Production of *hhjj* at the LHC

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Until now, a phenomenologically complete analysis of the $hh + 2j$ channel at the LHC has been missing. This is mostly due to the high complexity of the involved one-loop gluon fusion contribution and the fact that a reliable estimate thereof cannot be obtained through simplified calculations in the $m_t \to \infty$ limit. In this Letter, we report on the LHC's potential to access di-Higgs production in association with two jets in a fully showered hadron-level analysis. Our study includes the finite top and bottom mass dependencies for the gluon fusion contribution.

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After the recent discovery of the Higgs boson [1] at the LHC [2] and subsequent analyses of its interactions with known matter [3], a coarse-grained picture of consistency with the Standard Model (SM) expectation appears to be emerging. As yet, there is also no direct evidence of TeV-scale new physics. If this situation prevails, the Higgs sector is our *only* way to probe new physics effects that also directly link to naturalness and the mechanism of electroweak symmetry breaking.

Crucial to Higgs sector-induced electroweak symmetry breaking is the presence in the potential of higher order monomials (in particular, self-interactions and quartic gauge-Higgs interactions) of the Higgs field which misalign the Higgs field from the electroweak symmetry-preserving direction. However, the values of these terms are currently unknown, and it is experimentally unclear whether they exist at all. The discovery of the Higgs boson with SM-compatible W and Z couplings does not provide any additional information other than the mere existence of a symmetry-breaking vacuum and the size of the curvature of the potential around the local minimum at zero external momentum. Both are generic symmetry-breaking properties.

In the minimal approach of the SM, the potential reads

$$\mathcal{L} \supset (D_\mu H)^\dagger D^\mu H - [\mu^2 H^\dagger H + \eta (H^\dagger H)^2] \rightsquigarrow \frac{1}{2}(D_\mu h)^\dagger D^\mu h - \left(\frac{1}{2} m_h^2 h^2 + \sqrt{\frac{\eta}{2}} m_h h^3 + \frac{\eta}{4} h^4\right) \tag{1}$$

[after the Higgs doublet field is expanded around its vacuum expectation value in the unitary gauge]. Therefore, in the SM, the quartic and trilinear Higgs couplings are directly related to the Higgs pole mass m_h and the vacuum expectation value v (as set by the W mass and the electroweak coupling, for instance): $2\eta v^2 = m_h^2$.

These facts are the main motivation to study the LHC's potential to reconstruct the Higgs trilinear coupling $\lambda_{\rm SM} = \sqrt{\eta/2} m_h$ through measuring di-Higgs production cross sections [4,5]. (A measurement of the quartic Higgs interactions from triple Higgs final states appears impossible due to the tiny signal cross section [6].) Given the small production cross section of inclusive di-Higgs production at the LHC, it is imperative to apply state-of-the-art reconstruction and background rejection techniques for di-Higgs final states. For instance, the use of boosted $h \to b\bar{b}$ reconstruction techniques as discussed in Ref. [7] has revealed a potentially large sensitivity to the trilinear coupling in the $b\bar{b}\tau^+\tau^-$ final states [8,9]. In addition to these new analysis strategies focusing on diverse phase space regions, it is also mandatory to extend the list of available hadron collider processes which can be included into a combined limit across various Higgs decay channels [10].

Nonetheless, analyses of $pp \to hh + X$ do not fully constrain the Higgs sector when thought of as a general effective theory: Even when the trilinear Higgs interactions are known, we do not have information about the $WWhh$ and $ZZhh$ couplings in Eq. (1), which are as important for naturalness considerations as the Higgs boson itself.

Therefore, Higgs pair production in association with two jets via weak boson fusion (WBF) $\mathcal{O}(\alpha^4)$ is of outstanding theoretical relevance. This contribution to $pp \to hhjj + X$ production at the LHC is particularly interesting, because the WBF component involves the quartic $VV^\dagger hh$ ($V = W^\pm, Z$) vertices. (At a hypothetical linear collider, measurements are rather straightforward [11], but it is entirely unclear if there is potential at the LHC.) This has motivated high precision QCD calculations [23] for the WBF component, although it remained entirely unclear whether these contributions are, in fact, relevant. Although it is theoretically imperative to discuss to what extent we can access the $VV^{\dagger}hh$ couplings via direct measurements, a comprehensive signal vs background investigation of the $hhjj$ final state and an analysis of the expected sensitivity to the $VV^{\dagger}hh$ and trilinear Higgs couplings have not been performed so far. The purpose of this Letter is in not only providing the first complete and concise analysis of the $hhjj$ final state at the LHC ever, but we also show that previous work is largely irrelevant *unless* a large contribution from new physics is present. In this sense this work is not just a continuation of the earlier $pp \to hh$ program, as we give a realistic sensitivity estimate to new contributions not reflected in $pp \to hh$ analyses, putting existing work in a proper context.

One reason for the lack of phenomenologically complete studies of this particular final state is the highly involved modeling and (until now) unknown size of the gluon fusion (GF) contribution to $pp \to hhjj + X$ at $\mathcal{O}(\alpha_s^4\alpha^2)$. While applying low-energy effective theorems to gluon-Higgs interactions [12] $\mathcal{L}_{\rm eff} = \alpha_s/(12\pi)G^a_{\mu\nu}G^{a\mu\nu}\log(1+h/v)$ is fairly simple, momentum transfers $p_{T,h} \sim m_t$ probe the kinematic region where interference with the Higgs trilinear diagrams becomes relevant for the integrated cross section [8,13], so that integrating out the top quark cannot be justified in phenomenological investigations. Realistic modeling of these interference effects is at the heart of any attempt to extract the relevant couplings. A reliable targeted phenomenological analysis of the di-Higgs final state must therefore not be based on effective theory methods.

Keeping the full quark mass dependencies in the gluon fusion component of $pp \to hhjj$ is a computationally intense task at the frontier of one-loop multileg calculations. Given the high complexity of this process, we obtain a calculation time of up to ~1 min per phase space point and per massive fermion in the loop for the pure gluon case $gg \to hhgg$, which exhibits the largest complexity with around 1000 diagrams (details below). Clearly, traditional Monte Carlo event generation approaches do not promise a successful outcome unless the calculation time is significantly improved. In the following, we perform a phase space point-dependent reweighting of the effective theory to overcome this predicament. This allows us to provide a first analysis of the $hhjj$ final state at the LHC. We also present results on modifications of the Higgs trilinear and $VV^{\dagger}hh$ couplings on the resulting $pp \to hhjj + X$ phenomenology.

Elements of the analysis.—An apparent difference compared to single Higgs production studies in the two-jet category is the small cross section that is expected for $pp \to hhjj + X$ of inclusive $\mathcal{O}(10\text{fb})$. Typical GF and QCD background suppression tools for a 125 GeV Higgs boson such as, e.g., a central jet veto are not applicable, because in order to observe a signal in the first place we have to rely on large Higgs branching ratios to bottom quarks, hadronically decaying W's, and tau leptons. All these decay modes give rise to hadronic activity in the central detector region. (Semi)leptonic Z boson decays are too limited by small branching ratios to be of any phenomenological relevance in this case.

Relaxing the central jet veto criterion in favor of a large invariant mass cut on the tagging jets [14] is insufficient to tame the background contributions and is troubled by large combinatorial uncertainties and small statistics (see below). The most promising avenue is therefore a generalization of the boosted final state analysis of Ref. [8] to a lower p_T two-jet category: On the one hand, the signal cross section remains large by focusing on the $hh \to b\bar{b}\tau^+\tau^-$ final state and combinatorial issues can be avoided (i.e., through boosted kinematics and substructure techniques).

We generate signal events with MadEvent v4 [15] and v5 [16] for the WBF and GF contributions, respectively. The former event generation includes a straightforward add-on that allows us to include the effect of modified Higgs trilinear coupling. The GF event generation employs the FeynRules/Ufo [17] tool chain to implement the higher dimensional operators relevant for GF-induced $hhjj$ production in the $m_t \to \infty$ limit. We pass the events to Herwig++ [18] for showering and hadronization. For background samples we use Sherpa [19] and MadEvent v5, considering tth, $t\bar{t}jj$, $ZWWjj$, $ZHjj$, and $ZZjj$. As in the hh and hhj cases, the dominant background is due to $t\bar{t}$. We normalize the background samples by using next-to-leading-order (NLO) K factors, namely, 0.611 pb for tth [20] and 300.5 pb for $t\bar{t}jj$ [21]. We adopt a flat K factor of 1.2 for $Zh + 2j$ motivated from Ref. [22]. We have checked that all other backgrounds are completely negligible. The QCD corrections for the signal are known to be small for the WBF contribution [23]. We remain conservative and do not include a NLO K factor guess for the GF contribution.

We correct the deficiencies of the GF event generation in the $m_t \to \infty$ limit via an in-house reweighting library which is called at run time of the analysis for the weighted events. Based on the unweighting efficiency of $pp \to W^+W^-jj$ as implemented in Ref. [24], we estimate a speed improvement of a factor of at least 10^3 of our approach. (Similar reweighting techniques are used by the LHC experiments for calibrating Monte Carlo data against subsidiary measurements in control regions, e.g., for W+jets in $t\bar{t}$ analyses [25].) We include the effects of finite top and bottom quark masses, which are treated as complex parameters. The value of the Higgs trilinear coupling can be steered externally. For the generation of the matrix elements we used GoSam [26], a publicly available package for the automated generation of one-loop amplitudes. It is based on a Feynman diagrammatic approach using QGRAF [27] and FORM [28] for the diagram generation and Spinney [29], Haggies [30], and FORM to write an optimized Fortran output. The reduction of the one-loop amplitudes was done by using Samurai [31], which uses a d-dimensional integrand level decomposition based on unitarity methods [32]. The remaining scalar integrals have been evaluated by using OneLoop [33]. Alternatively, GoSam offers a reduction based on tensorial decomposition as contained in the Golem95 library [34]. The GoSam framework has been used recently for the calculation of signal and background processes important for Higgs boson searches at the LHC [35].

The maximum transverse momentum of the Higgs bosons is a good variable to compare effective with full theory. For inclusive $hhjj$ production we show a reweighted distribution in Fig. 1. Qualitatively, the reweighting pattern follows the behavior anticipated from $pp \to hhj$ production [8] and $pp \to hjj$ [14]. As expected, the shortcomings of the effective calculation for double Higgs production are more pronounced than for single Higgs production: Already for low-momentum transfers the effective theory deviates from the full theory by factors of 2, making the correction relevant even for low momenta, where one might expect the effective theory to be in reasonably good shape. It is precisely the competing and m_t-dependent contributions alluded to earlier which are not reflected in the effective theory causing this deviation. When the effective operators are probed at larger momentum transfers (and the massive quark loops are resolved in the full theory calculation), the effective theory overestimates the gluon fusion contribution by an order of magnitude. (A dedicated comparison of the full matrix element with the effective theory is an interesting question in itself, which we save for a separate study [36].) Figure 1 also demonstrates that the phase space is well covered by the effective theory Monte Carlo implementation and phase space coverage does not result in an issue for our procedure (we find similar coverage for angular distributions).

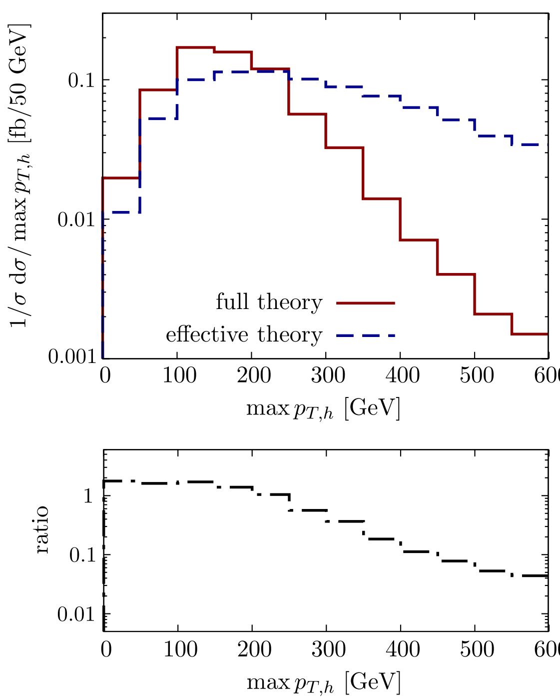

FIG. 1 (color online). max $p_{T,h}$ distribution and effective theory vs full theory comparison as a function of the maximum Higgs transverse momentum of the fully showered and hadronized gluon fusion sample (satisfying the parton-level generator cuts $p_{T,j} \geq 20$ GeV and $|\eta_j| < 4.5$).

Because of the particular shape of the reweighting in Fig. 1, we can always find a set of selection cuts for which effective theory and full calculation agree at the cross section level. Such an agreement, however, is purely accidental, as it trades off a suppression against an excess in two distinct phase space regions. An effective field theoretic treatment of $hhjj$ production without performing the described reweighting must never be trusted for either inclusive or more exclusive analyses.

In the hadron-level analysis we cluster jets from the final state by using FastJet [37], with $R = 0.4$, $p_T \geq 25$ GeV, and $|\eta_j| \leq 4.5$, and require at least two jets. We double b tag the event (70% acceptance, 1% fake) and require the invariant mass of the b jets to lie within 15 GeV of the Higgs boson mass of 125 GeV.

To keep matters transparent in the context of the highly involved $h \to \tau^+\tau^-$ reconstruction, we assume a perfect efficiency of 1 for demonstration purposes throughout. (We find the tau leptons to be rather hard, which can be used to trigger the event via the two-tau trigger with little signal loss.) We ask for two tau leptons that reproduce the Higgs boson mass of 125 GeV within ± 25 GeV. The precise efficiencies for leptons in the busy hadronic environment of the considered process at a 14 TeV high luminosity are currently unknown, but we expect the signal and background to be affected in a similar fashion. We remind the reader that no additional requirements on missing energy or m_{T2} are imposed, which are known to reconcile a smaller τ efficiency in the overall S/B [9].

The b jets are removed from the event, and jets that overlap with the above taus are not considered either. We require at least two additional jets which are termed "tagging jets" of the $hhjj$ event.

Results.—The cut flow of the outlined analysis can be found in Table I. There we also include analyses of signal samples with changed trilinear and $VV^\dagger hh$ couplings.

As can be seen from Table I, the $hhjj$ analysis in the $b\bar{b}\tau^+\tau^- jj$ channel will be challenging. However, we remind the reader that no additional selection criteria have been employed that are known to improve S/B in "ordinary" $hh \to b\bar{b}\tau^+\tau^-$ analysis [8,9]. The arguably straightforward strategy documented in Table I should rather be considered as establishing a baseline for a more exhaustive investigation [36] than the final verdict on $pp \to hhjj + X$ production.

The gluon fusion contribution dominates the signal component in the signal region, rendering the WBF contribution almost completely negligible for analysis with

TABLE I. Cross sections in femtobarns of the hadron-level analysis described in the text, including results with modified Higgs trilinear and $VV^{\dagger}hh$ couplings. Signal cross sections already include the branching ratios to the $h \to b\bar{b}, \tau^+\tau^-$ final states. The top four rows refer to the WBF sample, and the last line includes the reweighted GF contribution. For details, see the text.

	Signal with $\xi \times \lambda$			Background		S/B
	$\xi = 0$	$\xi = 1$	$\xi = 2$	$t\bar{t}jj$	Other	ratio to $\xi = 1$
Tau selection cuts	0.212	0.091	0.100	3101.0	57.06	0.026×10^{-3}
Higgs rec. from taus	0.212	0.091	0.100	683.5	31.92	0.115×10^{-3}
Higgs rec. from b jets	0.041	0.016	0.017	7.444	0.303	1.82×10^{-3}
2 tag jets	0.024	0.010	0.012	5.284	0.236	1.65×10^{-3}
Incl. GF after cuts/reweighting	0.181	0.099	0.067	5.284	0.236	$1/61.76$

	Signal with $\zeta \times \{g_{WWhh}, g_{ZZhh}\}$			Background	
	$\zeta = 0$	$\zeta = 1$	$\zeta = 2$	$t\bar{t}jj$	Other
Tau selection cuts	1.353	0.091	0.841	3101.0	57.06
Higgs rec. from taus	1.352	0.091	0.840	683.5	31.92
Higgs rec. from b jets	0.321	0.016	0.207	7.444	0.303
2 tag jets/reweighting	0.184	0.010	0.126	5.284	0.236
Incl. GF after cuts/reweighting	0.273	0.099	0.214	5.284	0.236

standard $VV^{\dagger}hh$ coupling choices. The behavior of the cross section as a function of the Higgs trilinear interaction results from destructive interference as is anticipated for studies in $pp \to hh + X$ [8,23].

With only about 30 expected WBF events in 3/ab, there is little leverage in the invariant dijet mass distribution to purify the selection towards WBF without jeopardizing statistical power. On the other hand, depending on the mechanism of electroweak symmetry breaking, a large enhancement of the WBF contribution can outrun the dominant GF events. On a more positive note, if a trilinear Higgs coupling measurement is obtained from other channels such as $pp \to hh + X$, this information can in principle be used in the above analysis to obtain a confidence level interval for the quartic Higgs-gauge couplings in a simple hypothesis test.

A dedicated analysis which employs techniques motivated recently by di-Higgs final states [9], as well as methods to separate WBF from GF based on energy momentum flow observables and kinematic information, jet substructure [8], and/or matrix elements, is likely to significantly enhance S/B. This is true even when limiting factors such b- and tau-tagging, smearing, and trigger issues are treated more realistically. We are therefore optimistic that such techniques will eventually allow us to not only add $pp \to hhjj + X$ to the list of measurable di-Higgs final states but also provide an additional handle to measure the Higgs trilinear and quartic Higgs-gauge couplings at a high luminosity LHC.

Summary, Conclusions and Outlook.—A crucial part of the electroweak physics agenda after the Higgs boson discovery is to reconstruct the symmetry-breaking potential, as well as to precisely unravel the new particle's role in TeV-scale physics. Measurements of the Higgs trilinear and the quartic Higgs-gauge couplings are highly sensitive parameters in this context, as they provide a clear picture of the Higgs sector dynamics and an independent cross-check of the mechanism that enforces unitarity.

This Letter summarizes the beginning of the $pp \to hhjj + X$ program. We have presented the first complete and coherent phenomenological analysis of di-Higgs production in association with two jets. Employing the full theory and not relying on the effective theory is not only extremely challenging from a calculational point of view, it is also phenomenologically absolutely crucial for a realistic description of this process. Only with a precise understanding of this process might we be able to get a handle on the $VV^{\dagger}hh$ coupling, which is important for a measurement of the quartic Higgs couplings. Exploiting the full bandwidth of state-of-the-art Monte Carlo tools, we have focused on what is probably the phenomenologically most attractive final state in terms of reconstruction potential, combinatorial limitations, relatively high signal yield, and comparably large background rejection as a first step towards a more dedicated analysis. Indeed, we find that WBF plays a completely subdominant role compared to GF, with little statistical handle to change this by using traditional techniques even at high luminosity.

Also, we have showed that, independent of the particular phase space region that a dedicated analysis targets, a reliable modeling of the signal crucially depends on the realistic generation of the gluon fusion signal contribution. Gluon fusion must not be based on effective field theory methods without applying a proper fully differential correction procedure.

Our results indicate that such an analysis at the LHC will be challenging but not hopeless. In particular, recent developments in the context of multi-Higgs production have not been exploited in the present Letter. We leave this to future work [36].

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