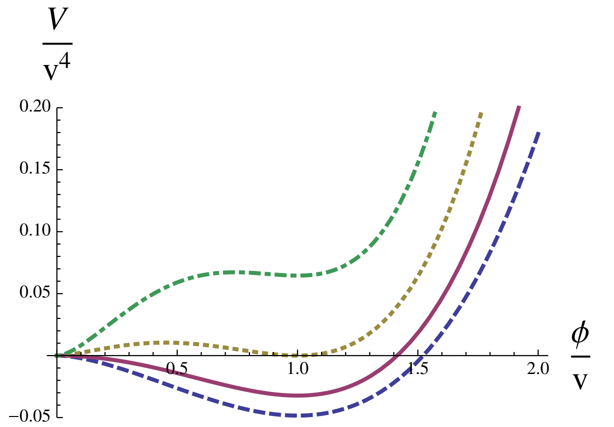

FIG. 3 (color online). Higgs potential with $m_h^2 = 125^2$, $\langle\phi\rangle \equiv v = 246$, and $\lambda = (\frac{1}{4}, \frac{1}{2}, 1, 2)\frac{m_h^2}{\langle\phi\rangle^2}$.

perturbative hidden sector, but there are many alternative possibilities. These may include unification at the UV fixed point, and as mentioned may involve strong coupling (in which case one would naturally use Randall Sundrum (RS1) with f_c corresponding to the position of the IR brane, and the Cs being given by the propagators of bulk gauge fields).

We can identify three generic phenomenological patterns. These are represented in Fig. 3 which shows the potential for the four prototypical values: $\lambda = (\frac{1}{4}, \frac{1}{2}, 1, 2)\frac{m_h^2}{\langle\phi\rangle^2}$. We shall now briefly describe them.

(a) *Degenerate or metastable electroweak symmetry breaking*: These configurations are attained for $\lambda \geq \frac{m_h^2}{\langle\phi\rangle^2} \simeq \frac{1}{4}$. The green and yellow lines correspond to $\lambda = 2\frac{m_h^2}{\langle\phi\rangle^2} \simeq \frac{1}{2}$ and $\lambda = \frac{m_h^2}{\langle\phi\rangle^2} \simeq \frac{1}{4}$, respectively. Both these values have $\frac{m_h^2}{2\langle\phi\rangle^2} < \lambda < \frac{13}{8}g_2^2 \simeq 0.7$ so that $\mathcal{A}_2 > 0$ and they can therefore be achieved in the minimal model presented in Sec. II A. The metastable case shown is relatively short lived, but by choosing values of λ closer to the critical value one can give it a lifetime longer than the age of the Universe. It is interesting to derive the resulting limits on λ coming from the decay rate from the metastable vacuum. An estimate of the O_4 symmetric bounce action is [70]

$$S_{O_4} \sim \frac{2\pi^2\langle\phi\rangle^4}{V(\langle\phi\rangle) - V(0)}. \tag{39}$$

A value of the action $S \gtrsim 400$ is generally sufficient to prevent vacuum decay within the lifetime of the Universe. Defining $\lambda = \frac{m_h^2}{\langle\phi\rangle^2} + \kappa$, the metastable minimum has the vacuum energy raised by $V(\langle\phi\rangle) - V(0) = \frac{\kappa}{4}\langle\phi\rangle^4$. Hence, the vacuum is metastable but has insignificant decay if $0 < \kappa \lesssim \pi^2/50$, or in other words if $0.25 < \lambda \lesssim 0.45$. Note that it requires almost no fine-tuning of λ to protect the metastable vacuum. (On the contrary rapid vacuum decay is actually rather hard to achieve.) It would be interesting to investigate the cosmological consequences of such a potential. We leave this as a subject for future study. Finally, the case with $\lambda = \frac{m_h^2}{\langle\phi\rangle^2}$ leads to two degenerate vacua, both at zero vacuum energy. The expansion of Eq. (37) around the minimum as $\phi = \langle\phi\rangle + h$ [given for the general case in Eq. (2)] is

$$V = \frac{m_h^2}{2}h^2 + \frac{5m_h^2}{6\langle\phi\rangle}h^3 + \frac{7m_h^2}{24\langle\phi\rangle^2}h^4 + \mathcal{O}(h^5). \quad (40)$$

(b) *Standard model with Higgs portal*: The red line in Fig. 3 has the standard model value of the quartic coupling, namely $\lambda = \frac{m_h^2}{2\langle\phi\rangle^2}$. Clearly for this value, the logarithm in the potential (37) cancels, the quadratic term is negative, and the conventional standard model picture follows. The GMESB mechanism is then simply providing a UV completion for the usual Higgs portal model, with the portal coupling to the hidden sector spurion $\langle\chi\rangle = f_c$ being generated radiatively by gauge mediation. This case can be realized with $\mathcal{A}_3 > 0$ and $\mathcal{A}_2 = 0$. In other words the standard model is a general prediction of gauge mediation of spontaneous scale breaking in which the mediation is dominated by colored degrees of freedom only. Since $\mathcal{A}_3$ is positive it may be achieved with a minimal configuration. For comparison, the potential expanded around the minimum is

$$V = -\frac{m_h^2\langle\phi\rangle^2}{8} + \frac{m_h^2}{2}h^2 + \frac{m_h^2}{2\langle\phi\rangle}h^3 + \frac{m_h^2}{8\langle\phi\rangle^2}h^4. \quad (41)$$

(c) *Logarithmic Higgs potential*: Finally, in Fig. 3, the blue line shows an example where the quartic term is becoming negligible and the minimization is dominated by the logarithm. In the limit that λ is negligible in (37) with the potential taking the approximate form,

$$V = \frac{1}{4}m_h^2\phi^2\left(\log\frac{\phi^2}{\langle\phi\rangle^2} - 1\right), \quad (42)$$

the Higgs self-couplings are given by

$$V = -\frac{m_h^2\langle\phi\rangle^2}{4} + \frac{m_h^2}{2}h^2 + \frac{m_h^2}{6\langle\phi\rangle}h^3 - \frac{m_h^2}{24\langle\phi\rangle^2}h^4 + \mathcal{O}(h^5). \quad (43)$$

In summary we see that GMESB leads to a more general Higgs potential that can have unusual deviations from the conventional standard model self-couplings. In all other respects, however, in particular in its couplings to the matter fields and gauge bosons, the Higgs boson behaves precisely as it would do in the conventional SM. Note that this is markedly different from the usual phenomenology of a Higgs-like dilaton (essentially because the Higgs is not actually the real dilaton): in the conventional notation of e.g. Ref. [71], only the c_3 operator is affected.

It is also interesting to note that, in common with GMSB, the flavor problem would be greatly ameliorated. As we have mentioned, depending on the manner in which scale invariance is restored in the UV, there could conceivably be light states of the order of a few TeV in nonminimal realizations. These states would be expected to receive order TeV masses much like the sfermions in GMSB, but (as in GMSB) these masses would be degenerate because they depend only on the gauge charges of the fields. Thus, any mixing angles could only arise from the Cabibbo-Kobayashi-Maskawa matrix, and one would expect the Glashow-Iliopoulos-Maiani mechanism to still be in operation.

Finally, we can estimate the mass of the dilaton in GMESB (identified with the field that spontaneously breaks scale invariance) which is rather heavy. Its mass follows from the usual partially conserved dilatation current relation, $f_c^2 m_\sigma^2 = -\langle T_\mu^\mu\rangle = -16\epsilon_{\rm vac}$, where $\epsilon_{\rm vac}$ is the contribution to the vacuum energy density from the hidden sector (see for example the discussion in Ref. [72]). Thus, a perturbative hidden sector would have a dilaton with the mass $m_\sigma \sim f_c/4\pi \sim$ 1–10 TeV, while a nonperturbative one would have a dilaton with the mass $m_\sigma \sim f_c \sim 10$–$10^3$ TeV.

ACKNOWLEDGMENTS

We thank Riccardo Argurio, James Barnard, Matt Buican, Zohar Komargodski, Valya Khoze, and Diego Redigolo for interesting dicussions. S. A. A. thanks the CERN theory division, the Galileo Galilei Institute for Theoretical Physics, and the INFN for hospitality and partial support during the final stages of this work. A. M. acknowledges funding by a Durham International Junior Research Fellowship. We acknowledge funding from the STFC.

Note added in proof.—During the final stages of publication Ref. [80] appeared on the ArXiV. The authors of that paper were concerned that the presence of the logarithmic mass term would not be consistent with the decoupling theorem. We argue that these concerns do not apply to the present case, which is indeed in accord with it. Although at first sight the example discussed in Ref. [80] resembles the one that we have presented, the crucial difference is that here the scale f_c cannot be decoupled without also decoupling the Higgs (whereupon the logarithmic terms dutifully disappear). More generally f_c represents the deviation from a scale invariant fixed point so there can be no parametrically large hierarchy of mass scales that would allow it to be decoupled while leaving the Higgs light. For example, we see this explicitly in the minimal GMESB case where the ratio of the Higgs mass to f_c is fixed by the particle content and anomalous dimensions at the UV fixed point. The only hierarchies of mass scales in the present theory are generated by loop factors (much as in Coleman-Weinberg) which is why f_c is constrained to be no more than 1000 TeV: indeed increasing it to much higher values would have been an attractive proposition, but it is alas impossible.

APPENDIX A: WHY SCALE INVARIANCE DOES NOT USUALLY PROTECT THE HIGGS MASS

It is instructive to ask when a restoration of scale invariance in the UV can be sufficient to protect the Higgs mass. Here, we shall discuss this question by examining a naïve perturbative example, in which one attempts to restore scale invariance simply through the addition of extra states or resonances. At the same time this gives the opportunity to present some expressions required for the main text.

Consider the SM, but turn off all the Yukawa couplings and $SU(3)_c \times U(1)_Y$ gauge couplings and also neglect the Higgs self-coupling so that the $SU(2)_W$ coupling dominates. Now make the theory run perturbatively to a UV fixed point at a high scale. Since $SU(2)_W$ has a nonzero β function in the SM, this requires extra degrees of freedom, and we can take them to be simply heavy $SU(2)$ fermion doublets, η_{L_i}, η_{R_i}, and scalar doublets, $\tilde{\eta}_{L_a}, \tilde{\eta}_{R_a}$. This general case includes the supersymmetric system which is an interesting point of reference. As we shall show in this appendix and in Appendix B, generally such a theory still suffers from the hierarchy problem because UV sensitivity is not removed and therefore it is not possible to take the continuum limit, i.e. to send the UV cutoff Λ_∞ to infinity. We will see that UV sensitivity is removed entirely in a perturbative theory when the poles of heavy states form a multipole (in momentum space) of sufficiently high order. Nonperturbatively, and in the Wilsonian language, one would say that the theory has to lie on a "renormalizable trajectory" (i.e. a self-similar RG trajectory that emanates from a repulsive UV fixed point—see for example the review of [73]).

Using dimensional regularization, we can compute the radiative contributions to the Higgs mass in two stages. First, the contribution to the gauge propagator can be captured in the current-current correlators for $SU(2)$:

$$\langle j_\mu(p) j_\nu(-p)\rangle = -(\eta_{\mu\nu}p^2 - p_\mu p_\nu)C\left(\frac{p^2}{\mu^2}, \epsilon\right), \quad \text{(A1)}$$

with the effective Lagragian being given by

$$\delta\mathcal{L} = -\frac{1}{4}g^2 C F_{\mu\nu}F^{\mu\nu}. \quad \text{(A2)}$$

The subsequent contribution to the potential (in Euclidean coordinates) is

$$\delta V = 3g^2 \int \frac{p^2 d^4p}{(2\pi)^4 p^2 + m_W^2}\left(C\left(\frac{p^2}{\mu^2}, \epsilon\right) - \Delta_Z\right), \quad \text{(A3)}$$

with Δ_Z being the counterterms. It is useful to define parameters scaled by f_c with hats, so that

$$\delta V = 3g^2 f_c^4 \int \frac{\hat{p}^2 d^4\hat{p}}{(2\pi)^4 \hat{p}^2 + \hat{m}_W^2}(C(\hat{p}^2, \hat{\mu}^2, \epsilon) - \Delta_Z). \quad \text{(A4)}$$

The dimensionless parameter C can be evaluated in general terms: Defining the scalar and fermion masses as M_a and M_i, respectively, we find contributions of the form

$$iC(\hat{p}^2, \hat{\mu}^2, \epsilon) = \frac{2g^2 C_R}{3}(I(p, M_a) + 2I(p, M_i)) + \frac{8g^2 C_R}{3p^2}[K(p, M_a) - K(p, M_i)], \quad \text{(A5)}$$

where

$$K(p, m) = m^2 I(p, m) + J(m) + \frac{im^2}{16\pi^2} \quad \text{(A6)}$$

and where

$$\begin{aligned} I(p, m) &= i\int \frac{d^4q}{(2\pi)^4}\frac{1}{(q^2 + m^2)((q + p)^2 + m^2)} \\ &= i\frac{1}{16\pi^2}\left(\frac{2}{\epsilon} + \Gamma'(1) + \log\left[\frac{4\pi\mu^2}{m^2}\right] - \int_0^1 dx \log\left[1 + \frac{p^2}{m^2}x(1 - x)\right]\right) \end{aligned} \quad \text{(A7)}$$

$$\begin{aligned} J(m) &= i\int \frac{d^4q}{(2\pi)^4}\frac{1}{(q^2 + m^2)} \\ &= -i\frac{m^2}{16\pi^2}\left(\frac{2}{\epsilon} + \Gamma'(1) + 1 + \log\left[\frac{4\pi\mu^2}{m^2}\right]\right), \end{aligned} \quad \text{(A8)}$$

so that

$$K(p, m) = -i\frac{m^2}{16\pi^2}\int_0^1 dx \log\left[1 + \frac{p^2}{m^2}x(1 - x)\right].$$

Scaling violation is parametrized by μ, which is directly attributed to RG-flow, whereas infinities appear as poles in ϵ. In a fully scale invariant theory (i.e. in the deep UV) the total dependence on μ should vanish which means that ϵ poles also cancel. In a cutoff regulated theory (c.f. Appendix B) poles in ϵ are replaced by dependence on the UV cutoff Λ_∞. Full scale invariance means that there is no remaining dependence on Λ_∞, so that one can take the continuum limit, $\Lambda_\infty \to \infty$, as desired. Including the rest of the standard model and Higgs bosons, and assuming only $SU(2)$ fundamentals, we find that the first piece in Eq. (A5) in conjunction with the obvious gauge loops and one-loop vertex correction, gives the β function

$$\begin{aligned}\beta &= -\frac{g^3}{16\pi^2}\left(\frac{11}{3}C_G - \frac{4}{3}(N_{\text{SM-ferm}} + N_\eta)C_{\text{fund}}\right.\\ &\quad\left. - \frac{2}{3}(N_h + N_{\tilde{\eta}})C_{\text{fund}}\right)\\ &= -\frac{g^3}{16\pi^2}\left(\frac{11}{3}N_c - \frac{2}{3}(N_{\text{SM-ferm}} + N_\eta)\right.\\ &\quad\left. - \frac{1}{3}(N_h + N_{\tilde{\eta}})\right), \end{aligned} \tag{A9}$$

where the Ns count the number of complete Dirac flavors, with each pair of complex scalars (in a hang-on from supersymmetry) counting one to $N_{\tilde{\eta}}$. In the SM there are 12 fermion doublets ($N_{\text{SM-ferm}} = 6$) and a single Higgs boson ($N_h = 1/2$) giving $\beta_{UV} = -\frac{g^3}{16\pi^2}(\frac{19}{6} - \frac{1}{3}(2N_\eta + N_{\tilde{\eta}}))$. In order to get conformal behavior in the UV, we could then simply choose the heavy states such that $(2N_\eta + N_{\tilde{\eta}}) = 3b^{(\text{SM})}_{SU(2)} = \frac{19}{2}$ (noting that scalars contribute $1/2$).

A useful renormalization scheme for a theory that flows to a fixed point in the UV is the so-called "sliding-scale scheme" [74] where $C_1(a, a, \epsilon) - \Delta_Z = 0$, because one does not need to perform matching and introduce thresholds by hand. Indeed, this renormalization scheme is the one that most closely mimics Wilsonian renormalization, as it effectively subtracts contributions from modes above the RG scale, μ. Defining $\zeta(x) = \sqrt{-4/x - 1}$, from the above discussion we find that the net contribution to the vacuum polarization of a single boson or Weyl fermion is

$$\begin{aligned} C_m(\hat{p}^2, \hat{\mu}^2) &= \frac{g^2}{16\pi^2}[H_\pm(\hat{p}^2) - H_\pm(\hat{\mu}^2)]\\ H_\pm(z) &= -\frac{4}{3}\left(\left(\kappa_\pm \pm \frac{2}{z}\right)\zeta(z)\tan^{-1}\left[\frac{1}{\zeta(z)}\right] \mp 2z^{-1}\right), \end{aligned}$$

where μ is the RG scale and $\hat{p} = p/m$, $\hat{\mu} = \mu/m$, $\pm$ refers to the spin statistics, and where $\kappa_\pm = 1$ or $\frac{1}{2}$, respectively. The function $H_\pm$ has the limiting behavior

$$\lim_{z\to 0} H_\pm(z) = \text{const.}$$

$$H_\pm(z) \xrightarrow{z\to\infty} \begin{cases} -\frac{1}{3}\log z - \frac{2}{z}(1 - \log z) + \mathcal{O}(z^{-2}) & : \text{boson} \\ -\frac{2}{3}\log z - \frac{4}{z} + \mathcal{O}(z^{-2}) & : \text{fermion.} \end{cases} \tag{A10}$$

There are then three interesting limits: $\mu, p \gg m$ ($\hat{\mu}, \hat{p} \gg 1$), which is when the mass is negligible, $\mu, p \ll m$ ($\hat{\mu}, \hat{p} \ll 1$), when the state is heavier than both the momentum and the RG scale, and $\mu \gg m \gg p$ ($\hat{\mu} \gg 1 \gg \hat{p}$), which is when the RG scale is larger than the mass but the momentum is small. Defining $\Delta b = -\frac{1}{3}, -\frac{2}{3}$ to be the contribution of the state to the β-function coefficient, to the leading order these limits give

$$\frac{16\pi^2}{g^2}C_m(\hat{p}^2, \hat{\mu}^2) \to \begin{cases} \Delta b \log\frac{p^2}{\mu^2} & \mu, p \gg m \\ 0 & \mu, p \ll m \\ \Delta b \log\frac{m^2}{\mu^2} & \mu \gg m \gg p \end{cases}. \tag{A11}$$

In the present context the contribution to the total polarization from the SM fields is canceled in the UV by the states of mass f_c which for convenience we assume are degenerate (and we also neglect the SM masses). Therefore, we must choose N_η and $N_{\tilde{\eta}}$ accordingly (as above) such that the total vacuum polarization including the SM states is

$$\begin{aligned}\frac{16\pi^2}{g^2}C_{\text{tot}}(\hat{p}^2, \hat{\mu}^2) &= b'^{(\text{SM})}_{SU(2)}\log\left[\frac{\hat{p}^2}{\hat{\mu}^2}\right] - b^{(\text{SM})}_{SU(2)}\int_0^1 dx \log\left[\frac{1 + \hat{p}^2x(1-x)}{1 + \hat{\mu}^2x(1-x)}\right]\\ &\quad - \frac{4(N_{\tilde{\eta}} - N_\eta)}{3}\int_0^1 dx \frac{1}{\hat{p}^2}\log[1 + \hat{p}^2x(1-x)] - \frac{1}{\hat{\mu}^2}\log[1 + \hat{\mu}^2x(1-x)], \end{aligned} \tag{A12}$$

where $b^{(\text{SM})}_{SU(2)}$ is the total β-function coefficient of the SM, and $b'^{(\text{SM})}_{SU(2)}$ is the β-function coefficient of the SM coming from vacuum polarization diagrams (there is a piece that can be attributed to the one-loop vertex diagram as well). Note that $C_{1-\text{tot}}(a, a) = 0$ which is the sliding-scale condition satisfied by construction. From (A11), the various limits of the vacuum polarization are

$$\frac{16\pi^2}{g^2}C_{\rm tot}(\hat{p}^2,\hat{\mu}^2) \rightarrow \begin{cases} (b'^{(\rm SM)}_{SU(2)} - b^{(\rm SM)}_{SU(2)})\log\left[\frac{p^2}{\mu^2}\right] & \mu, p \gg f_c \\ b'^{(\rm SM)}_{SU(2)}\log\left[\frac{p^2}{\mu^2}\right] & \mu, p \ll f_c \\ (b'^{(\rm SM)}_{SU(2)} - b^{(\rm SM)}_{SU(2)})\log\left[\frac{p^2}{\mu^2}\right] + b^{(\rm SM)}_{SU(2)}\log\left[\frac{p^2}{f_c{}^2}\right] & \mu \gg f_c \gg p \end{cases}. \tag{A13}$$

These limits have the following interpretation. The first $\mu, p \gg f_c$ limit is when the theory is in the UV scale invariant regime. Thus, the vacuum polarization contribution to the β function is such that it is precisely canceled by the SM vertex correction. The second limit corresponds to the effective theory in the IR and is just the normal SM vacuum polarization contribution to the β function. The third limit, $\mu \gg f_c \gg p$, corresponds to working in the complete theory with all the heavy f_c degrees of freedom still present. Again, the first term is precisely canceled by the SM vertex correction, but the second term corresponds to the finite logarithmic correction to the β function that one would expect to arise between the scale f_c (below which scale invariance is first broken) and the momentum, p.

Next we can insert this vacuum polarization into the two-loop W boson contribution to the Higgs mass. But upon performing the integral over p we now see (by expanding in $m_W^2 = g_W^2\phi^2$) that the Higgs mass generated by (A4) is still logarithmically divergent [75]. Indeed, the two-loop contribution to the potential can be written as

$$\delta V = \frac{3g^4}{16\pi^2}f_c^4\left[b_{SU(2)}J_1 - \frac{4(N_{\tilde{\eta}} - N_\eta)}{3}K_1\right], \tag{A14}$$

where

$$J_1 = \frac{1}{(4\pi)^{d/2}\Gamma(d/2)}\int_0^\infty ds\frac{s^{d/2}}{(s+\hat{m}_W^2)}\left\{2\zeta(s)\tan^{-1}\left[\frac{1}{\zeta(s)}\right] - \log s - 2\zeta(\hat{\mu}^2)\tan^{-1}\left[\frac{1}{\zeta(\hat{\mu}^2)}\right] + \log\hat{\mu}^2\right\}$$
$$K_1 = \frac{1}{(4\pi)^{d/2}\Gamma(d/2)}\int_0^\infty ds\frac{s^{d/2}}{(s+\hat{m}_W^2)}\left\{\frac{2}{s}\left(\zeta(s)\tan^{-1}\left[\frac{1}{\zeta(s)}\right] - 1\right) - \frac{2}{\hat{\mu}^2}\left(\zeta(\hat{\mu}^2)\tan^{-1}\left[\frac{1}{\zeta(\hat{\mu}^2)}\right] - 1\right)\right\}. \tag{A15}$$

Expanding in $\hat{m}_W^2/s$ and using the above limits in Eq. (A10), we see that the Higgs mass, m_W^2, piece of the potential is proportional to $\delta V \sim f_c^2 m_W^2 \log\hat{\mu}$. Thus, the hierarchy problem is not solved in this theory, although the divergence is rendered logarithmic thanks to the UV scale symmetry. As described in Appendix B, the UV scale invariance is equivalent to providing a single canceling residue that reduces the degree of divergence of the diagram but doesn't quite render the integral finite. While that may seem like an improvement, generally if this divergence is allowed to persist it means that physics is still not truly scale invariant above f_c. There is one exception to this rule (which does not apply in this case), which is when the divergence is associated with the nonzero anomalous dimension of the Higgs field at the UV fixed point. Then one would expect the coefficient of logarithmically divergent Higgs mass-squared terms to precisely match the coefficient of logarithmically divergent kinetic terms.

Such divergences can be made harmless (as in the main body of the text) if one assumes that the theory runs to a fixed point with the fields having the small finite anomalous dimension γ. Then one expects terms that go like $1/\gamma$. Alternatively and more brutally one can (as in Appendix B) add additional states that remove this divergence entirely. [The former case is physically similar to dimensional regularization, while the latter is similar to parity violating (PV) regularization.] In either case, there must be additional degrees of freedom to render the Higgs mass truly scale invariant in the UV allowing one to take the continuum limit. Consequently, the hierarchy problem remains.

In terms of the exact renormalization group, the interpretation would be as follows [73]. We are envisaging a flow emanating from a critical surface along a renormalizable trajectory. Such a flow would begin at a fixed point in which the action $S[\varphi,\mu,\Lambda_\infty] = S^*[\varphi] + \mathcal{O}(\mu/\Lambda_\infty)$, with a necessary (but not sufficient) condition for the theory to be renormalizable being that $\lim_{\Lambda_\infty\to\infty}(S[\varphi,\mu,\Lambda_\infty]) = S^*[\varphi]$ [76]. In order for the theory to approach the fixed point perturbatively, additional resonances have to remove any $\Lambda_\infty \to \infty$ divergences. Of course the usual arguments of asymptotic safety would then counter that maybe the theory just happens to flow close by a perturbative fixed point—about which we are doing our perturbation theory—and that it actually emanates from a different fixed point higher up; the conclusion of a lack of renormalizability is therefore incorrect. Unfortunately, while that may be true, in the present context this argument does not save us because it would imply a rapid change in the gauge β functions around the scale of the Gaussian fixed point where the theory is diverted to the second, higher, fixed point; one would expect this to be gauge mediated to the visible sector, so that the RG scale of the intermediate fixed point, rather than f_c, would become the typical scale of relevant operators induced in the IR theory.

APPENDIX B: FROM DIMENSIONAL TO PV TO CUTOFF REGULARIZATION

For clarification (and as background to the discussion in Appendix A) it is useful to consider the relation between different regularization schemes, and to ask if there is a preferred choice for scale invariant theories. As a warm-up, let us take the basic (Wick rotated) one-loop integral

$$J = \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2+m^2)^n}. \tag{B1}$$

In dimensional regularization in $d = 4 - \epsilon$ dimensions, and by replacing $p^2 \to s$, this becomes

$$J = \frac{\pi^{d/2}}{(2\pi)^d \Gamma(d/2)} \int_0^\infty ds \frac{s^{1-\epsilon/2}}{(s+m^2)^{n+1}}. \tag{B2}$$

The integrand has a pole at $s = -m^2$, and a branch-point at $s = 0$, so one way to evaluate it is to use the "keyhole" contour integral in Fig. 4 with a branch-cut placed along the positive real axis. Because of the branch-cut,

$$\int_{B\oplus D} ds \frac{s^{1-\epsilon/2}}{(s+m^2)^{n+1}} = (1-e^{-\pi i \epsilon}) \int_0^{s_\infty} ds \frac{s^{1-\epsilon/2}}{(s+m^2)^{n+1}}.$$

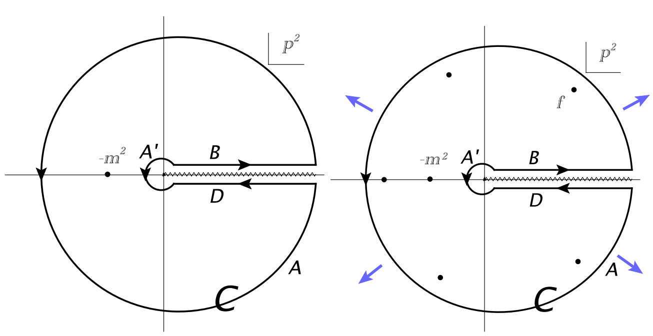

FIG. 4 (color online). Keyhole contour for turning dimensional regularization into cutoff (left), and (right) additional canceling singularities appearing in a hypothetical UV complete theory allowing the arc to be taken to infinity.

The A' integral vanishes for any n, while the integral from the arc of the radius s_∞ (denoted by A) is of the order of $s_\infty^{1-n-\epsilon/2}$.

The order in which limits are taken is the central issue. Suppose we take the $s_\infty \to \infty$ limit first, setting the arc integral to zero (for a large enough ϵ). We then have

$$(1-e^{-\pi i \epsilon}) J = \frac{\pi^{d/2}}{(2\pi)^d \Gamma(d/2)} \oint_C ds \frac{s^{d/2-1}}{(s+m^2)^{n+1}}. \tag{B3}$$

The integral J can then be easily evaluated by the residue theorem, and the relevant ϵ-pole is already explicit in Eq. (B3). For example, when $n = 1$ we have

$$\begin{aligned} J &= \frac{2\pi i}{(1-e^{-\pi i \epsilon})} \frac{1}{(4\pi)^{2-\epsilon/2}\Gamma(1-\epsilon/2)} (-m^2)^{-\epsilon/2} \\ &= \frac{1}{16\pi^2}\left[\frac{2}{\epsilon} - \gamma_E - \log\left(\frac{m^2}{4\pi}\right) + \mathcal{O}(\epsilon)\right], \end{aligned} \tag{B4}$$

as usual.

So we see that ϵ-poles in dimensional regularization are inevitable because of the branch-cut. However, what about finite integrals such as

$$J = \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2+m^2)} \left(\frac{f_c^2}{p^2+f_c^2}\right)^{n+1}, \tag{B5}$$

where $n \in Z$, $n > 0$ and $f_c \gg m$? As an example, such an integral could conceivably occur if one is considering contributions to β functions that have multiple cancellations of both leading (as in Appendix A) and subleading terms. The integral is already regulated by the second factor in the integrand, but the above argument tells us that there will be ϵ-poles anyway. In fact we now have contributions from two residues, one from the pole at $s = -m^2$, and one from the pole at $s = -f_c^2$. In total we find

$$\begin{aligned} (1-e^{-\pi i \epsilon}) J = 2\pi i \frac{\pi^{\frac{d}{2}} f_c^{2n}}{(2\pi)^d \Gamma(d/2)} &\left\{ \frac{1}{n!} \partial_s^n \frac{s^{\frac{d}{2}-1}}{(s+m^2)} \Big|_{s=-f_c^2} \right. \\ &\left. + \frac{s^{\frac{d}{2}-1}}{(s+f_c^2)^{n+1}} \Big|_{s=-m^2} \right\} \\ &= \mathcal{O}(\epsilon). \end{aligned} \tag{B6}$$

The integrand is a multipole in which the residues cancel at the leading order in ϵ, leaving no ϵ-pole for J but just a trail of finite pieces from the regularization. This is always the case if one chooses a regulating function that does not introduce essential singularities or branch-cuts, and is similar to Pauli Villars–type regularization.

Let us now return to the arc (both mathematically and metaphorically) of the previous divergent example, and suppose that we instead expand the propagator in m^2/s, but do not take the $s_\infty \to \infty$ limit before the $\epsilon \to 0$ limit. For the $n = 1$ integral, we then have in total

$$J = \frac{1}{16\pi^2}\left[\frac{2}{\epsilon} - \gamma_E - \log\left(\frac{m^2}{4\pi}\right) + \mathcal{O}(\epsilon)\right] - J_A, \qquad (B7)$$

where J_A is the additional contribution from the arc, given by

$$J_A = \frac{2\pi i}{(1-e^{-i\pi\epsilon})}\frac{\pi^{\frac{d}{2}}}{(2\pi)^d\Gamma(d/2)} s_\infty^{\frac{d}{2}-1-n}\int_0^{2\pi}\frac{d\theta}{2\pi}e^{i(\theta(\frac{d}{2}-n-1)}$$
$$= \frac{1}{16\pi^2}\left[\frac{2}{\epsilon} + 1 - \gamma_E - \log\left(\frac{s_\infty}{4\pi}\right) + \mathcal{O}(\epsilon)\right]. \qquad (B8)$$

This precisely cancels the pole in ϵ giving

$$J = \frac{1}{16\pi^2}\left[\log\left(\frac{\Lambda_\infty^2}{m^2}\right) + \mathcal{O}(1)\right], \qquad (B9)$$

and turning the dimensional regularization into cutof. Likewise, it is easy to check that the familiar quadratically divergent $n = 0$ integral in dimensional regularization,

$$J = \frac{2}{\epsilon}(1 + i\pi\epsilon/2)\frac{1}{(4\pi)^{2-\epsilon/2}\Gamma(2-\epsilon/2)}(-m^2)^{1-\epsilon/2}$$
$$= -\frac{m^2}{16\pi^2}\left[\frac{2}{\epsilon} - \gamma_E + 1 - \log\left(\frac{m^2}{4\pi}\right) + \mathcal{O}(\epsilon)\right], \qquad (B10)$$

is, upon reversing the order of limits, turned into the cutoff answer

$$J = \frac{\Lambda_\infty^2}{16\pi^2} + \frac{m^2}{16\pi^2}\log\left(\frac{m^2}{\Lambda_\infty^2}\right). \qquad (B11)$$

The reason this geometric picture is useful to bear in mind is that the residues encode the contribution to the integral from both the low energy theory and from states of mass, f_c, that we explicitly add into the theory in order to achieve UV scale invariance, while the arc encodes the remaining UV sensitivity (which we would obviously like to vanish). For example, with the regulated integral example of (B5) the latter contribution vanishes as the radius is taken to infinity, as in Fig. 4.

Suppose now that we have a theory that emanates from a fixed point in the UV, so we decide to measure all our couplings at some high scale, $\mu \gg f_c$, above which we consider g to be roughly constant. Then in the sliding-scale scheme discussed in Appendix A we would write an expression like Eq. (A12) with C_{tot} (or more precisely the β function) vanishing at $\hat{p} = \hat{\mu}$. The difference between the result at μ and the would-be continuum result is then given by subtracting one contour integral from the other which leaves a contour integral around an annulus (with a branch-cut) with the inner radius $s_\mu = \hat{\mu}^2$ and outer radius $s_\infty = \hat{\Lambda}_\infty^2$. In order to be able to take the continuum limit one would require this integral to converge to a constant as $\Lambda_\infty \to \infty$ with $\mu \gtrsim f_c$. Conversely, if the arc integral blows up in the $\Lambda_\infty \to \infty$ limit, a continuum limit does not exist within the perturbative description being considered: despite the theory becoming scale invariant in the UV, the higher the scale at which one measures the couplings, the more precisely one has to do it in order to have control over the low energy theory. From the point of view of the exact renormalization group, one would say that the theory never reaches the perturbative (but nontrivial) fixed point we had in mind, but flows past it, perhaps to some other interacting fixed point.

In addition we conclude that dimensional regularization has no preferred status in theories based on exact UV scale invariance. Indeed, suppose that one actually *had* a UV completion in which all integrals were canceled by resonances and the arc really could be taken to infinity. In such a theory the $s_\infty \to \infty$ and $\epsilon \to 0$ limits would have to commute since regularization is unnecessary. The only conceivable advantage of dimensional regularization would then be if, in conjunction with the Coleman-Weinberg prescription of setting tree level mass-terms to zero, what was left after the arc is taken to infinity were somehow a better approximation to the IR physics than simply putting a cutoff at f_c. As is evident from the main body of the text, this is generally not the case, so we conclude that "classical scale invariance" is not a good guiding principle for discussing the phenomenology of spontaneously broken exact scale invariance; all of which is not to say that the Coleman-Weinberg mechanism has no place, but just that it is a different program from that of exact UV scale invariance.

[1] A. Salam and J. A. Strathdee, Phys. Rev. **184**, 1760 (1969); J. R. Ellis, Nucl. Phys. **B22**, 478 (1970).
[2] S. R. Coleman and E. J. Weinberg, Phys. Rev. D **7**, 1888 (1973).
[3] E. Gildener and S. Weinberg, Phys. Rev. D **13**, 3333 (1976).
[4] W. A. Bardeen, Report No. FERMILAB-CONF-95-391-T.
[5] K. A. Meissner and H. Nicolai, Phys. Lett. B **648**, 312 (2007).
[6] W.-F. Chang, J. N. Ng, and J. M. S. Wu, Phys. Rev. D **75**, 115016 (2007).
[7] T. Hambye and M. H. G. Tytgat, Phys. Lett. B **659**, 651 (2008).
[8] W. D. Goldberger, B. Grinstein, and W. Skiba, Phys. Rev. Lett. **100**, 111802 (2008).
[9] R. Foot, A. Kobakhidze, K. L. McDonald, and R. R. Volkas, Phys. Rev. D **77**, 035006 (2008).

[10] M. Shaposhnikov and D. Zenhausern, Phys. Lett. B **671**, 162 (2009).
[11] S. Iso, N. Okada, and Y. Orikasa, Phys. Lett. B **676**, 81 (2009).
[12] K. A. Meissner and H. Nicolai, Phys. Rev. D **80**, 086005 (2009).
[13] M. Holthausen, M. Lindner, and M. A. Schmidt, Phys. Rev. D **82**, 055002 (2010).
[14] L. Alexander-Nunneley and A. Pilaftsis, J. High Energy Phys. 09 (2010) 021.
[15] T. Hur and P. Ko, Phys. Rev. Lett. **106**, 141802 (2011).
[16] A. R. Vieira, B. Hiller, M. D. R. Sampaio, and M. C. Nemes, Int. J. Theor. Phys. **52**, 3494 (2013).
[17] Z. Chacko and R. K. Mishra, Phys. Rev. D **87**, 115006 (2013).
[18] S. Iso and Y. Orikasa, Prog. Theor. Exp. Phys. **2013**, 023B08 (2013).
[19] C. Englert, J. Jaeckel, V. V. Khoze, and M. Spannowsky, J. High Energy Phys. 04 (2013) 060.
[20] M. Farina, D. Pappadopulo, and A. Strumia, J. High Energy Phys. 08 (2013) 022.
[21] M. Fabbrichesi and S. Petcov, Eur. Phys. J. C **74**, 2774 (2014).
[22] E. J. Chun, S. Jung, and H. M. Lee, Phys. Lett. B **725**, 158 (2013).
[23] M. Heikinheimo, A. Racioppi, M. Raidal, C. Spethmann, and K. Tuominen, Mod. Phys. Lett. A **29**, 1450077 (2014).
[24] I. Oda, arXiv:1305.0884.
[25] S. Dubovsky, V. Gorbenko, and M. Mirbabayi, J. High Energy Phys. 09 (2013) 045.
[26] T. Hambye and A. Strumia, Phys. Rev. D **88**, 055022 (2013).
[27] V. V. Khoze and G. Ro, J. High Energy Phys. 10 (2013) 075.
[28] D. Gorbunov and A. Tokareva, arXiv:1307.5298.
[29] C. D. Carone and R. Ramos, Phys. Rev. D **88**, 055020 (2013).
[30] G. Marques Tavares, M. Schmaltz, and W. Skiba, Phys. Rev. D **89**, 015009 (2014).
[31] A. Farzinnia, H. -J. He, and J. Ren, Phys. Lett. B **727**, 141 (2013).
[32] R. Dermisek, T. H. Jung, and H. D. Kim, arXiv:1308.0891.
[33] S. Y. Choi, C. Englert, and P. M. Zerwas, Eur. Phys. J. C **73**, 2643 (2013).
[34] V. V. Khoze, J. High Energy Phys. 11 (2013) 215.
[35] C. Tamarit, J. High Energy Phys. 12 (2013) 098.
[36] E. Gabrielli, M. Heikinheimo, K. Kannike, A. Racioppi, M. Raidal, and C. Spethmann, Phys. Rev. D **89**, 015017 (2014).
[37] R. Foot, A. Kobakhidze, K. L. McDonald, and R. R. Volkas, arXiv:1310.0223.
[38] O. Antipin, M. Mojaza, and F. Sannino, Phys. Rev. D **89**, 085015 (2014).
[39] T. G. Steele, Z. -W. Wang, D. Contreras, and R. B. Mann, Phys. Rev. Lett. **112**, 171602 (2014).
[40] M. Hashimoto, S. Iso, and Y. Orikasa, Phys. Rev. D **89**, 016019 (2014).
[41] M. Holthausen, J. Kubo, K. S. Lim, and M. Lindner, J. High Energy Phys. 12 (2013) 076.
[42] More precisely it is difficult to see what principle in a UV complete theory could guarantee a low energy theory with classical scale invariance when exact scale invariance does not.
[43] S. Weinberg, Report No. C76-07-23.1 HUTP-76/160.
[44] D. F. Litim, Phil. Trans. R. Soc. A **369**, 2759 (2011).
[45] H. Gies, J. Jaeckel, and C. Wetterich, Phys. Rev. D **69**, 105008 (2004).
[46] H. Gies, S. Rechenberger, and M. M. Scherer, Eur. Phys. J. C **66**, 403 (2010).
[47] J. Braun, H. Gies, and D. D. Scherer, Phys. Rev. D **83**, 085012 (2011).
[48] H. Gies, S. Rechenberger, M. M. Scherer, and L. Zambelli, Eur. Phys. J. C **73**, 2652 (2013).
[49] F. Bazzocchi, M. Fabbrichesi, R. Percacci, A. Tonero, and L. Vecchi, Phys. Lett. B **705**, 388 (2011).
[50] J. Barnard, T. Gherghetta, and T. S. Ray, J. High Energy Phys. 02 (2014) 002.
[51] J. Ellis and T. You, J. High Energy Phys. 06 (2013) 103.
[52] G. F. Giudice and R. Rattazzi, Phys. Rep. **322**, 419 (1999).
[53] C. Wetterich, Phys. Lett. B **718**, 573 (2012).
[54] We should add a word about nomenclature; here and throughout, by "hidden" we will mean fields that couple only to the gauge bosons of the standard model, or not at all. When we refer specifically to the former, we will call them messengers, in the tradition of GMSB.
[55] Note that the couplings should all take their UV fixed-point values. Any deviation would also be a function of ϕ and f_c so would be automatically included in this perturbative expansion.
[56] There are two possible reasons for the portal coupling to vanish there. The first is simply that one solution to all β functions vanishing has a zero portal coupling. The second would be a symmetry argument. For this one might consider the fact that in the limit $f_c \to 0$ and $g_{\mathrm{SM}} \to 0$ there is an effective enhancement of conformal symmetry to two conformal symmetries in the two sectors, much in the spirit of [37,77–79].
[57] We work in Euclidean space and the unitary gauge.
[58] D. Y. Bardin and G. Passarino, *The Standard Model in the Making: Precision Study of the Electroweak Interactions*, International Series of Monographs on Physics Vol. 104 (Oxford University Press, New York, 1999).
[59] Note that the situation is different from the one described in Ref. [3] because in that case the two scales ϕ and f_c appear at one loop as the overall "dilaton" $\sqrt{f_c^2 + \phi^2}$. Here, such one-loop terms are necessarily absent.
[60] S. P. Martin, Phys. Rev. D **65**, 116003 (2002).
[61] M. Shaposhnikov and D. Zenhausern, Phys. Lett. B **671**, 162 (2009); Z. Chacko and R. K. Mishra, Phys. Rev. D **87**, 115006 (2013).
[62] As in the previous section, a simple generalization would be to take the different mass scales $f_c^{(2)}$ and $f_c^{(3)}$ for the different gauge groups. In addition, if the hidden sector is strongly coupled, one might expect a tower of resonances separated by an interval, f_c, rather than a universal mass. This could presumably be modeled using bulk propagators for gauge bosons in RS1. We leave these possibilities for future work.
[63] That we consider massless for simplicity.

[64] We are working in the unitary gauge.
[65] Note that $2N_c - b'_{\rm SM} = b_0^{\rm SM}$ where $b_0^{\rm SM}$ is the SM one-loop β function coefficient.
[66] W. E. Caswell, Phys. Rev. Lett. **33**, 244 (1974).
[67] T. Banks and A. Zaks, Nucl. Phys. **B196**, 189 (1982).
[68] S. P. Martin and J. D. Wells, Phys. Rev. D **64**, 036010 (2001).
[69] O. Antipin, M. Gillioz, E. Mølgaard, and F. Sannino, Phys. Rev. D **87**, 125017 (2013).
[70] S. R. Coleman, Phys. Rev. D **15**, 2929 (1977); **16**1248 (1977).
[71] J. Serra, Eur. Phys. J. Web Conf. **60**, 17005 (2013).
[72] C. Coriano, L. Delle Rose, A. Quintavalle, and M. Serino, J. High Energy Phys. 06 (2013) 077.
[73] O. J. Rosten, Phys. Rep. **511**, 177 (2012).
[74] C. Scrucca, http://itp.epfl.ch/webdav/site /itp/users/181759/public/aqft.pdf.
[75] Note the advantage that supersymmetry has: there, reducing all divergences to logarithmic ones is consistent with a solution to the hierarchy problem.
[76] There is an additional requirement that the theory should also be self similar, for which one should also include renormalons in the $\Lambda_\infty \to \infty$ limit (see [73]).
[77] K. Benakli and C. Moura, Nucl. Phys. **B791**, 125 (2008).
[78] C. Cheung, Y. Nomura, and J. Thaler, J. High Energy Phys. 03 (2010) 073.
[79] R. Argurio, Z. Komargodski, and A. Mariotti, Phys. Rev. Lett. **107**, 061601 (2011).
[80] J. Elias-Miro, J. R. Espinosa, and T. Konstandin, arXiv:1406.2652.