Match-Fixing in a Monopoly Betting Market[*]

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January 23, 2016

Abstract

A monopolist bookmaker may set betting odds on a fairly even contest to induce match-fixing by an influential corrupt punter. His loss to the corrupt punter is more than made up for by enticing enough ordinary punters to bet on the losing team. This result is in sharp contrast to competitive bookmaking, where even contests have been shown to be immune to fixing.

The analysis also reveals a surprising result that the incidence of match-fixing can dramatically fall when match-fixing opportunities rise. This is shown by comparing two scenarios – when only one team is corruptible and when both are corruptible. For both teams corruptible, the bookmaker is uncertain about to which team the influential punter will have access, so carefully maneuvering the odds to induce match-fixing is too costly.

***JEL* Classification:** D42, K42. **Key Words:** Sports contest, gambling markets, bookie, monopoly pricing, punters, betting syndicate, match-fixing, privileged information, enforcement.

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1   Introduction

Match-fixing and betting related corruption seem widespread if one goes by media headlines, and legal scholars are beginning to pay attention.[1] Yet formal research on this issue is relatively lacking.[2] If a small group of bettors with financial muscles, to be referred as the *influential punter* (in short, IP or fixer), can access player(s) who, by underperforming, can tilt the outcome of a sports contest that has high visibility and on which many people bet, the competitive show is hit at its foundation.

In an earlier work (Bag and Saha, 2011), we presented a formal model of match-fixing in the context of a (Bertrand) competitive fixed odds betting market. There the bookmakers' desire to steal business by undercutting rivals' odds exposes the contest to fixing, whereby a favorite deliberately underperforms. Such destructive effects of competition are felt mostly in uneven contests; even contests are generally safe. One question then is what happens if a bookmaker has market power. Will he try to protect the market and his profit from the corrupt punter? Or, will he abuse his power to steer the market in the direction of odds rigging? We address this question here, by restricting our attention mainly to even or near-even contests.

In our model a monopolist bookmaker posts betting odds, or equivalently sets the 'prices' of bets, for a contest between two teams to attract bets from a mass of ordinary bettors. An anonymous influential punter may be able to bribe some members of one of the teams to underperform, and then place a large sum of bets on the other team. Such possibilities as well as the ordinary bettors' response are factored in by the bookie, at the time of posting the odds.

Bettors' responses (including IP's) depend on their position in the information hierarchy. We assume that the ordinary bettors have exogenous and heterogenous beliefs and are unaware of the risk of match-fixing. So they respond to market prices naively. The bookie and the IP, on the other hand, have superior information; they both know the teams' true win probabilities. Further, the IP can rise to the top of the information hierarchy, if he can fix the match.

Aware of the fact that IP will never have *less* information than him, the bookie cannot expect to make profit from the IP. But he needs to be mindful of how IP's incentives will be affected by the prices he will set. If the prices are so high that IP's betting becomes unprofitable despite match-fixing, the integrity of the contest will be protected, but only after sacrificing a sizeable

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[1]See an edited volume (Haberfeld and Sheehan, 2013) titled, *Match-Fixing in International Sports: Existing Processes, Law Enforcement, and Prevention Strategies.*

Betting related match-fixing makes frequent headlines. Latest is the allegation of an entire soccer league being fixed in Canada ("Revealed: Entire 'rogue league corrupted by match-fixing'" – October 14, 2015 news at The Telegraph, http://www.telegraph.co.uk/sport/football/11932437/Revealed-Entire-rogue-league-corrupted-by-match-fixing.html). The high profile *Indian Premier League* (or *IPL*) in which many top international cricketers participate was rocked by betting and fixing allegations in the recent past – see the report "Fixing? It's people like us doing it" on May 22, 2013 at http://www.espncricinfo.com/magazine/content/story/637034.html. For fixing allegations in other sports including soccer in Europe, ATP tennis tournaments, basketball, horse races etc., see links to various news reports in Bag and Saha (2011).

[2]There are some empirical papers on sports corruption. Duggan and Levitt (2002) study corruption in Sumo wrestling but not betting. Strumpf (2003), Wolfers (2006), Winter and Kukuk (2008) etc. study betting-related corruption in sports. See also two very recent contributions on detecting match-fixing – De Muinck and Quatacker (2013) for soccer, and Rodenberg and Feustel (2014) for tennis.

chunk of the ordinary bettors' market. On the other hand, if the prices are set appropriately low, IP can find match-fixing profitable; if he gets to bribe one team, his return from the bet on the other team will be large enough to cover the cost of bribery. In this case, the bookie faces a clear loss to IP, but the loss may be partially (if not fully) offset by attracting a significant number of ordinary bettors. So his profits from these two alternative price regimes can go either way. Since which price regime is his own choice, the question is whether he would go for the first regime, which we call *bribe prevention* or the second regime, which we call *bribe inducement*. In contests where teams are (near) evenly matched, which regime would he choose?

When only one team is corruptible, we find that he would choose bribe inducement for any positive chance of secret liaison with the corruptible team (given some standard assumptions).[3] This is because bribe prevention is too costly; it requires giving up too much on the ordinary punters' betting. In some extreme instances, when the probability of accessing the corrupt team is very high, the bookie actually welcomes the fixing role of the IP. With his tacit help he can drastically increase the losing chance of the corrupt team and steer most of the ordinary bettors to bet on it. Here, the loss to the IP is much smaller than the substantial gain of capturing (nearly) the whole of the unsuspecting bettors' market.

Surprisingly though, this perverse incentive loses its force when both teams are corruptible. In that case, though match-fixing opportunities are greater, the bookie is unsure of which team the IP will be able to bribe, and therefore, he needs to make concession on the bet prices of both teams, which means yielding a much greater loss to not only IP but also a section of ordinary bettors; lowering both prices does not help to tilt the market in one way or other. Therefore, bribe inducement will be preferred less frequently.

The finding of the monopoly setup is in sharp contrast to the competitive case analyzed in Bag and Saha (2011), where even contests are generally immune. Either the competitive prices are above the bribery threshold level, or the bookmakers (non-cooperatively) coordinate on the threshold prices for fear of letting the IP in. Under monopoly, despite his full market power, the bookie may not try to protect the market. The underlying market microstructure of our model assumes that the median bettor always believes the contest to be dead even. So when the contest is truly an even (or a near even) contest, the bookie's informational superiority disappears, and his expected profit will be minimum. In such cases, an anonymous fixer can come handy to make the contest lop-sided and in anticipation of that the bookie can induce the ordinary bettors *en masse* to back the losing team. But whether this will be optimal depends on how large is the expected cost of fixing, which can significantly vary depending on whether one team or both teams are corruptible.

Though our main analysis is presented assuming 'uniform' distribution of the ordinary bettors' beliefs, we show that our key insight that the bookie will be most keen to orchestrate match-fixing in even or near even contests will hold under general (but symmetric) belief distributions. However, the other assumptions regarding the bettors, such as their exogenous beliefs and their naivety of not

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[3]We assume that the corruptible team's identity is known. Teams with good reputation or regimentation and discipline are likely to be immune to outside influence.

suspecting foul play, are important. That bettors have diverse beliefs and less precise information than the bookie are somewhat essential to organize betting.[4] There is also considerable evidence on leisure betting, which is most likely to be based on naive beliefs (Saunders and Turner, 1987; Bruce and Johnson, 1992; Winter and Kukuk, 2008). But if we allow the bettors to be suspicious of match-fixing, when they see the posted odds that are *too good to be true* (based on their initial beliefs), market manipulation will backfire; the bookie will be better off by driving the fixer out of the market. We accommodate this argument, *albeit* less formally; but the message is clear: the more unsuspecting the bettors are, the stronger the incentives to induce match-fixing.

At a theoretical level, our exercise is also useful for extending the literature from insider betting to fixing and betting. The insider betting model of Shin (1991), from which we adapt the basic setup, is similar to insider trading (Glosten and Milgrom, 1985; Kyle, 1985; Seyhun, 1992), where some agents use privileged information which might have been randomly or costlessly acquired. Match-fixing on the other hand is about *generating* privileged information at a positive cost and then using it.[5] This is also related to the idea of market manipulation in finance where even an uninformed trader may buy a stock to bid up its price with the intent of selling the stock at a later date and profit from it (e.g., Allen and Gorton, 1992; Chakraborty and Yilmaz, 2004). Further, our study provides insights into the extent by which sabotages can be controlled through market prices – a relatively neglected issue so far in the industrial organization research.

From an empirical point of view, we question a common perception of the bookmakers as victims of match-fixing.[6] We argue that in many plausible scenarios they may indirectly benefit from match-fixing. Should the authority go after only the fixer, or the bookmaker as well? When the bookmaker is merely trying to avoid losses he cannot be blamed; but when he profits from fixing, he should be held culpable.[7]

We acknowledge that the insights from our model should not be directly applied to parimutuel betting markets. Such markets are differently organized and the bookmaker's role is different with very little power in setting market odds, as analyzed in a number of works by Ottaviani and Sorensen (2005; 2008; 2009; 2010). Fixed odds are practised by bookmakers in the UK and Europe, e.g., ChoiceOdds, Ladbrokes, Paddy Power, and William Hill.

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[4]Diversity of beliefs naturally follow from our assumption of exogenous beliefs; allowing correlated beliefs will largely eliminate demand for bets.

[5]A similar theme, namely sabotage, has been studied in contest models with completely different concerns than ours. There the focus is on the saboteur's incentives; see, for instance, Konrad (2000), Chen (2003).

[6]In the famous 2007 tennis episode involving Davydenko and Vassallo Arguello, the internet betting company, Betfair, voided all its bets following suspicious betting patterns – the market odds as reflected in bets drifted significantly against 4th seeded Davydenko while ahead in the match. In its December 2011 review the UK gambling commission requires licensed sports bet operators to provide the relevant sports governing body with information that they suspect may lead the gambling commission to consider making an order to void a bet. See Section 15.1 of the Gambling Commission document available at http://www.gamblingcommission.gov.uk/PDF/LCCP%20consolidated%20version%20-%20December%202011.pdf .

[7]See "Football match-fixing: How betting odds gives the game away" at http://www.bbc.co.uk/news/world-europe-11789671.

The paper is organized as follows. In Section 2, we present the model and an analysis of betting and bribery decisions. Sections 3 and 4 contain the main results, with some related examples and simulations appearing in Section 5. Sections 6 and 7 extend the earlier analysis to more general environments, followed by conclusions in Section 8. The formal proofs appear in the Appendix.

2   The Model

The setup is similar to that of Bag and Saha (2011), which in turn draws upon Shin (1991). A monopolist bookmaker, the bookie, sets the odds on each of two teams winning a sports match (i.e. sets the prices of two tickets); the match being drawn is not a possibility. Ticket i with price π_i yields a dollar if team i wins and yields nothing if team i loses. There are a continuum of naive punters, parameterized by individual and exogenous belief, the probability q, that team 1 will win; q is distributed 'uniformly' over $(0, 1)$.

The assumption of uniform distribution is justified on the ground of the bettors having well spread-out beliefs. However, later with a general distribution function of beliefs we show that our key results are likely to hold, if the distribution is continuous and symmetric with mean $1/2$.

Bettors' beliefs are not only exogenous and unrelated to the nature's draw of a team's win probability, but also naive in the sense that bettors are unaware of the risk of match-fixing. Later we relax the naivety assumption and show how the results might change if the bettors are 'rational'.

In the absence of any external influence, the probability that team i will win is $0 < p_i < 1$. External influence is exerted through bribery of a (few) corruptible (members of a) team to underperform. If team i is bribed, its probability of winning is secretly lowered from p_i to $\lambda_i p_i$ ($0 \leq \lambda_i < 1$). We assume λ_i to be exogenous.[8]

We assume the bookmaker to be honest. There is an anonymous influential punter, IP, who may bribe a team and bet against it. He can access team i with probability $0 \leq \mu_i \leq 1$. Note that the probabilistic access makes the prospect of match-fixing uncertain, even when $\lambda_i = 0$.

The bookie, IP, and the team players – all initially observe the draw p_1. The law enforcement authority investigates the losing team i with an exogenous probability $0 < \alpha_i < 1$.[9] On conviction, the corrupt players are fined $\mathfrak{f}$ in total and the match-fixer $\mathfrak{f}_I$.

The distribution of the ordinary bettors' wealth is 'uniform' over $[0, 1]$, with 1 dollar each and a collective wealth of y dollars. The wealth of IP is $z = 1 - y$ dollars. All agents are risk-neutral.

■ Betting and bribe-taking

Ordinary bettors' betting decision. The ordinary bettors adopt the following betting rule: Bet on team 1 if and only if $\frac{q}{\pi_1} \geq \max\{\frac{1-q}{\pi_2}, 1\}$; bet on team 2 if and only if $\frac{1-q}{\pi_2} \geq \max\{\frac{q}{\pi_1}, 1\}$.

Player incentives for bribe-taking and sabotage. Suppose a corrupt player (or a group of corrupt players) of team i is contacted by IP with probability μ_i and a deal is struck, by which the corrupt

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[8]The team may have honest players who cannot be bribed, and therefore, $\lambda_i = 0$ is not appropriate in a team game. One can make λ_i sensitive to bribe at the cost of some complexity.

[9]It does not employ sophisticated game-theoretic inferences about match-fixing by observing π_i or p_i.

player is promised a bribe b_i to underperform. The bribe will be paid only if the team loses. In the event team i wins, regardless of the player's underperformance the bribe money is not paid; the corrupt player in that case collects only the team reward w.

Given any belief p_i and α_i, a bribe deal involving b_i is accepted <u>and</u> honored by the corrupt player, if the expected payoff from underperformance is at least as great as the expected payoff from honest performance. That is to say, the deal is struck

$$\text{if and only if} \quad \lambda_i p_i w + (1 - \lambda_i p_i)(b_i - \alpha_i f) \geq p_i w + (1 - p_i)(b_i - \alpha_i f) \quad (1)$$

$$\text{i.e., if and only if} \quad b_i \geq w + \alpha_i f.$$

The inequality (1) makes underperformance incentive compatible and the agreement self-enforcing.[10] The minimum bribe required to entice underperformance is $\underline{b}_i = w + \alpha_i f$. We assume that IP holds all the bargaining power so that $b_i = \underline{b}_i$.

We should emphasize that if the players of two teams had different beliefs regarding the teams' chances (different from p_1), inequality (1) could be easily replicated for those beliefs, because (1) corresponds to the players' beliefs, and the lower bound on b_i is not sensitive to any belief. Therefore, divergence in players' beliefs from each other's and/or from true p_1 does not alter the analysis. If b_i were set in excess of $\underline{b}_i$, say via Nash bargaining of the surplus, then IP's belief (i.e. p_1) would be relevant, but not the players' beliefs.

■ **Bribery game Γ.** There are two key players, the bookmaker and IP, in the bribery game.

STAGE 1. Nature draws p_1 and reveals it to the bookie, IP and the players; the ordinary punters draw their respective private signals q. The bookmaker sets the prices (π_1, π_2) for the tickets on respective teams' win, where $0 \leq \pi_1, \pi_2 \leq 1$.

STAGE 2. IP secretly finds out if he could access team 1 or team 2 or neither,[11] and decides, on gaining access, whether to bribe the team to influence the contest outcome.

STAGE 3. All punters including IP place bets according to their 'eventual' beliefs. The match is played out according to the teams' winning probabilities $(p_1, 1 - p_1)$ or $(\lambda_1 p_1, 1 - \lambda_1 p_1)$ (when team 1 is bribed), or $(1 - \lambda_2 p_2, \lambda_2 p_2)$ (when team 2 is bribed) and the match takes place.

STAGE 4. Finally, the enforcement authority follows its investigation policy, α_i. On successful investigation, fines are imposed on the corrupt player(s) and IP. ‖

■ **Influential punter's betting and bribery incentives.** Before we analyze IP's incentives, it should be obvious that IP will hold either superior or same information as the bookie. The bookie can thus never make positive expected profit from IP. In particular, when IP has superior information (on the fixing) the bookie's expected profit from IP will be negative.

This is a consequence of our assumption that IP has precise knowledge of p_i (just like the bookie). Admittedly, this is a somewhat strong assumption, but it helps to underscore his role as a

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[10]The IP will honor his promise with the bribe, as is commonly assumed in corruption models.

[11]The timing of IP's access to teams (after or before the odds are posted) is immaterial.

fixer. We make the bookie's choice difficult by introducing a certain cost through this assumption. One can allow greater realism by making IP's information set coarse, which we do not study here.

Now, consider IP's incentives. Having learnt p_i and observed π_i ($i = 1, 2$), when $p_i \leq \pi_i$ for all $i = 1, 2$, the influential punter can profitably bet only if he succeeds in bribing one of the teams, because otherwise his expected gains from betting is zero. Further, to permit bribery of team i and betting on team j, it must be the case that $1 - \lambda_i p_i \geq \pi_j$. That is, the eventual probability of team j winning must not fall below π_j. Assuming that is the case, his expected profit from bribing team i is calculated as $E\Pi^I(b_i) = (1 - \lambda_i p_i)\left[\frac{z}{\pi_j} - \underline{b}_i - \alpha_i f_I\right] - z$. Substituting $\underline{b} = w + \alpha_i f$,

$$E\Pi^I(b_i) = (1 - \lambda_i p_i)z\left[\frac{1}{\pi_j} - \Omega_i\right] - z = (1 - \lambda_i p_i)z\left[\frac{1}{\pi_j} - \frac{1}{\phi_i}\right],$$

$$\text{where} \quad \Omega_i = \frac{w + \alpha_i(f + f_I)}{z}, \quad \text{and} \quad \phi_i = \frac{1 - \lambda_i p_i}{1 + (1 - \lambda_i p_i)\Omega_i}.$$

Clearly, for $E\Pi^I$ to be positive π_j must be smaller than ϕ_i. By corollary then IP is prevented from bribing team i if $\pi_j > \phi_i$. In the event of $\pi_i = \phi_j$, we make a tie-breaking rule to ensure that the bookie's optimal bribery program always has a solution.

Assumption 1 (Tie-breaking rule) *If 'bribing and betting' and 'betting without bribing' yield zero expected profit, then* IP *will choose an option (including 'not betting at all') in accordance with the bookie's preference.*

Given Assumption 1, if the bookie wishes to *induce match-fixing* of team i, he needs to set π_j subject to the following constraint (along with setting $\pi_i \geq p_i$ for both teams):

$$\pi_j \leq \frac{1 - \lambda_i p_i}{1 + (1 - \lambda_i p_i)\Omega_i} \equiv \phi_i. \tag{2}$$

What will be the IP's incentives, if $p_i > \pi_i$ were set instead? In this instance, IP can make positive gains from 'honest' betting, and therefore there is an opportunity cost to fixing. ϕ_i will no longer be the threshold price for π_j. In Bag and Saha (2011) we have characterized these cases exhaustively. In this paper we restrict our attention only to the case where $p_i \leq \pi_i$ for both teams. Setting *both* π_1 and π_2 below p_1 and p_2, respectively, would imply money pump scenario. We impose the following (no) *Dutch-book restriction* to rule it out:

Assumption 2 *The bookie must always choose prices $0 \leq \pi_1, \pi_2 \leq 1$ such that $\pi_1 + \pi_2 \geq 1$.*

3 Prevent, tolerate or encourage match-fixing?

Our main interest is to see whether the bookie would choose not to prevent match-fixing, and if so in which type of contests that is most likely to happen. To develop an early insight we first consider the benchmark case of 'honest' betting where match-fixing is not possible by assumption.

■ **No IP scenario.** Suppose there is no IP so that the teams' winning probabilities are maintained at (p_1, p_2). Given Assumption 2, bettors with beliefs $q \in [0, 1 - \pi_2]$ will buy ticket 2 and $q \in [\pi_1, 1]$ will buy ticket 1; punters with q in$(1 - \pi_2, \pi_1)$ will buy neither. Thus the bookie's objective is:

$$\max_{\pi_1, \pi_2} \ \text{E}\Pi \ = \ \underbrace{\Big[\int_{\pi_1}^{1} y(1 - \frac{p_1}{\pi_1}) \, dq \Big]}_{\text{ticket 1 profit}} + \underbrace{\Big[\int_{0}^{1 - \pi_2} y(1 - \frac{p_2}{\pi_2}) \, dq \Big]}_{\text{ticket 2 profit}}$$

$$= \ y \Big[3 - \pi_1 - \pi_2 - \frac{p_1}{\pi_1} - \frac{p_2}{\pi_2} \Big]. \tag{3}$$

The solution to this problem is $\pi_1^0 = \sqrt{p_1}$ and $\pi_2^0 = \sqrt{p_2}$, resulting in expected profit:

$$\text{E}\Pi_0 = y \Big[3 - 2\sqrt{p_1} - 2\sqrt{1 - p_1} \Big]. \tag{4}$$

Fig. 1 depicts the profit curve, perfectly symmetric and U-shaped. The bookie makes the most when the contest is highly uneven. By setting the price of the weak team's ticket very low, he induces most naive punters to bet on it and earns nearly y. On the other hand, at the perfectly even contest his profit is the lowest. Here, those bettors who believe that the contest is even or nearly even abstain from betting.
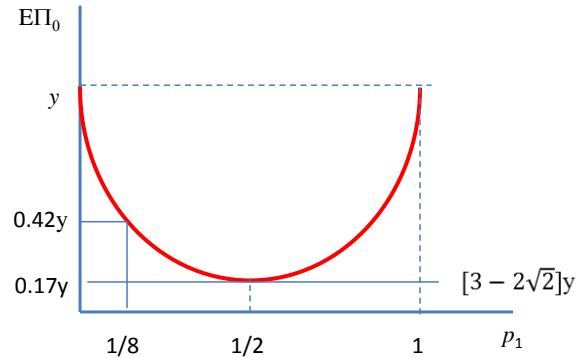

Figure 1: Expected profit when there is no IP

It is around the even contest the bookie would wish that p_1 were different from $1/2$, say for example $1/8$. Then the bookie's expected profit would rise from $0.17y$ to $0.42y$. Indeed, the same gains in profit would be achieved, if the bookie could engage the IP for a fixed fee (not more than $0.25y$) to bribe team 1 with probability $4/5$ and reduce p_1 from $1/2$ to $1/32$. The ex-ante probability of team 1 winning would then be exactly $1/8$. Of course, when the bookie has no link with IP, the only way he can engage the IP is by inducing him through market prices, which clearly will have to be distorted from what is shown in Fig. 1. Nevertheless, the intuition is that the monopolist's incentive to manipulate the market is the strongest around the even contest.

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This observation is, of course, made on the basis of the uniform distribution of bettors' (naive) beliefs with mean $1/2$. That being the case, the intuition is likely to be more general. We show later on with a general distribution function that as long as it is symmetric with mean $1/2$, even contests are likely to be rife for fixing. However, if the mean belief diverges from $1/2$, the minimum profit point will also diverge from $1/2$, but probably not stray too far from $1/2$. For the uniform distribution, the minimum profit point will be between the mean belief and $1/2$. We show this in Section 6. The main point is that the minimum profit point will be fairly close to the dead even contest, and hence the bookie's incentive to induce fixing will also be stronger here.

■ **Bribe prevention program.** We now return to our game Γ, and first analyze the bookie's bribe prevention strategy. Formally, the bookie solves the following bribe-prevention program:

$$\max_{\pi_1,\pi_2} \ E\Pi_{BP} = y\Big[3 - \pi_1 - \pi_2 - \frac{p_1}{\pi_1} - \frac{p_2}{\pi_2}\Big],$$
$$\text{subject to:} \quad \pi_1 + \pi_2 \geq 1, \quad \pi_1 \geq \max\{p_1, \phi_2\}, \quad \text{and} \quad \pi_2 \geq \max\{p_2, \phi_1\}.$$

Lemma 1 (Optimal bribe prevention prices) *If* IP *can potentially bribe team* j *to bet on team* i, *bribe prevention price of ticket* i *at any given* p_i, *is* $\pi_i^{BP} = \max\{\sqrt{p_i}, \phi_j\}$, $i, j = 1, 2$ *and* $i \neq j$.

It is easy to verify that ϕ_2 is an increasing function of p_1 and it intersects the curve $\sqrt{p_1}$ from above at a unique point. Let p_1^0 be this point of intersection. Clearly, at all $p_1 < p_i^0$ the bribe prevention constraint binds on π_1, implying that profit on ticket 1 will have to be sacrificed if bribery of team 2 is to be prevented. Likewise, ϕ_1 is downward sloping in p_1 like $\sqrt{p_2}$ and ϕ_1 intersects $\sqrt{p_2}$ from below. Let this intersection point be p_1^1. That means, at all $p_1 > p_1^1$ the bribe prevention constraint binds on π_2 implying loss of profit from ticket 2, if bribery of team 1 is to be prevented. Combining these two observations the following can be stated.

Lemma 2 (Costless or costly prevention of match-fixing) *If* $p_1^0 < p_1^1$ *then at all* $p_1 \in [p_1^0, p_1^1]$ *bribe prevention is costless (on either tickets), relative to the 'no* IP*' case. Alternatively, if* $p_1^1 < p_1^0$ *then at all* $p_1 \in [p_1^1, p_1^0]$ *bribe prevention is costly.*

These two regimes have different implications for the bookie. When trying to prevent fixing is costly, he has a hard choice to make. If he chooses 'not to prevent', it should be seen as a 'loss minimizing' strategy in a corrupted environment over which he has no control. But when prevention of fixing is costless, he is certainly on a safer ground. But if he still chooses not to prevent fixing, it *must be* due to his intention to profit from the evil influence of the fixer. Though both decisions favor bribe inducement, and are based on similar profit comparisons, they are *qualitatively* different. To underscore their difference we will call the decision to induce bribery in the first scenario *bribe tolerance* and in the second scenario *bribe encouragement*. In the first case, there is very little sense to go after the bookie, while in the second case he can be held guilty of wrongdoing.

■ **Bribe inducement: only one team corruptible.** We will first analyze the case where only one team, namely team 1, is corruptible. That means, to induce bribery the bookie needs to

set π_2 below the threshold level ϕ_1 as given by (2). No such constraint will apply to the price of ticket 1, because team 2 is not corruptible. The formal statement of the bribe inducement (BI) program is as follows:

$$\max_{\pi_1,\pi_2} E\Pi_{BI} = \underbrace{\left[\int_{\pi_1}^{1} y(1 - \frac{p_1\lambda_1^*}{\pi_1})\,dq\right]}_{\text{ticket 1 profit}} + \underbrace{\left[\int_0^{1-\pi_2} y(1 - \frac{1-p_1\lambda_1^*}{\pi_2})\,dq + \mu_1 z(1 - \frac{1-\lambda_1 p_1}{\pi_2})\right]}_{\text{ticket 2 profit}}$$

$$= y\left[3 - \pi_1 - \pi_2 - \frac{p_1\lambda_1^*}{\pi_1} - \frac{1-p_1\lambda_1^*}{\pi_2}\right] - \mu_1 z\left[\frac{1-\lambda_1 p_1}{\pi_2} - 1\right] \qquad (5)$$

$$\text{subject to} \quad p_2 \le \pi_2 \le \phi_1 \; (< 1 - \lambda_1 p_1), \quad \pi_1 \ge p_1, \quad \pi_1 + \pi_2 \ge 1,$$

where $\lambda_1^* = \mu_1\lambda_1 + (1-\mu_1)$ is the dampening impact of bribery on team 1's winning chances.

Let us note that the solution to the BI problem is

$$\pi_1 = \begin{cases} \sqrt{p_1\lambda_1^*}, & \forall \mu_1 \in (0, \frac{p_2}{1-\lambda_1}] \\ p_1, & \forall \mu_1 \in (\frac{p_2}{1-\lambda_1}, 1), \end{cases} \qquad \pi_2 = \begin{cases} \sqrt{1 - p_1\lambda_1^* + \mu_1\frac{z}{y}(1-\lambda_1 p_1)}, & \forall \mu_1 \in (0, \tilde{\mu}_1] \\ \phi_1(p_1), & \forall \mu_1 \in (\tilde{\mu}_1, 1], \end{cases} \qquad (6)$$

where by setting $\sqrt{1 - p_1\lambda_1^* + \mu_1(z/y)(1-\lambda_1 p_1)} = \phi_1$, we obtain

$$\tilde{\mu}_1 = \frac{y}{1 - \lambda_1 p_1 - y p_2}\left[\phi_1^2 - p_2\right]. \qquad (7)$$

□ *Even contests.* While the above solution is given for any given p_1, for the rest of this section we will concentrate on a specific value of p_1, namely $1/2$, for our suspicion that it is at and around the even contest the bookie's incentive to induce fixing is strongest. If we find bribe inducement optimal at $p_1 = 1/2$, then by continuity at all p_1 close to $1/2$ also bribe inducement will be optimal.

Now we set out to compare the profit from bribe prevention with that from bribe inducement at $p_1 = \frac{1}{2}$. Given that only team 1 can be bribed, if $1/2 < p_1^1$ then at $p_1 = 1/2$ bribe prevention is costless (relative to the benchmark case), and the bookie's profit will be $E\Pi_{BP} = y[3 - 2\sqrt{2}] \equiv E\Pi_0$. Alternatively, if $p_1^1 < 1/2$, then his bribe prevention profit will be lower:

$$E\Pi_{BP} = y\left[3 - \sqrt{2} - \phi_1 - \frac{1}{2\phi_1}\right] < E\Pi_0. \qquad (8)$$

Turning our attention to bribe inducement profit, we see from Eq. (6) that at any given p_1, such as $1/2$, the expected profit critically depends on the range of μ_1, i.e. whether μ_1 falls in between or below/above the two critical values of μ_1, namely $\tilde{\mu}_1$ and $\frac{p_2}{1-\lambda_1}$ which is simply $\frac{1}{2(1-\lambda_1)}$ at $p_1 = 1/2$. However, *a priori* we cannot guarantee if $\tilde{\mu}_1(\frac{1}{2}) >$ or $< \frac{1}{2(1-\lambda_1)}$, and of the two cases the first one is less tractable. Hence, we focus on $\tilde{\mu}_1(\frac{1}{2}) < \frac{1}{2(1-\lambda_1)}$ by assuming the following:

Assumption 3 *For any given λ, y and Ω are such that $\tilde{\mu}_1(\frac{1}{2}) < \min\left\{1, \frac{1}{2(1-\lambda_1)}\right\}$.*

If $\lambda_1 \geq \frac{1}{2}$, our assumption automatically holds, because $\tilde{\mu}_1 < 1$.[12]

We now write the bookie's profit function for bribe inducement for three ranges of μ_1.[13] At each range of μ_1 total profit is broken into two terms, showing profit from each ticket, as follows:

$$E\Pi_{BI}^1 = y\left(1 - \sqrt{\frac{\lambda_1^*}{2}}\right)^2 + y\left[\frac{3}{2} + \mu_1 \cdot \frac{2 - y(1+\lambda_1)}{2y} - \sqrt{2\{1 + \mu_1(\frac{2-\lambda_1}{y} - 1)\}}\right]$$
$$\forall\ \mu_1 \in (0, \tilde{\mu}_1], \qquad (9)$$

$$E\Pi_{BI}^2 = y\left(1 - \sqrt{\frac{\lambda_1^*}{2}}\right)^2 + y\left[\frac{3}{2} + \mu_1 \cdot \frac{2 - y(1+\lambda_1)}{2y} - \phi_1 - \frac{1}{\phi_1}\left\{\frac{1}{2} + \frac{\mu_1}{2y}(2 - y - \lambda_1)\right\}\right]$$
$$\forall\ \mu_1 \in (\tilde{\mu}_1, \frac{1}{2(1-\lambda_1)}], \qquad (10)$$

$$E\Pi_{BI}^3 = y\left[\frac{\mu_1(1-\lambda_1)}{2}\right] + y\left[\frac{3}{2} + \mu_1 \cdot \frac{2 - y(1+\lambda_1)}{2y} - \phi_1 - \frac{1}{\phi_1}\left\{\frac{1}{2} + \frac{\mu_1}{2y}(2 - y - \lambda_1)\right\}\right]$$
$$\forall\ \mu_1 \in (\frac{1}{2(1-\lambda_1)}, 1]. \qquad (11)$$

From the above we can make the following observations.

At sufficiently small values of μ_1, bribe inducement profit is arbitrarily close to the benchmark (i.e. no IP) profit. As $\mu_1 \to 0$, from (9) we see that $E\Pi_{BI}^1 \to E\Pi_0$ (with $\lambda_1^* \to 1$). Now if $p_1^1 < \frac{1}{2}$ (so that $E\Pi_{BP} < E\Pi_0$), then we can claim that at all sufficiently small values of μ_1 bribe inducement is optimal. This is our first result, and it does not depend on any additional assumptions.

The reason is that for a very small chance of bribery, prices do not need to be distorted much from their unconstrained levels in order to induce bribery. But to do the contrary, price of ticket 2 has to be discretely raised to ϕ_1 which forces a significant loss. Therefore, *tolerating* a small chance of match-fixing is preferable.

At moderate or higher values of μ_1, the optimality of bribe inducement depends on how long $E\Pi_{BI}$ continues to be greater than $E\Pi_{BP}$. If $E\Pi_{BI}^1$, $E\Pi_{BI}^2$ and $E\Pi_{BI}^3$ were all declining in μ_1, then the prediction is straightforward. After a critical value of μ_1 the cost of inducing bribery would be far greater than the cost of preventing bribery, and hence match-fixing will be prevented. Furthermore, $E\Pi_{BI}$ will never exceed $E\Pi_0$, and hence *bribe encouragement* will never happen.

But if $E\Pi_{BI}^2$ and $E\Pi_{BI}^3$ are increasing in μ_1 then some interesting possibilities arise. In Fig. 2 we present this scenario. It is then possible that the whole profit curve $E\Pi_{BI}$ can be above $E\Pi_{BP}$, which means that at all values of μ_1 bribe inducement is preferred (provided $E\Pi_{BP} < E\Pi_0$). Indeed, sufficient conditions can be specified to ensure this possibility. A key requirement for this to happen is to make sure that $E\Pi_{BI}^2$ is increasing in μ_1. It can be verified that as $\mu_1 \to 0$, $E\Pi_{BI}^2 \to E\Pi_{BP}$. Further, imposing a positive slope condition on $E\Pi_{BI}^2$ ensures that $E\Pi_{BI}$ would be above $E\Pi_{BP}$ for at least up to $\mu_1 = \frac{1}{2(1-\lambda_1)}$, and also guarantees positive slope of $E\Pi_{BI}^3$.

Formally, the sufficient condition comes down to restricting the wealth of the ordinary punters

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[12]But if $\lambda_1 < \frac{1}{2}$, then we need $w + \alpha_1(\mathfrak{f} + \mathfrak{f}_I) > \max\left\{2(1-y)\left[\sqrt{\frac{y(1-\lambda_1)}{y(1-2\lambda_1)+2-\lambda_1}} - \frac{1}{2-\lambda_1}\right],\ 0\right\}$.

[13]Implicitly, for $E\Pi_{BI}^3$ we are assuming $\lambda_1 < \frac{1}{2}$, otherwise the interval $[\frac{1}{2(1-\lambda_1)}, 1]$ would be empty.

within a critical range, which we denote by Σ_1 introduced in Definition 1 below (see Lemma A1 in Appendix for the derivation of Σ_1; some simulations are also included).

Definition 1 *Suppose* $J(y) \equiv y - \frac{2(1-\phi_1)-\lambda_1}{1-\sqrt{2}[\sqrt{2}-(1-\lambda_1)]\phi_1} = 0$ *at two values of* y, *say* y_0 *and* y_1.[14] *Then define* Σ_1 *to be the set of all* $y \in (y_0, y_1)$, *such that at all* $y \in \Sigma_1$ *we have* $J(y) > 0$.

The condition says that y should not be too high or too small. The reason for this lies in the fact that the bookie must do a tricky balancing act if he were to induce bribery. On the one hand, he must reduce the price of ticket 1 to entice the naive punters to bet on team 1. Hence their wealth and consequently their wagers on team 1 should not be too small. However, higher y also releases some countervailing effect via ticket 2. As IP is to be conceded enough gains to finance his fixed cost of bribery, the price of ticket 2 must be so adjusted that iIf IP's wealth (z) falls, the price of ticket 2 must be lowered appropriately to enable him to generate the same surplus as before to cover the cost of bribery. Thus, lower z means lower π_2 (via lower ϕ_1), which in turn attracts many naive punters as well to bet on team 2 – something the bookie must try to avoid. Essentially, enabling the IP to fix the match creates a *free riding* opportunity for some ordinary bettors. Hence, z should not be too small, i.e. y should not be too high. We now state our first set of results.
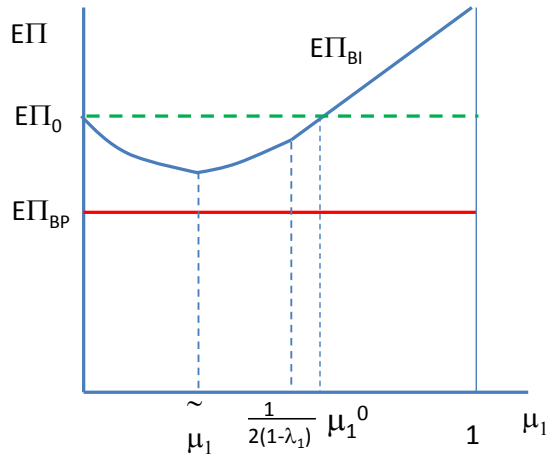

Figure 2: Inducement of bribery at all $\mu_1 \in (0, 1]$

Proposition 1 (Inducing match-fixing) *Consider even and near even contests, and suppose* $p_1^1 < \frac{1}{2}$, *so that bribe prevention is costly.*

(i) *Then there is always a range of* μ_1 *starting from* $\mu_1 = 0$ *where inducing match-fixing is optimal.*

(ii) *Suppose Assumption 3 holds. If* $y \in \Sigma_1$, *inducing match-fixing will be optimal at* _all_ $\mu_1 \in (0, 1]$.

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[14]$J(y) = 0$ is quadratic in y (ϕ_1 is a function of $z = 1 - y$).

Thus, for the even and near-even contests match-fixing is very likely. While that could be due to the high cost of preventing match-fixing, our next result will show that the bookie may have good reasons to even *encourage* match-fixing, because $E\Pi_{BI}$ can potentially exceed $E\Pi_0$ at some high value of μ_1, as shown in Fig. 2. For this to happen we need two things: (i) $E\Pi_{BI}^3$ must be increasing in μ_1, and (ii) $E\Pi_{BI}^3$ must exceed $E\Pi_0$ at μ_1 sufficiently close to 1. It turns out that both requirements are met if we ensure $E\Pi_{BI}^3 > E\Pi_0$ (as $\mu_1 \to 1$), which boils down to another critical range of y, denoted as Σ_2.[15] See Appendix for some simulations of Σ_2.

Since Σ_2 is defined in terms of $E\Pi_{BI}^3$ which presupposes $\lambda < \frac{1}{2}$, we need to consider $\lambda \geq 1/2$ as well, in which case $E\Pi_{BI}^2$ applies to all $\mu_1 \geq \tilde{\mu}_1$. There, we define the critical region of y in terms of the limiting value of $E\Pi_{BI}^2$ (exceeding $E\Pi_0$) as Σ_2^a.

Definition 2 *Define*

$$L(y) \equiv (2 - \lambda_1) \left[\frac{y}{2 + (2 - \lambda_1)\Omega_1} + \frac{\Omega_1}{2} \right].$$

(i) Suppose $\lambda_1 < \frac{1}{2}$, and let Σ_2 be the set of all y such that $y[2\sqrt{2} - \frac{3}{2} - \lambda_1] - L(y) > 0$.
(ii) Suppose $\lambda_1 \geq \frac{1}{2}$, and let Σ_2^a be the set of all y such that $y[2\sqrt{2} - \frac{3}{2} - \lambda_1] + y\left(\sqrt{\lambda_1} - \frac{1}{\sqrt{2}}\right)^2 - L(y) > 0$.

If $y \in \Sigma_2$ when $\lambda_1 < \frac{1}{2}$, or if $y \in \Sigma_2^a$ when $\lambda_1 \geq \frac{1}{2}$, then by definition $E\Pi_{BI} > E\Pi_0$ as $\mu_1 \to 1$. Once again the critical range of y specifies the size of the ordinary punters' wealth to be neither too high nor too low. The explanation is same as before. If it is too high, match-fixing is potentially rewarding, but the scope of free riding (by some ordinary punters) is also great. But if it is too low, the potential gain from bribery is small.

Now in Proposition 2 we claim that if $y \in \Sigma_2$ or Σ_2^a, then above a critical value of μ_1, not only is bribe inducement optimal, but it is of the 'bribe encouragement' type.

Proposition 2 (Actively encourage bribery) *Consider the even or a near-even contest and suppose Assumption 3 holds. Then there exists a critical value of μ_1, say $\mu_1^0 \in (\tilde{\mu}_1, 1)$, such that at all $\mu_1 > \mu_1^0$ the expected profit from bribe inducement exceeds the expected profit of the 'no IP' case, if (i) $y \in \Sigma_2$, when $\lambda_1 < 1/2$, <u>or</u> (ii) $y \in \Sigma_2^a$, when $\lambda_1 \geq 1/2$.*

It is notable that we have not insisted on $p_1^1 < \frac{1}{2}$ and/or $y \in \Sigma_1$ in Proposition 2. While those assumptions along with the condition specified in Proposition 1 give rise to the graph we have presented in Fig. 2, they are not necessary for bribe encouragement. As long as $y \in \Sigma_2$, $E\Pi_{BI}$ will cross $E\Pi_0$ at some high μ_1, and bribe encouragement will take place. So, Propositions 1 and 2 focus on two different aspects – general bribe inducement for unrestricted μ_1 (Proposition 1) vs. a specific type of bribe inducement for a smaller range of μ_1 (Proposition 2) – and it is not always possible to rank the corresponding y-value sets. Table 1 provides a comparison of results.

The main message of Propositions 1 and 2 is that when contests are close the bookmaker may prefer to orchestrate match-fixing (via market prices). Though in most cases, such orchestration is

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[15]Let $\mu_1 \to 1$ in $E\Pi_{BI}$ and write the expression for $E\Pi_{BI} - E\Pi_0 > 0$, which gives $y[2\sqrt{2} - \frac{1}{2} - p_1 - \lambda_1] > y\phi_1 + (1 - \lambda_1 p_1)\Omega_1$. Substitute $p_1 = 1/2$ on the right-hand side to obtain $L(y)$.

	Proposition 1	Proposition 2
Type of contests	Even	Even
Bribe prevention (relative to the benchmark case)	Costly	May or may not be costly
Bribe inducement is preferred	At all μ_1	At high values of μ_1
Nature of bribe inducement highlighted	Bribe tolerance	Bribe encouragement
The required range of y	Relatively narrow	Relatively wide

Table 1: Comparison of results

merely a strategy of cutting losses, we cannot rule out a perverse motive of market manipulation with the tacit help of an anonymous fixer, quite legally.

4   Both teams corruptible

When both teams are corruptible, how does the prospect of match-fixing change? We can see that the bookie now has a choice of targeting a specific team or either teams (to bribe). If a specific team is bribed, the analysis is similar to that of the previous section.[16] So we extend our analysis to bribery of either teams. *Prima facie* the scope for match-fixing increases under this option. But as the bookie does not know which team the IP might get access to, he will have to concede sufficient rents on either ticket. This makes bribe inducement quite costly and less often preferable.

To simplify the analysis, we impose symmetry: $0 < \mu_1 = \mu_2 = \mu \leq 1/2$, $\lambda_1 = \lambda_2 = \lambda \geq 0$, $\alpha_1 = \alpha_2 = \alpha$ and $\Omega_1 = \Omega_2 = \Omega$. As before, we assume that at most one team can be accessed (with probability 2μ).

Even contests. We focus on $p_1 = 1/2$, and recall from Lemma 1 that the optimal bribe prevention prices are $\pi_i = \max\{\sqrt{p_i}, \phi_j\}$. At $p_1 = 1/2$, if the price constraints don't bind (i.e $p_1^0 < \frac{1}{2} < p_1^1$), $E\Pi_{BP} = E\Pi_0 = y\left[3 - 2\sqrt{2}\right]$; but if the constraints bind (i.e. $p_1^1 < \frac{1}{2} < p_1^0$), the bribe prevention profit (at $p_1 = 1/2$) is

$$E\Pi_{BP} = y[3 - 2\phi - \frac{1}{\phi}]. \tag{12}$$

For bribe inducement, we need to consider first the ex-ante probability of each team's win. For team 1 it is $r_1 = \mu(\lambda p_1 + (1 - \lambda p_2)) + (1 - 2\mu)p_1 = p_1 + \mu(1 - \lambda)(1 - 2p_1)$; for team 2 it is

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[16]There is one difference though. The team which is not targeted must be protected from bribery.

$r_2 = p_2 + \mu(1-\lambda)(1-2p_2)$. Curiously, at $p_1 = 1/2$ we have $r_1 = r_2 = 1/2$. The prospect of bribing either team cancels out each other's gains in winning probability. However, an exact offset occurs only at $p_1 = 1/2$; at $p_1 > 1/2$, $r_1 < p_1$ and consequently $r_2 > p_2$. So in general bribery of either team reduces the gap between the favorite and the longshot, making the contest more even.

If at the perfectly even contest the probability of win does not change with bribery, then what is the potential benefit of inducing bribery? The simple answer is: none, unless bribe prevention is costly. In other words, in this environment there is no hope for getting higher profit than the 'no IP' case. Bribe encouragement cannot occur here; at best there may be bribe tolerance.

The general statement of the bribe inducement problem is

$$\max_{\pi_1,\pi_2} \; E\Pi_{BI} = y\Big[3 - \pi_1 - \pi_2 - \frac{r_1}{\pi_1} - \frac{1-r_1}{\pi_2}\Big] - z\Big[\frac{\mu(1-\lambda p_2)}{\pi_1} + \frac{\mu(1-\lambda p_1)}{\pi_2} - 2\mu\Big] \tag{13}$$
$$\text{subject to} \quad p_2 \leq \pi_2 \leq \phi_1 \; (< 1 - \lambda p_1), \quad p_1 \leq \pi_1 \leq \phi_2 \; (< 1 - \lambda p_2), \quad \pi_1 + \pi_2 \geq 1.$$

The unconstrained solution is given by

$$\check{\pi}_1 = \sqrt{r_1 + \mu(z/y)(1 - \lambda p_2)}, \qquad \check{\pi}_2 = \sqrt{1 - r_1 + \mu(z/y)(1 - \lambda p_1)}, \tag{14}$$

At $p_1 = \frac{1}{2}$, the symmetric unconstrained solution is $\check{\pi}_1 = \check{\pi}_2 = \sqrt{\frac{1}{2} + \mu\frac{z}{y}\frac{(2-\lambda)}{2}}$, leading to profit

$$E\Pi_{BI}^{U} = y[3 - 4\check{\pi}] + 2\mu z = 3y - 2\sqrt{2y}\sqrt{y + \mu z(2 - \lambda)} + 2\mu z. \tag{15}$$

On the other hand, if the constraints bind the profit is restricted to be

$$E\Pi_{BI}^{C} = y[3 - 2\phi - \frac{1}{\phi}] + \mu z[2 - \frac{2-\lambda}{\phi}] = E\Pi_{BP} - (2 - \lambda)\Omega\mu z. \tag{16}$$

Clearly, bribery is not preferred. Further, if bribe prevention is costless, i.e. $E\Pi_{BP} = E\Pi_0$, bribe inducement will be out of question. Therefore, the best prospect for bribe inducement arises when $\pi_i = \check{\pi}_i < \phi$, in addition to $E\Pi_{BP} < E\Pi_0$ and we would be looking for the scenario $E\Pi_{BP} \leq E\Pi_{BI}^{U} < E\Pi_0$. We find that the condition $\check{\pi} < \phi$ (at $p_1 = 1/2$) is satisfied if y belongs to a critical range, which we call Σ_3.[17]

In the Appendix we show through simulation that under suitable parameter specifications Σ_3 is nonempty (see Table 6). With higher μ and λ the set Σ_3 shrinks. Intuitively, the expected payout to IP increases with μ. So the bookie will try to optimize on this loss, by raising the unconstrained prices, but then the likelihood of the constraint binding also increases.

Assuming $y \in \Sigma_3$, we are guaranteed to have unconstrained prices for the BI problem. Now we try to identify the values of μ at which we will get $E\Pi_{BI}^{U} > E\Pi_{BP}$. We note that $E\Pi_{BI}^{U}$ *is a decreasing*

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[17]It can be checked that $\check{\pi}$ is declining in y (with the second-order derivative positive), and drops from infinity (at $y = 0$) to $\sqrt{1/2}$ at $y = 1$. Likewise, ϕ is also declining in y (but with the second-order derivative being negative) and drops from a fraction at $y = 0$ to zero at $y = 1$. If the two curves cross each other, they will cross twice. Let these two intersection points be denoted as $\hat{y}_0(> 0)$ and $\hat{y}_1(< 1)$. Let us denote all the values of y strictly lying between the two points by $\Sigma_3 = \{y | \hat{y}_0 < y < \hat{y}_1\}$.

function of μ, as

$$\frac{\partial E\Pi_{BI}^U}{\partial \mu} = 2z \left[1 - \frac{2-\lambda}{\check{\pi}} \right] < 0 \qquad since \quad \check{\pi} < \phi = \frac{2-\lambda}{2 + (2-\lambda)\Omega} < 2 - \lambda.$$

From Eq. (15) we also see that $E\Pi_{BI}^U$ approaches $E\Pi_0$ when $\mu \to 0$, and thereby at all μ sufficiently small we must have $E\Pi_{BI}^U > E\Pi_{BP}$. At the other end of μ, i.e. $\mu = 1/2$, we impose $E\Pi_{BI}^U < E\Pi_{BP}$ with the help of the following assumption.

Assumption 4 *Let* $2\sqrt{\frac{2-\lambda z}{y}} - \frac{z}{y} > 2\phi + \frac{1}{\phi}$.

Now we state the main result of this section.

Proposition 3 (More match-fixing possibilities imply less match-fixing) *Suppose* $p_1^1 < \frac{1}{2} < p_1^0$ *(so that* $E\Pi_{BP} < E\Pi_0$*),* $y \in \Sigma_3$ *(so that* $E\Pi_{BI} = E\Pi_{BI}^U$*), and Assumption 4 holds. Then at all even and near-even contests bribe inducement of either team is preferred to bribe prevention if* μ *is less than a critical value* $\hat{\mu} < 1/2$. *For* μ *above* $\hat{\mu}$ *match-fixing will be prevented. Bribery in the range* $\mu \leq \hat{\mu}$ *will be of the 'bribe tolerance' variety.*

The message of Proposition 3 is strikingly different from that of Propositions 1 and 2. No longer does bribery yield higher profit than the 'no IP' case. So the bookie has no incentive to 'welcome' the IP. He can tolerate him but only up to a certain level of μ. The intuition is that, to induce bribery the bookie must incur losses to IP on both tickets. This loss steadily rises with μ outweighing the relative cost of bribe prevention, rendering bribe inducement quickly suboptimal.

It can also be shown that if λ is sufficiently small the $E\Pi_{BI}^U$ curve will be U-shaped and symmetric (against p_1), which will shift down with μ, as illustrated in Fig. 4 for an example. An implication is that even when the incentive to induce bribery disappears at the perfectly even contest, it may still persist at nearby contests. Example 2 in the next section makes this point sharper.

5 Examples and simulations

Example 1. We present some simulations on the basis of an example assuming only team 1 is corruptible. Here, the entire range of p_1 is considered, though our main interest lies around $p_1 = 1/2$. As our simulations show, the theoretical results are generalizable to a large set of p_1.

We set $\lambda_1 = 0$, $w + \alpha(f + f_I) = 0.05$ and $y = 0.8$, which implies $z = 0.2$ and therefore $\Omega = 0.25$. Since $\lambda = 0$, $\phi_1 = \phi_2 = 1/(1+\Omega) = 0.8$ at all p_1; we can also calculate the upper limit of y to be 0.88 that supports Assumption 3. Further, it is clear that $y = 0.8$ belongs to the set $\Sigma_1 = (0.29, 0.83)$ (see Table 4 in the Appendix). Hence the conditions for Proposition 1 are satisfied. So we can expect bribe inducement to be optimal at all μ_1 at $p_1 = 1/2$.

μ_1	p_{10}	$\hat{p_1}$	p_{11}	$\frac{E\Pi_{BI}-E\Pi_{BP}}{E\Pi_{BP}}$ at $p_1 = 0.5$ (%)	$\frac{E\Pi_{BI}-E\Pi_0}{E\Pi_0}$ at $p_1 = 0.5$ (%)
0.05	0.245	0.450	1	3.87	
0.10	0.198	0.465	1	3.10	
0.20	0.194	0.505	1	6.98	0.73
0.30	0.189	0.540	0.660	14.73	8.02
0.40	0.183	0.579	0.695	27.13	19.70
0.50	0.178	0.610	0.710	43.41	35.04
0.60	0.171	0.632	0.720	62.79	53.28
0.70	0.169	0.646	0.725	82.17	71.53
0.80	0.168	0.656	0.728	101.55	89.78
0.90	0.166	0.665	0.732	120.93	108.02
0.95	0.166	0.669	0.733	131.00	118.25

Table 2: Bribe inducement range of p_1

In Table 2 we present several values of μ_1 over which bribe inducement is optimal.[18] p_{10} is the critical probability at which $E\Pi_{BI} = E\Pi_0$ (also $= E\Pi_{BP}$). $\hat{p}_1$ is the critical value of p_1 at which $E\Pi_{BI} = E\Pi_0(> E\Pi_{BP})$. p_{11} is the probability at which $E\Pi_{BI} = E\Pi_{BP}$. So at all $p_1 < p_{10}$ and at all $p_1 > p_{11}$ (provided $p_{11} < 1$), bribe inducement is not optimal. Alternatively, bribe inducement is optimal at all $p_1 \in [p_{10}, p_{11}]$, of which at all $p_1 \in [p_{10}, \hat{p}_1]$ bribery is of the 'bribe encouragement' type, and at all $p_1 \in [\hat{p}_1, p_{11}]$ bribery is of the 'bribe tolerance' type.

Columns 2-4 of Table 2 give these critical values of p_1 against μ_1. As can be seen, with successive increases in μ_1, p_{10} declines and $\hat{p}_1$ increases; thus, clearly the region of bribe encouragement expands. As p_{11} also decreases with μ the region of bribe tolerance contracts. It is also seen that when team 1 is a strong favorite, bribery is not optimal at $\mu_1 = 0.3$. On the other hand, when team 1 is moderately underdog (p_1 slightly greater than p_{10}) not only is bribery optimal, but it gives higher profit than the benchmark case.

More importantly, bribery is always optimal at the perfectly even contest ($p_1 = 1/2$), a confirmation of Proposition 1. At $\mu_1 = 0.20$ or higher values, we have bribe encouragement at $p_1 = 1/2$. Columns 5 and 6 give the percentage of profit gained from bribe inducement, over and above the bribe prevention profit and the benchmark profit respectively, at the perfectly even contest. We see that these proportions also rise with μ_1. With $\mu_1 = 0.9$ the profit gains from bribery exceed 100 percent. For Column 6, we should note that the condition for Proposition 2 is met; $y = 0.8 \in \Sigma_2 = (0.16, 0.92)$ (for which see Table 5 in the Appendix).

Figs. 3a, 3b and 3c show the profit curves for three values of μ_1. The red curve represents profit from bribe inducement and the blue curve profit from bribe prevention. The yellow curve is the expected profit from the 'no IP' case. Three sets of graphs are based on the same parameter values except μ_1. In Fig. 3a, we assume $\mu_1 = 0.05$, in Fig. 3b, $\mu_1 = 0.3$ and in Fig. 3c $\mu_1 = 0.7$.

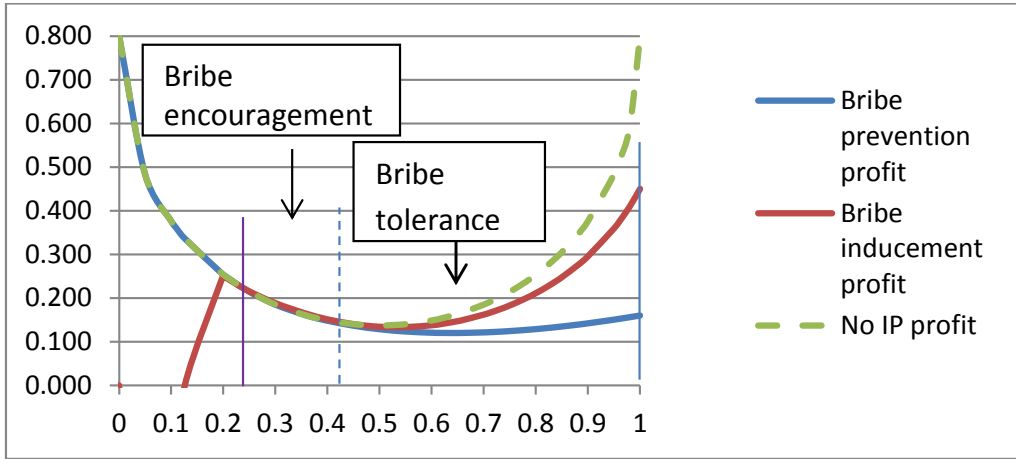[18] All the simulation work in the paper has been done in Microsoft Excel.

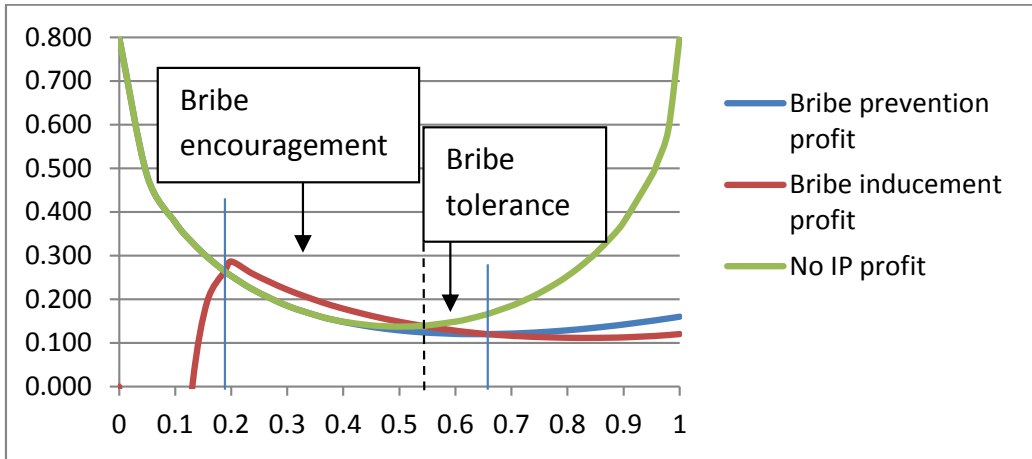Fig. 3a: Bribe inducement is optimal at $p_1=0.245$ onward at all p_1; $\mu_1=0.05$

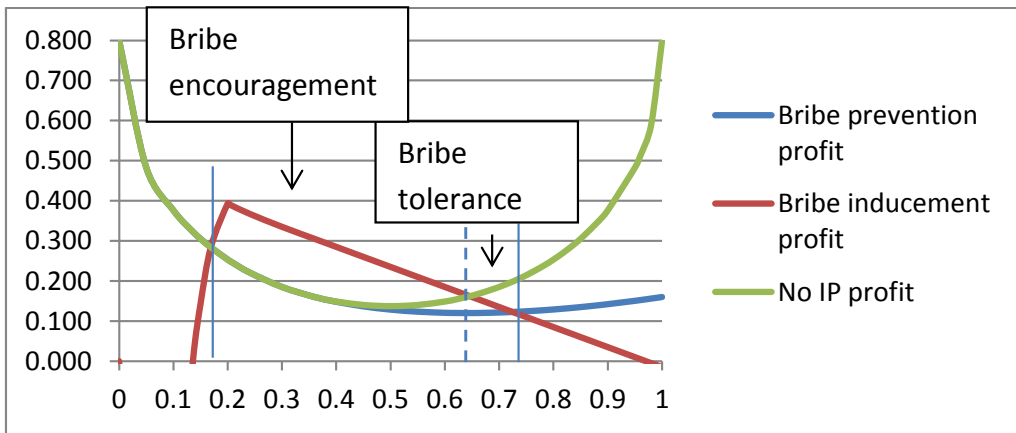Fig. 3b: Bribe inducement is optimal at all p_1 in [0.189, 0.66]; $\mu_1=0.3$

Fig. 3c: Bribe inducement is optimal at all p_1 in [0.136, 0.725]; $\mu_1=0.7$

Figure 3: Data common to three figures above: $\lambda_1 = 0$, $\Omega_1 = 0.25$, $\phi_1 = 0.8$, $y = 0.8$

μ	$E\Pi_{BI}$ at $p_1 = 0.5$	$\frac{E\Pi_{BI}-E\Pi_{BP}}{E\Pi_{BP}}$ at $p_1 = 0.5$	$E\Pi_{BI} > E\Pi_{BP}$ at p_1
0	0.137	14.16	$\in [0.36, 0.64]$
0.03	0.132	10.00	$\in [0.36, 0.64]$
0.05	0.129	7.50	$\in [0.36, 0.64]$
0.08	0.124	3.33	$\in [0.36, 0.64]$
0.10	0.121	0.83	$\in [0.36, 0.64]$
0.11	0.120	0	$\in [0.36, 0.64]$
0.13	0.117	-2.50	$\in [0.36, 0.42]$ and $[0.58, 0.64]$
0.15	0.114	-5.00	$\in [0.36, 0.39]$ and $[0.61, 0.64]$
0.17	0.111	-7.5	$= 0.36$, and $=0.64$

Table 3: Bribe tolerance range of p_1

As should be obvious, the bribery incentive is concentrated around $p_1 = 1/2$. When μ_1 is as low as 0.05 (Fig. 3a), bribe inducement profit actually exceeds the 'no IP' profit between $p_1 = 0.245$ and $p_1 = 0.45$. But as μ_1 increases (see Figs. 3b or 3c) the match-fixing incentive disappears from the higher end of p_1. The bribing incentives generally increase in moderately uneven contests with team 1 being weak. The even contests remain vulnerable and the payoff from match-fixing around these contests keeps increasing with μ_1. In fact, higher the value of μ_1, stronger the incentive to fix the match (around the even contest). This heightened incentive spreads to uneven contests (moving leftward) where team 1 is an underdog. This also confirms our discussion based on Fig. 1 that the bookie's incentive to induce bribery is strongest at $p_1 = 1/2$.

Example 2. This example is for the case of both teams corruptible. We assume symmetry and consider similar parameter values, $\lambda_1 = \lambda_2 = 0$, $\Omega_1 = \Omega_2 = 0.25$ and $y = 0.8$; but we restrict our attention to $p_1 \in [\frac{1}{(1+\Omega)^2}, 1 - \frac{1}{(1+\Omega)^2}] = [0.36, 0.64]$. Also verify from Table 6 in the Appendix that $y = 0.8$ belongs to Σ_3 for any $\mu \leq 0.15$, given $\lambda = 0$. Hence the unconstrained solution will hold for the BI problem. Further, Assumption 4, required for Proposition 3, is also satisfied.

Fig. 4 shows two expected profit curves from bribe inducement, at $\mu = 0.05$ and $\mu = 0.15$, and the benchmark and the bribe prevention profit over the interval of p_1, $[0.36, 0.64]$. Here, $\sqrt{p_i} < \phi(= 1/(1+\Omega))$ for both $i = 1, 2$. That is, bribe prevention is costly at $p_1 = 1/2$. $E\Pi_{BP}$ is constant at 0.120. The benchmark profit $E\Pi_0$ is 0.137 at $p_1 = 1/2$, but higher elsewhere.

How the BI profit varies with successive changes in μ at $p_1 = 1/2$ (in reference to Proposition 3) is given in Table 3 by simulating the BI and BP profits for a range of μ. We see that $E\Pi_{BI}$ is equal to $E\Pi_0$ only at $\mu = 0$. As μ rises, $E\Pi_{BI}$ falls (see Column 2 of Table 3); but still bribe inducement is preferred at all $\mu \leq 0.11$, as shown by column 3 of Table 3.

From Fig. 4 we see that the optimality of bribe inducement holds over the entire interval $[0.36, 0.64]$ at $\mu = 0.05$. At $\mu = 0.15$, no longer is bribery optimal at the perfectly even contest (see

Table 3). However, contests with $p_1 \in (0.36, 0.39)$ and $p_1 \in (0.61, 0.64)$ are still suitable for fixing. These two intervals gradually shrink and disappear at $\mu > 0.17$ (see column 4 in Table 3).

If we compare the maximum incidence of match-fixing at the perfectly even contest ($p_1 = 1/2$) between the two examples, it falls from 1 in Example 1 to $2\mu = 0.22$ in Example 2. Greater scope of bribery drastically reduces the incidence of bribery.
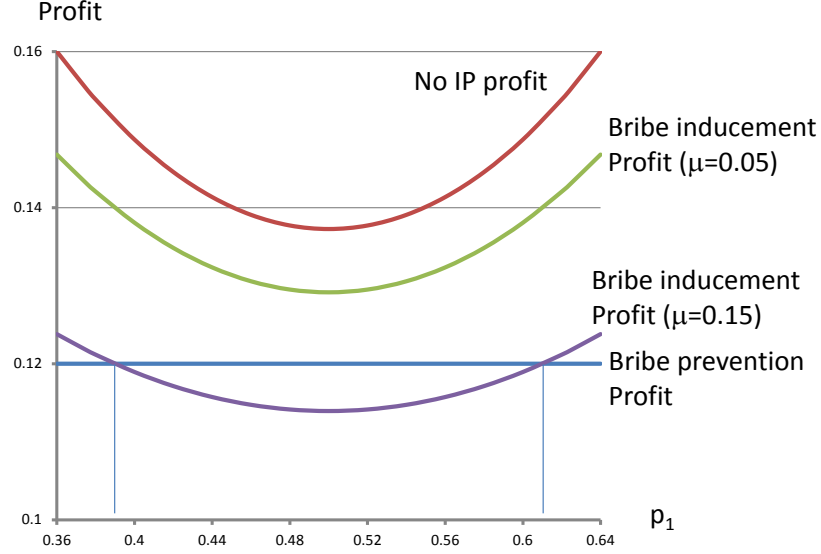

Figure 4: Bribe tolerance in even contests

6 More general distribution functions of bettors' beliefs

We now examine whether our analysis of the bookie's incentives is specific to uniform beliefs assumption. One way to do that would be to extend the 'no IP' case to a class of more general distribution functions, and see if the bookie's profit is still minimum around even contests.

Suppose the bettors' beliefs are given by a probability density function $g(q)$ over $q \in [a, b]$ with $0 \leq a < \frac{1}{2}$ and $\frac{1}{2} < b \leq 1$; $g(q) > 0$ at all $q \in (a, b)$. The cumulative density function is $G(q)$. One advantage of this formulation is that we can allow the bettors' beliefs to be correlated with the draw of p_1. For example, we may have $a = \underline{a} > 0$ with $b = 1$ for any draw of $p_1 > \frac{1}{2}$, $b = \underline{b} < 1$ with $a = 0$ for any draw of $p_1 < \frac{1}{2}$, and ($a = 0$, $b = 1$) if $p_1 = \frac{1}{2}$.

Bettors with beliefs $q \in [\pi_1, b]$ buy ticket 1, and with beliefs $(1 - q) > \pi_2$ (or $q \in [a, 1 - \pi_2]$) buy ticket 2. They each have one dollar to bet. Restricting $\pi_1 < b$ and $\pi_2 < 1 - a$ for positive sale of tickets, the bookie maximizes the following, subject to (no) Dutch-book, i.e., Assumption 2:

$$
\begin{aligned}
E\Pi_0 &= y \int_{\pi_1}^{b} (1 - \frac{p_1}{\pi_1}) g(q) dq + y \int_{a}^{1 - \pi_2} (1 - \frac{p_2}{\pi_2}) g(q) dq \\
&= y \left[1 - G(\pi_1) - \frac{p_1}{\pi_1} + p_1 \frac{G(\pi_1)}{\pi_1} + G(1 - \pi_2) - p_2 \frac{G(1 - \pi_2)}{\pi_2} \right].
\end{aligned}
$$

19

The first-order conditions for profit maximization are:

$$\frac{\partial E\Pi_0}{\partial \pi_1} = y \cdot \frac{p_1 g(\pi_1)}{\pi_1^2} \left[\frac{1 - G(\pi_1)}{g(\pi_1)} - \frac{(\pi_1 - p_1)}{p_1} \pi_1 \right] = 0, \tag{17}$$

$$\frac{\partial E\Pi_0}{\partial \pi_2} = y \cdot \frac{p_2 g(1 - \pi_2)}{\pi_2^2} \left[\frac{G(1 - \pi_2)}{g(1 - \pi_2)} - \frac{(\pi_2 - p_2)}{p_2} \pi_2 \right] = 0. \tag{18}$$

The second-order conditions are satisfied if the following assumption is made, which also helps to ensure monotonicity of the optimal prices.[19]

Assumption 5 *The hazard rate $\frac{1 - G(x)}{g(x)}$ is decreasing, and $\frac{G(x)}{g(x)}$ is increasing, in x.*

It is easy to see that the above assumption will be satisfied by densities symmetric around $\frac{(a+b)}{2} = 1/2$ with support $[a, b]$ where $0 \leq a < 1/2 < b \leq 1$, and also for asymmetric distributions generated by mildly moving away from the symmetric distributions.

The following lemma shows that the bookie's problem has a unique interior solution and prices are (weakly) monotonic in p_1. Moreover, the bookie will never set π_1 below a and π_2 below $1 - b$ because by raising π_1 up to a and π_2 up to $1 - b$, profit can be raised without losing ticket sales.

Lemma 3 (Monotonic prices) *Optimal prices (π_1^0, π_2^0) solving the bookie's problem are unique: $\pi_1^0(p_1) \geq \max\{p_1, a\}$, $\pi_2^0(p_2) \geq \max\{p_2, 1-b\}$. π_1^0 is non-decreasing in p_1 and π_2^0 is non-increasing in p_1. Further, the 'price markup' on ticket i (i.e. $\frac{\pi_i - p_i}{p_i}$) is decreasing in p_i. At $p_1 > b$ ticket 1 is not sold and at $p_1 < a$ ticket 2 is not sold.*

The markup behavior noted above is implied by Assumption 5.

Substituting (π_1^0, π_2^0) into the profit function, we derive $E\Pi_0(p_1)$. There are two aspects of interest – the level of $E\Pi_0$ at extreme values of p_1, and the slope of $E\Pi_0$. At $p_1 < a$, when only ticket 1 is sold, $E\Pi_0 = y(1 - \frac{p_1}{a})$ when $\pi_1 = a$. That is, the expected profit is close to y. Similarly, at $p_1 > b$, expected profit is again close to y, as $E\Pi_0 = y(1 - \frac{p_2}{1-b})$ when $\pi_2 = 1 - b$. The slope of $E\Pi_0$ is negative at $p_1 < a$ and positive at $p_1 > b$. But at $p_1 \in [a, b]$ that we are primarily interested in, the slope can vary from negative to positive, as can be seen from the following expression, on the basis of which we arrive at our claim (Proposition 4) about the shape of the profit function:

$$k(p_1) \equiv \frac{\partial E\Pi_0}{\partial p_1} = y \left[-\frac{1 - G(\pi_1^0(p_1))}{\pi_1^0(p_1)} + \frac{G(1 - \pi_2^0(p_2))}{\pi_2^0(p_2)} \right]. \tag{19}$$

---

[19]Second-order conditions are: $\frac{\partial^2 E\Pi_0}{\partial \pi_1^2} = -\frac{[g(\pi_1)]^2 + (1 - G(\pi_1))g'(\pi_1)}{[g(\pi_1)]^2} - \frac{2\pi_1 - p_1}{p_1} < 0$, $\frac{\partial^2 E\Pi_0}{\partial \pi_2^2} = -\frac{[g(1 - \pi_2)]^2 - G(1 - \pi_2)g'(1 - \pi_2)}{[g(1 - \pi_2)]^2} - \frac{2\pi_2 - p_2}{p_2} < 0$. The first terms (inclusive of the negative sign) are the derivatives of $\frac{1 - G(\pi_1)}{g(\pi_1)}$ and $\frac{G(1 - \pi_2)}{g(1 - \pi_2)}$ that are negative by Assumption 5. Since $\pi_i > p_i$ for positive profit, second-order conditions follow.

Proposition 4 (A class of distribution functions & corruption-free betting) (i) *The bookie's profit $E\Pi_0(p_1)$ is (nearly) U-shaped with a unique minimum at p_1^*, which solves $k(p_1) = 0$ in* (19).

(ii) *Further, if $g(q)$ is symmetric and $a + b = 1$, then $p_1^* = 1/2$.*

(iii) *Suppose $g(q)$ is symmetric. Also assume that at $p_1 = \frac{a+b}{2}$, the price markups satisfy*

$$\frac{\pi_1}{p_1} \leq (>)\frac{\pi_2}{p_2} \quad \text{if } a + b \geq (<)1. \tag{20}$$

Then, for $a + b > 1$, $p_1^* \in (\frac{1}{2}, \frac{a+b}{2})$ if

$$(\text{ at } p_1 = \frac{1}{2}) \quad \pi_1 - \pi_2 \leq a + b - 1 \leq \pi_1 - \pi_2 \quad (\text{ at } p_1 = \frac{a+b}{2}); \tag{21}$$

and for $a + b < 1$, $p_1^* \in (\frac{a+b}{2}, \frac{1}{2})$ if

$$(\text{ at } p_1 = \frac{a+b}{2}) \quad \pi_1 - \pi_2 \leq a + b - 1 \leq \pi_1 - \pi_2 \quad (\text{ at } p_1 = \frac{1}{2}). \tag{22}$$

(The second assumption in part **(iii)**, i.e. condition (20), suggests that when team 1 is already a favorite, the price markup on ticket 1 should be smaller than that on ticket 2, reflecting the difficulty of making a profit from a team that has a greater chance to win.)

As an illustration we derive the minimum profit point for *'unbalanced'* uniform distribution, which automatically satisfies Assumption 5, the second assumption in part **(iii)**, and conditions (21)–(22). Optimal prices are

$$\pi_1^* = \min\{b, \max\{a, \sqrt{bp_1}\}\}, \qquad \pi_2^* = \min\{(1-a), \max\{1-b, \sqrt{(1-a)p_2}\}\}.$$

Substituting $\pi_1 = \sqrt{bp_1}$ and $\pi_2 = \sqrt{(1-a)p_2}$ into Eq. (19) we obtain the minimum profit point as $\sqrt{p_1} = \sqrt{\frac{b}{1-a}p_2}$, or $p_1^* = \frac{b}{1+b-a}$. If $b = 1 - a$ (symmetry) then at $p_1^* = \frac{1}{2}$. But if $a + b < 1$, $p_1^* = \frac{b}{1+b-a} \in (\frac{a+b}{2}, \frac{1}{2})$, and if $a + b > 1$, $\frac{b}{1+b-a} \in (\frac{1}{2}, \frac{a+b}{2})$.

The main message of Proposition 4 is that for a symmetric distribution the minimum profit point will diverge from the even contest, only if the support is 'unbalanced' (i.e. $a + b \neq 1$). Even then, p_1^* will not stray too far from $1/2$, as long as the prices observe some regularity conditions (i.e. (21) or (22)). Therefore, our insight into the bookie's desire to orchestrate fixing at even and near-even contests remains valid for a class of (general) distribution functions.

7 Partially rational bettors

So far our analysis relied on naive bettors assumption. Here we briefly consider the case of *rational bettors*. Intuitively, like in markets for financial assets involving noise and sophisticated/informed

traders where the latter group benefits at the expense of the former, in our model naive bettors create an environment in which unscrupulous fixers and monopolist bookmaker can implicitly collude or coordinate to the detriment of fair contest and fair betting. It is to be expected that if the market is populated more by rational bettors, the monopolist's hands will be somewhat tied limiting his profitable match-fixing opportunities. We will verify this simple intuition using a plausible description of rational bettors.

There are at least two counts on which the bettors' rationality can be improved. First, bettors may become aware that there is a possibility of match-fixing, and second, they can try to extract information about the teams' winning chances from the posted odds.

If we were to admit rationality only on the first count, then our model can be easily extended by allowing the bettors to revise their q as $q[\mu_1\lambda_1 + (1 - \mu_1)]$ when only team 1 is corrupt and $[\mu\lambda q + \mu(1 - \lambda(1 - q)) + (1 - 2\mu)q]$ when both teams are corrupt. Bettors may apply this belief regardless of the bookie's strategy. This belief revision will effectively compress the support of the bettors' belief distribution.[20] Then using the insight of Proposition 4 we can tell the bookie's expected profit curve (ignoring his potential loss to the fixer, if any) will certainly be somewhat dampened at very low and very high values of p_1, but will still have a minimum point at or near $p_1 = \frac{1}{2}$, which suggests that the bookie's match-fixing incentive may weaken, but will not disappear.

An alternative would be to allow for rationality on both counts. Admittedly, modeling information extraction from the posted odds is a challenging problem and it is beyond the scope of this paper. In our setting, by observing the bet prices the bettors cannot infer p_1 (even if they know all parameters of the model), simply because they cannot be sure which strategy the bookie is pursuing – bribe prevention or inducement. Therefore, we introduce a simple assumption on the bettors' behavior, which makes the bettors at least partially rational. We assume that the knowledge of the presence of a fixer makes them suspicious of foul play, if they see the posted odds are really 'too far away' from the odds induced by their initial belief. Our assumption is couched as a 'behavioral rule' as follows:

If the market odds are **too good to be true***, a 'partially rational' bettor would apply* **extreme caution** *and may revise his prior* q *before placing any bets. If the posted prices offer a rate of return in excess of an exogenous threshold* $R > 0$ *on a particular ticket, say ticket 1, then the rational bettor reasons that it is too good to be true; team 1 is going to be bribed, and it is wise not to bet on it at all.*

Suppose for a bettor with belief q*, ticket 1 is a preferred bet:* $\max\{\frac{1-q-\pi_2}{\pi_2}, 0\} \leq \frac{q-\pi_1}{\pi_1}$*. As long as* $\frac{q-\pi_1}{\pi_1} \leq R$*, he sticks to his belief* q *and bets on team 1. But if* $\frac{q-\pi_1}{\pi_1} > R$*, he completely withdraws from betting (i.e. revises* $q = 0$*). Similar rule applies for a bettor when ticket 2 is his preferred bet.*

There are several justifications for the above strategy. First, in reality, most investors are weary of projects that promise too high a return, when they are conscious of the possibility of foul plays. In gambling too, betting odds that are 'too good to be true' should make them nervous. It is also

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[20]In the first case the support is $[0, \mu_1\lambda_1 + (1 - \mu_1)]$, and in the second case, it is $[\mu(1 - \lambda), 1 - \mu(1 - \lambda)]$.

nearly impossible for the bettors to do sophisticated guesswork and extract precise information from posted odds, due to cognitive limitations, bounded rationality, or just lack of expertise. A simple cut-off strategy can be the most practical way to make decisions in such environments.

Second, though our assumption pertains to a sudden withdrawal from betting, one could alternatively model gradual withdrawal. Discontinuous participation in markets populated by rogue elements is similar in spirit to how prices may not smooth transactions in markets with asymmetric information, as shown in the credit-rationing model of Stiglitz and Weiss (1981). The theoretical basis for the cut-off strategy can also be traced to studies on market manipulation in finance, where insiders' profitable trading opportunities become restricted for fear of unfavorable response by uninformed participants if prices signal informed trading; see, for instance, Chakraborty and Yilmaz (2004).

Third, our behavioral assumption can be seen as an alternative to allowing bettors' beliefs to be correlated with the nature's initial draw (which makes them 'less naive', a point we mentioned in Section 6). Even without such information precision, the bettors may bet conservatively (for other reasons), which in turn can limit the monopolist's hands in inducing match-fixing.

Fourth, the real world betting market is likely to have a mix of both types of bettors – naive and rational (full or partial, whichever way it is defined). The first group, called leisure/amateur bettors, takes a punt of a few dollars on a sport that they love for the *enjoyment of gambling* (Saunders and Turner (1992), Bruce and Johnson (1987, p.3), Winter and Kukuk (2008)). The second type of bettors, who may not be fully rational but are instinctively watchful, are very likely to conform to our behavioral assumption. In a simple tractable way, we aim to incorporate this mix of bettors in our model. Admittedly, an ideal treatment of rational bettors requires a richer model borrowing insights from recent developments in the market manipulation and learning literature.

In what follows, we will refer to our 'partially rational' bettors simply as 'rational' bettors.

Betting rule for rational bettors. Given the above definition of rationality, a rational bettor bets on team 1 if and only if $\pi_1 \leq q \leq \pi_1(1 + R)$, and bets on team 2 if and only if $\pi_2 \leq 1 - q \leq \pi_2(1 + R)$, or $1 - \pi_2(1 + R) \leq q \leq 1 - \pi_2$.

The above rule implies that there are now two additional groups of bettors who will not bet. All (rational) bettors with $q < 1 - \pi_2(1 + R)$ and $q > \pi_1(1 + R)$ will not bet, in addition to bettors with $q \in (1 - \pi_2, \pi_1)$. That also means under no circumstances, the bookie will be able to induce all bettors to bet in a preferred direction and collect the entire fund y. This is probably an early indication of the difficulty of defrauding a rational bettor.

We now present an analysis of the bookie's problem for the case where both teams are corrupt, and μ_i, λ_i, and Ω_i are symmetric ($\mu < \frac{1}{2}$). The analysis is comparable with Section 4 for $p_1 = \frac{1}{2}$, and for better comparison we will assume that the ordinary bettors are of two types: naive as in previous sections and rational as defined above; ρ proportion of the ordinary bettors is naive and $1 - \rho$ proportion is rational. Their prior beliefs are 'uniform'.

Given any p_1 (ignoring bribery), the bookie's profit from rational bettors is calculated as:

$$(1-\rho)\{\underbrace{\left[\int_{\pi_1}^{\pi_1(1+R)} y(1-\frac{p_1}{\pi_1})\,dq\right]}_{\text{ticket 1 profit}} + \underbrace{\left[\int_{1-\pi_2(1+R)}^{1-\pi_2} y(1-\frac{p_2}{\pi_2})\,dq\right]}_{\text{ticket 2 profit}}\} = (1-\rho)yR[\pi_1+\pi_2-1].$$

Note that, the expected profit from the rational bettors becomes independent of p_1. The expected profit from the naive bettors is unchanged from the previous section(s). Combining all bets (including IP's) we write bookie's aggregate profit and the bribe inducement program as:

$$\max_{\pi_1,\pi_2} \ E\Pi = (1-\rho)yR[\pi_1+\pi_2-1] + \rho\left[3-\pi_1-\pi_2-\frac{r_1}{\pi_1}-\frac{r_2}{\pi_2}\right] + \mu z\left[2-\frac{1-\lambda p_1}{\pi_2}-\frac{1-\lambda p_2}{\pi_1}\right],$$

$$\text{subject to} \quad p_2 \le \pi_2 \le \phi_1 \ (<1-\lambda p_1), \quad p_1 \le \pi_1 \le \phi_2 \ (<1-\lambda p_2), \quad \pi_1+\pi_2 \ge 1,$$

where $r_1 = q + \mu(1-\lambda)(1-2q)$ and $r_2 = 1-r_1$. Differentiating $E\Pi$ with respect to π_i yields:

$$\frac{\partial E\Pi}{\partial \pi_i} = (1-\rho)yR + \rho y\left[-1+\frac{r_i}{\pi_i^2}\right] + \mu z(1-\lambda p_j)\frac{1}{\pi_i^2}, \ \ i \ne j.$$

Since $\rho < 1$, π_i has to be potentially much larger now for an interior optimum. In particular, consider the special case of $\rho = 0$ (all bettors are rational). Then the above derivative is strictly positive, and by (2), the bookie should set $\pi_1 = \phi_2$ and $\pi_2 = \phi_1$, yielding match-fixing inducement profits:

$$E\Pi_{BI} = yR[\phi_1+\phi_2-1] - \mu z\Omega[2-\lambda].$$

On the other hand, in this special case if the bookie were to prevent bribery, his profits would be at least:

$$E\Pi_{BP} = yR[\phi_1+\phi_2-1].$$

Thus, we have the following result:

Proposition 5 (Rational bettors and no match-fixing) *When almost all ordinary bettors are rational (i.e. $\rho \to 0$), the monopolist bookie prefers bribe prevention over match-fixing. With a mix of bettors, match-fixing may be optimal less often than when all bettors are naive.*

The above result shows that rational bettors are an obstacle to orchestrating match-fixing. While the complete elimination of match-fixing result (obtained in the special case) is rather extreme, we should keep in mind that the underlying assumptions are also extreme, such as complete withdrawal from betting, and both teams being corruptible.[21] Also, with all rational bettors, if withdrawal from betting is gradual, then match-fixing is likely to become optimal in some cases.

An additional consideration that we have ignored due to the one-shot nature of the game is the bookie's reputation effect. If match-fixing is uncovered, ordinary bettors would be reluctant to bet

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[21]Recall, for naive bettors, both teams being corruptible may imply less match-fixing.

in the future. This will work against bribe inducement.

Does Proposition 5 cast a serious doubt on our model of match-fixing? We definitely think not. In real-life sports betting, markets are likely to have a mix of naive and rational bettors. So long as there are considerable number of naive bettors, who follow the game with their exogenous beliefs and completely unmindful of the dark side of betting, unscrupulous fixers will always have their negative influence.

8 Enforcement issues and concluding remarks

We have presented a model of monopoly betting with the risk of match-fixing, and shown that the market outcome is vastly different from the competitive case for even contests. The monopolist bookmaker abuses his market power sometimes and encourages match-fixing. But other times, he tolerates match-fixing as more of a survival strategy. The undesirable outcome of course depends on several market microstructure elements, such as the bettors' beliefs, information hierarchy, market concentration and the extent to which fixing can tilt the outcome.

One key element of our analysis is exogenous enforcement. Clearly strategic enforcement would add a particular type of interaction between the bookie and anti-corruption authority. But in light of our present analysis it can be said that even if enforcement is made clever, it would hardly pick an even contest to investigate, because its outcome is 'meant' to go either way. We can also see that in many cases of interest, the optimal odds by the bookie give away whether the bookie is trying to orchestrate match-fixing. Shouldn't then the bookie be prosecuted as well? Particularly when match-fixing is *encouraged*, he is as much culpable as the match-fixer.

It is also well known, and often discussed in the public media, how bookmakers can tell whether some unusually high wagers or betting activity and even the odds pattern (where odds are generated endogenously as in parimutuel setting) can be the tell-tale sign of fixing (see footnote 7). But then shouldn't the enforcement's investigation policy be accordingly more sophisticated to extract information from market data?

In the real world, there may also be a lack of will, and appropriate regulation and coordination among different agencies. A random report on the internet about football governance by the sport's highest body, FIFA, expresses disappointment in the following
(source: http://www.fifa.com/aboutfifa/organisation/footballgovernance/news/newsid=2001014/index.html):

"In football, a national association can sanction a member of the football family if they are found guilty of contravening the legal, football framework....But for people outside of football, currently the custodial sentences imposed are too weak, and offer little to deter someone from getting involved in match-fixing."

Our analysis highlights that enforcement in sports betting is a complex task requiring sophisticated guesswork about what the bookie is trying to do and what he knows, and there are also legislative challenges.

Appendix A

Proof of Lemma 1. First consider the case of no IP. Maximizing the bookie's payoff as given in (3) we obtain the following first-order conditions:

$$\frac{\partial E\Pi}{\partial \pi_1} = y\left[-1 + \frac{p_1}{\pi_1^2}\right] = 0, \quad \text{and} \quad \frac{\partial E\Pi_0}{\partial \pi_2} = y\left[-1 + \frac{p_2}{\pi_2^2}\right] = 0.$$

These determine the **unconstrained optimal prices** for the bookie: $\pi_1^* = \sqrt{p_1}$, $\pi_2^* = \sqrt{p_2}$. Clearly, the Dutch-book constraint is satisfied, as $\sqrt{p_1} \geq p_1$, $\sqrt{p_2} \geq p_2$. If team 1 is corruptible, then π_2^* must be at least ϕ_1, hence $\pi_2^* = \max\{\sqrt{p_2}, \phi_1\}$. If not, then $\pi_2^* = \sqrt{p_2}$. **Q.E.D.**

Derivation of bribe inducement profit when only team 1 is corruptible. From Eq. (5) we can write the ticket 1 profit as $E\Pi_{BI}(\text{Ticket } 1) = y(1 - \pi_1)(1 - \frac{\lambda_1^* p_1}{\pi_1})$. From Eq. (6) substitute $\pi_1 = \sqrt{p_1 \lambda_1^*}$ assuming $\sqrt{\lambda_1^* p_1} > p_1$ (i.e. the constraint on π_1 does not bind), and obtain

$$E\Pi_{BI}(\text{Ticket } 1) = y(1 - \sqrt{\lambda_1^* p_1})^2 ; \tag{23}$$

when $\sqrt{\lambda_1^* p_1} < p_1$ (i.e. the constraint on π_1 does bind), we have

$$E\Pi_{BI}(\text{Ticket } 1) = y\mu_1 p_2(1 - \lambda_1). \tag{24}$$

On the other hand, as the IP bets on team 2, ticket 2 profit is

$$
\begin{aligned}
E\Pi_{BI}(\text{Ticket } 2) &= y(1 - \pi_2)\left(1 - \frac{1 - \lambda_1^* p_1}{\pi_2}\right) + z\mu_1\left(1 - \frac{1 - \lambda_1 p_1}{\pi_2}\right) \\
&= y\left[1 - \pi_2 - \frac{1 - \lambda_1^* p_1 + \mu_1(z/y)(1 - \lambda_1 p_1)}{\pi_2} + \left(1 - \lambda_1^* p_1 + \mu_1 \frac{z}{y}\right)\right].
\end{aligned}
$$

Substituting $\lambda_1^* = \mu_1 \lambda_1 + (1 - \mu_1)$ and $z = 1 - y$ in the third term above rewrite it as

$$E\Pi_{BI}(\text{Ticket } 2) = y\left[(1 - \pi_2) - \frac{1 - \lambda_1^* p_1 + \mu_1(z/y)(1 - \lambda_1 p_1)}{\pi_2} + \left(p_2 + \mu_1\left\{\frac{1 - y}{y} + p_1(1 - \lambda_1)\right\}\right)\right].$$

From Eq. (6) substitute $\pi_2 = \sqrt{1 - \lambda_1^* p_1 + \mu_1(z/y)(1 - \lambda_1 p_1)}$ into the above and obtain

$$E\Pi_{BI}(\text{Ticket } 2) = y\left[1 - 2\sqrt{1 - \lambda_1^* p_1 + \mu_1 \frac{z}{y}(1 - \lambda_1 p_1)} + \left(p_2 + \mu_1\left\{\frac{1 - y}{y} + p_1(1 - \lambda_1)\right\}\right)\right].$$

Further, we can simplify $1 - \lambda_1^* p_1 + \mu_1 \frac{z}{y}(1 - \lambda_1 p_1)$ to be $p_2 + \frac{\mu_1}{y}\{p_1(y - \lambda_1) + 1 - y\}$ and write

$$E\Pi_{BI}(\text{Ticket } 2) = y\left[1 - 2\sqrt{p_2 + \frac{\mu_1}{y}\{p_1(y - \lambda_1) + 1 - y\}} + \left(p_2 + \mu_1\left\{\frac{1 - y}{y} + p_1(1 - \lambda_1)\right\}\right)\right]. \tag{25}$$

Now suppose π_2 is constrained to be ϕ_1, then ticket 2 profit is

$$E\Pi_{BI}(\text{Ticket 2}) = y\left[1 - \phi_1 - \frac{p_2 + \frac{\mu_1}{y}\{p_1(y-\lambda_1)+1-y\}}{\phi_1} + \left(p_2 + \mu_1\left\{\frac{1-y}{y} + p_1(1-\lambda_1)\right\}\right)\right]. \quad (26)$$

In order to calculate total profit we need to consider two cases depending on the size of λ_1.

Case 1: $\lambda_1 < \frac{1}{2}$ and $\tilde{\mu}_1 < \frac{1}{2(1-\lambda_1)} < 1$. This is the case presented in Eqs. (9)–(11). When neither of the two prices are constrained, the total bribe inducement profit is given by the sum of the profit expressions given in Eqs. (23) and (25) as follows:

$$E\Pi_{BI} = y(1 - \sqrt{\lambda_1^* p_1})^2 + y\left[1 - 2\sqrt{p_2 + \frac{\mu_1}{y}\{p_1(y-\lambda_1)+1-y\}} + \left(p_2 + \mu_1\left\{\frac{1-y}{y} + p_1(1-\lambda_1)\right\}\right)\right]. \quad (27)$$

Substitute $p_1 = 1/2$, and with some rearrangement of terms obtain Eq. (9), referred as $E\Pi_{BI}^1$.

Next, consider π_1 to be unconstrained and π_2 to be constrained. The total profit is now given by the sum of (23) and (26), which yields

$$E\Pi_{BI} = y(1 - \sqrt{\lambda_1^* p_1})^2 + y\left[1 - \phi_1 - \frac{p_2 + \frac{\mu_1}{y}\{p_1(y-\lambda_1)+1-y\}}{\phi_1} + \left(p_2 + \mu_1\left\{\frac{1-y}{y} + p_1(1-\lambda_1)\right\}\right)\right]. \quad (28)$$

Once again substitute $p_1 = p_2 = 1/2$ and rearrange terms to obtain $E\Pi_{BI}^2$ as given in Eq. (10).

Finally, when both prices are constrained we need to sum up the profits given in Eqs. (24) and (26) to derive the total profit, which is

$$E\Pi_{BI} = y\mu_1 p_2(1-\lambda_1) + y\left[1 - \phi_1 - \frac{p_2 + \frac{\mu_1}{y}\{p_1(y-\lambda_1)+1-y\}}{\phi_1} + \left(p_2 + \mu_1\left\{\frac{1-y}{y} + p_1(1-\lambda_1)\right\}\right)\right]. \quad (29)$$

Set $p_1 = 1/2$ in the above to obtain $E\Pi_{BI}^3$ in Eq. (11). This completes the derivation of Eqs. (9)–(11).

Case 2: $\lambda \geq \frac{1}{2}$ or $\tilde{\mu}_1 < 1 \leq \frac{1}{2(1-\lambda_1)}$. This case arises if $\lambda_1 \geq \frac{1}{2}$. Here the constraint on π_1 never binds. Total profit from bribe inducement will be $E\Pi_{BI} = E\Pi_{BI}^1$ at all $\mu_1 \in (0, \tilde{\mu}_1]$ as before, and $E\Pi_{BI} = E\Pi_{BI}^2$ at all $\mu_1 \in (\tilde{\mu}_1, 1]$.

There are two other possible cases. The case of $\frac{1}{2(1-\lambda_1)} < 1 < \tilde{\mu}_1$ does not apply, because the constraint on π_2 will always bind at some μ_1 (given $p_1 = 1/2$). Hence $\tilde{\mu}_1$ is always less than 1. Finally, the case of $\frac{1}{2(1-\lambda_1)} < \tilde{\mu}_1 < 1$ is ruled out by Assumption 3. $\parallel$

Next, we develop two lemmas for subsequent use in the proof of Proposition 1.

Lemma A1. *If $y \in \Sigma_1$, then the profit expression $E\Pi_{BI}^2$ is increasing in $\mu_1 \in (0, \frac{1}{2(1-\lambda_1)}]$ when $\lambda_1 < 1/2$, and increasing over the entire interval $\mu_1 \in (0, 1]$ when $\lambda_1 \geq 1/2$. Also, at $\tilde{\mu}_1$, $E\Pi_{BI}^2 > E\Pi_{BP}$.*

Proof of Lemma A1. First note that $E\Pi_{BI}^2$ is the highest BI profit at $\mu_1 \in [\tilde{\mu}_1, \min\{\frac{1}{2(1-\lambda_1)}, 1\}]$. But $E\Pi_{BI}^2$ is also feasible over $\mu_1 \in (0, \tilde{\mu}_1)$, because setting $\pi_2 = \phi_1$ induces bribery at this range of μ_1; but the bookie does better by setting $\pi_2 = \sqrt{1 - \lambda_1^* p_1}$ $(< \phi_1)$. Hence we can check the slope of $E\Pi_{BI}^2$ over its entire feasible range.

Consider $E\Pi_{BI}^2$ as given in Eq. (10) and differentiate with respect to μ_1:

$$\frac{E\Pi_{BI}^2}{\partial\mu_1} = y\left[(1-\lambda_1)\left(\frac{1}{\sqrt{2\lambda_1^*}} - \frac{1}{2}\right) + \frac{2-y(1+\lambda_1)}{2y} - \frac{2-y-\lambda_1}{2y\phi_1}\right].$$

This derivative is positive if

$$\frac{(1-\lambda_1)\sqrt{2}y\phi_1}{y(2\phi_1-1) + 2(1-\phi_1) - \lambda_1} > \sqrt{\lambda_1^*}.$$

The right-hand side expression is largest $(=1)$ when $\mu_1 \to 0$. So we try to identify the values of y such that the left-hand side expression is strictly greater than 1. If such values of y are found, then clearly at any μ_1 the above inequality will hold. For that we need

$$y > \frac{2(1-\phi_1) - \lambda_1}{1 - \phi_1\sqrt{2}[\sqrt{2} - (1-\lambda_1)]}.$$

But note that ϕ_1 is also a function of y (via Ω_1 which depends on z). Let us denote the right-hand side expression as $K(y)$, and write $J(y) = y - K(y)$.

Higher y means lower z and higher Ω_1, which in turn implies lower ϕ_1. In particular at $y = 0$, $0 < K(0) < 1$; on the other hand, when $y \to 1$, $\phi_1 \to \infty$ and by L'Hospital's theorem $K(1) = \frac{2}{2-\sqrt{2}(1-\lambda_1)} > 1$. So by Intermediate value theorem there must be at least two values of y, say y_0 and y_1, such that for all $y \in \Sigma_1 = (y_0, y_1)$ we must have $J(y) > 0$ (see Definition 1); in fact, as footnote 14 asserts, the $\{y_0, y_1\}$ pair is unique. Thus, if $y \in \Sigma_1$, at all μ_1, $E\Pi_{BI}^2$ will be increasing.

Now note from Eq. (10) that, if $E\Pi_{BI}^2$ were evaluated at $\mu_1 = 0$, it would be just equal to $E\Pi_{BP}$. Then by the positive slope of $E\Pi_{BI}^2$ it is implied that at $\tilde{\mu}_1$ we must have $E\Pi_{BI}^2 > E\Pi_{BP}$.   **Q.E.D.**

Lemma A2. *If* $y \in \Sigma_1$, *then* $E\Pi_{BI}$ *is increasing in* μ_1 *at all* $\mu_1 > \tilde{\mu}_1$.

Proof of Lemma A2. Consider first $\tilde{\mu}_1 \le \frac{1}{2(1-\lambda_1)}$ (< 1), where $E\Pi_{BI}$ is given by $E\Pi_{BI}^2$ as in Eq. (10). By Lemma A1 we know $E\Pi_{BI}^2$ is increasing in μ_1 if $y \in \Sigma_1$.

Next, consider $\mu_1 \in (\frac{1}{2(1-\lambda_1)}, 1]$, where $E\Pi_{BI}$ is given by $E\Pi_{BI}^3$ as in Eq. (11). We know $E\Pi_{BI}^3 = E\Pi_{BI}^2$ at $\mu_1 = 1/[2(1-\lambda_1)]$. Therefore, all we need is to ensure that $E\Pi_{BI}^3$ is increasing in μ_1 in this range. We obtain

$$\frac{\partial E\Pi_{BI}^3}{\partial\mu_1} = y\left[\frac{1}{2\phi_1} - 2\right] + 1 - \frac{2-\lambda_1}{2\phi_1} > 0, \quad \text{if} \quad y > \frac{2 - 2\phi_1 - \lambda_1}{1 - 2\lambda_1\phi_1}.$$

This restriction is automatically satisfied if $y \in \Sigma_1$, because $\frac{2-2\phi_1-\lambda_1}{1-2\lambda_1\phi_1} < \frac{2(1-\phi_1)-\lambda_1}{1-\phi_1\sqrt{2}[\sqrt{2}-(1-\lambda_1)]}$. Therefore, $E\Pi_{BI}^3$ is also upward-sloping if $y \in \Sigma_1$.

The case of $\lambda_1 \ge 1/2$ is trivial. Here $E\Pi_{BI}^3$ is not relevant; $E\Pi_{BI}^2$ applies to the whole range of $[\tilde{\mu}_1, 1]$. As $y \in \Sigma_1$ guarantees increasing $E\Pi_{BI}^2$, the claim is true.   **Q.E.D.**

Proof of Proposition 1. **(i)** The proof of this part involves only $E\Pi_{BI}^1$. Recall,

$$E\Pi_{BI}^1 = y\left(1 - \sqrt{\frac{\lambda_1^*}{2}}\right)^2 + y\left[\frac{3}{2} + \mu_1 \frac{2 - y(1 + \lambda_1)}{2y} - \sqrt{2\{1 + \mu_1(\frac{2 - \lambda_1}{y} - 1)\}}\right] \quad \forall \ \mu_1 \in (0, \tilde{\mu}_1].$$

As $\mu_1 \to 0$, $E\Pi_{BI}^1 \to y(3 - 2\sqrt{2}) = E\Pi_0$. Now since $p_1^1 < \frac{1}{2}$, we must have $E\Pi_{BP} < E\Pi_0$. Then as $\mu_1 \to 0$, $E\Pi_{BI}^1 > E\Pi_{BP}$. Hence, at all μ_1 quite close to zero, $E\Pi_{BI}^1 > E\Pi_{BP}$. That proves part (i).

(ii) There are two ranges of λ_1 to consider: $\lambda_1 < \frac{1}{2}$ and $\lambda_1 \geq \frac{1}{2}$. Before we consider each of these cases, let us first note that $E\Pi_{BI}^1$ is decreasing in μ_1 at sufficiently small values of μ_1.

Deriving $\frac{\partial E\Pi_{BI}^1}{\partial \mu_1}$ and then letting $\mu_1 \to 0$ (i.e. $\lambda_1^* \to 1$) we obtain (with some simplification):

$$\left.\frac{\partial E\Pi_{BI}^1}{\partial \mu_1}\right|_{\mu_1 \to 0} = \frac{1 - y}{2\sqrt{2}}\left[2\lambda_1 - (4 - 2\sqrt{2})\right] < 0 \ \text{ at all } \lambda_1 \in [0, 1].$$

Case 1: $\lambda_1 < \frac{1}{2}$ and $\tilde{\mu}_1 < \frac{1}{2(1-\lambda_1)}$ (< 1). The expected profit from bribe prevention at $p_1 = 1/2$ is given in Eq. (8). The BI profit is given in Eqs. (9)–(11), with the underlying prices as

$$\text{For } \mu_1 \in (0, \tilde{\mu}_1], \quad \pi_1 = \sqrt{\lambda_1^*/2}, \ \ \pi_2 = \sqrt{\frac{2 - \lambda_1^*}{2} + \mu_1 \frac{z}{y}\frac{(2 - \lambda_1)}{2}} \ ;$$

$$\text{For } \mu_1 \in (\tilde{\mu}_1, \frac{1}{2(1 - \lambda_1)}], \quad \pi_1 = \sqrt{\lambda_1^*/2}, \ \ \pi_2 = \phi_1(1/2) \ ;$$

$$\text{For } \mu_1 \in (\frac{1}{2(1 - \lambda_1)}, 1], \quad \pi_1 = 1/2, \ \ \pi_2 = \phi_1(1/2) \ .$$

Now recall Eqs. (9)–(11). It turns out that comparing $E\Pi_{BI}^1$ with $E\Pi_{BP}$ is cumbersome; therefore, we will take an indirect route by comparing $E\Pi_{BI}^2$ with $E\Pi_{BP}$ and then compare $E\Pi_{BI}^2$ with $E\Pi_{BI}^1$ (for $\mu_1 \in (0, \tilde{\mu}_1]$). We should note that when $\mu_1 \in (0, \tilde{\mu}_1]$ the bookie could have chosen for ticket 2 any price from the interval $[p_2, \phi_1]$, but he chose $\pi_2 = \sqrt{1 - p_1\lambda_1^* + \mu_1\frac{z}{y}(1 - \lambda_1 p_1)} < \phi_1$. Therefore, it must be that $E\Pi_{BI}^1 \geq E\Pi_{BI}^2$ at all $\mu_1 \in (0, \tilde{\mu}_1]$, if $E\Pi_{BI}^2$ was evaluated in this interval.

Comparing $E\Pi_{BI}^2$ and $E\Pi_{BP}$ we see that the latter is constant in μ_1, whereas the former is increasing in μ_1 (refer to Lemma A1). Moreover, as $\mu_1 \to 0$ we have

$$E\Pi_{BI}^2 = E\Pi_{BP} = y\left[3 - \sqrt{2} - \phi_1 - \frac{1}{2\phi_1}\right].$$

Thus, $E\Pi_{BI}^2 > E\Pi_{BP}$ at all $\mu_1 \in (0, \tilde{\mu}_1]$. Then as $E\Pi_{BI}^1 \geq E\Pi_{BI}^2$ at all $\mu_1 \in (0, \tilde{\mu}_1]$, we must have $E\Pi_{BI} > E\Pi_{BP}$. Further, by Lemma A2 $E\Pi_{BI}$ is increasing at all $\mu_1 > \tilde{\mu}_1$. Therefore, $E\Pi_{BI} > E\Pi_{BP}$ at all $\mu_1 > \tilde{\mu}_1$ as well.

Case 2: $\lambda_1 \geq \frac{1}{2}$ or $\tilde{\mu}_1 < 1 \leq \frac{1}{2(1-\lambda_1)}$. In this case, $E\Pi_{BI} = E\Pi_{BI}^2$ at all $\mu_1 \in (\tilde{\mu}_1, 1]$. Here $E\Pi_{BI}^3$ does not apply. We know that if $y \in \Sigma_1$, $E\Pi_{BI}^2$ will be an increasing function of μ_1 (refer to Lemma A1), and $E\Pi_{BI}^2 > E\Pi_{BP}$ at all $\mu_1 \geq \tilde{\mu}_1$. At $\mu_1 < \tilde{\mu}_1$, we must also have $E\Pi_{BI} \equiv E\Pi_{BI}^1 > E\Pi_{BP}$ by the same reasoning given in case 1 above. Hence, at all μ_1 bribe inducement will be optimal. **Q.E.D.**

$w + \alpha_1(f + f_I) = 0.05$			$w + \alpha_1(f + f_I) = 0.06$		
λ_1	Σ_1	$\bar{y}$	λ_1	Σ_1	$\bar{y}$
0	$(0.29, 0.83)$	0.88	0	$(0.33, 0.77)$	0.855
0.05	$(0.32, 0.81)$	0.87	0.05	$(0.44, 0.725)$	0.845
0.10	$(0.37, 0.78)$	0.86	0.10	$(0.57, 0.62)$	0.83
0.15	$(0.44, 0.73)$	0.85	0.15	null	0.82
0.19	$(0.60, 0.62)$	0.838	0.19	null	0.805
0.20	null	0.835	0.20	null	0.805

Table 4: Critical range of y: Σ_1

Proof of Proposition 2. (i) Suppose $\lambda_1 < \frac{1}{2}$ and consider $E\Pi_{BI}^3 - E\Pi_0$ at $\mu_1 = 1$. After simplification the expression for $E\Pi_{BI}^3 - E\Pi_0 > 0$ (i.e. $E\Pi_{BI} - E\Pi_0 > 0$) becomes

$$y[2\sqrt{2} - 1 - p_1 - \lambda_1] > y\phi_1 + (1 - \lambda_1 p_1)\Omega_1.$$

At $p_1 = \frac{1}{2}$ the LHS expression becomes $y[2\sqrt{2} - \frac{3}{2} - \lambda_1]$, and the RHS expression becomes

$$L(y) = (2 - \lambda_1)\left[\frac{y}{2 + (2 - \lambda_1)\Omega_1} + \frac{\Omega_1}{2}\right],$$

which we have introduced in Definition 2. So if $y \in \Sigma_2$ then the above inequality holds. We know from Proposition 1 that at $\tilde{\mu}_1$, we have $E\Pi_{BI} < E\Pi_0$. Then by the intermediate value theorem there must exist a critical μ_1, namely $\hat{\mu}_1 \in (\tilde{\mu}_1, 1)$ such that at all $\mu_1 > \hat{\mu}_1$ we have $E\Pi_{BI} > E\Pi_0$. That is, bribe encouragement takes place.

(ii) For $\lambda_1 \geq \frac{1}{2}$ the proof is identical, except that now we consider $E\Pi_{BI}^2 - E\Pi_0$ at $\mu_1 = 1$. **Q.E.D.**

Proof of Proposition 3. Given $p_1^1 < \frac{1}{2} < p_1^0$, we have $E\Pi_{BP} < E\Pi_0$. Further, given $y \in \Sigma_3$, the unconstrained prices given in Eq. (14) solve the BI problem. The resultant BI profit at $p_1 = 1/2$ is given by Eq. (15), while the BP profit is given by Eq. (12). We have already checked that $E\Pi_{BI}^U$ is declining in μ, and at μ close to 0, $E\Pi_{BI}^U > E\Pi_{BP}$. At $\mu = 1/2$ by setting $E\Pi_{BI}^U < E\Pi_{BP}$ and rearranging terms we get $2\sqrt{\frac{2 - \lambda z}{y}} - \frac{z}{y} > 2\phi + \frac{1}{\phi}$, which is precisely our Assumption 4.

Hence, there must exist a unique critical value $\hat{\mu} \in (0, 1/2)$, such that at all $\mu \leq \hat{\mu}$ we have $E\Pi_{BI} \geq E\Pi_{BP}$ and at all $\mu > \hat{\mu}$ we have $E\Pi_{BI} < E\Pi_{BP}$. BI is optimal up to $\hat{\mu}$. **Q.E.D.**

Simulation of the critical ranges of y – refer Tables 4, 5 and 6.

Proof of Lemma 3. It is obvious that it will never be optimal to set $\pi_1 < a$ and $\pi_2 < 1 - b$. If π_1 is set below a, the expected profit from ticket 1 is $y(1 - \frac{p_1}{\pi_1})$ which can be increased by raising π_1 towards a. Similarly, if $\pi_2 < 1 - b$, $E\Pi_0 = y(1 - \frac{p_2}{\pi_2})$ can be improved by raising π_2. Hence, $\pi_1 \geq \max\{p_1, a\}$ and $\pi_2 \geq \max\{p_2, 1 - b\}$. Next, to show existence and uniqueness, we need to consider three cases, depending on the range of p_1.

$w + \alpha_1(f + f_I) = 0.05$		$w + \alpha_1(f + f_I) = 0.06$	
λ_1	Σ_2	λ_1	Σ_2
0	(0.16, 0.925)	0	(0.19, 0.909)
0.05	(0.17, 0.922)	0.05	(0.2, 0.904)
0.10	(0.18, 0.917)	0.10	(0.215, 0.895)
0.15	(0.19, 0.91)	0.15	(0.23, 0.886)
0.20	(0.2, 0.9)	0.20	(0.27, 0.878)

Table 5: Critical range of y: Σ_2

$w + \alpha_1(f + f_I) = 0.05$			
λ	Σ_3 at $\mu = 0.05$	Σ_3 at $\mu = 0.10$	Σ_3 at $\mu = 0.15$
0	(0.11, 0.88)	(0.21, 0.87)	(0.29, 0.87)
0.10	(0.13, 0.86)	(0.24, 0.85)	(0.33, 0.84)
0.15	(0.14, 0.85)	(0.26, 0.84)	(0.36, 0.83)
0.30	(0.22, 0.76)	(0.40, 0.75)	(0.55, 0.70)
0.40	(0.37, 0.61)	null	null
0.45	null	null	null

Table 6: Critical range of y: Σ_3

Case 1 [$p_1 \in [a, b]$]. Consider Eq. (17). For any given p_1 neither $\pi_1 = a$ nor $\pi_1 = b$ can be a solution. If $\pi_1 = a$, the bracketed expression is $\frac{1-G(a)}{g(a)} + \frac{(p_1-a)a}{p_1} = \frac{1}{g(a)} + \frac{(p_1-a)a}{p_1} > 0$ (and infinitely large if $g(a)$ is arbitrarily small). Similarly, if $\pi_1 = b$, the bracketed expression is $\frac{1-G(b)}{g(b)}) - \frac{(b-p_1)b}{p_1}$. Since $b > p_1$ and $G(b) = 1$ the sign of the bracketed term depends on whether $g(b) > 0$ or $g(b) = 0$. If $g(b) > 0$ then the term is clearly negative. If $g(b) = 0$ then by L'Hospital rule $\lim_{x \to b} \frac{1-G(x)}{g(x)} = -\lim_{x \to b} \frac{g(b)}{g'(b)} = 0$ because $g'(b) > 0$ due to our assumption that $g(q) > 0$ at all $q \in (a, b)$. Hence, here too the bracketed term is negative. Hence, by the intermediate value theorem, there must be a unique $\pi_1 \in (a, b)$ such that $\frac{\partial E\Pi_0}{\partial \pi_1} = 0$. Similar reasoning applies for the unique interior solution of π_2 with respect to Eq. (18).

Case 2 [$p_1 < a$]. Here ticket 2 will not be sold, because $p_2 > 1 - a$. For ticket 1, π_1 is either given by Eq. (17) or $\pi_1 = a$ whichever is greater.

Case 3 [$p_1 > b$]. Here ticket 1 sale is zero. For ticket 2, the solution given by Eq. (18) must not fall below $1 - b$, and the maximum of the two will be optimal.

Next, we consider (weak) monotonicity of prices. We know that each price is constant over at some values of p_1. Specifically, at $p_1 < a$ we have $\pi_1^0 = a$ over a range of p_1, and at $p_1 > b$ we have $\pi_2 = 1 - b$ over a range of p_1. But when (π_1^0, π_2^0) is given by Eqs. (17) and (18), strict monotonicity

holds due to Assumption 5, which we show as follows. From Eq. (17) we can derive:

$$\frac{\partial^2 E\Pi_0}{\partial \pi_1^2}\frac{\partial \pi_1^0}{\partial p_1} + \frac{\partial^2 E\Pi_0}{\partial \pi_1 \partial p_1} = 0, \quad \text{or} \quad \frac{\partial^2 E\Pi_0}{\partial \pi_1^2}\frac{\partial \pi_1^0}{\partial p_1} + \frac{(\pi_1^h)^2}{p_1^2} = 0.$$

Since $\frac{\partial^2 E\Pi}{\partial \pi_1^2} < 0$ by the second-order condition, we must have $\frac{\partial \pi_1^0}{\partial p_1} > 0$.

Symmetric argument establishes $\frac{\partial \pi_2^0}{\partial p_2} > 0$ from Eq. (18).

For the markups consider Eqs. (17)–(18). By Assumption 5, $\frac{1-G(\pi_1)}{g(\pi_1)}$ decreases with p_1. Then it follows that $\frac{(\pi_1-p_1)}{p_1}\pi_1$ also decreases in p_1. Since π_1 is increasing in p_1, $\frac{\pi_1-p_1}{p_1}$ must be falling. Similarly, from Eq. (18) it follows that $\frac{(\pi_2-p_2)}{p_2}\pi_2$ must fall if π_2 rises, and π_2 rises if p_2 rises; therefore, the fall in $\frac{(\pi_2-p_2)}{p_2}\pi_2$ must be attributable to $\frac{\pi_2-p_2}{p_2}$. This completes the proof. **Q.E.D.**

Proof of Proposition 4. **(i)** To establish that $E\Pi_0(p_1)$ is (nearly) U-shaped we determine its slope in three ranges of p_1: $p_1 < a$, $p_1 > b$ and $p_1 \in [a, b]$. It is straightforward to check that at $p_1 < a$, $E\Pi_0'(p_1) = -y/a$ if $\pi_1 = a$ and $E\Pi_0'(p_1) = -y(1-\pi_1^0)\pi_1^0$ if $\pi_1 = \pi_1^0$. Similarly, at $p_1 > b$, $E\Pi_0'(p_1) = y/(1-b)$ if $\pi_2 = 1-b$ and $E\Pi_0'(p_1) = y(1-\pi_2^0)\pi_2^0$ if $\pi_2 = \pi_2^0$.

For $p_1 \in [a, b]$, consider the slope function $k(p_1)$ (Eq. (19)). At $p_1 = a$, $k(p_1) = -\frac{1-G(\pi_1^0(a))}{\pi_1^0(a)} - \frac{G(a)}{1-a} = -\frac{1-G(\pi_1^0(a))}{\pi_1^0(a)} < 0$ as $\pi_2 = 1-a$. At $p_1 = b$, $k(p_1) = -\frac{1-G(b)}{b} + \frac{G(1-\pi_2^0(b))}{\pi_2^0(b)} = \frac{G(1-\pi_2^0(b))}{\pi_2^0(b)} > 0$.

Next, to show that $k(p_1)$ is an increasing function, we differentiate the terms given in (19):

$$\frac{\partial}{\partial p_1}\left(\frac{1-G(\pi_1^0)}{\pi_1^0}\right) = -\frac{\pi_1^0 g(\pi_1^0) + (1-G(\pi_1^0))}{(\pi_1^0)^2}\pi_1^{0\prime}(p_1) < 0,$$

$$\frac{\partial}{\partial p_1}\left(\frac{G(1-\pi_2^0)}{\pi_2^0}\right) = -\frac{\pi_2^0 g(1-\pi_2^0) + (G(1-\pi_2^0))}{(\pi_2^0)^2}\pi_2^{0\prime}(p_1) > 0,$$

since $\pi_1'(p_1) > 0$ and $\pi_2'(p_1) < 0$. By combining these two expressions, we obtain:

$$k'(p_1) \equiv \frac{\partial^2 E\Pi(\pi_1^h, \pi_2^h)}{\partial p_1^2} = -\frac{\partial}{\partial p_1}\left(\frac{1-G(\pi_1^h)}{\pi_1^h}\right) + \frac{\partial}{\partial p_1}\left(\frac{G(1-\pi_2^h)}{\pi_2^h}\right) > 0.$$

Now, as $k(p_1)$ is strictly increasing in p_1, and $k(a) < 0$ and $k(b) > 0$, there must be a unique p_1 between a and b at which $k(p_1) = 0$. This proves the claim that p_1^* is unique. It gives minimum profit because $E\Pi_0(.)$ is convex (as confirmed by $k'(p_1) > 0$).

Assume symmetry of $g(q)$ around $\frac{a+b}{2}$, and consider three cases.

(ii) *Case* $a + b = 1$. $g(q)$ is symmetric around $\frac{1}{2}$. At $p_1 = \frac{1}{2}$, by symmetry $\pi_1 = \pi_2$ and also $g(\pi_1) = g(1-\pi_2)$ and therefore, $1-G(\pi_1) = G(1-\pi_2)$; hence we must have $k(p_1) = 0 \Rightarrow p_1^* = 1/2$.

(iii) For the remaining two cases, impose condition (20). The proof here involves evaluating $k(p_1)$ at two points – $p_1 = \frac{1}{2}$ and $p_1 = \frac{a+b}{2}$. For that, we make use of the optimal price conditions by substituting $\frac{1-G(\pi_1)}{\pi_1} = \frac{(\pi_1-p_1)}{p_1}g(\pi_1)$ from Eq. (17), and $\frac{G(1-\pi_2)}{\pi_2} = \frac{(\pi_2-p_2)}{p_2}g(1-\pi_2)$ from Eq. (18),

into Eq. (19), to rewrite $k(p_1)$ as

$$k(p_1) = y\left[-\frac{(\pi_1 - p_1)}{p_1}g(\pi_1) + \frac{(\pi_2 - p_2)}{p_2}g(1 - \pi_2)\right].$$

We also note that from the symmetry of $g(.)$ it follows that $g(\pi_1) = g(a + b - \pi_1)$ and $g(1 - \pi_2) = g(a + b - 1 + \pi_2)$. Now consider each case.

Case $a+b > 1$. Here the even contest $(p_1 = \frac{1}{2})$ is to the left of the midpoint of the distribution, as majority of the bettors believe team 1 to be favorite. Therefore, $\pi_1 = \pi_2$ at some $p_1 < \frac{1}{2}$ and $\pi_1 > \pi_2$ thereafter. Now first evaluate $k(p_1)$ at $p_1 = \frac{1}{2}$, where

$$\text{sign of } k(p_1) = \text{sign of}\left[-(\pi_1 - p_1)g(\pi_1) + (\pi_2 - p_2)g(1 - \pi_2)\right].$$

As $\pi_1 > \pi_2$, the sign of $k(p_1)$ critically depends on whether $g(\pi_1) > g(1 - \pi_2)$. Since $\pi_1 > \pi_2 > \frac{1}{2}$, we must have $1 - \pi_2 < \frac{1}{2}$ where $g'(.) > 0$, because $p_1 = \frac{1}{2}$ is to the left of the midpoint $\frac{a+b}{2}$. This implies that due to the symmetry of $g(.)$ at $p_1 = a + b - (1 - \pi_2)$, the density function $g(.)$ must be decreasing. However, we cannot ascertain whether π_1 would be greater or less than $\frac{a+b}{2}$. As long as $\pi_1 \leq a + b - 1 + \pi_2$, or $\pi_1 - \pi_2 < a + b - 1$, $g(\pi_1) \geq g(1 - \pi_2)$. Then $k(p_1) < 0$ at $p_1 = \frac{1}{2}$. This confirms the upper bound on $\pi_1 - \pi_2$ as specified in (21).

Now evaluate $k(p_1)$ at $p_1 = \frac{a+b}{2}$, at which $p_1 > p_2$. Here,

$$\text{sign of } k(p_1) = \text{sign of}\left[-\frac{(\pi_1 - p_1)}{p_1}g(\pi_1) + \frac{(\pi_2 - p_2)}{p_2}g(1 - \pi_2)\right].$$

By our assumption, $\frac{(\pi_1 - p_1)}{p_1} < \frac{(\pi_2 - p_2)}{p_2}$. Since $\pi_1 > \frac{a+b}{2}$, $g(\pi_1)$ must be declining at π_1, while $g(1 - \pi_2)$ must be upward-sloping at $1 - \pi_2$ (since $\pi_2 > 1 - \frac{a+b}{2}$). Also as $g(1 - \pi_2) = g(a + b - 1 + \pi_2)$ by the symmetry of $g(q)$, $g(\pi_1) < g(1 - \pi_2)$ if $\pi_1 > a + b - 1 + \pi_2$. This is what is given as the lower bound on $\pi_1 - \pi_2$ in (21). Then at $p_1 = \frac{a+b}{2}$, $k(p_1) > 0$. Hence, p_1^* must lie between $\frac{1}{2}$ and $\frac{a+b}{2}$.

Case $a + b < 1$. Here $\frac{a+b}{2} < \frac{1}{2}$ and $\pi_1 < \pi_2$ at all $p_1 \leq \frac{1}{2}$. Follow the same reasoning as above and make use of the markup assumption. At $p_1 = \frac{a+b}{2}$, $g(\pi_1) > g(1 - \pi_2)$ if $\pi_1 - \pi_2 < a + b - 1$; this makes $k(p_1) < 0$ at $p_1 = \frac{a+b}{2}$ as specified in condition (22). Similarly, at $p_1 = \frac{1}{2}$, $g(\pi_1) < g(1 - \pi_2)$ if $\pi_1 - \pi_2 > a + b - 1$; then $k(p_1) > 0$. Therefore, $k(p_1) = 0$ at some $p_1^* \in (\frac{a+b}{2}, \frac{1}{2})$.   **Q.E.D.**

References

[1] Allen, F. and Gorton, G. (1992). Stock Price Manipulation, Market Microstructure and Asymmetric Information. *European Economic Review* **36**, 624-630.

[2] Bag, P.K. and Saha, B. (2011). Match-Fixing under Competitive Odds. *Games and Economic Behavior* **73**, 318-344.

[3] Bruce, A.C. and Johnson, J.E.V. (1992). Toward an Explanation of Betting as a Leisure Pursuit. *Leisure Studies* **11**, 201-218.

[4] Chakraborty, A. and Yilmaz, B. (2004). Informed Manipulation. *Journal of Economic Theory* **114**, 132-152.

[5] Chen, K.-P. (2003). Sabotage in Promotion Tournament. *Journal of Law, Economics and Organization* **19**, 119-140.

[6] De Muinck, P. and Quatacker, J. (2013). "Corruption in Sports: Match-Fixing with Gambling Purposes," http://lib.ugent.be/fulltxt/RUG01/002/164/534/RUG01-002164534_2014_0001_AC.pdf.

[7] Duggan, M. and Levitt, S.D. (2002). Winning isn't Everything: Corruption in Sumo Wrestling. *American Economic Review* **92**, 1594-1605.

[8] Glosten, L. and Milgrom, P. (1985). Bid, Ask and Transaction Prices in a Specialist Market with Heterogenously Informed Traders. *Journal of Financial Economics* **14**, 71-100.

[9] Haberfeld, M.R. and Sheehan, D. (2013). *Match-Fixing in International Sports: Existing Processes, Law Enforcement, and Prevention Strategies*, Edited volume, Springers.

[10] Konrad, K. (2000). Sabotage in Rent-seeking Contests. *Journal of Law, Economics and Organization* **16**, 155-165.

[11] Kyle, A. (1985). Continuous Auctions and Insider Trading. *Econometrica* **53**, 1315-1335.

[12] Levitt, S.D. (2004). Why are Gambling Markets Organised so Differently from Financial Markets? *Economic Journal* **114**, 223-246.

[13] Ottaviani, M. and Sorensen, P.N. (2005). "Parimutuel versus Fixed-odds Markets," http://didattica.unibocconi.eu/myigier/index.php?IdUte=48832&idr=16546&lingua=eng&comando=Apri.

[14] Ottaviani, M. and Sorensen, P.N. (2008). The Favorite-longshot Bias: An Overview of the Main Explanations. In *Handbook of sports and lottery markets*, ed. Donald B. Hausch and William T. Ziemba, 83-101. Elsevier North Holland.

[15] Ottaviani, M. and Sorensen, P.N. (2009). Surprised by the Parimutuel Odds? *American Economic Review* **99**, 2129-2134.

[16] Ottaviani, M. and Sorensen, P.N. (2010). Noise, Information, and the Favorite-Longshot Bias in Parimutuel Predictions. *American Economic Journal: Microeconomics* **2**, 58-85.

[17] Rodenberg, R. and Feustel, E.D. (2014). Forensic Sports Analytics: Detecting and Predicting Match-Fixing in Tennis. *Journal of Prediction Markets*, **8**, 77-95.

[18] Saunders, D.M. and D.E. Turner. (1987). Gambling and Leisure: The Case of Racing. *Leisure Studies* **3**, 281-299.

[19] Seyhun, H.N. (1992). Why Does Aggregate Insider Trading Predict Future Stock Returns? *Quarterly Journal of Economics* **107**, 1303-1331.

[20] Shin, H. (1991). Optimal Betting Odds Against Insider Traders. *Economic Journal* **101**, 1179-1185.

[21] Shin, H. (1992). Prices of State Contingent Claims with Insider Traders, and the Favourite-longshot Bias. *Economic Journal* **102**, 426-435.

[22] Stiglitz, J. and Weiss, A. (1981). Credit Rationing in Markets with Imperfect Information. *American Economic Review* **71**, 393-410.

[23] Strumpf, K.S. (2003). "Illegal Sports Bookmakers," http://www.unc.edu/~cigar/.

[24] Winter, S. and Kukuk, M. (2008). Do Horses like Vodka and Sponging? On Market Manipulation and the Favourite-longshot Bias. *Applied Economics* **40**, 75-87.

[25] Wolfers, J. (2006). Point Shaving: Corruption in NCAA Basketball. *American Economic Review* (*Papers and Proceedings*) **96**, 279-283.