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Semi-local quantum criticality in string/M-theory

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Abstract

Semi-local quantum critical behaviour in $D-1$ spacetime dimensions can be holographically described by metrics that are conformal to $AdS_2 \times \mathbb{R}^{D-2}$, with the conformal factor characterised by a parameter η. We analyse such "η-geometries" in a top-down setting by focussing on the $U(1)^4$ truncation of $D = 4$ $N = 8$ gauged supergravity. The model has extremal black hole solutions carrying three non-zero electric or magnetic charges which approach AdS_4 in the UV and an $\eta = 1$ geometry in the IR. Adding a fourth charge provides a mechanism to resolve the singularity of the η-geometry, replacing it with an $AdS_2 \times \mathbb{R}^2$ factor in the IR, while maintaining a large region where the η-geometry scaling is approximately valid. Some of the magnetically charged black hole solutions preserve supersymmetry while others just preserve it in the IR. Finally, we show that η-geometries, with various values of η, can be obtained from the dimensional reduction of geometries consisting of AdS or Lifshitz geometries with flat directions.

1 Introduction

A locally quantum critical fixed point exhibits a scaling of space and time of the form $t \to \lambda^z t$, $\mathbf{x} \to \lambda \mathbf{x}$ in the limit $z \to \infty$. Such scaling arises very naturally within the context of holography. The simplest example is the standard electrically charge AdS-RN black brane solution of Einstein-Maxwell theory in D spacetime dimensions. At zero temperature, $T = 0$, this solution interpolates between AdS_D in the UV and an $AdS_2 \times \mathbb{R}^{D-2}$ fixed point solution in the IR. From the dual perspective, this AdS-RN black hole solution describes a CFT in $D-1$ dimensions at finite charge density and the $T = 0$ solution, providing it is stable, describes an emergent locally quantum critical fixed point in the far IR dual to the $AdS_2 \times \mathbb{R}^{D-2}$ solution. The correlators obtained from the $AdS_2 \times \mathbb{R}^{D-2}$ solution only scale with energy, but since they still depend on the momentum we will follow [1] and call this semi-local quantum criticality. In the special case of $D = 4$ the AdS-RN black hole geometry can also be supported by magnetic fields or, more generally, by both electric and magnetic fields, with similar behaviour in the IR.

One interesting feature of the AdS-RN black holes is that they can have fermionic spectral functions with novel behaviour [2–9]. Another striking, and related, feature is that they have finite entropy density at $T = 0$. A natural interpretation is that this is indicating that the system is becoming unstable at low temperatures and indeed, depending on the details of the gravitational system, there are a variety of possible superfluid and spatially modulated instabilities that can arise, both in bottom-up and top-down settings including [10–22]. It is worth noting, however, that it has recently been shown that there is at least one top-down setting where the semi-local quantum critical ground state is known to be stable[1] via the preservation of supersymmetry [23].

More recently it has been emphasised in [24] that there is a broader framework to realise holographically the notion of semi-local quantum criticality. The idea is to consider geometries that are conformally related to $AdS_2 \times \mathbb{R}^{D-2}$. Specifically, we shall define the η-geometry by the line element

$$ds^2 = \frac{1}{\rho^{2\eta/(D-2)}} \left(-\frac{dt^2}{\rho^2} + \ell^2 \frac{d\rho^2}{\rho^2} + dx_i dx_i \right) , \tag{1.1}$$

where $i = 1, \ldots, D-2$, the number of spatial dimensions in the dual field theory, and η and ℓ are constants. Under the scaling $t \to \zeta t, x_i \to x_i, \rho \to \zeta \rho$ the line element scales as $ds \to \zeta^{-\eta/(D-2)} ds$. The special case when $\eta = 0$ is just $AdS_2 \times \mathbb{R}^{D-2}$ with

[1] At least in the strict $N \to \infty$ limit of the dual CFT.

the conformal UV boundary located at $\rho \to 0$. When $\eta > 0$ the geometry is singular when $\rho \to \infty$. Note that when $\eta = 0$, ℓ is the radius of the AdS_2 factor and that when $\eta \neq 0$, ℓ can be set to one by scaling the coordinates. One way to think about these geometries [24] is as a limit of the "hyperscaling violating" geometries considered in [25–27] labelled, in the notation of [27], by θ, z in the limit that $\theta \to -\infty$, $z \to \infty$ with $\eta \equiv -z/\theta$ held fixed. Geometries with $\eta = 1$ were also discussed earlier in [28] and one of our aims will be to generalise and extend the results of that pioneering paper (for related early work see [29, 30] and more recent work [31, 32]).

An interesting property of the η-geometries is that the finite temperature generalisations have an entropy density that depends on the temperature via $s \propto T^\eta$ and hence, when $\eta > 0$, they have $s \to 0$ as $T \to 0$ [24, 30]. The case of $\eta = 1$ is particularly interesting since the temperature dependence of s is linear, corresponding to linear specific heat [24, 28]. Another interesting feature when $\eta \geq 0$ is that there can be spectral weight that is not exponentially suppressed at low-energies and finite momentum, as one expects for physics associated with Fermi surfaces [24, 33].

In this paper we will explore several aspects of the η-geometries within top-down settings. The main focus will be on the $U(1)^4$ truncation of $D = 4$ $N = 8$ gauged supergravity [34], studied earlier in this context in [28]. Although this model is not quite a consistent truncation of $D = 11$ supergravity on S^7, all of the solutions we consider can be uplifted to obtain exact solutions in $D = 11$. We show that for the class of analytic black hole geometries with four electric charges found in [34], or the magnetic analogues which we write down here, we can obtain an $\eta = 1$ geometry in the far IR as $T \to 0$ after setting one of the charges to zero. In addition we show that introducing a small fourth charge resolves the singularity with an $AdS_2 \times \mathbb{R}^2$ geometry in the far IR, with an intermediate scaling region associated with an $\eta = 1$ geometry. This is reminiscent of the resolution of singularities in string theory by the addition of fluxes that have been considered in other contexts [35] and also analogous to the resolution of the singularities of the Lifshitz geometries discussed in [36] (related work appears in [37–39]).

We also consider the solutions after they are uplifted to $D = 11$ on an S^7. For the uplifted electrically charged solutions we find the $\eta = 1$ geometry region uplifts to a $D = 11$ solution with an AdS_3 factor, generalising what was seen in [28]. The presence of the AdS_3 factor provides an understanding of the linear specific heat [28]. However, this is not the full story since, by contrast, we show that there is no such AdS_3 factor in the uplifted magnetically charged solutions.

We will not carry out a detailed stability analysis of these analytic black hole

solutions here. However, based on [40] and on the analysis of the stability properties of the $AdS_2 \times \mathbb{R}^2$ geometries presented in [41], we expect that many of the analytic black hole solutions, carrying either electric or magnetic charges, are unstable. Such instabilities are certainly interesting since they are associated with new branches of black hole solutions appearing at finite temperature, corresponding to new phases. However, such instabilities also mean that many of the $\eta = 1$ geometries will probably not correspond to the true ground states at zero temperature.

On the other hand there is a particularly interesting subclass of the analytic black hole solutions where the instabilities are ameliorated by the presence of an emergent supersymmetry in the IR. This subclass has four non-vanishing magnetic charges and while in the extremal $T = 0$ limit they are not supersymmetric solutions they nevertheless approach supersymmetric $AdS_2 \times \mathbb{R}^2$ geometries in the IR of the type constructed in [41, 42] (building on [43]). This emergent supersymmetry implies that the near horizon region is free from instabilities and suggests that the full solutions themselves may also be stable. If this is the case, these solutions would provide the first examples of stable, non-supersymmetric black brane solutions with finite entropy at zero-temperature. Moreover, these solutions can exist with an approximate intermediary $\eta = 1$ geometry scaling region which dominates the IR when one of the charges is set to zero.

Another result of this paper is that the same $D = 4$ $U(1)^4$ theory also admits a new class of solutions carrying purely magnetic charges with analogous properties to those described in the previous paragraph, but preserving supersymmetry everywhere. In particular, we numerically construct supersymmetric solutions interpolating between AdS_4 in the UV and η-geometries with $\eta = 1$ in the IR. Adding small amounts of a fourth charge again provides a natural singularity resolution mechanism with an intermediate $\eta = 1$ geometry scaling region and a supersymmetric $AdS_2 \times \mathbb{R}^2$ solution in the far IR. A duality transformation maps these supersymmetric magnetic solutions to a new class of non-supersymmetric electric solutions.

Finally, in a quite different direction, we conclude the paper by briefly showing that a simple way to construct η-geometries is from the dimensional reduction of the product of AdS or Lifshitz geometries with some flat directions. Similar observations were made earlier (independently) in the context of specific classes of models in [30].

The plan of the rest of the paper is as follows. In section 2 we introduce the $D = 4$ $U(1)^4$ model that we mostly consider, and also recall the magnetic and electrically charged $AdS_2 \times \mathbb{R}^2$ solutions of [41]. In section 3 we analyse the analytic class of asymptotically AdS_4 black brane solutions carrying electric charges found in [34]. We

discuss the analytic magnetically charged black holes in section 4 and the numerically constructed supersymmetric magnetic solutions in section 5. We conclude in section 6 by obtaining the η-geometries via dimensional reduction.

2 The $D = 4$ gauged supergravity theory

We consider the $U(1)^4$ truncation of $N = 8$ $D = 4$ gauged supergravity given in [34] that keeps three neutral scalar fields ϕ_a and four gauge fields A^i. Solutions of this theory will be the major focus of the paper. The Lagrangian is given by

$$\mathcal{L} = \frac{1}{2}R - \frac{1}{4}\sum_{a=1}^{3}(\partial\phi_a)^2 - \sum_{i=1}^{4} X_i^{-2}\left(F^i\right)_{\mu\nu}\left(F^i\right)^{\mu\nu} - V\left(X_i\right), \tag{2.1}$$

where

$$X_1 = e^{\frac{1}{2}(-\phi_1-\phi_2-\phi_3)}, \quad X_2 = e^{\frac{1}{2}(-\phi_1+\phi_2+\phi_3)}, \quad X_3 = e^{\frac{1}{2}(\phi_1-\phi_2+\phi_3)}, \quad X_4 = e^{\frac{1}{2}(\phi_1+\phi_2-\phi_3)} \tag{2.2}$$

and the potential is given by

$$V\left(X_i\right) = -\frac{1}{2}\sum_{i\neq j} X_i X_j = -2\left(\cosh\phi_1 + \cosh\phi_2 + \cosh\phi_3\right). \tag{2.3}$$

Any solution of this theory that satisfies $F^i \wedge F^j = 0$ can be uplifted[2] to $D = 11$ on an S^7 using the formulae in [34]; all of the solutions that we consider satisfy this condition.

Note that the equations of motion for this model exhibit the electric-magnetic duality transformation given by

$$F^i \to X_i^{-2} * F^i, \qquad \phi_a \to -\phi_a\,, \tag{2.4}$$

with the metric unchanged.

In the following we will sometimes utilise the fact that the equations of motion for (2.1) can be consistently truncated to theories involving a smaller numbers of fields. For example it is consistent to further truncate by setting

$$\begin{aligned} &\phi_2 = -\phi_3, \qquad i.e. \qquad X_1 = X_2\,, \\ &F^1 = F^2\,, \end{aligned} \tag{2.5}$$

[2]To do this we should set $g^2 = 1/2$ in eq. (3.8) of [34] and identify $(F^i)^{there} = 2\sqrt{2}(F^i)^{here}$. It is also worth noting that we are using the same conventions as in [44] setting $g = 1$ there.

to obtain a theory with two scalar fields and three vector fields. This should be a sector of an $SU(2) \times U(1) \times U(1)$ invariant subsector of $SO(8)$ gauged supergravity. On the other hand we can set

$$\begin{aligned} &\phi_1 = \phi_2 = -\phi_3, \qquad i.e. \qquad X_1 = X_2 = X_3\,, \\ &F^1 = F^2 = F^3\,, \end{aligned} \tag{2.6}$$

to obtain a theory involving one scalar field and two gauge-fields. In fact this is a sector of the $SU(3)$ invariant subsector of $SO(8)$ gauged supergravity [45] [46] and the corresponding uplifted solutions will have $SU(3) \times U(1)^2$ symmetry.

This theory has an AdS_4 vacuum, with with $\phi_a = 0$ radius squared $1/2$, which can be uplifted to $D = 11$ to give the $AdS_4 \times S^7$ solution. In this AdS_4 vacuum the three neutral scalars have $m^2 = -4$ and hence can be quantised as $\Delta = 1$ or $\Delta = 2$ operators. For the $AdS_4 \times S^7$ solution to be consistent with supersymmetry, they should be quantised so that $\Delta = 1$ (for more discussion see e.g. [47]). There may be sub-truncations and/or other uplifts where it is appropriate to quantise as a $\Delta = 2$ operator, but we will continue assuming $\Delta = 1$.

2.1 Analytic $AdS_2 \times \mathbb{R}^2$ solutions

We briefly review the $AdS_2 \times \mathbb{R}^2$ solutions supported by magnetic or electric charges that were studied in [41, 42] as they will appear in the subsequent analysis.

2.1.1 Magnetic $AdS_2 \times \mathbb{R}^2$ solutions

The solutions supported by magnetic flux are given by

$$\begin{aligned} ds^2 &= L^2\, ds^2\,(AdS_2) + dx_1^2 + dx_2^2\,, \\ F^i &= \tfrac{1}{2} q_i\, dx_1 \wedge dx_2\,, \\ \phi_1 &= f_1, \quad \phi_2 = f_2, \quad \phi_3 = f_3\,, \end{aligned} \tag{2.7}$$

where q_i, f_a are constants and L is the AdS_2 radius. Defining the on-shell quantities

$$\bar{X}_1 = e^{\frac{1}{2}(-f_1-f_2-f_3)}, \quad \bar{X}_2 = e^{\frac{1}{2}(-f_1+f_2+f_3)}, \quad \bar{X}_3 = e^{\frac{1}{2}(f_1-f_2+f_3)}, \quad \bar{X}_4 = e^{\frac{1}{2}(f_1+f_2-f_3)}\,, \tag{2.8}$$

there is a three parameter family of solutions specified by arbitrary values of (f_1, f_2, f_3) with

$$q_i^2 = \frac{\bar{X}_i^2}{2} \sum_{j \neq k \neq i} \bar{X}_j \bar{X}_k, \qquad L^{-2} = -2\, V\left(\bar{X}_i\right)\,. \tag{2.9}$$

Note that the q_i can be chosen to have either sign. In order for these solutions to preserve supersymmetry it is necessary that one of the following conditions is satisfied:

$$\begin{aligned} &q_1+q_2+q_3+q_4=0, \qquad q_1+q_2-q_3-q_4=0,\\ &q_1-q_2+q_3-q_4=0, \qquad q_1-q_2-q_3+q_4=0. \end{aligned} \tag{2.10}$$

Furthermore, it was shown in [41] that this implies that the supersymmetry locus is given by the conditions

$$2\sum \bar{X}_i^2 = \left(\sum_i \bar{X}_i\right)^2 . \tag{2.11}$$

In fact, conversely, this condition, along with (2.9) implies the preservation of supersymmetry. Indeed, (2.11) and (2.9) imply that $2q_i = \pm\bar{X}_i(-2\bar{X}_i + \sum_j \bar{X}_j)$ which, along with demanding one of the conditions in (2.10), is sufficient for preservation of supersymmetry.

2.1.2 Electric $AdS_2 \times \mathbb{R}^2$ solutions

The solutions supported by electric flux can be obtained from the duality transformation (2.4). Explicitly they are given by

$$\begin{aligned} ds^2 &= L^2\, ds^2\,(AdS_2) + dx_1^2 + dx_2^2\,,\\ F^i &= \tfrac{1}{2} q_i L^2 Vol(AdS_2)\,,\\ \phi_1 &= f_1, \quad \phi_2 = f_2, \quad \phi_3 = f_3\,, \end{aligned} \tag{2.12}$$

where

$$q_i = \bar{X}_i^3 \sum_{j\neq i} \bar{X}_j, \qquad L^{-2} = -2\,V\left(\bar{X}_i\right)\,, \tag{2.13}$$

and the $\bar{X}_i$ are the on-shell quantities defined in (2.8). These solutions do not preserve supersymmetry.

3 Electric black hole solutions and η-geometries

We begin with the analytic class of asymptotically AdS_4 black brane solutions carrying four electric charges [34]

$$\begin{aligned} ds^2 &= -f\,\Pi^{-1}\,dt^2 + f^{-1}\,\Pi\,dr^2 + r^2\,\Pi\,\left(dx_1^2+dx_2^2\right)\\ A^i &= \frac{\varepsilon_i}{2}\left(\mu_i + \frac{1}{\sqrt{2\,Q_i}}\left(1-H_i^{-1}\right)\right)dt, \qquad X_i = H_i^{-1}\,\Pi^{1/2}, \end{aligned} \tag{3.1}$$

where

$$f = -\frac{b}{r} + 2r^2\Pi^2, \qquad H_i = 1 + \frac{bQ_i}{r}, \qquad \Pi = (H_1H_2H_3H_4)^{1/2}\ . \tag{3.2}$$

We have $Q_i \geq 0$, $b \geq 0$ and $\varepsilon_i = \pm 1$. As $r \to \infty$ the solutions approach AdS_4. The black hole event horizon is located at $r = r_h \geq 0$ where r_h is the largest root of the equation

$$(\Pi^{-1} f)(r_h) = 0\,. \tag{3.3}$$

The chemical potentials, μ_i, for the four $U(1)$'s are given by

$$\mu_i = \frac{1}{\sqrt{2\,Q_i}}\,\left(H_i^{-1}\,(r_h) - 1\right)\,, \tag{3.4}$$

ensuring regularity of the gauge-potentials at the horizon. Note that b is the "emblackening" parameter (we will not denote it as μ, as is often done, to avoid confusion with the chemical potentials in the dual CFT).

In order to analyse the asymptotic UV behaviour of the scalar fields, it is useful to introduce a new radial coordinate $\rho^2 = r^2\Pi$. We then find as $\rho \to \infty$

$$\begin{aligned}
\phi_1 &= \frac{b(Q_1+Q_2-Q_3-Q_4)}{2\rho} - \frac{b^2(Q_1-Q_2+Q_3-Q_4)(Q_1-Q_2-Q_3+Q_4)}{8\rho^2} + \dots \\
\phi_2 &= \frac{b(Q_1-Q_2+Q_3-Q_4)}{2\rho} - \frac{b^2(Q_1+Q_2-Q_3-Q_4)(Q_1-Q_2-Q_3+Q_4)}{8\rho^2} + \dots \\
\phi_3 &= \frac{b(Q_1-Q_2-Q_3+Q_4)}{2\rho} - \frac{b^2(Q_1+Q_2-Q_3-Q_4)(Q_1-Q_2+Q_3-Q_4)}{8\rho^2} + \dots
\end{aligned} \tag{3.5}$$

Thus for the $\Delta = 1$ quantisation relevant for maximal supersymmetry, we see that, generically, there are non-zero deformations, corresponding to the $1/\rho^2$ pieces and non-zero expectation values, corresponding to the $1/\rho$ pieces (assuming that there is no mixing). It is also worth noting that if the Q_i are chosen so that one of the deformation parameters vanishes, then both of the other two expectation values do as well.

Notice that these analytic black hole solutions depend on 5 independent parameters: four μ_i and b (the Q_i are fixed by regularity at the black hole event horizon). If we stay within a static, spatially homogeneous and isotropic context, and with electric charges only, the most general solutions should depend on 8 parameters (and there could be discrete families of solutions). These can be viewed as the temperature, four chemical potentials μ_i and three deformations for the $\Delta = 1$ operator (or the $\Delta = 2$ operator in the other quantisation). In section 5 we will numerically construct some new solutions outside of the analytic family.

3.1 Four $Q_i \neq 0$: $AdS_2 \times \mathbb{R}^2$ in the IR at $T = 0$

Let us first consider the generic case when all four of the Q_i are non-zero. We will show that in the extremal, $T = 0$, limit the black hole solutions all approach smooth domain wall solutions interpolating between AdS_4 in the UV and $AdS_2 \times \mathbb{R}^2$ in the IR. In particular, all of these black hole solutions have finite entropy at $T = 0$. Furthermore, we will see that the entire moduli space of electrically charged $AdS_2 \times \mathbb{R}^2$ solutions given in section 2.1.2 can be obtained.

To begin with we rescale the radial coordinate via $r \to b\,\rho$. For an extremal black hole event horizon, in addition to (3.3) we have $(\Pi^{-1} f)'(r_h) = 0$. These conditions imply the relations

$$\begin{aligned} b^2 &= \frac{\rho_h}{2\,(Q_1+\rho_h)\,(Q_2+\rho_h)\,(Q_3+\rho_h)\,(Q_4+\rho_h)}\,, \\ Q_4 &= \rho_h^2 \frac{Q_1Q_2 + Q_1Q_3 + Q_2Q_3 + 2\rho_h(Q_1+Q_2+Q_3) + 3\rho_h^2}{Q_1Q_2Q_3 - \rho_h^2\,(Q_1+Q_2+Q_3) - 2\rho_h^3}\,, \end{aligned} \tag{3.6}$$

The second equation fixes Q_4 in terms of Q_1, Q_2, Q_3 and also the location of the extremal horizon at $\rho = \rho_h$. It is convenient now to rescale the charges

$$Q_i = \rho_h \bar{q}_i^2\,, \tag{3.7}$$

and upon evaluating the scalars on the horizon we find

$$\begin{aligned} e^{-2\phi_1} &= \frac{(1+\bar{q}_3^2)^2}{\bar{q}_1^2\bar{q}_2^2\bar{q}_3^2 - 2 - \bar{q}_1^2 - \bar{q}_2^2 - \bar{q}_3^2}\,, \\ e^{-2\phi_2} &= \frac{(1+\bar{q}_2^2)^2}{\bar{q}_1^2\bar{q}_2^2\bar{q}_3^2 - 2 - \bar{q}_1^2 - \bar{q}_2^2 - \bar{q}_3^2}\,, \\ e^{2\phi_3} &= \frac{(1+\bar{q}_1^2)^2}{\bar{q}_1^2\bar{q}_2^2\bar{q}_3^2 - 2 - \bar{q}_1^2 - \bar{q}_2^2 - \bar{q}_3^2}\,. \end{aligned} \tag{3.8}$$

Notice that the condition for the positivity of Q_4 (i.e. the reality of $\bar{q}_4$) is the same as that for the reality of ϕ_a in (3.8). It is now easy to invert equation (3.8) and express the constants $\bar{q}_1$, $\bar{q}_2$, $\bar{q}_3$ in terms of the scalars ϕ_a and we find

$$\begin{aligned} \bar{q}_1^2 &= \frac{1}{X_1}\,(X_2 + X_3 + X_4)\,, \\ \bar{q}_2^2 &= \frac{1}{X_2}\,(X_1 + X_3 + X_4)\,, \\ \bar{q}_3^2 &= \frac{1}{X_3}\,(X_1 + X_2 + X_4)\,. \end{aligned} \tag{3.9}$$

Analysing the behaviour of the metric we obtain

$$ds^2 = -\frac{b^2(\rho-\rho_h)^2}{L^2}dt^2 + \frac{L^2}{(\rho-\rho_h)^2}d\rho^2 + \frac{b\rho_h^{1/2}}{\sqrt{2}}(dx_1^2+dx_2^2)\,, \tag{3.10}$$

with L^{-2} as in (2.13). Analysing the flux in the near horizon limit we also obtain the same expression in (2.13) after identifying

$$q_i = X_1^2\bar{q}_i\,. \tag{3.11}$$

Finally, we can check that the conditions (3.9) are now precisely as in (2.13). In other words we have shown that we can obtain the full moduli space of electric $AdS_2\times\mathbb{R}^2$ solutions of [41].

3.2 Three $Q_i\neq 0$: Geometries with $\eta=1$ in the IR at $T=0$

Next we consider the special case that one of the four charges is zero. For definiteness we choose $Q_4=0$. As we can see from (3.6), the extremal $T=0$ limit is achieved when $b=\frac{1}{\sqrt{2Q_1Q_2Q_3}}$ with $r_h\to 0$. In the near horizon limit, as $r\to 0$, the geometry now approaches

$$\begin{aligned} ds^2 &\approx -U_0\, r^{3/2}\, dt^2 + U_0^{-1}\,\frac{dr^2}{r^{3/2}} + \frac{r^{1/2}}{(8Q_1Q_2Q_3)^{1/4}}\,\left(dx_1^2+dx_2^2\right)\,,\\ U_0 &= \frac{Q_1Q_2+Q_1Q_3+Q_2Q_3}{\left(\frac{1}{2}Q_1Q_2Q_3\right)^{3/4}}\,, \end{aligned} \tag{3.12}$$

while the scalars approach

$$\begin{aligned} \phi_1 &\approx \frac{1}{4}\,\ln\left(\frac{Q_1Q_2}{2Q_3^3}\right) - \frac{1}{2}\,\ln r\,,\\ \phi_2 &\approx \frac{1}{4}\,\ln\left(\frac{Q_1Q_3}{2Q_2^3}\right) - \frac{1}{2}\,\ln r\,,\\ \phi_3 &\approx -\frac{1}{4}\,\ln\left(\frac{Q_2Q_3}{2Q_1^3}\right) + \frac{1}{2}\,\ln r\,. \end{aligned} \tag{3.13}$$

We also find that the three non-trivial gauge-fields can be written

$$A_i \approx -\frac{\varepsilon_i}{2}\,\frac{Q_1Q_2Q_3}{Q_i^{3/2}} r\, dt\,, \qquad i=1,2,3\,, \tag{3.14}$$

where we have used the fact that when $Q_4=0$ the chemical potentials defined in (3.4) are simply $\mu_i=-1/(2Q_i)^{1/2}$. After the coordinate change

$$t\to\frac{8}{U_0^2}\,t, \qquad r\to\frac{U_0^2}{16}\,\rho^{-2}, \qquad x_i\to\frac{2\,(8Q_1Q_2Q_3)^{1/8}}{U^{1/2}}\,x_i\,, \tag{3.15}$$

we see that we get a semi-local quantum critical metric with $\eta = 1$:

$$ds^2 \approx \frac{1}{\rho}\left(-\frac{dt^2}{\rho^2} + \frac{d\rho^2}{\rho^2} + dx_1^2 + dx_2^2\right). \tag{3.16}$$

It is worth emphasising that the $\eta = 1$ geometry is not an exact solution of the equations of motion.

We expect that the $Q_4 = 0$ solutions at finite temperature have an entropy that behaves as $s \to T$, for low temperatures [24], corresponding to linear specific heat. We can see this behaviour as follows. For small temperatures the horizon will be located at $r = \delta r_h$. Since we require that the chemical potentials (3.4) are fixed we deduce that

$$\delta Q_i = -2\sqrt{2Q_1Q_2Q_3}\,\delta r_h\,, \qquad i = 1, 2, 3\,. \tag{3.17}$$

On the other hand, using this and the definition of the location of the black hole event horizon (3.3), we conclude that we should vary b according to

$$\delta b = \frac{Q_1Q_2 + Q_1Q_3 + Q_2Q_3}{2Q_1Q_2Q_3}\,\delta r_h\,. \tag{3.18}$$

Recalling the definitions of the temperature and entropy density (with $16\pi G = 2$)

$$T = \left.\frac{(f\,\Pi^{-1})'}{4\pi}\right|_{r=r_h}, \qquad s = 2\pi\ r^2\Pi\big|_{r=r_h}\,, \tag{3.19}$$

we deduce, at leading order in the variations, that

$$\delta T = \frac{Q_1Q_2 + Q_1Q_3 + Q_2Q_3}{2^{5/4}\pi\,(Q_1Q_2Q_3)^{3/4}}\,\sqrt{\delta r_h}\,, \qquad \delta s = \frac{2^{1/4}\pi}{(Q_1Q_2Q_3)^{1/4}}\,\sqrt{\delta r_h}\,, \tag{3.20}$$

and hence

$$\delta s = \frac{2\sqrt{2}\pi^2\sqrt{Q_1Q_2Q_3}}{Q_1Q_2 + Q_1Q_3 + Q_2Q_3}\,\delta T\,, \tag{3.21}$$

as claimed.

3.3 AdS_3 in the uplift

If we uplift this entire class of geometries with $Q_4 = 0$, we find that the $\eta = 1$ geometry appearing in the IR at $T = 0$ uplifts to a locally AdS_3 region. This generalises the result of [28] which considered the special case $Q_1 = Q_2 = Q_3$. It was also pointed out in [28] that the AdS_3 factor provides a natural interpretation of the behaviour $s \propto T$ that we saw in the last subsection.

Specifically, if we uplift the $\eta = 1$ limiting IR geometry (3.12), (3.13), (3.14) to eleven dimensions using [34] (see footnote 1) we obtain, as $r \to 0$,

$$ds_{11}^2 \approx \frac{1}{(8Q_1Q_2Q_3)^{1/12}}\,\mu_4^{4/3}\,\left(-U_0 r\,dt^2 + U_0^{-1}\,\frac{dr^2}{r^2} + 2\,(8Q_1Q_2Q_3)^{1/4}\,r\,d\phi_4^2\right)$$
$$+\,\frac{\mu_4^{4/3}}{(8Q_1Q_2Q_3)^{1/3}}\,\left(dx_1^2+dx_2^2\right) + \frac{2}{(Q_1Q_2Q_3)^{1/3}}\,\mu_4^{-2/3}\,\sum_{i=1}^{3} Q_i\,\left(d\mu_i^2 + \mu_i^2\,(d\phi_i + 2A_i\,dt)^2\right), \tag{3.22}$$

where A_i are given in (3.14). It is interesting to point out that the chemical potentials for the gauge-fields that we have used, which arose from regularity at the event horizon at finite temperature, imply that the metrics are free of closed time-like curves in $D = 11$, in contrast to the gauge used in [28].

3.4 Charge as a resolution mechanism and intermediate scaling

We have shown that the class of solutions with $Q_4 = 0$ at $T = 0$ all approach an $\eta = 1$ geometry in the IR and hence are singular. On the other hand we showed in section 3.1 that when all four charges are non-zero the solutions approach $AdS_2 \times \mathbb{R}^2$ in the IR. It is thus clear that that adding a small fourth charge, $Q_4 \neq 0$, will resolve the η-geometry singularity. In addition, for small Q_4 we expect to obtain an intermediate scaling regime where the geometry is essentially the η-geometry and then very far in the IR, it approaches the $AdS_2 \times \mathbb{R}^2$ solution. This is analogous to the singularity resolution of Lifshitz geometries discussed in [36].

To illustrate this point in more detail, for simplicity we now focus on the sub-class of extremal solutions with $Q_1 = Q_2 = Q_3 \equiv Q/b$ and $Q_4 \equiv q/b$, with $b = 2^{-1/2}Q_1^{-3/2}$. The fourth charge, Q_4, will be much smaller than the other three if $q << Q$. It is convenient to parametrise the family of solutions in terms of the location of the extremal horizon, $r = r_h$. Doing so we obtain the relation

$$q = \frac{3\,r_h^2}{Q - 2\,r_h}\,, \tag{3.23}$$

while the metric reads

$$
\begin{aligned}
ds^2 &= -U\,dt^2 + U^{-1}\,dr^2 + W\,\left(dx_1^2 + dx_2^2\right)\,, \\
W &= (Q+r)^{3/2}\,\left(\frac{3\,r_h^2}{Q-2r_h} + r\right)^{1/2}\,, \\
U &= 2\,(r-r_h)^2\,\frac{3Q^3 + 3Q^2 r + Qr\,(r-4\,r_h) - r\,r_h\,(2r+r_h)}{(Q-2r_h)\,(Q+r)^{3/2}\left(\frac{3\,r_h^2}{Q-2r_h}+r\right)^{1/2}}\,.
\end{aligned}
\tag{3.24}
$$

We see that when $r_h > 0$ we have an $AdS_2 \times \mathbb{R}^2$ geometry in the IR. On the other hand when $r_h = 0$, we have $Q_4 = 0$ and we are back in the situation that we described in section 3 for the special case of three equal non-vanishing charges. In particular we obtain the $\eta = 1$ geometry (3.12) as $r \to 0$.

In order to illustrate the intermediate scaling region it is illuminating to define the functions

$$
p_1 = (r-r_h)\,\frac{U'}{U}\,, \qquad p_2 = (r-r_h)\,\frac{W'}{W}\,,
\tag{3.25}
$$

and explicitly we have

$$
\begin{aligned}
p_1 =& 2 + \frac{3(Q+r_h)}{2(Q+r_h+y)} + \frac{r_h(Q+r_h)}{2r_h(r_h-2y)+2Q(r_h+y)} \\
&- \frac{(Q+r_h)\,(6Q^2+3Qy-r_h(6r_h+5y))}{3(Q-r_h)(Q+r_h)^2+(3Q-5r_h)(Q+r_h)y+(Q-2r_h)y^2}\,, \\
p_2 =& \frac{y\,((Q+r_h)^2+4(Q-2\,r_h)y)}{2(Q+r_h+y)(r_h(r_h-2\,y)+Q(r_h+y))}\,,
\end{aligned}
\tag{3.26}
$$

with $y = r - r_h$. We now focus on three different scaling regions obtaining

$$
\begin{aligned}
p_1 &\approx \begin{cases} 2, & y \to 0 \\ 3/2, & r_h << y << Q \\ 2, & y >> Q >> r_h \end{cases} \\
p_2 &\approx \begin{cases} 0, & y \to 0 \\ 1/2, & r_h << y << Q \\ 2, & y >> Q >> r_h\,. \end{cases}
\end{aligned}
\tag{3.27}
$$

For $r_h \neq 0$, as $y \to 0$ we see the scaling behaviour of the $AdS_2 \times \mathbb{R}^2$ geometry. Similarly for very large y we see the scaling associated with the asymptotic AdS_4 geometry. Finally, when an intermediate region with $r_h << y << Q$ exists, the

metric has the scaling behaviour of an $\eta = 1$ geometry (see (3.12)). As expected such a region exists for $q << Q$.

Similar observations also hold for the three scalar fields. To see this we first recall from (2.6) that the sub-class of solutions with three equal charges that we are focussing on are actually solutions of a consistent truncation of the equations of motion of (2.1) to a theory with a single scalar field: $\phi = \phi_1 = \phi_2 = -\phi_3$ and two vector fields. We therefore examine the quantity

$$p_3 = (r - r_h)\ \phi' = -\frac{(Q - 3r_h)(Q + r_h)y}{2(Q + r_h + y)(r_h(r_h - 2y) + Q(r_h + y))}\,, \tag{3.28}$$

which has the behaviour

$$p_3 \approx \begin{cases} 0, & y \to 0 \\ -1/2, & r_h << y << Q \\ 0, & y >> Q >> r_h\,. \end{cases} \tag{3.29}$$

In the intermediate scaling region we again see the behaviour expected for an $\eta = 1$ geometry (see (3.12),(3.13)).

4 Analytic magnetically charged black holes

The magnetic version of the analytic black hole solutions is easily obtained from the analytic electric solutions (3.1) using the duality transformation (2.4). Explicitly we have

$$\begin{aligned} ds^2 &= -f\,\Pi^{-1}\,dt^2 + f^{-1}\,\Pi\,dr^2 + r^2\,\Pi\,\left(dx_1^2 + dx_2^2\right)\,, \\ F_i &= -\varepsilon_i\,\frac{\sqrt{Q_i}b}{2\sqrt{2}}\,dx_1 \wedge dx_2\,, \qquad X_i = H_i\,\Pi^{-1/2}, \end{aligned} \tag{4.1}$$

where f, H_i and Π are the same as (3.2). Since the metric is unchanged, many of the properties we saw in the previous section for the electric solutions follow straightforwardly.

In particular, when three magnetic charges are non-zero we obtain η-geometries with $\eta = 1$ at $T = 0$ in the far IR. Furthermore, when we switch on a small fourth magnetic charge we obtain solutions at $T = 0$ that have an intermediate scaling region associated with an $\eta = 1$ geometry and then in the far IR approach a magnetically charged $AdS_2 \times \mathbb{R}^2$ solution of section 2.1.1.

Using the supersymmetry transformations given in section 5, we can deduce that, as for the electric solutions discussed in the last section, these analytic magnetic solutions do not preserve any supersymmetry. However, they can exhibit an interesting emergent supersymmetry at $T = 0$ in the far IR as we now explain.

When all four magnetic charges are non-zero the analysis of the near horizon limit in the extremal $T = 0$ case is almost identical to the electric case that we considered in section 3.1. Rescaling $r \to b\,\rho$ the extremal $T = 0$ limit of the solutions (4.1) again lead to the conditions (3.6). We next scale the magnetic charges via

$$Q_i = \rho_h q_i^2 \,, \tag{4.2}$$

where now q_i are the magnetic charges appearing in the magnetic $AdS_2 \times \mathbb{R}^2$ solutions given in (2.7). Evaluating the scalars on the horizon we have

$$\begin{aligned} e^{2\phi_1} &= \frac{(1+q_3^2)^2}{q_1^2 q_2^2 q_3^2 - 2 - q_1^2 - q_2^2 - q_3^2} \,, \\ e^{2\phi_2} &= \frac{(1+q_2^2)^2}{q_1^2 q_2^2 q_3^2 - 2 - q_1^2 - q_2^2 - q_3^2} \,, \\ e^{-2\phi_3} &= \frac{(1+q_1^2)^2}{q_1^2 q_2^2 q_3^2 - 2 - q_1^2 - q_2^2 - q_3^2} \,, \end{aligned} \tag{4.3}$$

and hence

$$\begin{aligned} q_1^2 &= \frac{X_1}{X_4} + \frac{X_1}{X_2} + \frac{X_1}{X_3} \,, \\ q_2^2 &= \frac{X_2}{X_1} + \frac{X_2}{X_4} + \frac{X_2}{X_3} \,, \\ q_3^2 &= \frac{X_3}{X_2} + \frac{X_3}{X_1} + \frac{X_3}{X_4} \,. \end{aligned} \tag{4.4}$$

We observe that these are precisely the same conditions appearing in (2.9). Since the metric is as in (3.10), we conclude that we can obtain all of the magnetic $AdS_2 \times \mathbb{R}^2$ solutions of [41] in the IR.

In particular, the sub-locus of the magnetic $AdS_2 \times \mathbb{R}^2$ solutions of [41] that preserve supersymmetry, i.e. satisfying (2.9) and (2.11), can be obtained as near horizon limits of non-supersymmetric extremal black hole solutions. This emergent supersymmetry is interesting. One consequence is that the black hole solutions must be stable in the IR. While this leaves open the possibility that there are instabilities not localised in the IR (for example, instabilities of the type studied by Gubser-Mitra [40,48]), it is possible that for certain charges these are absent as well. These solutions would then provide the first top-down examples of stable non-supersymmetric

solutions with non-vanishing entropy in the IR. Note also that we can switch off one of the charges, leading to an η=1 geometry in the far IR in which there is also an emergent supersymmetry.

4.1 Uplifted magnetic $\eta = 1$ geometries

We can uplift to $D = 11$ the limiting $\eta = 1$ geometry that appears at $T = 0$ in the far IR. We again write $Q = Q_1Q_2Q_3$ and find that as $r \to 0$

$$ds_{11}^2 \approx \frac{\delta^{2/3}}{2^{1/12}Q^{1/4}} r^{1/3} \left[-U_0 r\, dt^2 + U_0^{-1} \frac{dr^2}{r^2} + \frac{1}{(8Q)^{1/4}} \left(dx_1^2 + dx_2^2 \right) \right.$$
$$\left. + \frac{2^{3/4}Q^{1/4}}{\delta}\, r^{-1} \left(d\mu_4^2 + \mu_4^2\, d\phi_4^2 \right) + \frac{2^{5/4}Q^{3/4}}{\delta} \sum_{i=1}^{3} Q_i^{-1} \left(d\mu_i^2 + \mu_i^2 \left(d\phi_i + 2A_i \right)^2 \right) \right] \quad (4.5)$$

where we have defined $\delta = \sum_{i=1}^{3} Q_i \mu_i^2$ and the three non-vanishing magnetic gauge fields are given by $A_i = -\varepsilon_i \frac{Q_i}{8\sqrt{Q}} \left(x_1\, dx_2 - x_2\, dx_1 \right)$ for $i = 1, 2, 3$. In contrast to the uplifted electric solutions given in (3.22) we no longer see an AdS_3 factor.

5 Supersymmetric magnetically charged black holes

In this section we will discuss supersymmetric solutions of the $U(1)^4$ theory (2.1) carrying three non-vanishing magnetic charges which approach η-geometries with $\eta = 1$ in the far IR. Furthermore, these geometries can be resolved, while preserving supersymmetry, by the addition of a fourth magnetic charge. If the fourth charge is small the solutions will have a large an intermediate $\eta = 1$ geometry scaling regime before approaching $AdS_2 \times \mathbb{R}^2$ in the far IR. Being supersymmetric these solutions are expected to be stable.

We consider magnetically charged solutions within the ansatz

$$\begin{aligned} ds^2 &= -e^{2W}\, dt^2 + dr^2 + e^{2U} \left(dx_1^2 + dx_2^2 \right) , \\ F^i &= \tfrac{1}{2} \lambda q_i\, dx_1 \wedge dx_2 \,, \\ \phi_a &= \phi_a(r) \,, \end{aligned} \quad (5.1)$$

where λ is a constant, given below, chosen to simplify some expressions. In order to obtain supersymmetric solutions, following [41, 44] we will restrict our attention to solutions with

$$\sum_i q_i = 0 \,. \quad (5.2)$$

It will be convenient to write

$$q_1 = Q + Z - \epsilon, \quad q_2 = Q - Z - \epsilon$$
$$q_3 = -2Q - \epsilon, \qquad q_4 = 3\,\epsilon\,. \tag{5.3}$$

As discussed in [41] the supersymmetry variations lead to the first order system of equations given by

$$\begin{aligned}
-W' + \frac{1}{2\sqrt{2}}\sum_i X_i + \frac{\alpha\lambda\lambda}{2\sqrt{2}} e^{-2U} \sum_i X_i^{-1} q_i = 0\,,\\
-U' + \frac{1}{2\sqrt{2}}\sum_i X_i - \frac{\alpha\lambda}{2\sqrt{2}} e^{-2U} \sum_i X_i^{-1} q_i = 0\,,\\
-\sqrt{2}\phi_a' - 2\sum_j \partial_{\phi_a} X_j + 2\alpha\lambda e^{-2U} \sum_j q_j \partial_{\phi_a} X_j^{-1} = 0\,,
\end{aligned} \tag{5.4}$$

where $\alpha = \pm 1$.

We next recall that when all four charges are non-zero, there is a locus of supersymmetric magnetic $AdS_2 \times \mathbb{R}^2$ solutions that we summarised in section (2.1.1).

5.1 Supersymmetric $\eta = 1$ geometries in the IR

Setting $\epsilon = 0$ in (5.3) we have three non-vanishing charges and we can construct a supersymmetric domain wall that approaches AdS_4 in the UV and an $\eta = 1$ geometry in the IR. To see this we can set up an approximate IR expansion to the equations (5.4) of the form

$$\begin{aligned}
U &= \ln r + \dots\,, \qquad\qquad W = 3\,\ln r + \dots\,,\\
\phi_1 &= \ln\left(\frac{\left(3Q^2+Z^2\right)^2}{8\left(Q^2-Z^2\right)^2}\right) + 2\,\ln r + \dots\,,\\
\phi_2 &= \ln\left(\frac{\left(3Q^2+Z^2\right)^2}{32Q^2\left(Q+Z\right)^2}\right) + 2\,\ln r + \dots\,,\\
\phi_3 &= -\ln\left(\frac{\left(3Q^2+Z^2\right)^2}{32Q^2\left(Q-Z\right)^2}\right) - 2\,\ln r + \dots\,,
\end{aligned} \tag{5.5}$$

where we have chosen the constant λ

$$\lambda = \frac{16\,Q\,\left(Q-Z\right)\left(Q+Z\right)}{\left(3Q^2+Z^2\right)^2\,\alpha}\,. \tag{5.6}$$

This expansion yields the approximate metric behaviour

$$ds_4^2 \approx -r^6\,dt^2 + dr^2 + r^2\,\left(dx_1^2 + dx_2^2\right)\,, \tag{5.7}$$

and after the coordinate transformation $r \to 2\rho^{-1/2}$ we obtain the metric (1.1) with $\eta = 1$.

Using this expansion it is possible to construct supersymmetric solutions that approach AdS_4 in the UV and this $\eta = 1$ geometry in the IR. While this can be done directly, such solutions can also be obtained as a limit of the solutions that we construct in the next subsection.

5.2 Intermediate scaling in supersymmetric magnetic solutions

We now construct supersymmetric magnetic solutions carrying four magnetic charges which approach AdS_4 in the UV and $AdS_2 \times \mathbb{R}^2$ in the IR with an intermediate $\eta = 1$ geometry scaling region. We consider flows with the magnetic fluxes constrained as in (5.3) and we take $Z = 0$ for simplicity. Recall from (2.5) that these solutions lie within a consistent truncation with two scalars ϕ_1 and $\phi_2 = -\phi_3$ and three gauge-fields. When $\epsilon \neq 0$ we have four non-vanishing magnetic charges and we expect supersymmetric domain walls approaching AdS_4 in the UV and $AdS_2 \times \mathbb{R}^2$ in the IR. When ϵ is small there should be a large intermediate scaling $\eta = 1$ geometry regime

The relevant supersymmetric $AdS_2 \times \mathbb{R}^2$ solutions are given by

$$\begin{aligned} W &= r/L, \qquad\qquad L^2 = \frac{2e^{f_2}(1-e^{2f_2})^2(1+e^{2f_2})}{(3+2e^{2f_2}+3e^{4f_2})^2} \\ e^{U_0} &= \sqrt{6\lambda Q\alpha}\,\frac{e^{f_2/2}\sqrt{1-e^{2f_2}}}{\sqrt{(3+6e^{2f_2}-e^{4f_2})\,g}}\,, \\ e^{f_1} &= 2\frac{\cosh(f_2)}{\sinh^2(f_2)}, \qquad \epsilon = \frac{\left(3+e^{2f_2}\right)Q}{3+\cosh(2f_2)-2\sinh(2f_2)}\,, \end{aligned} \tag{5.8}$$

where L is the radius of the AdS_2 and f_1, f_2 are the constant values of the scalars ϕ_1, ϕ_2, respectively. This is a one-parameter family of solutions which we can take to be specified by f_2 (or by ϵ). Note that we will focus on $f_2 < 0$. These solutions have a "universal" irrelevant operator with dimension $\Delta = 2$. There is also another irrelevant operator of dimension

$$\Delta_{IR} = \frac{6+2\,\cosh(2f_2)+\sqrt{-26+24\,\cosh(2f_2)+66\,\cosh(4f_2)}}{4+12\,\cosh(2f_2)}\,. \tag{5.9}$$

We want to construct domain wall solutions that interpolate between these $AdS_2 \times$

$\mathbb{R}^2$ solution in the IR and AdS_4 in the UV. In the UV we have the expansion

$$\begin{aligned} W &= \frac{r}{L_{UV}} - \frac{1}{16}\left(2\,c_2^2 + c_1^2\right)\, e^{-2r/L_{UV}} + \dots \\ U &= \frac{r}{L_{UV}} - \frac{1}{16}\left(2\,c_2^2 + c_1^2\right)\, e^{-2r/L_{UV}} + \dots \\ f_1 &= c_1\, e^{-r/L_{UV}} + \dots \\ f_2 &= c_2\, e^{-r/L_{UV}} + \dots \end{aligned} \tag{5.10}$$

where $L_{UV} = 1/\sqrt{2}$ and c_i are two constants of integration. For the IR expansion we have

$$\begin{aligned} W &= W_0 + r/L + c\, e^{r/L} + \dots \\ U &= U_0 - \frac{9 + 20\,\cosh(2f_2) + 3\,\cosh(4f_2)}{50 + 8\,\cosh(2f_2) + 6\,\cosh(4\,f_2)}\, c\, e^{r/L} + \dots \\ \phi_1 &= f_1 + \frac{7 + 28\,\cosh(2\,f_2) - 3\,\cosh(4f_2)}{25 + 4\,\cosh(2f_2) + 3\,\cosh(4f_2)}\, c\, e^{r/L} + \dots \\ \phi_2 &= f_2 + 8\sinh^3(f_2)\, \frac{1 + 3\cosh(2f_2)}{54\,\cosh(f_2) + 7\,\cosh(3f_2) + 3\,\cosh(5f_2)}\, c\, e^{r/L} + \dots \end{aligned} \tag{5.11}$$

where the constant W_0 corresponds to simple scalings of the time coordinate t and c is a deformation due to an irrelevant operator with $\Delta = 2$. In this expansion we have not included the possibility for a deformation of the operator with dimension given in (5.9). We could do this giving rise to additional domain wall solutions.

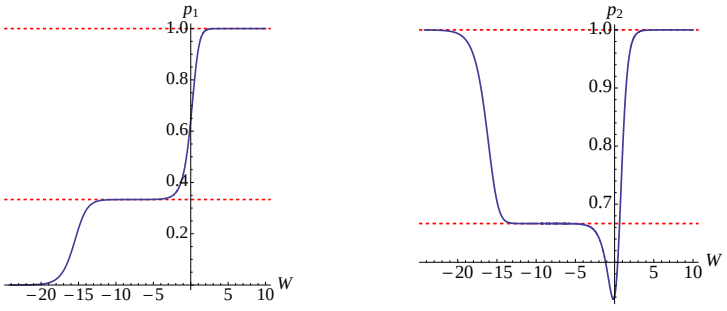
We choose $\epsilon/Q = 2 \times 10^{-10}$. We have two integration constants in the IR and two integration constants in the UV. Since we have set $\phi_2 = -\phi_3$ we have four first order BPS equations to solve, given in (5.4), and so we expect to find a unique solution. Indeed we constructed such a solution numerically. To discuss the scaling properties of the solution it is convenient to define

$$\begin{aligned} p_1 &= \frac{U'}{W'}, \qquad p_2 = 1 + \frac{W''}{W'^2}\,, \\ p_3 &= \frac{\phi_2'}{W'}, \qquad p_4 = \frac{\phi_1'}{W'}\,, \end{aligned} \tag{5.12}$$

and consider the p_i to be functions of W, which is natural if we decided to use W as a radial coordinate instead of ρ in our ansatz (5.1). Corresponding to the three different scaling regimes we expect to see

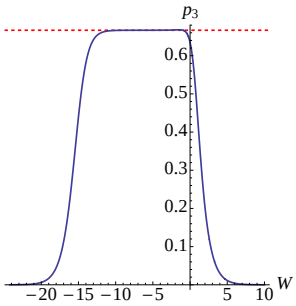
- $AdS_2 \times \mathbb{R}^2$

$$p_1 = 0, \quad p_2 = 1 \quad p_3 = p_4 = 0$$

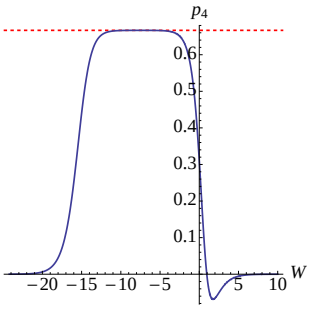


(a) Plot of p_1

(b) Plot of p_2



(c) Plot of p_3



(d) Plot of p_4

Figure 1: A plot of the four functions p_i, defined in (5.12), as a function of the radial coordinate W for supersymmetric magnetically charged solutions. The plots reveal three scaling regimes, corresponding to AdS_4 for large W, an $\eta = 1$ geometry for intermediate W and $AdS_2 \times \mathbb{R}^2$ for small W.

- $\eta = 1$ geometry

$$p_1 = 1/3, \quad p_2 = 2/3, \quad p_3 = p_4 = 2/3$$

- AdS_4

$$p_1 = 1, \quad p_2 = 1, \quad p_3 = p_4 = 0\,. \tag{5.13}$$

In figure 1 we have plotted the functions $p_i(W)$, which clearly reveals these three regimes.

We conclude this section by noting that all of the supersymmetric solutions with magnetic charges that we have constructed in this section have electrically charged analogues obtained from the duality transformation (2.4). However, these electrically charged solutions will not be supersymmetric.

6 η-geometries from dimensional reduction

We finish this paper by describing some simple ways in which the η-geometries (1.1) can be obtained via Klauza-Klein reduction. Similar observations were made earlier (independently) in the context of specific classes of models in [30]. For example, to obtain η geometries in $D = 4$ we start with $AdS_{2+k} \times \mathbb{R}^2$, given in Poincaré type coordinates by

$$ds^2 = L^2 \left[-\frac{dt^2}{r^2} + \frac{dr^2}{r^2} + \frac{dy_a dy_a}{r^2} \right] + dx_1^2 + dx_2^2 \,, \tag{6.1}$$

where L is the radius of the AdS_{2+k} and $a = 1, \ldots, k$. We now perform a dimensional reduction on the k spatial dimensions y_a. To do this we rewrite the metric in the form

$$ds^2 = r^k \left(\frac{1}{r^k} \left\{ L^2 \left[-\frac{dt^2}{r^2} + \frac{dr^2}{r^2} \right] + dx_1^2 + dx_2^2 \right\} \right) + \frac{L^2}{r^2} dy_a dy_a \,. \tag{6.2}$$

A straightforward calculation shows that the metric in the round braces is the $D = 4$ Einstein-frame metric, and we see an η-geometry with $\eta = k$ and $\ell = L$. A simple extension is to replace AdS_{2+k} with a Lifshitz geometry with dynamical exponent z. We then have

$$ds^2 = r^k \left(\frac{1}{r^k} \left\{ L^2 \left[-\frac{dt^2}{r^{2z}} + \frac{dr^2}{r^2} \right] + dx_1^2 + dx_2^2 \right\} \right) + \frac{L^2}{r^2} dy_a dy_a \,. \tag{6.3}$$

We again reduce on the k spatial dimensions y_a and perform a coordinate transformation to find an η geometry in $D = 4$ with $\eta = k/z$ and $\ell = L/z$.

These constructions immediately provide rich top-down constructions. For example, we can start with the $AdS_3 \times \mathbb{R}^2$ solutions of $D = 5$ maximal gauged supergravity that are supported by magnetic fields and, in general, scalar fields which were studied in [41, 43]. These can be uplifted on an S^5 to obtain exact solutions of type IIB supergravity. A subclass of solutions are also solutions of Romans $D = 5$ gauged supergravity and furthermore there is a unique solution which is a solution of $D = 5$ minimal gauged supergravity, and these can be uplifted to both type IIB and $D = 11$ in infinite numbers of ways [49–51] . After dimensional reduction on a spatial dimension contained within the AdS_3 factor we obtain infinite top-down examples of η geometries in $D = 4$ with $\eta = 1$ that are supported by magnetic fields as well as other scalar fields. Interestingly, for the solutions in maximal gauged supergravity and in Romans supergravity there is a supersymmetric locus of solutions and this provides supersymmetric examples of $\eta = 1$ geometries in $D = 4$.

These constructions might provide a helpful framework for obtaining useful insights into the holographic dictionary for η-geometries along the lines of [52].

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