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Investigating bounds on decoherence in quantum mechanics via B and D-mixing

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Abstract

We investigate bounds on decoherence in quantum mechanics by studying B and D-mixing observables, making use of many precise new measurements, particularly from the LHC and B factories. In that respect we show that the stringent bounds obtained by a different group in 2013 rely on unjustified assumptions. Finally, we point out which experimental measurements could improve the decoherence bounds considerably.

1. Introduction

The interpretation of quantum mechanics is still an open problem, see for example the very recent discussion of the quantum pigeonhole in [1]. Many of the related questions go back to the effect of entanglement that was studied in 1935 by Einstein, Podolsky and Rosen [2]. Numerous different systems have been investigated in that respect. It is also interesting to test the validity of the foundations of quantum mechanics in systems that are usually used to search for physics beyond the standard model. Therefore we discuss here the mixing of neutral mesons, which is a well-known and well-studied quantum mechanical effect, see e.g. [3] for an early discussion

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of B-mesons in that respect. It leads to the fact that the neutral mesons that are propagating in space–time are described by the mass eigenstates, e.g. B_H and B_L, which are linear combinations of the flavour eigenstates defined by the quark content, e.g. $B_d = (\bar{b}d)$ and $\bar{B}_d = (b\bar{d})$:

$$B_H = pB_d - q\bar{B}_d, \tag{1}$$

$$B_L = pB_d + q\bar{B}_d. \tag{2}$$

In these systems we have the following observables: the mass difference $\Delta M = M(B_H) - M(B_L)$, the decay rate difference $\Delta\Gamma = \Gamma(B_L) - \Gamma(B_H)$ and semi-leptonic CP asymmetries, which can be expressed as $a_{sl} = 2(1 - |q/p|)$.

For the two neutral B-meson systems, we now have quite precise experimental numbers for the mixing observables. The HFAG 2014 [4] values read:

	B_s	B_d
ΔM	$(17.761 \pm 0.022)\ \text{ps}^{-1}$	$(0.510 \pm 0.003)\ \text{ps}^{-1}$
$\Delta\Gamma$	$(0.091 \pm 0.008)\ \text{ps}^{-1}$	$(0.001 \pm 0.010)\ \Gamma$
Γ	$\frac{1}{1.512\pm0.007}\ \text{ps}^{-1}$	$\frac{1}{1.519\pm0.005}\ \text{ps}^{-1}$
a_{sl}	-0.0077 ± 0.0042	-0.0009 ± 0.0021

(3)

where Γ denotes the total decay rate of the neutral B-mesons. The experimental numbers for ΔM_d, ΔM_s and $\Delta\Gamma_s$ are now dominated by LHC measurements. The most precise values were obtained for ΔM_d by LHCb [5], for ΔM_s by LHCb [6] and for $\Delta\Gamma_s$ by ATLAS [7], CMS [8] and LHCb [9]. Here no entanglement effects are expected to occur, hence we use these values as independent inputs for our investigations of decoherence. For the semi-leptonic asymmetries and $\Delta\Gamma_d$ we do not yet have clear experimental evidence for a non-zero value; only some bounds are available. Thus we give for completeness the corresponding standard model predictions [10–14] of these quantities:

	B_s	B_d
$\Delta\Gamma$	$(0.087 \pm 0.021)\ \text{ps}^{-1}$	$(0.0029 \pm 0.0007)\ \text{ps}^{-1}$
a_{sl}	$(1.9 \pm 0.3) \cdot 10^{-5}$	$(-4.1 \pm 0.6) \cdot 10^{-4}$

(4)

In the neutral D-meson system, typically the mixing parameters x, y and $|p/q|$ are determined directly [4]:

$$x := \frac{\Delta M}{\Gamma} = 0.41^{+0.14}_{-0.15}, \tag{5}$$

$$y := \frac{\Delta\Gamma}{2\Gamma} = 0.63^{+0.07}_{-0.08}, \tag{6}$$

$$\left|\frac{p}{q}\right| = 0.93^{+0.09}_{-0.08}. \tag{7}$$

2. Decoherence in B-mixing

At the B-factories B-mesons were typically produced via the decay of the $\Upsilon(4s)$ resonance, thus also producing entangled $\bar{B}_d B_d$ pairs. To describe decoherence in B_d-mixing, semileptonic decays of the neutral B-mesons were investigated, e.g. $\bar{B}_d \to l^- \bar{\nu}_l X$ and $B_d \to l^+ \nu_l X$. If no mixing occurs, one gets from semi-leptonic decays events with one positively charged and one negatively charged lepton – so-called opposite-sign leptons. If mixing is taken into account one can also get events with two positively or two negatively charged leptons, so-called same-sign leptons. Following [15] we define the ratio of like-sign dilepton decays of a neutral B-meson to opposite-sign dilepton events, R (based on the investigations in [16,17]), as

$$R = \frac{N^{++} + N^{--}}{N^{+-} + N^{-+}}. \tag{8}$$

N^{++} denotes the events with two positively charged leptons in the final states and so on. In [15] Bertlmann and Grimus used the parameter ζ to describe decoherence effects in quantum mechanics in a phenomenological manner. $\zeta = 0$ corresponds to the familiar case of quantum mechanics, while $\zeta = 1$ describes a case where no quantum mechanical interference effects are occurring at all – corresponding to Furry's hypothesis [18]. The general expression for R in terms of the mixing parameters x, y and $|p/q|$ then reads [15]

$$R = \frac{1}{2}\left(\left|\frac{p}{q}\right|^2 + \left|\frac{q}{p}\right|^2\right) \frac{x^2 + y^2 + \zeta\left[y^2\frac{1+x^2}{1-y^2} + x^2\frac{1-y^2}{1+x^2}\right]}{2 + x^2 - y^2 + \zeta\left[y^2\frac{1+x^2}{1-y^2} - x^2\frac{1-y^2}{1+x^2}\right]}. \tag{9}$$

This formula can be written more in the more compact form

$$R = R_0 \frac{1}{\sqrt{1 - a_{sl}^2}} \frac{1 + \alpha\zeta}{1 + \beta\zeta}, \tag{10}$$

with

$$R_0 = \frac{x^2 + y^2}{2 + x^2 - y^2}, \tag{11}$$

$$\alpha = \frac{y^2(1 + x^2)^2 + x^2(1 - y^2)^2}{(x^2 + y^2)(1 + x^2)(1 - y^2)}, \tag{12}$$

$$\beta = \frac{y^2(1 + x^2)^2 - x^2(1 - y^2)^2}{(2 + x^2 - y^2)(1 + x^2)(1 - y^2)}, \tag{13}$$

$$a_{sl} = \frac{N^{++} - N^{--}}{N^{++} + N^{--}} = \frac{\left|\frac{p}{q}\right|^2 - \left|\frac{q}{p}\right|^2}{\left|\frac{p}{q}\right|^2 + \left|\frac{q}{p}\right|^2}. \tag{14}$$

R_0 can also be expressed in terms of the mixing probability

$$\chi = \frac{x^2 + y^2}{2(1 + x^2)}, \tag{15}$$

as

$$R_0 = \frac{\chi}{1 - \chi}. \tag{16}$$

The current values of χ from PDG [19] or HFAG [4] are not measured directly but derived via Eq. (15) from the direct measurements of ΔM, $\Delta\Gamma$ and Γ. Historically R was approximated by R_0 and used to extract the mixing probability, see e.g. [20,21].

Eq. (10) represents our master equation. The numerical values of the coefficients in this equation can be calculated quite precisely by using the most recent experimental numbers from HFAG [4]. For the heavy neutral mesons we get:

	$B_s = (\bar{b}s)$	$B_d = (\bar{b}d)$	$D^0 = (c\bar{u})$
R_0	$0.997247\ (1 \pm 2.7 \cdot 10^{-5})$	$0.2308\ (1 \pm 0.010)$	$0.319\ (1^{+0.27}_{-0.25})$
α	$6.14^{+0.88}_{-0.80} \cdot 10^{-3}$	$0.6249^{+0.0032}_{-0.0032}$	$1.51^{+0.31}_{-0.24}$
β	$3.37^{+0.88}_{-0.80} \cdot 10^{-3}$	$-0.14424^{+0.00077}_{-0.00076}$	$0.39^{+0.24}_{-0.17}$
$\frac{1}{\sqrt{1-a_{sl}^2}}$	$1 + 3.0^{+4.1}_{-2.4} \cdot 10^{-5}$	$1 + 4.1^{+41.0}_{-4.1} \cdot 10^{-7}$	$1.011^{+0.043}_{-0.010}$

(17)

The above coefficients are known quite precisely for the neutral B-system, while there are still sizable uncertainties in the D-meson systems. If quantum mechanics holds, i.e. $\zeta = 0$, we predict (by using the measured values of $\Delta\Gamma$, ΔM, Γ and a_{sl}) the following values for the ratio R:

	B_s	B_d	D^0
R^{QM}	$0.997277^{+0.000049}_{-0.000036}$	0.2308 ± 0.0024	$0.322^{+0.089}_{-0.079}$

(18)

These numbers can be compared to the measured values of R stemming from ARGUS (1993) [20] and CLEO (1993) [21] for the B_d-system. To our knowledge there are no measurements available for the B_s or the D-meson system:

	B_s	B_d	D^0
R	–	$0.194(1 \pm 0.424)$ $0.187(1^{+0.278}_{-0.239})$	–

(19)

The first entry in the table is from ARGUS [20], while the second is from CLEO (1993) [21]. A significant deviation of R^{QM} from the measured value R would point towards a violation of quantum mechanics, that could be described by a non-vanishing value of ζ. This can be expressed as

$$\zeta = \frac{\frac{R}{R_0}\sqrt{1 - a_{sl}^2} - 1}{\alpha - \beta \frac{R}{R_0}\sqrt{1 - a_{sl}^2}}. \tag{20}$$

If quantum mechanics holds, then Eq. (10) states $R = R_0/\sqrt{1 - a_{sl}^2}$ and thus Eq. (20) gives $\zeta = 0$, as expected. Using the experimental values for R from [20] or [21] we get from Eq. (20) the following bounds on ζ:

$$\zeta = -0.26^{+0.30}_{-0.28}, \tag{21}$$

$$\zeta = -0.21^{+0.46}_{-0.53}. \tag{22}$$

Eq. (21) has been calculated using the value of R measured by ARGUS [20], whereas Eq. (22) is derived from the CLEO value [21]. The central value for ζ is slightly negative, but the value $\zeta = 0$ is within the one standard deviation region of the measured value of R, thus no decoherence effects can be seen yet in the neutral B_d-system. Total decoherence, i.e. $\zeta = 1$, is thus excluded by about four standard deviations. The uncertainty of the extracted value for ζ is completely dominated by the uncertainty in R. At this point, however, some caution is necessary. It was shown in [22,23] that bounds on ζ depend on the basis (flavour or mass basis) used for describing the neutral B-mesons. Thus quantitative statements describing the deviation from decoherence have to be taken with some care.

Using the old experimental inputs from Bertlmann and Grimus [15] we get

$$\zeta = -0.16^{+0.30}_{-0.31}. \tag{23}$$

The shift in the central value stems from the fact that at that time all the mixing parameters were known much less precisely. The uncertainty, which is dominated by R, stayed more or less the same, because we use the same value for R. Unfortunately there are no new measurements of the ratio R in the B_d-system available and there exists no measurement at all in the B_s or D-system. Thus we are limited by the experimental accuracy stemming from 1993. Here any new experimental investigation of R would be very helpful, using for example the huge data set collected by the B-factories. In that respect it is of course interesting to ask, what experimental precision in R would result in what bound on ζ? We find:

δR	$\pm 25\%$	$\pm 10\%$	$\pm 5\%$	$\pm 2\%$	$\pm 1\%$	$\pm 0.5\%$
$\delta\zeta$	$^{+1.15}_{-1.07}$	$^{+0.45}_{-0.44}$	$^{+0.23}_{-0.22}$	± 0.10	± 0.06	± 0.05

(24)

With a precision of 1% in R, the current uncertainties in x, y and $|p/q|$ dominate the uncertainty in ζ.

The Belle Collaboration performed in [24] a time-dependent analysis of semi-leptonic B-decays and obtained a very strong bound on decoherence:

$$\zeta^{\text{Belle}} = 0.029 \pm 0.057. \tag{25}$$

It is, however, not completely clear how to relate ζ^{Belle} to our time-integrated analysis. Moreover, Belle neglected $\Delta\Gamma_d$ and a_{sl}, whose experimental values can still be in the percentage range, which is comparable to the uncertainty in ζ^{Belle}.

3. The analysis of Alok and Banerjee from 2013

In [25] it was tried to avoid the experimental short-comings related to the almost unknown value of R by a trick: because Eq. (10) seems to indicate $R \approx R_0$, it was assumed in Eq. (20) that $R = R_0$. This is equivalent to starting from

$$R = \frac{\chi}{1-\chi}, \tag{26}$$

with χ defined in Eq. (15), in order to investigate bounds on decoherence. With that approximation we get

$$\zeta = \frac{\sqrt{1-a_{sl}^2} - 1}{\alpha - \beta\sqrt{1-a_{sl}^2}}, \tag{27}$$

an equation that does not depend at all on R and has all coefficients quite precisely known. Taking Eq. (27) as a starting point we obtain the following strong bounds on ζ:

$$\zeta(B_d) = -0.53^{+0.53}_{-5.32} \cdot 10^{-6}, \tag{28}$$

$$\zeta(B_s) = -0.0107^{+0.0085}_{-0.0149}. \tag{29}$$

Thus Eq. (27) leads to very stringent constraints on decoherence in quantum mechanics, which are orders of magnitude better than the ones obtained in Eq. (21). Alok and Banerjee found that total decoherence is excluded by 34 standard deviations in the B_d-system and by 24–31 standard deviations in the B_s-system.

However, the first thing to notice when studying Eq. (27) is that now the value of ζ depends purely on the experimental value of a_{sl} – a result which is in contradiction with the definition of a_{sl}, which is independent of ζ.

Secondly, one now obtains also a non-vanishing ζ-value if one takes the standard model values for $a_{sl}^{s,d}$, which of course assume the validity of quantum mechanics:

$$\zeta\left(a_{sl}^{d,\mathrm{SM}}\right) = -1.09^{+0.30}_{-0.34} \cdot 10^{-7}, \tag{30}$$

$$\zeta\left(a_{sl}^{s,\mathrm{SM}}\right) = -6.53^{+2.87}_{-4.62} \cdot 10^{-8}. \tag{31}$$

So we get a violation of quantum mechanics – albeit a tiny one – even if we take standard model predictions for the semi-leptonic asymmetries. This is clearly a contradiction and points to this method being invalid.

Thirdly, Alok and Banerjee also found a bound in the B_s-system, even though there was no measurement at all of R available in this system and R is the only quantity sensitive to decoherence effects.

Finally, it is clear that one cannot simply equate R and R_0 without taking into account the accuracy of this approximation as well as the independent uncertainties of R and R_0. These uncertainties can be written as

$$R_0 = \bar{R}_0(1 \pm \delta_{R_0}), \tag{32}$$

$$R = \bar{R}(1 \pm \delta_R). \tag{33}$$

The central value $\bar{R}_0$ and the relative error δ_{R_0} are given in Eq. (17), R and δ_R are given by the experimental value from 1993, see Eq. (19). Concerning the equality of R and R_0: Eq. (10) shows that the relation between R and R_0 reads

$$R = R_0(1+\epsilon)\left(1^{+\delta_+}_{-\delta_-}\right), \tag{34}$$

with $1+\epsilon = 1/\sqrt{1-a_{sl}^2}$. The tiny value of ϵ can thus be read off from Eq. (17), while δ_+ and δ_- can be estimated by varying ζ in the range of -1 to $+1$. One finds sizable values, that clearly cannot be neglected, because they present by far the dominant uncertainties:

	B_d	B_s
δ_+	0.90	0.0027
δ_-	0.67	0.0028

(35)

Hence the correctly modified version of Eq. (20) reads

$$\zeta = \frac{(1+\epsilon)\left(1^{+\delta_+}_{-\delta_-}\right)\sqrt{1-a_{sl}^2} - 1}{\alpha - \beta(1+\epsilon)\left(1^{+\delta_+}_{-\delta_-}\right)\sqrt{1-a_{sl}^2}}. \tag{36}$$

It is evident that Alok and Banerjee have simply set ϵ, δ_+ and δ_- to zero in order to obtain Eq. (27). Taking the finite values of ϵ, δ_+ and δ_- into account one gets instead

$$\zeta = \frac{0^{+\delta_+}_{-\delta_-}}{\alpha - \beta\left(1^{+\delta_+}_{-\delta_-}\right)}. \tag{37}$$

Firstly, we see that the dependence on a_{sl} has disappeared, as it should. Next, taking finite values of $\delta_{+,-}$ from Eq. (35) into account we find using Eq. (37) simply that ζ lies in between -1 and $+1$, which was what we initially assumed in order to obtain the values in Eq. (35). Therefore by rewriting Eq. (20), in order to avoid using the experimental value of R, one learns nothing new.

To summarise: by neglecting the dominant effect of $\delta_{+,-}$ the authors of [25] artificially created a very precise relation, given in Eq. (27), which does not depend at all on R. We have shown that this approximation is unjustified and leads to a false conclusion.

4. Conclusion

In this paper we have investigated decoherence in B-mixing using the most recent values for the mixing observables. Within current experimental uncertainties we find no hint for any decoherence effect and total decoherence is excluded by about four standard deviations in the B_d-system. The current precision is, however, strongly limited by the very imprecise value of the ratio of like-sign dilepton events to opposite-sign dilepton events, R. The most recent experimental number for R stems from 1993. Here any updated measurements, using, for example, the large data set of the B factories, would be very desirable. Moreover, first measurements of R for the B_s-system (e.g. from the $\Upsilon(5s)$ data set of Belle) and the D-system, e.g. from BES would be very interesting.

Finally, we have also shown that the analysis in [25], which yields very precise bounds on possible decoherence is incorrect, as unjustified assumptions were made and the dominant uncertainty was simply neglected.

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