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Lifetime difference of B_s mesons - Theory status‡

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Abstract. We give an update on the lifetime difference of B_s mesons which accounts for recent lattice bag parameter results, and obtain $(\Delta\Gamma/\Gamma)_{B_s} = (9.3^{+3.4}_{-4.6})\%$. We then review the current theoretical uncertainties and conclude with a pessimistic perspective on further improvements.

1. Status of theory

The width difference in the B_s system is expected to be the largest rate difference in the B hadron sector [1]. Some experimental bounds on this quantity from LEP [2] and CDF [3] exist, and in the near future $\Delta\Gamma_{B_s}$ will be measured quite precisely [3]. Several factors contribute to the interest in $\Delta\Gamma_{B_s}$: a large value of the width difference opens up the possibility for novel studies of CP violation without the need for tagging [4]. Moreover, an experimental value of $\Delta\Gamma_{B_s}$ would give information about the mass difference in the B_s system [5] (although at this moment it appears that the mass difference will be measured sooner than the width difference). Another interesting point is that new physics can only lead to a decrease of the width difference compared to the standard model value [6]. An experimental number which is considerably smaller than the theoretical lower bound, would thus be a hint for new physics that affects B_s-$\bar{B}_s$ mixing. Besides the need for a reliable theoretical prediction of $\Delta\Gamma_s$ in order to fulfill the above physics program it is of conceptual interest to compare experiment and theory in order to test local quark-hadron duality, which is the underlying assumption

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in calculating heavy quark decay rates. One can show that duality holds exactly in the limit $\Lambda_{\mathrm{QCD}} \ll m_b - 2m_c \ll m_b$ and $N_c \to \infty$ [7]. So far no deviation from duality has been conclusively demonstrated experimentally and theoretical models of duality violation in B decays tend to predict rather small effects [8].

During the last year there has been remarkable interest in the lifetime difference of B_s mesons. Besides several experimental studies, many lattice calculations of the relevant nonperturbative constants were done [9], [10], [11]. This improvement in theory input motivates the present update of the result presented in [12]. We also clarify the origin of seemingly disagreeing recent evaluations of $\Delta\Gamma_{B_s}$.

The theoretical status of the lifetime difference computation is as follows: the heavy quark expansion (HQE) allows us to expand the decay rate of heavy mesons in inverse powers of the heavy quark mass. The leading term is described by the decay of a free quark (the so-called parton model) and therefore equal for all B hadrons. The next term is suppressed by two powers of the heavy quark mass; it is related to the kinetic and the chromomagnetic operator. The so-called weak annihilation and Pauli interference diagrams contribute first at third order. Operators involving the spectator quark begin to appear at this order and these are mainly responsible for the lifetime differences of B hadrons. (A smaller contribution comes from the matrix elements of the kinetic and chromomagnetic operator; for the lifetime difference of B_s mesons these terms vanish identically.) We can write for the decay rate difference of two mesons

$$\Delta\Gamma = \frac{\Lambda^3}{m_b^3}\left[\left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \dots\right) + \frac{\Lambda}{m_b}\left(\Gamma_4^{(0)} + \dots +\right) + \dots\right] \ .$$

The Γ_i's are products of perturbatively calculable Wilson coefficients and matrix elements, which have to be determined by some non-perturbative methods like lattice-QCD or sum rules. For a comparison of experiment and theory we need besides the experimental value several theoretical ingredients. First, the perturbative prediction in leading order ($\Gamma_3^{(0)}$), second, corrections due to the strong interaction ($\Gamma_3^{(1)}$: NLO-QCD), third, subleading $1/m_b$-corrections ($\Gamma_4^{(0)}$) and last, but not least, the determination of the appearing non-perturbative parameters. For $(\Delta\Gamma/\Gamma)_{B_s}$ we have all these pieces: the leading term was calculated in [13] in the factorization approximation; $1/m_b$-corrections were determined in [14]; the α_s-corrections are given in [12]; and the lattice values for the decay constant f_{B_s} and the bag parameters B can be taken from computations directed at the mass difference. B_S, which is specific to the width difference was calculated in [9], [10], [11]. (The matrix elements of the dimension-7-operators, which emerge in Γ_4, have been estimated up to now only in vacuum insertion approximation [14].) As the calculation of $\Gamma_3^{(1)}$ was the first to consider QCD corrections to spectator effects, there is also a conceptual point of interest. Soft gluon emission from the spectator s quark leads to power-like infrared divergences, which would spoil the HQE. In [12] the infrared safety of $\Delta\Gamma_s$ was explicitly shown, in agreement with the theoretical argument of [15].

Compared to $\Delta\Gamma_s$ our knowledge about the lifetimes ratios $\tau(B^+)/\tau(B_d)$ and $\tau(\Lambda_b)/\tau(B_d)$ is much poorer. Here only $\Gamma_3^{(0)}$ is known [16], while the calculation of $\Gamma_3^{(1)}$ and $\Gamma_4^{(0)}$ is still missing. Moreover we only have preliminary lattice studies for the

bag parameters in Γ_3 [17]. Results from sum rules for the bag parameters are discussed in [18]. For these lifetime ratios more work needs to be done.

2. Numerical update and uncertainties

The following update is based on the NLO expressions given in [12]. The width difference is normalized to the semi-leptonic B_s branching fraction; numerically we find

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{230\,\text{MeV}}\right)^2 \left[0.007B(m_b) + 0.132\frac{M_{B_s}^2 B_S(m_b)}{(\bar{m}_b + \bar{m}_s)^2} - 0.078\right] \quad (1)$$

We factored out the decay constant and use B and B_S to parametrize the matrix elements

$$\langle \bar{B}_s|(\bar{b}_i s_i)_{V-A}(\bar{b}_j s_j)_{V-A}|B_s\rangle = \frac{8}{3}f_{B_s}^2 M_{B_s}^2 B, \quad (2)$$

$$\langle \bar{B}_s|(\bar{b}_i s_i)_{S-P}(\bar{b}_j s_j)_{S-P}|B_s\rangle = -\frac{5}{3}f_{B_s}^2 M_{B_s}^2 \frac{M_{B_s}^2}{(\bar{m}_b + \bar{m}_s)^2}B_S. \quad (3)$$

Here $\bar{m}_q$ denote $\overline{\text{MS}}$ quark masses at the scale $m_b = 4.8\,\text{GeV}$. The third term in square brackets in Eq. (1) is an estimate of the $1/m_b$ correction in the factorization approximation [14]. (This correction is slightly larger than in [12], because we now use $\bar{m}_s = 0.1\,\text{GeV}$. We will also use $\bar{m}_b = 4.2\,\text{GeV}$.) We note that

i) the term involving the parameter $B(m_b)$, which also appears in the mass difference, is negligible;

ii) the NLO correction to the coefficient is large and reduces the width difference. In LO we obtain the coefficients 0.011 and 0.203 instead of 0.007 and 0.132, respectively;

iii) the $1/m_b$ correction is also large and negative, and its importance is amplified by the negative NLO correction.

As a consequence recent estimates of the width difference tend to be significantly smaller than the leading order estimate of [14] and those based on the factorization approximation ($B = B_S = 1$).

There is another way of representing the result (1) [10] based on the observation that $B(m_b)$ appears in the mass difference an the fact the mass difference for B_d mesons is accurately determined experimentally. Then

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\tau_{B_s}\Delta m_{B_d}\frac{M_{B_s}}{M_{B_d}}\right)\left|\frac{V_{ts}}{V_{td}}\right|^2 K\xi^2\left[0.030 - 0.937R_S(m_b) - \frac{0.35}{B}\right]. (4)$$

In this representation the value of the first bracket can be taken from experiments, and

$$K = \frac{4\pi}{3}\frac{m_b^2}{m_W^2}\left|\frac{V_{cb}V_{cs}}{V_{ts}V_{tb}}\right|^2\frac{1}{\eta_B(m_b)S_0(x_t)} \quad (5)$$

is a known factor, if we assume that the CKM matrix is unitary. The advantage of this representation is that hadronic uncertainties enter only in ratios, i.e. in

$$\xi^2 = \frac{f_{B_s}^2 B_{B_s}(m_b)}{f_{B_d}^2 B_{B_d}(m_b)}, \quad R_S(m_b) = -\frac{5}{8}\frac{M_{B_s}^2}{(\bar{m}_b + \bar{m}_s)^2}\frac{B_S(m_b)}{B(m_b)}, \quad (6)$$

	$B(m_b)$	$\bar{B}_S(m_b)$	f_{B_s}/MeV	$(\Delta\Gamma/\Gamma)_{B_s}$
BBGLN98 [12]	0.90	1.07	210	6.0%
Hashimoto *et al.* [9]	0.85 ± 0.11	1.24 ± 0.16	245	10.7%
Becirevic *et al.* [10]	$0.91(3)^{+0.00}_{-0.06}$	$1.32(3)^{+0.03}_{-0.04}$	uses Eq. (4)	4.7%
Gimenez/Reyes [11]	0.83 ± 0.08	1.25 ± 0.14	uses Eq. (4)	5.1%
this summary	0.9 ± 0.1	1.25 ± 0.1	$230 \pm 10\%$	9.3%

Table 1. Summary of recent evaluations of $(\Delta\Gamma/\Gamma)_{B_s}$. For theoretical uncertainties on $(\Delta\Gamma/\Gamma)_{B_s}$ consult text. $\bar{B}_S(m_b) \equiv M_{B_s}^2 B_S(m_b)/(\bar{m}_b + \bar{m}_s)^2$.

and these are believed to be better known than f_{B_s} and $B_S(m_b)$. This advantage is more than compensated by the need to know $|V_{ts}/V_{td}|$ in Eq. (4), which makes the prediction for $\Delta\Gamma_{B_s}$ sensitive to the global fits to the unitarity triangle and the theoretical assumptions that go into it. In particular, the prediction for $\Delta\Gamma_{B_s}$ now depends on the assumption that the standard model describes flavour mixing correctly, even though the decay of B_s meson is unlikely to be affected by new physics in mixing. For this reason we would rather discourage the use of this method to obtain $\Delta\Gamma_{B_s}$.

In Table 1 we present a summary of recent estimates for $\Delta\Gamma_{B_s}$, based on either Eq. (1) (default) or Eq. (4). (We use here only estimates based on lattice calculations of $B(m_b)$ and $B_S(m_b)$.) It is evident that there is remarkable agreement on the hadronic parameters $B(m_b)$ and $B_S(m_b)$, and that the spread of results is mainly caused by different values of f_{B_s} or the use of Eq. (4). In the following we first assess the theoretical uncertainties of the NLO calculation and then present our best evaluation, given as "this summary" in Table 1. The current theoretical uncertainties are as follows:

i) *Residual scale dependence.* This dependence arises from residual effects in the matching of the NLO Wilson coefficients for $\Delta\Gamma_{B_s}$ to the parameters B and B_S computed on the lattice, and from the matching of the NLO Wilson coefficients for $\Delta\Gamma_{B_s}$ to the Wilson coefficients of the $\Delta B = 1$ weak effective Hamiltonian. Here we estimate only the second source of scale dependence, using the coefficients for scales $m_b/2$ and $2m_b$ ($m_b = 4.8\,\mathrm{GeV}$) given in [12]. We then find

$$\delta\left(\frac{\Delta\Gamma}{\Gamma}\right)_{\mathrm{scale}} = \binom{+1.1\%}{-2.7\%} \cdot \left(\frac{f_{B_s}}{230\,\mathrm{MeV}}\right)^2 \frac{M_{B_s}^2 B_S(m_b)}{(\bar{m}_b + \bar{m}_s)^2}. \tag{7}$$

ii) *Normalization.* There is an overall normalization error in Eq. (1), which results from the uncertainty in the measurement of the semi-leptonic branching fraction and from the value of the b quark mass. We estimate this error to be 10%.

iii) $1/m_b$-*correction.* The second major source of theoretical uncertainty originates from the term "−0.078" in Eq. (1), which has been obtained using the factorization

assumption for the power-suppressed four-quark matrix elements [14]. To estimate this uncertainty, we introduce four parameters that measure the deviation from the factorization approximation, corresponding to the four independent operators at order $1/m_b$. We find that all $1/m_b$ effects add essentially constructively, so that there is no reason to suspect an amplification of corrections to the factorization approximation as a result of cancellations in the factorized expression. We then find

$$\delta \left(\frac{\Delta\Gamma}{\Gamma}\right)_{1/m_b} = (\pm 6\% \, \Delta r) \cdot \left(\frac{f_{B_s}}{230 \, \text{MeV}}\right)^2, \tag{8}$$

where Δr parametrizes a typical deviation from the factorization approximation, corresponding to quantities like $B - 1$, and $B_S - 1$ in leading power. There exists no reliable information on violations of the factorization assumption for the matrix elements in question, but experience with leading order matrix elements suggests that $\Delta r \approx 0.3$ is a reasonable upper limit.

iv) $B(m_b)$ *and* $B_S(m_b)$. Substantial work has been invested in controlling the nonperturbative parameters at leading power. Table 1 shows that the results are rather consistent with each other (the number 1.07 in the first line is obsolete) and suggests that

$$B(m_b) = 0.9 \pm 0.1, \qquad \bar{B}_S(m_b) \equiv \frac{M_{B_s}^2 B_S(m_b)}{(\bar{m}_b + \bar{m}_s)^2} = 1.25 \pm 0.10. \tag{9}$$

We note that at the present stage the uncertainty in B and B_S has become a minor factor, the dominant ones coming from residual scale dependence and $1/m_b$ corrections.

We can now combine these uncertainties with Eq. (1) to obtain our final result

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{230 \, \text{MeV}}\right)^2 \left(1 \pm 0.1\right) \Big[(13.2^{+1.1}_{-2.7})\% \, \bar{B}_S(m_b) + 0.7\% \, B(m_b)$$
$$- (7.8 \pm 1.8\%) \Big]$$
$$= \left(9.3^{+3.4}_{-4.6}\right)\%. \tag{10}$$

The final number is obtained by adding all errors in squares and assigning a 10% uncertainty to $f_{B_s} = 230 \, \text{MeV}$. It is worth noting that this number is considerably larger than the result obtained in [10, 11] for two reasons: first, the overall normalization obtained from Eq. (4) is about 25% smaller compared to that of Eq. (1) with $f_{B_s} = 230 \, \text{MeV}$, since the global CKM fit prefers a smaller value of the B meson decay constant; second, Ref. [10] uses $m_b = 4.6 \, \text{GeV}$, which increases the $1/m_b$ correction by almost a factor 1.5.

3. Prospects for improvement

Despite (or because of?) extensive work on radiative and $1/m_b$ corrections the theoretical prediction of the width difference of the B_s mass eigenstates remains rather uncertain.

This is due to an unfortunate conspiracy of negative corrections at next-to-leading order in α_s and in the heavy quark expansion. The large error exhibited by Eq. (10) makes it improbable that new physics would be first observed in the width difference, since a new physics contribution to the B_s-$\bar{B}_s$ mixing phase large enough to cause an effect larger than the theoretical uncertainty would rather been seen elsewhere, for instance as a time-dependent asymmetry in $B_s \to J/\psi\phi$.

Is further improvement possible? It is conceivable that future lattice calculations will reduce further the uncertainty in B and B_S, but the impact of the present uncertainty in these parameters on the total error is no longer dominant. Some improvement could be possible concerning the overall normalisation, but this effect can also not be substantial. The obvious targets for improvement are therefore

ii) scale dependence: its reduction would require the calculation or estimate of α_s^2 corrections. This appears to be a hard endeavour, and it would also require the computation of three-loop anomalous dimensions of the weak effective Hamiltonian.

ii) $1/m_b$ corrections: obviously, the relevant matrix elements should be computed on the lattice to rid the calculation from the factorization assumption. Such a calculation does not appear likely soon, given the familiar difficulties with higher dimension operators. Since the $1/m_b$ corrections are large, one may think of computing the α_s corrections to them. While this is feasible, the effort is probably too large as long as the matrix elements of the operators are not known accurately enough.

It therefore appears that $(\Delta\Gamma/\Gamma)_{B_s}$ will remain considerably uncertain for the foreseeable future.

Acknowledgments

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