

Characteristics of group velocities of backward waves in a hollow cylinder

Hanyin Cui, Weijun Lin, Hailan Zhang, Xiuming Wang

State Key Laboratory of Acoustics, Institute of Acoustics, Chinese Academy of Sciences, Beijing,

People's Republic of China, 100190

Jon Trevelyan

School of Engineering and Computing Sciences, Durham University, Durham, United Kingdom,

DH1 3LE

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Abstract

It is known that modes in axially uniform waveguides exhibit backward-propagation characteristics that group and phase velocities have opposite signs. For elastic plates, group velocities of backward Lamb waves depend only on Poisson's ratio. This paper investigates the way to achieve a large group velocity of a backward mode in hollow cylinders by changing the outer to inner radius ratio, in order that such a mode with strong backward-propagation characteristics may be used in acoustic logging tools. Dispersion spectra of guided waves in hollow cylinders with various radius ratios are numerically simulated to explore the existence of backward modes and to choose the clearly visible backward modes with high group velocities. Analyses of group velocity characteristics show that only a small number of low order backward modes are suitable for practical exploitation, and the radius ratio to reach the highest group velocity corresponds to the accidental degeneracy of neighboring pure transverse and compressional modes at the wavenumber $k = 0$. It is also shown that large group velocities of backward waves are achievable in hollow cylinders made of commonly encountered materials, which may bring cost benefits in the case of fabricating acoustic devices utilizing backward-propagation effects.

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I. INTRODUCTION

A backward wave refers to a wave for which phase and group velocities are oppositely directed.¹⁻² It is well established that backward waves, resulting from strong mode repulsions, exist in axially uniform (constant cross section) optical waveguides.³ In the frequency-wavenumber ($\omega-k$) spectra, backward wave branches often occur in the vicinity of the wavenumber $k = 0$, and they are noticeable by anomalous dispersion characteristics in that the frequency decreases with the wavenumber (i.e., the group velocity $V_g = \partial\omega/\partial k$ and the phase velocity $V = \omega/k$ have opposite signs) and the zero-group-velocity (ZGV) at nonzero wavenumber.¹⁻²

Applications of backward wave motions of Lamb waves in plate structures have been explored. For example, acoustic superlens investigations⁴⁻⁵ applying the negative refraction effect or the backward-propagation effect that the energy flux direction is opposite to the phase velocity direction; and the non-contact laser-based nondestructive evaluation and material characterization techniques using ZGV resonances of backward Lamb modes⁶⁻⁹. Generally, a large group velocity and a wide range of frequencies over which the backward mode exists (i.e., the backward-propagation effect is strong) is often required for the purpose of designing new acoustic devices utilizing backward waves.¹⁰⁻¹¹ For Lamb waves in elastic homogeneous plates, group velocities of backward wave branches are determined by only one factor - Poisson's ratio or the bulk velocity ratio¹², the bulk velocity ratio $\kappa = V_p/V_s$ is related to the Poisson's ratio ν by $\nu = (\kappa^2 - 2)/2(\kappa^2 - 1)$; and high group velocities appear in plates with bulk velocity ratios

being rational numbers, i.e., ratios between even integers and odd integers $\kappa = 2n/(2m+1)$ or $\kappa = (2m+1)/2n$, $n = 1, 2, 3, \dots$, and $m = 0, 1, 2, \dots$ ¹².

One application of interest is the design of an acoustic isolator of the Logging-While-Drilling (LWD) monopole acoustic tool utilizing the backward-propagation effect. The LWD monopole acoustic technology is aimed at the real-time measurement of compressional wave velocities of earth formations during drilling.¹³ A commonly used design of a LWD monopole tool involves an acoustic transmitter and receiver system assembled transversely to the longitudinal axis of the drilling collar which can be simplified as a cylindrical shell.^{13, 14} The transmitter and receivers are separated by the acoustic isolator. Acoustic isolation is essential for the accurate measurement of compressional wave velocities of formations using a LWD monopole tool.¹⁵ When the transmitter, which is mounted to the collar, starts to work, it excites motion in both the formation and the collar. Thus, signals received by the receivers are composed of the formation waves and the tool waves. The tool waves are axisymmetric guided waves propagating along the collar. They need to be suppressed through acoustic isolation, so that the receivers only receive the formation waves. The isolator can be made by cutting grooves on the collar in the range from the transmitter to the receivers. This grooved structure enhances the attenuation of the tool waves¹⁵; however, it is at a cost of a reduction in strength of the tool.

The authors seek to design an acoustic isolator with high mechanical strength, instead of using the grooved structure, so that the tool can function well during drilling. The proposed method is to decrease the propagation velocities of the tool waves by applying the interference effects of backward- and forward- propagating tool waves propagating along the cylindrical

collar, in order to separate the formation arrivals from the delayed tool waves. For planar structures, it has been reported that the self-interference between forward- and backward-propagating parts of a single Lamb mode could produce a stationary mode with finite standing wave ratio¹⁶. In particular, interference effects can lead to plate resonances in which Lamb waves have zero group velocity⁷. These works show the potential of using backward waves to manipulate the flow of acoustic energy in novel ways.⁴ For practical purpose, only the guided modes with strong backward- propagation effects (i.e., a high group velocity and a wide frequency range of existence) are useful in the design of the acoustic isolation part of the collar.

In this paper, influences of geometry and material properties on group velocities of backward modes propagating along the hollow cylinder (i.e., the simplified model of the metallic collar) are theoretically studied, in order to search for the modes with strong backward-propagation effects. In an elastic plate, group velocities of backward Lamb modes are determined only by material properties of the plate.¹² In a hollow cylinder, the authors seek to achieve high group velocities of backward modes by changing the external radius to internal radius ratio of the cylinder, b/a , instead of using a particular (possibly expensive) material to manufacture the cylinder. This is only a primary study to find out whether there are clearly manifest backward modes propagating along the simplified model of a drilling collar and how the radius ratio b/a affects the group velocity of a backward mode. There are still plenty of opportunities for further research, for instance, investigations of interference effects of backward- and forward-propagating parts of guided waves on the curved surface of the collar, to be carried out for the potential application of designing an acoustic isolator in a LWD acoustic tool.

As early as 1904 Lamb first discussed the possibility of the existence of backward waves.¹⁷ Mindlin¹ and Medick¹⁸ gave the complex frequency-wavenumber ($\omega-k$) spectra, for Lamb waves in plates having bulk velocity ratios which are or are not rational numbers, in which backward -propagation branches are clearly identified. In the $\omega-k$ plane, a typical backward wave branch starts from the frequency axis (at $k = 0$), descends with frequency, and terminates at a saddle point intersected by a complex branch, i.e., an evanescent wave. The saddle point with horizontal slope is named as the zero-group-velocity (ZGV) point corresponding to the vanished group velocity while it is with nonzero wavenumber. In 1965, Meitzler² reported on an experimental observation of backward waves propagating in elastic cylinders and plates, that is, the second real branch in dispersion spectra shows a backward-propagation region in which phase and group velocities have opposite signs.

Numerous research works have since reported properties of backward waves and ZGV points both numerically and experimentally. Most described Lamb waves in plates¹⁹⁻²³, multilayered thin films and coatings²⁴⁻²⁵. For example, Negishi¹⁹ numerically revealed that backward-propagation phenomena exist in the first order symmetric Lamb mode (S_1) for Poisson's ratio $\nu < 0.45$ and in the second order anti-symmetric Lamb mode (A_2) for $\nu < 0.31$. Werby and Uberall²³ theoretically demonstrated that S_1 modes for all elastic materials have a backward-propagation region. From the authors' numerical results, group velocities of backward-propagation regions of S_1 modes for $\nu > 0.451$ and of A_2 for $\nu > 0.317$ are too small to be applied in practice. In 2008, Prada *et al.*¹² numerically analyzed the existence conditions of ZGV-Lamb modes versus Poisson's ratio and concluded that ZGV modes are not a rare phenomenon. In 2012, Kausel²⁶

theoretically analyzed that for engineering requirements the number of effective ZGV modes in laminated media is in the single digits and those visible ZGVs often show up in low order modes.

Guided elastic waves in cylindrical structures, which are also commonly encountered as waveguides, also exhibit backward-propagation motions^{2-3, 27-29}. For example, for a thin wall pipe, Ces *et al.*²⁷ reported the variations of ZGV frequencies of the second order symmetric backward Lamb modes S_{2b} with the ratio of outer radius to inner radius b/a ($1.08 \leq b/a \leq 2$). However, little attention has been paid to the group velocity of backward waves in cylindrical structures. In this paper, for further understanding of group velocity characteristics of backward modes, phase and group velocities dispersion spectra of axisymmetric longitudinal guided modes in hollow cylinders with various radius ratios b/a are numerically simulated and analyzed in combination with a theoretical development. The radius ratio can take a value from unity to infinity, i.e., $1 < b/a < \infty$; limit cases are considered as $b/a \rightarrow \infty$ representing a solid free rod, and $b/a \rightarrow 1$ representing a thin plate. Properties of backward waves in these two limit cases have been reported^{1-3, 12}, and they provide useful cases with which one can test our numerical results. Two types of materials, having bulk velocity ratios which are or are not a rational number, respectively, are chosen for hollow cylinder models.

The objectives are firstly to understand the reasons for the existence of backward modes in relatively thick and thin walled hollow cylinders, respectively; secondly, to select those backward modes of engineering interest, exhibiting the two properties that are a high group velocity and a wide range of frequencies over which the backward mode exists; thirdly, to find the optimum radius ratios b/a of a pipe corresponding to the highest group velocity; finally, to

search for a guided mode that will be useful for engineering exploitation, by virtue of having clear backward-propagation properties, and in a hollow cylinder in which the radius ratio may be optimized, so that a high group velocity is achievable in such a cylinder made of a commonly available material.

II. DISPERSION EQUATIONS AND CUTOFF FREQUENCIES

A. Dispersion equations

The dispersion equation for axisymmetric guided modes (i.e., longitudinal modes) propagating in an infinite-length hollow cylinder with traction free boundary conditions is derived from the property that the determinant of a four by four matrix is equal to zero, as given by Gazis³⁰ in 1959. The equation is expressed as

$$\det(m_{ij}) = 0, \quad i, j = 1, 2, 3, 4. \quad (1)$$

where the matrix elements are listed in Appendix A. Phase velocity dispersion curves are obtained by tracing the real roots of the transcendental dispersion equation (1), which may be numerically solved by applying the bisection method. Group velocity dispersion spectra are calculated using the relation with the phase velocity,

$$V_g = \frac{V}{1 - \frac{\omega}{V} \frac{dV}{d\omega}}, \quad (2)$$

where ω is the angular frequency, and V is the phase velocity of guided modes.

In this paper, only the backward branches of axisymmetric guided modes are considered. For a LWD monopole acoustic tool, the transmitter is an axisymmetric source which will excite the

axisymmetric modes in the circular collar. According to Ibanescu *et al.*³, in a circular waveguide with constant cross-section, there is no interaction between an axisymmetric mode with its circumferential order $n = 0$ and a non-axisymmetric mode with $n \neq 0$. Due to the continuous rotational symmetry of a hollow cylinder, axisymmetric and non-axisymmetric guided modes are uncoupled for any wavenumber k .³ Hence, the backward branches of axisymmetric modes are not relevant to the non-axisymmetric modes. Future investigation of backward-propagation properties of non-axisymmetric modes may reveal potential applications of LWD dipole and quadrupole acoustic tools. The dipole and quadrupole sources could excite non-axisymmetric modes (i.e., flexural guided modes) with $n = 1$ and $n = 2$, respectively. The coupling between two modes with the same circumferential order n ($n \neq 0$) may generate a backward flexural mode.³ It is interesting to study the backward flexural modes for multi-pole acoustic loggings.

Dispersion spectra of guided modes in a solid cylinder and in a thin plate are also numerically simulated respectively. A cylinder and a plate can be considered as two limit cases of a hollow cylinder with radius ratios $b/a \rightarrow \infty$ and $b/a \rightarrow 1$, respectively. Properties of backward waves in the two limit cases have been investigated and reported^{1-3, 12}. These results are useful for further understanding of backward waves in hollow cylinders with $1 < b/a < \infty$. Expressions of dispersion equations for a cylinder and a plate are not included in this paper for brevity; instead, they can be found in Rose³¹.

B. Cutoff frequencies

At cutoff frequencies, i.e., the wavenumber $k = 0$, guided modes decouple into pure compressional P modes and pure transverse S modes. They are cylindrical waves of infinite

wavelength. The coupling between compressional and transverse type vibrations, which is caused by the boundary conditions, causes repulsions between neighboring dispersion curves of a given symmetry (symmetric or anti-symmetric) that prevent them from intersecting.³² Ibanescu *et al.*³ have obtained quantitative results for the behavior of the dispersion relation $\omega(k)$ in the vicinity of $k = 0$ by using perturbation theory for the eigenvalue equation for ω^2 in axially uniform waveguides. They found that the strength of repulsion is related to the frequency separation at $k = 0$ between two neighboring modes; and the rule is that the smaller the frequency separation at $k = 0$, the stronger the repulsion becomes. Prada *et al.*¹² and Kausel²⁶ found a similar rule to apply in the cases of acoustic waves in elastic plates and laminate media, respectively, that the smaller the frequency gap between a pair of S and P modes at $k = 0$, the stronger the repulsion. In the vicinity of $k = 0$, the strong repulsion between a pair of neighboring P and S modes may produce a backward wave.^{3, 12, 26} The extent of repulsion is related to the difference between the cutoff frequencies of the P and S modes, and the strongest repulsion corresponds to the case of identical cutoff frequencies.^{3, 12, 26} Hence, cutoff frequencies are required to firstly distinguish between S and P modes in dispersion spectra in which numerous modes occur, and secondly to assist the estimation of the extent of the mode repulsion. Cutoff frequencies are calculated from dispersion equations at $k = 0$. Three dispersion equations for a hollow cylinder, a solid cylinder, and a thin plate at $k = 0$ are considered below.

(1) A hollow cylinder

The motion at a cutoff frequency is independent of the axial coordinate z , i.e., the wavenumber k is equal to zero. The dispersion equation (1) for longitudinal modes in a hollow cylinder at $k =$

0 reduces to

$$(m_{22}m_{44} - m_{24}m_{42})(m_{11}m_{33} - m_{13}m_{31}) = 0, \quad (3)$$

where the matrix elements are

$$\begin{aligned} m_{22} &= x_1^2 I_1(x_1), & m_{24} &= x_1^2 K_1(x_1) \\ m_{42} &= x_2^2 I_1(x_2), & m_{44} &= x_2^2 K_1(x_2) \\ m_{11} &= -x_1^2 I_0(w_1) - 2w_1 I_1(w_1), \\ m_{13} &= -x_1^2 K_0(w_1) - 2w_1 K_1(w_1), \\ m_{31} &= -x_2^2 I_0(w_2) - 2w_2 I_1(w_2), \\ m_{33} &= -x_2^2 K_0(w_2) - 2w_2 K_1(w_2), \end{aligned} \quad (4)$$

where $I_n(x)$ and $K_n(x)$ in the above formulae are the n^{th} order modified Bessel functions of the first and second kinds, and the other parameters are defined as

$$x_1 = i \frac{\omega}{V_s} a, \quad x_2 = i \frac{\omega}{V_s} b, \quad w_1 = i \frac{\omega}{V_p} a, \quad w_2 = i \frac{\omega}{V_p} b, \quad (5)$$

where V_p and V_s are the longitudinal and shear bulk velocities, respectively; a and b are the inner and outer radii of the hollow cylinder; and ω is the angular frequency.

Longitudinal modes at cutoff frequencies $k = 0$ decouple into pure P and S modes. The four terms $m_{22}, m_{24}, m_{42}, m_{44}$ in the first parentheses of Equation (3) are related only to the shear bulk velocity V_s , while the terms $m_{11}, m_{13}, m_{31}, m_{33}$ in the second parentheses are functions of the longitudinal bulk velocity V_p and the Poisson's ratio ν .

Cutoff frequencies of pure S modes, f_{cs} , are numerically calculated from the equation that the expression in the first parentheses of Equation (3) is equal to zero, giving

$$I_1(x_1)K_1(x_2) - I_1(x_2)K_1(x_1) = 0. \quad (6)$$

The equation to calculate cutoff frequencies of pure P modes, f_{cp} , is that the expression in the second parentheses of Equation (3) is equal to zero, i.e.,

$$\begin{aligned} & [x_1^2 I_0(w_1) + 2w_1 I_1(w_1)][x_2^2 K_0(w_2) + 2w_2 K_1(w_2)] - \\ & [x_2^2 I_0(w_2) + 2w_2 I_1(w_2)][x_1^2 K_0(w_1) + 2w_1 K_1(w_1)] = 0. \end{aligned} \quad (7)$$

As shown in Equations (6) and (7), cutoff frequencies f_{cs} and f_{cp} depend not only on material properties but also on the radius ratio b/a of a hollow cylinder.

(2) A solid cylinder

In an infinitely-long solid cylinder with traction free boundary conditions, with the exception of the lowest order longitudinal mode, all the other longitudinal modes have finite cutoff frequencies. At cutoff frequencies with $k = 0$, the dispersion equation for longitudinal modes has the form of

$$[\alpha J_0(\alpha) - \frac{2V_s^2}{V_p^2} J_1(\alpha)]J_1(\beta) = 0, \quad (8)$$

where $J_n(x)$ is the n^{th} order Bessel function of the first kind, $\alpha = \omega b/V_p$, $\beta = \omega b/V_s$, and b is radius of the cylinder.

As shown in Equation (8), longitudinal modes could be broken down into two categories that are the axial-shear modes (S modes) and the radial modes (P modes), according to their motions at cutoff frequencies. For S modes, their motions at their cutoff frequencies are axial shear of the cylinder. The cutoff frequencies, f_{cs} , depend on the shear bulk velocity,

$$J_1(\beta) = 0. \quad (9)$$

For P modes, their z -independent motions at their cutoff frequencies are purely radial in the cylinder, and their cutoff frequencies f_{cp} depend on the longitudinal bulk velocity and Poisson's

ratio, i.e.,

$$\alpha J_0(\alpha) - \frac{2V_s^2}{V_p^2} J_1(\alpha) = 0. \quad (10)$$

(3) A thin plate

At cutoff frequencies, Lamb modes in a thin plate with thickness $d = b - a$ decouple into pure thickness-shear type (S modes) and pure thickness-stretch type (P modes). This paper follows the Lamb mode classification by Prada *et al.*¹² For symmetric modes, the cutoff frequencies are¹²

$$f_{cs}d = nV_s, \text{ mode } S_{2n}, \text{ and } f_{cp}d = (2m+1)V_p/2, \text{ mode } S_{2m+1}, \quad (11)$$

and for antisymmetric modes, they are¹²

$$f_{cs}d = (2m+1)V_s/2, \text{ mode } A_{2m+1}, \text{ and } f_{cp}d = nV_p, \text{ mode } A_{2n}, \quad (12)$$

where $n = 1, 2, 3, \dots$, and $m = 0, 1, 2, \dots$

III. NUMERICAL RESULTS AND ANALYSES

Characteristics of backward waves can be obtained from dispersion spectra derived from the roots of the dispersion equation. Only the real roots, i.e., those corresponding to propagating modes, are numerically calculated by applying the bisection technique to solve these dispersion equations. The roots are plotted as a set of dispersion curves in the dimensionless wavenumber ($kd/2\pi$) and frequency (fd/V_s) plane; where $d = b - a$, being the pipe wall thickness, and V_s is the shear bulk velocity. Further, cutoff frequencies of P and S modes are calculated using Equations (6-7) and (9-12). Their dimensionless values ($f_{cp}d/V_s$ and $f_{cs}d/V_s$) are marked using different symbols in the frequency coordinates of dispersion spectra, in order to distinguish P modes from S modes at $k = 0$. Our numerical results of dispersion spectra correlate well with the

reference solutions found in Rose³¹ and Sinha *et al.*³³

Influences of the material property and the geometry of a hollow cylinder on backward waves are now considered. Nickel pipes of bulk velocity ratio being a rational number and steel pipes of bulk velocity ratio not being a rational number are chosen for numerical simulations. Their material properties are: for nickel with Poisson's ratio $\nu = 0.333$, longitudinal and shear bulk velocities are $V_p = 6000$ m/s, $V_s = 3000$ m/s, and density $\rho = 8900$ kg/m³, with $V_p/V_s = 2$; for steel with $\nu = 0.292$, $V_p = 5900$ m/s, $V_s = 3200$ m/s, and $\rho = 7900$ kg/m³, with $V_p/V_s \approx 1.84$. For the analyses of sensitivity to waveguide geometry, the variable is the ratio of outer radius to inner radius b/a . The outer radius is fixed as $b = 10$ mm, and the radius ratio b/a varies from 1.02 to 100. In addition, dispersion spectra for solid cylinders and thin plates are also simulated to represent two limits of a hollow cylinder with $b/a \rightarrow \infty$ and $b/a \rightarrow 1$, respectively.

A. Existence of backward modes in hollow nickel cylinders

In the dimensionless frequency-wavenumber plane, dispersion curves of longitudinal modes in four nickel pipes with the radius ratio b/a being infinity (i.e., the solid cylinder), 50, 3.333, and 1.053 are displayed in Figures 1(a)-1(d), respectively, in order to illustrate the influence of the ratio b/a on the existence of backward modes. The cutoff frequencies of pure S and P modes are marked with dots and squares, respectively. In this figure, k denotes the wave number at which a particular guided mode propagates at any given frequency, f .

Backward regions of Lamb modes have different classifications. Werby and Uberall²³ classified that the first order symmetric Lamb mode S_1 has a double-valued phase velocity over a particular frequency range. Marston²⁹ suggested the region corresponding to “negative” group

velocity should be classified as a backward wave segment of the second order symmetric Lamb mode S_2 , which is designated as backward mode S_2^b . The reason is that modes at the same frequency having two phase velocities are orthogonal.³⁴⁻³⁵ This paper follows the mode classification by Marston²⁹ and Meitzler³⁶. For instance, as labeled in Fig. 1(b), longitudinal modes are denoted $L(0,m)$, where the modal order $m = 1,2,3,\dots$, and backward longitudinal modes are denoted $L(0,3)_b$, $L(0,6)_b$, ..., where the subscript b denotes backward mode.

From numerical results, the number and distribution of backward modes in a hollow cylinder depend on the radius ratio b/a and Poisson's ratio. It is different from the case of a thin plate that backward-propagation regions of Lamb waves depend only on Poisson's ratio.¹² A general observation from Fig. 1 is that, as the ratio b/a decreases, dispersion curves are gradually becoming more closely spaced in the vicinity of $k = 0$. It is expected that the number of backward modes in a thin-walled pipe with the relatively small ratio b/a is larger than that in a thick-walled pipe with the large b/a . However, in the frequency range where $fd/V_s \leq 3.25$, the number of backward modes, i.e., those labeled with subscript b in Fig. 1, does not monotonically increase with decreasing b/a .

(1) Thick-walled nickel pipes

Consider a hollow cylinder with a relatively large radius ratio b/a . Backward modes appear as in the limit case of backward modes in a solid cylinder with $b/a \rightarrow \infty$. For a thick-walled pipe or a solid cylinder, the waveguide has three symmetries: the continuous translational symmetry in the axial direction (i.e., the z -axis), the reflection symmetry through the cross sectional ($r\theta$) plane, and the continuous rotational symmetry that is symmetric with respect to reflection across

planes containing the z -axis. At $k = 0$, the difference between the cutoff frequencies of S and P modes, i.e., $|f_{cs} - f_{cp}|$, can be made arbitrarily small.³ At $k \neq 0$, the propagation along the $+z$ and $-z$ directions breaks the reflection symmetry through the $(r\theta)$ plane. The two modes are no longer pure S or P ones; a mode coupling occurs such that the mechanical displacements of two modes are composed of both compressional and transverse components. This leads to a repulsion between the two neighboring modes in the vicinity of $k = 0$ in the dispersion spectra. According to Ibanescu *et al.*³, the rule is that a stronger repulsion corresponds to a smaller $|f_{cs} - f_{cp}|$; and if the repulsion is sufficiently strong, a backward wave region with negative slope exists.

When comparing Fig. 1(a) with Fig. 1(b), a similarity is evident between the dispersion spectra of the longitudinal modes in the solid nickel cylinder with $b/a \rightarrow \infty$ and those in the thick-walled hollow cylinder with $b/a = 50$. At the vertical axis $k = 0$, S and P modes are distinguished using points and squares. In the vicinity of $k = 0$, as discussed in the above paragraph, the strong repulsion between the second and third branches $L(0,2)$ and $L(0,3)$, that are pure S and P modes at $k = 0$ respectively, generates a visible backward-propagation region $L(0,3)_b$. In this region, the frequency exhibits a negative gradient with respect to the wavenumber. There are other high order backward modes, such as the $L(0,6)_b$ in Fig. 1(a) and the $L(0,6)_b$ and $L(0,9)_b$ in Fig. 1(b). Their dispersion curves are relatively flat, i.e., the group velocities are rather small. For a thick-walled hollow cylinder, group velocity characteristics of the clearly visible backward mode $L(0,3)_b$ are discussed further below.

(2) Thin-walled nickel pipes

Consider a hollow cylinder with a relatively small radius ratio b/a . Backward modes appear as

in the limit case of backward Lamb modes in a thin plate. For instance, as shown in Fig. 1(d), the $L(0,5)_b$ and $L(0,11)_b$ backward modes in the thin nickel pipe with $b/a = 1.053$ are resulting from strong repulsions of two pairs of neighboring branches that are pure S and P modes at $k = 0$, i.e., $L(0,4)$ & $L(0,5)$ and $L(0,10)$ & $L(0,11)$, respectively. The two backward modes are similar as the second and sixth order symmetric backward Lamb modes, i.e., S_{2b} and S_{6b} in the thin nickel plate of thickness $d = b - a$, generated by the repulsions between two pairs of symmetric Lamb modes S_1 & S_2 and S_3 & S_6 , respectively.

Compared to a solid cylinder, an additional symmetry exists in a thin plate. Because of the symmetry through the median plane of the plate, symmetric and antisymmetric Lamb waves are uncoupled for any wavenumber.^{34, 37} The dispersion curves of some symmetric Lamb modes may intersect with the curves of antisymmetric Lamb modes.³⁷ Thus, no repulsion (i.e., no backward region) occurs between two neighboring modes if one mode is symmetric S_l and the other is antisymmetric A_p , where l and p are the modal orders $l, p = 1, 2, 3, \dots$ ¹² For example, in a nickel plate with thickness d , no backward region exists between the symmetric mode S_4 and the antisymmetric mode A_2 . The cutoff frequencies of S_4 (pure S mode at $k = 0$) and A_2 (pure P mode at $k = 0$) are equal, i.e., from Equations (11-12) that $f_{cs}d/V_s = f_{cp}d/V_s = 2$. Since S_4 and A_2 belong to different families (symmetries), even though $f_{cs} = f_{cp}$, the two modes are uncoupled for any wavenumber k , that is, no backward mode exists.

A similar phenomenon is exhibited in a thin-walled pipe. For instance, in Figures 1(c) and 1(d), there is no sign of a backward region of the two $L(0,7)$ modes, i.e., the slopes of the two $L(0,7)$ dispersion curves are both positive. Although the seventh and eighth branches, $L(0,7)$ and $L(0,8)$

modes, are very close at $k = 0$ and they are pure S and P modes respectively, the two branches do not repel each other in the vicinity of $k = 0$. The reason is that, for a mode with a short wavelength (or a high frequency), the curvature of a thin-walled pipe can be ignored³⁸⁻³⁹. In that case, the pipe resembles a thin plate having a symmetry through the median plane. As illustrated in Fig. 1(d), the $L(0,7)$ and $L(0,8)$ modes behave as the uncoupled Lamb modes S_4 and A_2 . Hence, no backward mode is generated between the uncoupled $L(0,7)$ and $L(0,8)$ modes.

The important result is that, in a thin-walled pipe, the difference between cutoff frequencies of two neighboring S and P branches $|f_{cs} - f_{cp}| \rightarrow 0$ is not a sufficient condition for the existence of a backward mode. The symmetry properties of the neighboring S and P modes through the median plane also need to be considered.

B. Influences of the radius ratio on backward modes

From the dispersion spectra, although numerous backward modes might exist in a nickel pipe, only a small number of low order modes are clearly visible, for instance, $L(0,3)_b$, $L(0,5)_b$, and $L(0,11)_b$. For practical purposes, attention is now paid to determining the effects of the radius ratio b/a on the group velocity and the frequency range of existence of the visible backward waves. The purpose is to search for a suitable radius ratio corresponding to a strong backward-propagation effect.

(1) Parameter definitions

For clarity, the authors here define the frequency range of existence and the maximum group velocity of a backward mode. To illustrate these parameters, Figures 2(a) and 2(b) show phase and group velocity dispersion curves of the $L(0,5)_b$ mode in a nickel pipe with radius ratio $b/a =$

3.33, respectively. The frequency range of existence of a backward mode is defined as

$$\Delta f = f_{up} - f_{down}. \quad (13)$$

As shown in Fig. 2, the cutoff frequency of the $L(0,5)_b$ mode, f_{up} , corresponds to the frequency at which phase velocity turns to infinity while wavenumber and group velocity become zero, respectively, i.e., $V = \infty, k = 0, V_g = 0$, except for the case of an accidental degeneracy $f_{cs} = f_{cp}$. With an accidental degeneracy, the group velocity at f_{up} is a finite value^{3, 26}. The value of f_{up} can be obtained from the cutoff frequency given by Equations (6-7) and (9-12). The f_{down} corresponds to the zero-group-velocity (ZGV) point at which the phase velocity is finite, the wavenumber is nonzero, and the group velocity is zero, i.e., $V \neq \infty, k \neq 0, V_g = 0$. The value of f_{down} can be determined from the zero crossings of the group velocity dispersion curves.

A backward mode is normally dispersive in that its phase and group velocities are changing with frequency, as shown in Fig. 2. The maximum group velocity of one backward mode, $V_{g,max}$, in the frequency range $f_{down} \leq f \leq f_{up}$ can be numerically obtained from group velocity dispersion spectra. It is marked in Fig. 2(b) for an illustration.

(2) Clearly visible backward modes

For engineering interests, quantitative analyses of three backward modes $L(0,3)_b$, $L(0,5)_b$, and $L(0,11)_b$ in nickel pipes are carried out. Numerical results of the variation of two dimensionless quantities, i.e., the maximum group velocity $V_{g,max}/V_s$ and the frequency range of existence $2\pi\Delta f(b-a)/V_s$, with the radius ratio b/a , are displayed in Figures 3(a) and 3(b), respectively.

As shown in Fig. 3, with the varying radius ratio, the frequency range of existence of a backward mode becomes wider as the maximum group velocity becomes larger. Moreover, for a

thick-walled nickel hollow cylinder with $b/a > 5$, the $L(0,3)_b$ mode is the dominant backward-propagation one having a relatively large $V_{g,max}/V_s$ and $2\pi\Delta f(b-a)/V_s$; while in the thin-walled pipe case of $b/a < 5$, the $L(0,5)_b$ mode is the dominant one.

For the other high order backward longitudinal modes, e.g., $L(0,11)_b$, their frequency ranges of existence and group velocities are rather small compared to those of $L(0,3)_b$ in the case of $b/a > 5$ or $L(0,5)_b$ at $b/a < 5$. These high order backward modes are not suitable for utilization in engineering devices, especially when the system attenuations need to be considered.

In Fig. 3, the largest values of both the maximum group velocity and the frequency range of existence of the $L(0,3)_b$ mode are exhibited in the case of $b/a = 8.25$, and those of the $L(0,5)_b$ mode appear at $b/a \rightarrow 1$, i.e., for the thin plate case. These peaks correspond to accidental frequency degeneracy phenomena $f_{cs} = f_{cp}$, as illustrated in Figures 4(a) and 4(b).

For instance, Fig. 4(c) shows the influence of the radius ratio b/a on the occurrence of the backward mode $L(0,3)_b$. The $L(0,3)_b$ is resulting from the strong repulsion between the neighboring $L(0,2)$ and $L(0,3)$ modes in the vicinity of $k = 0$; and the strength of repulsion becomes larger with the decreasing frequency separation at $k = 0$. As shown in Fig. 4(c), the frequency separation depends on the ratio b/a . (1) For the nickel pipe of $b/a = 2$, the difference between the cutoff frequencies of $L(0,2)$ and $L(0,3)$ modes is relatively large and the two modes are weakly interacting modes. (2) For $b/a = 3.33$, a stronger interaction leads to a flat backward-propagation region and a zero-group-velocity point of the lower branch. (3) For $b/a = 8.25$, the accidental degeneracy of the two branches at $k = 0$ corresponds to the strongest repulsion in the vicinity of $k = 0$. It gives rise to the highest group velocity and the widest range

of frequencies over which the backward mode $L(0,3)_b$ exists. When $b/a \leq 8.25$, $L(0,2)$ and $L(0,3)$ modes are pure P and S modes at $k = 0$, respectively; when $b/a > 8.25$, $L(0,2)$ and $L(0,3)$ are pure S and P modes at $k = 0$, respectively. (4) For $b/a = 20$, the weaker coupling between the two branches leads to a less pronounced trough, i.e., a weak backward-propagation effect.

(3) Monotonic variation relations

For most of the backward modes, variation relations between the maximum group velocity and the radius ratio b/a (or relations between the frequency range of existence and b/a) are complicated, as illustrated in Fig. 5(a). For instance, two variation curves for $L(0,3)_b$ in Fig. 3 are not monotonic. By contrast, for the $L(0,5)_b$ and $L(0,11)_b$ modes, the variation curves in Fig. 3 are monotonically increasing with the decreasing ratio b/a in the ranges of $1 < b/a \leq 10$ and $1 < b/a \leq 15$, respectively, and finally approaching their maxima at $b/a \rightarrow 1$.

The distinctive behavior is related to the ratio between bulk longitudinal and shear velocities V_p/V_s . For a nickel plate (i.e., the limit case of a nickel pipe with $b/a \rightarrow 1$) for which the bulk velocity ratio is a rational number $V_p/V_s = 2$, a coincidence of two cutoff frequencies (i.e., an accidental degeneracy) occurs occasionally. Originating from a coincident cutoff frequency, the repulsion between two neighboring branches with opposite slopes reaches its maximum.^{3, 12, 26} From Equation (11), the cutoff frequencies of S_1 and S_2 are identical, i.e., $f_{cs1}d/V_s = f_{cp1}d/V_s = 1$, and those of S_3 and S_6 are also equal, i.e., $f_{cs2}d/V_s = f_{cp2}d/V_s = 3$. Two symmetric backward Lamb modes S_{2b} and S_{6b} are generated from the strongest repulsions between S_1 & S_2 and S_3 & S_6 , respectively. Hence, in the limit case of $b/a \rightarrow 1$, both modes $L(0,5)_b$ and $L(0,11)_b$ reach their highest group velocities and the widest frequency ranges of existence, respectively.

For a thin-walled nickel pipe, the backward modes $L(0,5)_b$ and $L(0,11)_b$ respectively resemble the backward Lamb modes S_{2b} and S_{6b} in a thin nickel plate with thickness $d = b - a$. The gaps between the cutoff frequencies of $L(0,4)$ and $L(0,5)$ (which generates $L(0,5)_b$) and between those of $L(0,10)$ and $L(0,11)$ (which generates $L(0,11)_b$) both become small with decreasing b/a , and finally diminish to zero at $b/a \rightarrow 1$. The general rule is that the smaller the frequency gap at $k = 0$, the stronger the repulsion; and the strongest repulsion corresponds to the accidental frequency degeneracy.³ According to this rule, the smaller the radius ratio b/a , the higher the group velocities, and the wider the frequency ranges of existence of $L(0,5)_b$ and $L(0,11)_b$ modes in a thin-walled nickel pipe with $V_p/V_s = 2$, as illustrated in Fig. 5(b).

It is worth noting that there is no need to manufacture a very thin walled pipe to generate the highest group velocity of the $L(0,5)_b$ mode. As illustrated in Fig. 3(a), the curve of the maximum group velocity $V_{g,max}$ becomes flat when b/a approaches 1. For example, the value of $V_{g,max}$ at $b/a = 2$ is greater than 85% of that at $b/a \rightarrow 1$.

C. Bulk velocity ratio not being a rational number

In a thin plate, the repulsion between two Lamb waves reaches a maximum only if the bulk velocity ratio is a rational number.¹² For example, the maximum group velocity, $V_{g,max}$, of the backward Lamb mode S_{2b} in a nickel plate is about 1.681 times larger than that of the S_{2b} mode in a steel plate; and the frequency range of existence, $2\pi\Delta f(b-a)/V_s$, of the S_{2b} in the nickel plate is about 1.487 times wider than that of the same mode in the steel plate. The bulk velocity ratio of the nickel plate is a rational number, $V_p/V_s = 2$, while that of the steel plate is not a

rational number, as $V_p/V_s \approx 1.84$. Material is the only controllable parameter to obtain a backward mode of a large group velocity in a thin plate.

In a hollow cylinder, the radius ratio b/a obviously affects the behavior of backward modes, as discussed in Section III. B. It is interesting to study, in a hollow pipe with a bulk velocity ratio not being a rational number, whether we could control the ratio b/a to obtain a clearly visible backward mode exhibiting a comparable magnitude of group velocity to that of a pipe with bulk velocity ratio being a rational number.

From numerically predicted dispersion spectra of longitudinal modes in a large number of steel hollow cylinders with different ratios b/a , the clearly visible backward modes are the low order modes $L(0, 3)_b$, $L(0,5)_b$, and $L(0,7)_b$. For these three modes, their variation curves of the dimensionless group velocity $V_{g,max}/V_s$ and the frequency range of existence $2\pi\Delta f(b-a)/V_s$ with the variable b/a are traced in Figures 6(a) and 6(b), respectively.

As shown in Fig. 6, influences of the radius ratio b/a on the three backward modes are dramatic. Under the condition of $b/a < 6$, $L(0,5)_b$ is the most visible backward mode with relatively large group velocity and wide frequency range of existence; in the case of $b/a > 6$, the $L(0,3)_b$ is the most manifest one which backward-propagation effect is strong. The $L(0,7)_b$ mode is not attractive for practical interest. Moreover, in steel hollow cylinders with bulk velocity ratios not being a rational number, variation curves of the three modes are not monotonic. The peak values of each curve in Fig. 6 correspond to the strongest repulsions, originating from the accidental degeneracy of neighboring pure P and S modes at the wavenumber $k = 0$. Noticeably, the peak group velocity of the backward mode $L(0,5)_b$ in the steel pipe with radius ratio $b/a = 3.6$

is rising about 1.65 times larger than that in the steel plate with $b/a \rightarrow 1$.

In order to directly compare the clearly visible backward modes $L(0,3)_b$ and $L(0,5)_b$ in the nickel and steel hollow cylinders, the maximum group velocity, the frequency ranges of existence, as well as the radius ratios b/a corresponding to the strongest repulsions, are listed in Table I. Clearly, both group velocities and frequency ranges of existence of two backward modes in steel pipes are comparable with those in nickel pipes. It means that one can expect to achieve a backward mode with large group velocity in a hollow cylinder made of a commonly encountered material for which bulk velocity ratio is not a rational number.

IV. CONCLUSIONS AND DISCUSSIONS

This paper provides theoretical and numerical analyses on the problem of whether it is possible to achieve large group velocities of backward modes by changing the ratio of the outer radius to the inner radius of a hollow cylinder, b/a . The major findings are:

- (1) In a hollow cylinder, propagation properties of a backward mode are determined by two factors. They are the Poisson's ratio of the pipe material and the radius ratio b/a .
- (2) Generally, variation curves of both the maximum group velocity and the frequency range of existence of a backward mode are not monotonic with the ratio b/a . Except for the case of a thin-walled pipe with bulk velocity ratio being a rational number, for certain backward modes, variation curves are monotonically increasing with decreasing b/a and finally reaching their maxima at $b/a \rightarrow 1$.
- (3) The highest group velocity and the widest frequency range of existence of a backward mode, that resulting from the strongest repulsion, correspond to the accidental frequency

degeneracy of two neighboring pure transverse and compressional modes (S and P modes) at the wavenumber $k = 0$. The radius ratio b/a corresponding to the strongest repulsion can be numerically calculated by setting the cutoff frequency of the S mode to that of the neighboring P mode.

- (4) The third and fifth order longitudinal backward modes, i.e., $L(0,3)_b$ and $L(0,5)_b$, show strong backward-propagation effects in that they have large group velocities and wide frequency ranges of existence. They are promising for engineering applications as very few of the backward modes are clearly visible.
- (5) Comparing group velocities of backward modes in pipes made of different Poisson's ratios, it is concluded that a comparably large group velocity can be achieved using commonly available materials. It is known that, in a thin plate, the largest group velocity is possible only for particular materials for which bulk velocity ratios are rational numbers. In a hollow cylinder, by selecting the clearly visible backward mode and by manufacturing the suitable geometry of a pipe (i.e., the radius ratio b/a), it is possible to obtain the required large group velocity in common and inexpensive materials, resulting in cost benefits for the potential application of designing acoustic logging tools utilizing the backward-propagation effect.

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APPENDIX A: MATRIX ELEMENTS OF DISPERSION EQUATION

The dispersion equation of the axisymmetric guided modes in a hollow cylinder with free boundary conditions is the determinant of a 4×4 coefficient matrix, that is,

$$\det(m_{ij}) = 0, \quad i, j = 1, 2, 3, 4. \quad (\text{A1})$$

The matrix elements are

$$\begin{aligned} m_{11} &= (2Y_1^2 - x_1^2)I_0(w_1) - 2w_1I_1(w_1), \\ m_{12} &= 2Y_1[x_1I_0(x_1) - I_1(x_1)], \\ m_{13} &= (2Y_1^2 - x_1^2)K_0(w_1) - 2w_1K_1(w_1), \\ m_{14} &= 2Y_1[x_1K_0(x_1) - K_1(x_1)], \\ m_{21} &= 2w_1Y_1I_1(w_1), \\ m_{22} &= (Y_1^2 + x_1^2)I_1(x_1), \\ m_{23} &= 2w_1Y_1K_1(w_1), \\ m_{24} &= (Y_1^2 + x_1^2)K_1(x_1), \\ m_{31} &= (2Y_2^2 - x_2^2)I_0(w_2) - 2w_2I_1(w_2), \\ m_{32} &= 2Y_2[x_2I_0(x_2) - I_1(x_2)], \\ m_{33} &= (2Y_2^2 - x_2^2)K_0(w_2) - 2w_2K_1(w_2), \\ m_{34} &= 2Y_2[x_2K_0(x_2) - K_1(x_2)], \\ m_{41} &= 2w_2Y_2I_1(w_2), \\ m_{42} &= (Y_2^2 + x_2^2)I_1(x_2), \\ m_{43} &= 2w_2Y_2K_1(w_2), \\ m_{44} &= (Y_2^2 + x_2^2)K_1(x_2), \end{aligned} \quad (\text{A2})$$

respectively. $I_n(x)$ and $K_n(x)$ in the above formulae are the n^{th} order modified Bessel functions of the first and second kinds, and the other parameters are defined as

$$x_1 = \beta a, x_2 = \beta b, w_1 = \alpha a, w_2 = \alpha b, Y_1 = ka, Y_2 = kb, \quad (\text{A3})$$

where a and b are the inner and outer radii of the hollow cylinder; $k = \omega/V$ is the wavenumber;

ω is the angular frequency, and V is the phase velocity of guided modes propagating in the pipe.

The parameters α and β in equation (A3) are

$$\alpha = \sqrt{k^2 - \frac{\omega^2}{V_p^2}}, \beta = \sqrt{k^2 - \frac{\omega^2}{V_s^2}}, \quad (\text{A4})$$

where V_p and V_s are the longitudinal and shear bulk velocities, respectively, which are functions of the pipe's material.

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TABLE I. The dominant backward modes exist in the nickel and steel hollow cylinders, respectively. For each clearly visible backward mode, the largest group velocity $V_{g,max}$, the widest frequency range of existence $\Delta\omega d/V_s$, and the ratio of outer radius to inner radius b/a corresponding to the strongest repulsion are listed respectively. Here, $\kappa = V_p/V_s$ is the bulk velocity ratio, $\omega = 2\pi f$ is the angular frequency, and $d = b - a$ is the pipe wall thickness.

Material	Mode	b/a	$V_{g,max}$ (km/s)	$\frac{\Delta\omega d}{V_s}$
Nickel $\kappa = 2$	L(0,3) _b	8.25	1.92	0.339
	L(0,5) _b	1.00	1.91	0.580
Steel $\kappa \approx 1.84$	L(0,3) _b	20.0	1.83	0.329
	L(0,5) _b	3.60	1.88	0.625

Figure Captions

FIG. 1 (Color online) Predicted dispersion curves of longitudinal modes in the solid nickel cylinder (a) and those in three nickel hollow cylinders with the same outer radius $b = 10$ mm and different inner radii $a = 0.2, 3$, and 9.5 mm (b) (c) and (d), respectively. The vertical and horizontal axes are dimensionless frequency fd/V_s and wavenumber $kd/2\pi$, where $d = b - a$ is the pipe wall thickness, f is the frequency, $V_s = 3000$ m/s is the shear bulk velocity of nickel, the wavenumber $k = \omega/V = 2\pi f/V$, and V is the phase velocity of guided waves. Moreover, cutoff frequencies of pure S and P modes at $k = 0$ are marked with dots and hollow squares, respectively.

FIG. 2 (Color online) Phase velocity (a) and group velocity (b) dispersion curves of the backward mode $L(0,5)_b$ and the forward mode $L(0,4)$ in the nickel hollow cylinder with the bulk velocity ratio $V_p/V_s = 2$ and the radius ratio $b/a = 3.33$. The solid and dashed lines represent the $L(0,4)$ and $L(0,5)_b$ modes, respectively.

FIG. 3 (Color online) Variations, with the radius ratio b/a in the range of $1 < b/a \leq 100$, of the normalized maximum group velocity $V_{g,max}/V_s$ (a) and the frequency range of existence $\Delta\omega d/V_s$ (b) of the three backward-propagation modes in a set of hollow nickel pipes with the fixed outer radius b being equal to 10 mm. The ratio between longitudinal and shear bulk velocities is a rational number, i.e., $V_p/V_s = 2$. The shear bulk velocity $V_s = 3000$ m/s, and $d = b - a$ is the pipe wall thickness. The solid, dashed, and dotted lines represent variation curves of the $L(0,3)_b$, $L(0,5)_b$, and $L(0,11)_b$ longitudinal backward modes, respectively.

FIG. 4 (Color online) Dispersion spectra of longitudinal modes in the nickel hollow cylinders with radius ratio $b/a = 8.25$ (a) and $b/a = 1.02$ (b), respectively. The interactions between the L(0,2) and L(0,3) modes in four nickel pipes with radius ratios $b/a = 2, 3.33, 8.25$, and 20 , respectively, are shown in figure (c). The solid and dotted lines represent the forward and backward modes, respectively. At the wavenumber $k = 0$, one branch is pure transverse (S) wave, and the other is pure compressional (P) wave. The dots and hollow squares represent cutoff frequencies of pure S and P modes, respectively. For $b/a = 2$, weakly interacting modes (1). For $b/a = 3.33$, a stronger interaction leads to a flat backward-propagation region of the lower branch (2). For $b/a = 8.25$, an accidental degeneracy at $k = 0$ corresponds to the strongest repulsion which gives rise to the clearly visible backward mode L(0,3)_b with the strongest backward-propagation effect (3). For $b/a = 20$, a weaker repulsion leads to a less pronounced trough of the lower branch (4).

FIG. 5 (Color online) Group velocity dispersion curves of the L(0,3)_b modes in the nickel hollow pipes having their outer to inner radius ratios $b/a = \infty, 10, 8, 6.67$, and 5 , respectively (a); and those of the L(0,5)_b modes in the cases of $b/a = 1.02, 2, 5$, and 8 , respectively (b). For each backward mode, the influence of the radius ratio b/a on the frequency range of existence and that on the maximum group velocity are similar.

FIG. 6 (Color online) Variations, with the ratio between outer to inner radii in the range of $1 < b/a \leq 100$, of the maximum group velocity $V_{g,max}/V_s$ (a) and the frequency range of existence $\Delta\omega d/V_s$ (b) of the three backward modes in hollow steel pipes with the fixed outer radius b being equal to 10 mm. The ratio between longitudinal and shear bulk velocities of the

steel pipe is not a rational number, i.e., $V_p/V_s \approx 1.84$. The shear bulk velocity $V_s = 3200$ m/s, and $d = b - a$ is the pipe wall thickness. The solid, dashed, and dotted lines represent variations curves of the $L(0,3)_b$, $L(0,5)_b$, and $L(0,7)_b$ longitudinal backward modes, respectively.











