

# The physical properties of $z > 2$ Lyman limit systems: new constraints for feedback and accretion models

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## ABSTRACT

We study the physical properties of a homogeneous sample of 157 optically thick absorption line systems at redshifts  $\sim 1.8$ – $4.4$ , selected from a high-dispersion spectroscopic survey of Lyman limit systems (LLSs). By means of multiple ionization models and Bayesian techniques, we derive the posterior probability distribution functions for the density, metallicity, temperature and dust content of the absorbing gas. We find that  $z > 2$  LLSs are highly ionized with ionization parameters between  $-3 \lesssim \log U \lesssim -2$ , depending on the H I column density. LLSs are characterized by low temperatures ( $T < 5 \times 10^4$  K) and reside in dust-poor environments. Between  $z \sim 2.5$ – $3.5$ ,  $\sim 80$  per cent of the LLSs have physical densities between  $n_{\text{H}} \sim 10^{-3.5}$ – $10^{-2}$  cm $^{-3}$  for the assumed UV background, but we caution that a degeneracy between the ionization parameter and the intensity of the radiation field prevents robust inference on the density and sizes of LLSs. Conversely, metallicity estimates are less sensitive to the assumptions behind ionization corrections. LLSs at  $z > 2$  are characterized by a broad unimodal distribution over  $> 4$  orders of magnitude, with a peak at  $\log Z/Z_{\odot} \sim -2$ . LLSs are metal poor, significantly less enriched than DLAs, with  $\sim 70$  per cent of the metallicity PDF below  $\log Z/Z_{\odot} \leq -1.5$ . The median metallicity of super LLSs with  $\log N_{\text{H I}} \geq 19$  rapidly evolves with redshift, with a 10-fold increase between  $z \sim 2.1$ – $3.6$  ( $\sim 1.5$  Gyr). Based on this sample, we find that LLSs at  $z = 2.5$ – $3.5$  account for  $\sim 15$  per cent of all the metals produced by UV-selected galaxies. The implications for theories of cold gas accretion and metal ejection from galaxies are also discussed.

**Key words:** nuclear reactions, nucleosynthesis, abundances – radiative transfer – methods: statistical – galaxies: haloes – galaxies: high-redshift – quasars: absorption lines.

## 1 INTRODUCTION

A detailed census of the metal budget in the Universe as a function of redshift provides valuable integral constraints to some of the most fundamental astrophysical processes that regulate galaxy evolution, including star formation, galactic outflows and gas accretion (e.g. Fukugita, Hogan & Peebles 1998; Bouché et al. 2007; Peebles et al. 2014; Rafelski et al. 2014). Both at high redshift ( $z \sim 2.5$ ) and in the local Universe, approximately  $\sim 1/3$  of the metals can be found within galaxies (Bouché et al. 2007; Peebles et al. 2014), with the more diffuse intergalactic medium (IGM) and circumgalactic medium (CGM) being a significant repository of cosmic metals. Indeed, metals are ubiquitously detected in absorption line studies, albeit with the notable exception of a few pristine gas pockets (Fumagalli, O’Meara & Prochaska 2011a; Simcoe et al. 2012).

Thus, the study of metallicity in absorption provides valuable insight into the chemical enrichment of cosmic structures across a wide range of densities, from the modest overdensities traced by the Ly  $\alpha$  forest in the IGM (Schaye et al. 2003; Simcoe et al. 2004; Simcoe 2011), up to the largest overdensities traced by damped Ly $\alpha$  systems (DLAs) found against quasars or  $\gamma$  –ray bursts sightlines (Pettini et al. 1994; Prochaska et al. 2003; Rafelski et al. 2012; Cucchiara et al. 2015).

Until recently, a bottleneck in compiling a full census of cosmic metals was the lack of a systematic study of the metallicity in large samples of Lyman limit systems (LLSs), defined as optically thick clouds with neutral hydrogen column densities between  $17.2 \leq \log N_{\text{H I}} < 20.3$ .<sup>1</sup> Differently from DLAs and the

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<sup>1</sup> Throughout this paper, logarithmic column densities are expressed in units of cm $^{-2}$ .

IGM, which have been the subject of dedicated campaigns to characterize their metal content, LLSs have received far less attention in past years, with most previous studies focusing on small samples, often restricted to the subset of so-called super-LLSs (SLLSs) with  $19 \leq \log N_{\text{H I}} < 20.3$ . Part of this unbalance can be attributed to the difficulties in measuring  $N_{\text{H I}}$  in optically thick absorbers between  $17.2 \leq \log N_{\text{H I}} < 19$ , for which saturated hydrogen Lyman series lines and the lack of damping wings prevent precise estimates of the neutral hydrogen column density. Furthermore, differently from the study of metals in neutral DLAs, LLSs are significantly ionized, and observers can only access tracers of the underlying metal content of LLSs via metal lines. Detailed ionization modelling is thus needed to infer the intrinsic metal content of LLSs (see Section 4).

One of the first studies of the LLS metallicity was presented in Steidel (1990), who analysed eight systems between  $2.90 < z < 3.23$  with column densities  $17.0 < \log N_{\text{H I}} \leq 19.3$ , finding with photoionization calculations metallicity between  $-3.0 < \log Z/Z_{\odot} \leq -1.5$ . Since this study, however, the attention has focused mostly on SLLSs. For instance, Dessauges-Zavadsky et al. (2003) studied 12 SLLSs between  $z \sim 1.8\text{--}4.3$  and conducted photoionization modelling to assess the importance of ionization corrections (ICs), concluding that generally ICs are  $< 0.2$  dex for  $\log N_{\text{H I}} \geq 19.3$  and thus considered negligible. Péroux et al. (2007) and Péroux et al. (2008) studied the abundance of six SLLSs at  $z < 1.5$  and 13 SLLSs at  $z \geq 3$ . Again using photoionization modelling, they concluded that ICs are small compared to observational uncertainties, typically below  $\sim 0.2$  dex and not exceeding  $\sim 0.35$  dex. Together with values from the literature, they reported that the metallicity of SLLSs evolves more rapidly with redshift than for DLAs, and that SLLSs tend to have higher metallicity than DLAs especially at  $z < 2$  (cf. Kulkarni et al. 2007). Nevertheless, SLLSs do not appear to substantially contribute to the total metal budget of cosmic structures at  $z \sim 2.5$  (but see Prochaska et al. 2006). More recently, Som et al. (2013) presented the analysis of the abundances of five SLLSs between  $1.7 < z < 2.4$  with the aid of ionization models, finding that a varying degree of ionization correction was needed for individual systems. Together with a large compilation of data from the literature, they also concluded that SLLSs are on average more enriched than DLAs, and may evolve faster with redshift (see also Meiring et al. 2009b; Battisti et al. 2012), a result that could be partially explained by different selections as a function of redshift (Dessauges-Zavadsky, Ellison & Murphy 2009).

In recent years, there has been a renewed interest in probing the metal content of LLSs at lower column densities in the context of CGM studies and the postulated connection between LLSs and cold accretion (e.g. Faucher-Giguère & Kereš 2011; Fumagalli et al. 2011b, 2014; van de Voort et al. 2012). Fumagalli et al. (2013) presented the analysis of a composite spectrum of 38 LLSs at  $z \sim 2.8$  together with simple ionization modelling, finding evidence that LLSs have typical metallicity below  $\log Z/Z_{\odot} \sim -1.5$ . Lehner et al. (2013) studied 28 partial LLSs and LLSs with column densities  $16.2 \lesssim \log N_{\text{H I}} \lesssim 18.5$  at  $z \lesssim 1$ , uncovering a bimodal distribution with two branches peaking at  $\log Z/Z_{\odot} \sim -1.6$  and  $\log Z/Z_{\odot} \sim -0.3$ . Finally, Cooper et al. (2015) presented the analysis of 17 LLSs at  $z \sim 3.2\text{--}4.4$ , selected based on the lack of visible metal lines in Sloan Digital Sky Survey (SDSS) spectra. Together with the study of a small but representative sample of LLSs, after ionization corrections, they offered additional evidence that high-redshift LLSs span a range of metallicity between  $-3.0 < \log Z/Z_{\odot} \leq -1.5$ .

In this work, we aim to provide a coherent analysis of the metal content of a large sample of LLSs from the High Dispersion

Lyman Limit System (HD-LLS) survey (Prochaska et al. 2015), which includes 157 LLSs with column densities  $17.3 \leq \log N_{\text{H I}} < 20.3$  between  $z \sim 1.76\text{--}4.39$ . This study represents a 10-fold increase in the sample size for analyses of the metal abundances of LLSs derived from high-dispersion data (Section 2). By means of multiple grids of ionization models, combined with a Bayesian formalism and Markov Chain Monte Carlo (MCMC) techniques (Section 3), we derive posterior probability distribution functions (PDFs) for the metallicity and the physical density of LLSs, also assessing the robustness of ICs (Section 4), and exploring possible systematic effects arising from different model assumptions. We also include a homogeneous reanalysis of data for systems with  $17.2 \leq \log N_{\text{H I}} < 20.3$  from the literature, especially at  $z < 2$ . Finally, we discuss the physical properties of LLSs (Section 5), both in the context of the cosmic budget of metals (Section 5.3) and of the properties of the CGM (Section 5.4). Summary and conclusions follow in Section 6. The readers who are primarily interested in the astrophysical implications of our work may wish to focus mainly on Section 5. Throughout this work, we assume solar abundances from Asplund et al. (2009) for which  $Z_{\odot} = 0.0134$ , and the ‘Planck 2013’ cosmology (Planck Collaboration 2014) with Hubble constant  $H_0 = (67.8 \pm 0.78) \text{ km s}^{-1} \text{ Mpc}^{-1}$  and matter density parameter  $\Omega_{\text{m}} = 0.308 \pm 0.010$ .

## 2 OBSERVATIONS

Our primary data set includes 157 LLSs from the HD-LLS survey presented in Prochaska et al. (2015), which is composed by an H I selected sample of optically thick absorbers between  $z = 1.76$  and  $4.39$ . Systems from this sample have been observed at high resolution with echelle or echellette spectrographs at the Keck and Magellan telescopes. We refer the reader to the work by Prochaska et al. (2015) for additional details on the observations, data reduction and analysis, including the measurement of column densities for hydrogen and metal ions. Relevant to this analysis, column densities have been measured using the apparent optical depth method (Savage & Sembach 1991) focusing on transitions outside the Ly  $\alpha$  forest. This means that common transitions such as O VI, C III, Si III are not included for the majority of the systems analysed. The implications of this restriction will be discussed below.

In this study, we also collect 77 LLSs (strictly with  $17.3 \leq \log N_{\text{H I}} < 20.3$ ) from the literature, when ion column densities are available (see Table 1), bringing the total sample to 234 systems. Although we analyse literature data together with our own sample in a self-consistent manner, in the following we will primarily refer to the HD-LLS subset for a statistical analysis of the LLS population, thus avoiding possible selection biases from heterogeneous compilations of LLSs in the literature. A list of the LLSs included in this study and a summary of their redshifts and H I column densities is provided in Table 1 and Fig. 1. As highlighted in Fig. 1, by leveraging the HD-LLS sample, our analysis significantly augments the samples used in previous studies, especially for  $17.3 \leq \log N_{\text{H I}} < 19.0$ .

## 3 IONIZATION MODELLING AND PARAMETER ESTIMATION

Differently from the higher column-density DLAs, the bulk of the gas in SLLSs and LLSs is not fully neutral, and therefore, the observed ions are only tracers of the underlying chemical abundance of the gas. It is therefore necessary to apply ICs to translate the observed ion abundance into a measurement of the gas-phase

**Table 1.** List of the LLSs included in this study.

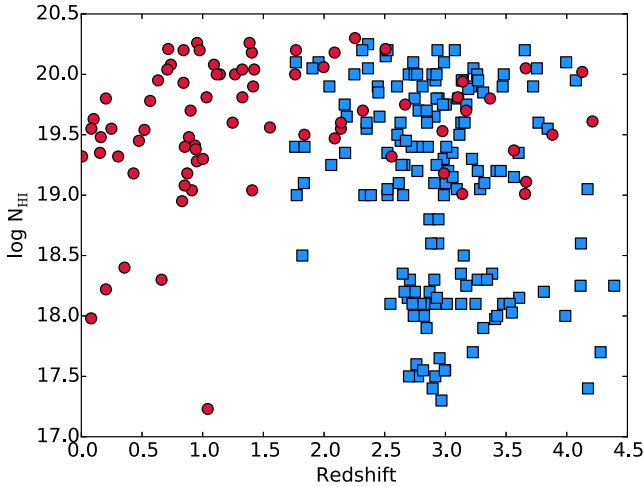
System	Redshift	$\log N_{\text{HI}}^a$	Reference	System	Redshift	$\log N_{\text{HI}}$	Reference
HDLLS statistical sample							
J000345–232346z2.187	2.187	19.65 ± 0.15	[P15]	J102509+045246z3.130	3.130	18.10 ± 0.50*	[P15]
J003454+163920z3.754	3.754	20.05 ± 0.20	[P15]	J102832–004607z2.824	2.824	18.00 ± 0.30	[P15]
J004049–402514z2.816	2.816	17.55 ± 0.15	[P15]	J103249+054118z2.761	2.761	17.60 ± 0.20	[P15]
J010355–300946z2.908	2.908	19.10 ± 0.15	[P15]	J103456+035859z2.849	2.849	19.60 ± 0.20	[P15]
J010516–184642z2.927	2.927	20.00 ± 0.20	[P15]	J103456+035859z3.003	3.003	19.10 ± 0.15	[P15]
J010619+004823z3.286	3.286	19.05 ± 0.25	[P15]	J103456+035859z3.059	3.059	19.15 ± 0.25	[P15]
J010619+004823z3.321	3.321	19.10 ± 0.20	[P15]	J103514+544040z2.457	2.457	19.65 ± 0.25	[P15]
J010619+004823z4.172	4.172	19.05 ± 0.20	[P15]	J103514+544040z2.846	2.846	19.70 ± 0.15	[P15]
J012156+144823z2.662	2.662	19.25 ± 0.20	[P15]	J104018+572448z3.266	3.266	18.30 ± 0.60*	[P15]
J012403+004432z3.078	3.078	20.20 ± 0.20	[P15]	J110325–264506z1.839	1.839	19.40 ± 0.15	[P15]
J012700–004559z2.944	2.944	19.80 ± 0.20	[P15]	J110708+043618z2.601	2.601	19.90 ± 0.20	[P15]
J013340+040059z3.995	3.995	20.10 ± 0.30	[P15]	J111008+024458z3.476	3.476	18.10 ± 0.40*	[P15]
J013340+040059z4.117	4.117	18.60 ± 0.80*	[P15]	J111113–080402z3.481	3.481	20.00 ± 0.20	[P15]
J013421+330756z4.279	4.279	17.70 ± 0.15	[P15]	J111113–080402z3.811	3.811	18.20 ± 0.50*	[P15]
J014850–090712z2.995	2.995	17.55 ± 0.15	[P15]	J113130+604420z2.362	2.362	20.05 ± 0.15	[P15]
J015741–010629z2.631	2.631	19.45 ± 0.20	[P15]	J113418+574204z3.410	3.410	17.97 ± 0.19	[P15]
J015741–010629z3.385	3.385	18.35 ± 0.75*	[P15]	J113621+005021z3.248	3.248	18.10 ± 0.60*	[P15]
J020455+364918z1.955	1.955	20.10 ± 0.20	[P15]	J115659+551308z2.616	2.616	19.10 ± 0.30	[P15]
J020455+364918z2.690	2.690	18.15 ± 0.65*	[P15]	J115906+133737z3.723	3.723	19.90 ± 0.15	[P15]
J020944+051717z3.988	3.988	18.00 ± 0.30	[P15]	J115940–003203z1.904	1.904	20.05 ± 0.15	[P15]
J020950–000506z2.523	2.523	19.05 ± 0.15	[P15]	J120331+152254z2.708	2.708	18.30 ± 0.70*	[P15]
J020951–000513z2.523	2.523	19.00 ± 0.20	[P15]	J120918+095427z2.363	2.363	20.25 ± 0.20	[P15]
J020951–000513z2.547	2.547	18.10 ± 0.50*	[P15]	J120918+095427z3.023	3.023	19.20 ± 0.30	[P15]
J023903–003850z2.868	2.868	18.80 ± 0.45*	[P15]	J121539+090608z2.523	2.523	20.20 ± 0.20	[P15]
J024122–363319z2.739	2.739	18.00 ± 0.70*	[P15]	J124820+311043z4.075	4.075	19.95 ± 0.15	[P15]
J030341–002322z2.443	2.443	19.90 ± 0.15	[P15]	J124957–015928z3.530	3.530	18.10 ± 0.40*	[P15]
J030341–002322z2.941	2.941	18.60 ± 0.30*	[P15]	J125336–022808z3.603	3.603	19.35 ± 0.20	[P15]
J033854–000520z2.746	2.746	20.00 ± 0.20	[P15]	J125759–011130z2.918	2.918	19.95 ± 0.20	[P15]
J033900–013318z3.116	3.116	19.50 ± 0.20	[P15]	J130756+042215z2.250	2.250	20.00 ± 0.15	[P15]
J034024–051909z2.174	2.174	19.35 ± 0.20	[P15]	J130756+042215z2.749	2.749	18.20 ± 0.60*	[P15]
J034227–260243z3.012	3.012	18.10 ± 0.30	[P15]	J132554+125546z3.767	3.767	19.60 ± 0.20	[P15]
J034402–065300z3.843	3.843	19.55 ± 0.15	[P15]	J132729+484500z3.058	3.058	19.35 ± 0.25	[P15]
J042610–220217z4.175	4.175	17.40 ± 0.15	[P15]	J133146+483826z2.910	2.910	19.65 ± 0.35*	[P15]
J043906–504740z2.796	2.796	18.10 ± 0.60*	[P15]	J133254+005251z3.421	3.421	19.20 ± 0.20	[P15]
J045142–132033z2.998	2.998	17.55 ± 0.15	[P15]	J133757+021820z3.270	3.270	19.95 ± 0.15	[P15]
J073149+285448z3.608	3.608	18.15 ± 0.45*	[P15]	J133942+054822z2.952	2.952	17.65 ± 0.25	[P15]
J073621+651312z2.909	2.909	18.30 ± 0.70*	[P15]	J134002+110630z2.508	2.508	20.15 ± 0.15	[P15]
J075155+451619z2.927	2.927	19.80 ± 0.20	[P15]	J134811+281802z2.448	2.448	19.85 ± 0.15	[P15]
J081054+460358z3.472	3.472	19.90 ± 0.30	[P15]	J134816–013509z2.883	2.883	18.60 ± 0.70*	[P15]
J081435+502946z3.004	3.004	19.75 ± 0.15	[P15]	J134939+124230z3.158	3.158	19.60 ± 0.30	[P15]
J081618+482328z3.343	3.343	18.30 ± 0.50*	[P15]	J135706–174401z3.007	3.007	19.40 ± 0.25	[P15]
J082619+314848z2.856	2.856	19.40 ± 0.20	[P15]	J140243+590958z2.986	2.986	19.30 ± 0.30	[P15]
J082849+085854z2.044	2.044	19.90 ± 0.10	[P15]	J140248+014634z3.456	3.456	19.20 ± 0.30	[P15]
J085959+020519z2.845	2.845	17.90 ± 0.60*	[P15]	J140747+645419z2.935	2.935	20.20 ± 0.20	[P15]
J091210+054742z2.522	2.522	19.35 ± 0.20	[P15]	J142903–014518z3.427	3.427	18.00 ± 0.40*	[P15]
J091546+054942z2.663	2.663	18.20 ± 0.70*	[P15]	J145408+511443z3.231	3.231	20.05 ± 0.15	[P15]
J092459+095103z3.219	3.219	19.30 ± 0.20	[P15]	J145649–193852z2.170	2.170	19.75 ± 0.20	[P15]
J092705+562114z1.775	1.775	19.00 ± 0.10	[P15]	J145649–193852z2.351	2.351	19.55 ± 0.20	[P15]
J093153–000051z2.927	2.927	19.25 ± 0.25	[P15]	J145807+120937z2.648	2.648	18.35 ± 1.05*	[P15]
J094253–110425z2.917	2.917	17.50 ± 0.15	[P15]	J145907+002401z2.767	2.767	20.00 ± 0.20	[P15]
J094932+033531z3.311	3.311	19.85 ± 0.15	[P15]	J150654+522005z4.114	4.114	18.25 ± 0.65*	[P15]
J095256+332939z3.144	3.144	19.95 ± 0.20	[P15]	J150932+111313z1.821	1.821	18.50 ± 0.50*	[P15]
J095256+332939z3.211	3.211	19.90 ± 0.20	[P15]	J151047+083535z2.722	2.722	19.40 ± 0.40*	[P15]
J095256+332939z3.262	3.262	20.00 ± 0.30	[P15]	J155036+053749z2.980	2.980	19.75 ± 0.25	[P15]
J095309+523029z1.768	1.768	20.10 ± 0.10	[P15]	J155103+090849z2.700	2.700	17.50 ± 0.20	[P15]
J095542+411655z2.812	2.812	19.90 ± 0.15	[P15]	J155556+480015z3.131	3.131	19.60 ± 0.15	[P15]
J100428+001825z2.746	2.746	19.80 ± 0.20	[P15]	J155738+232057z2.773	2.773	19.40 ± 0.40*	[P15]
J101539+111815z2.870	2.870	18.20 ± 0.70*	[P15]	J155810–003120z2.630	2.630	19.60 ± 0.20	[P15]
J101939+524627z1.834	1.834	19.10 ± 0.30	[P15]	J160843+071508z1.763	1.763	19.40 ± 0.30	[P15]
J161545+060852z2.988	2.988	19.00 ± 0.50*	[P15]	J212916+003756z2.735	2.735	20.10 ± 0.20	[P15]
J162116–004250z3.105	3.105	19.80 ± 0.20	[P15]	J212916+003756z2.917	2.917	18.10 ± 0.40*	[P15]
J171227+575506z2.315	2.315	20.20 ± 0.15	[P15]	J214144–384041z2.893	2.893	20.00 ± 0.15	[P15]
J171227+575506z2.849	2.849	18.10 ± 0.50*	[P15]	J220639–181846z2.698	2.698	20.00 ± 0.15	[P15]

Table 1 – continued

System	Redshift	$\log N_{\text{H I}}$	Reference	System	Redshift	$\log N_{\text{H I}}$	Reference
HDLLS statistical sample (continued)							
J172323+224358z4.391	4.391	$18.25 \pm 0.25$	[P15]	J223408+000001z2.652	2.652	$19.00 \pm 0.30^*$	[P15]
J173352+540030z2.779	2.779	$19.70 \pm 0.20$	[P15]	J223438+005730z2.604	2.604	$19.50 \pm 0.25$	[P15]
J173352+540030z3.151	3.151	$18.50 \pm 0.60^*$	[P15]	J223819–092106z3.128	3.128	$18.35 \pm 0.65^*$	[P15]
J183753–584809z2.729	2.729	$18.10 \pm 0.60^*$	[P15]	J224147+135203z3.654	3.654	$20.20 \pm 0.20$	[P15]
J200324–325144z3.172	3.172	$19.75 \pm 0.15$	[P15]	J230301–093930z3.312	3.312	$17.90 \pm 0.20$	[P15]
J200324–325144z3.188	3.188	$19.88 \pm 0.13$	[P15]	J231543+145606z2.943	2.943	$18.80 \pm 0.30$	[P15]
J200324–325144z3.548	3.548	$18.03 \pm 0.23$	[P15]	J231543+145606z3.135	3.135	$19.95 \pm 0.15$	[P15]
J205344–354652z2.332	2.332	$19.00 \pm 0.25$	[P15]	J231643–334912z2.386	2.386	$19.00 \pm 0.20$	[P15]
J205344–354652z2.350	2.350	$19.60 \pm 0.25$	[P15]	J231934–104036z2.675	2.675	$19.45 \pm 0.15$	[P15]
J205344–354652z2.989	2.989	$20.10 \pm 0.15$	[P15]	J232340+275800z3.267	3.267	$19.20 \pm 0.60^*$	[P15]
J205344–354652z3.094	3.094	$19.05 \pm 0.15$	[P15]	J232340+275800z3.565	3.565	$19.15 \pm 0.35^*$	[P15]
J205344–354652z3.172	3.172	$18.25 \pm 0.55^*$	[P15]	J233446–090812z3.226	3.226	$17.70 \pm 0.30$	[P15]
J212329–005052z2.059	2.059	$19.25 \pm 0.15$	[P15]	J234855–144436z2.775	2.775	$17.50 \pm 0.20$	[P15]
J212912–153841z2.638	2.638	$19.25 \pm 0.15$	[P15]	J235057–005209z2.930	2.930	$18.15 \pm 0.75^*$	[P15]
J212912–153841z2.769	2.769	$19.20 \pm 0.20$	[P15]	J235833–544042z2.895	2.895	$17.40 \pm 0.20$	[P15]
J212912–153841z2.968	2.968	$17.30 \pm 0.20$	[P15]				
Literature sample							
J000520+052410z0.851	0.851	$19.08 \pm 0.04$	[M09]	J122037–004032z0.975	0.975	$20.20 \pm 0.07$	[M08]
J001210–012207z1.386	1.386	$20.26 \pm 0.05$	[M09]	J122414+003709z1.266	1.266	$20.00 \pm 0.07$	[M07]
J002127+010420z1.326	1.326	$20.04 \pm 0.11$	[M09]	J122607+173649z2.557	2.556	$19.32 \pm 0.15$	[D03]
J002133+004300z0.520	0.520	$19.54 \pm 0.05$	[D09]	J122836+101841z0.938	0.938	$19.41 \pm 0.05$	[M08]
J002133+004300z0.942	0.942	$19.38 \pm 0.13$	[D09]	J131119–012030z1.762	1.762	$20.00 \pm 0.08$	[S13]
J011800+032000z4.128	4.128	$20.02 \pm 0.15$	[P07]	J131956+272808z0.661	0.661	$18.30 \pm 0.30$	[K12]
J012126+034707z2.976	2.976	$19.53 \pm 0.10$	[P07]	J132323–002155z0.716	0.716	$20.21 \pm 0.20$	[P06b]
J012403+004432z2.988	2.988	$19.18 \pm 0.10$	[P07]	J133007–205616z0.853	0.853	$19.40 \pm 0.05$	[M08]
J013340+040100z3.139	3.139	$19.01 \pm 0.10$	[P07]	J141217+091625z2.668	2.668	$19.75 \pm 0.10$	[D03]
J013405+005109z0.842	0.842	$19.93 \pm 0.15$	[P06a]	J143511+360437z0.203	0.203	$19.80 \pm 0.10$	[B12]
J013724–422417z3.101	3.101	$19.81 \pm 0.10$	[P07]	J143645–005150z0.738	0.738	$20.08 \pm 0.11$	[M08]
J013724–422417z3.101	3.665	$19.11 \pm 0.10$	[P07]	J144653+011355z2.087	2.087	$20.18 \pm 0.10$	[D03]
J015428+044818z0.160	0.160	$19.48 \pm 0.10$	[S15]	J145418+121054z2.255	2.255	$20.30 \pm 0.15$	[D03]
J015733–004824z1.416	1.416	$19.90 \pm 0.07$	[D09]	J145418+121054z3.171	3.171	$19.70 \pm 0.15$	[D03]
J021857+081728z1.769	1.769	$20.20 \pm 0.10$	[D09]	J145508–004507z1.093	1.093	$20.08 \pm 0.06$	[M08]
J031155–765151z0.203	0.203	$18.22 \pm 0.20$	[L09]	J151326+084850z2.088	2.088	$19.47 \pm 0.10$	[D03]
J035405–272425z1.405	1.405	$20.18 \pm 0.15$	[M07]	J152510+002633z0.567	0.567	$19.78 \pm 0.08$	[D09]
J042707–130253z1.408	1.408	$19.04 \pm 0.05$	[M09]	J155103+090849z2.320	2.320	$19.70 \pm 0.05$	[S13]
J044117–431343z0.101	0.101	$19.63 \pm 0.15$	[S15]	J155304+354828z0.083	0.083	$19.55 \pm 0.15$	[B12]
J045608–215909z0.474	0.474	$19.45 \pm 0.05$	[S15]	J163145+115602z0.900	0.900	$19.70 \pm 0.05$	[M09]
J082601–223026z0.911	0.911	$19.04 \pm 0.05$	[M07]	J163428+703132z1.041	1.041	$17.23 \pm 0.15$	[Z04]
J092554+400414z0.248	0.248	$19.55 \pm 0.15$	[B12]	J205145+195006z1.116	1.116	$20.00 \pm 0.15$	[M09]
J092837+602521z0.154	0.154	$19.35 \pm 0.15$	[B12]	J210244–355306z2.507	2.507	$20.21 \pm 0.10$	[D03]
J100102+594414z0.303	0.303	$19.32 \pm 0.10$	[B12]	J211927–353741z1.996	1.996	$20.06 \pm 0.10$	[D03]
J100902+071343z0.356	0.356	$18.40 \pm 0.41$	[T11]	J213135–120705z0.430	0.429	$19.18 \pm 0.05$	[S15]
J100930–002619z0.843	0.843	$20.20 \pm 0.06$	[M07]	J215145+213013z1.002	1.002	$19.30 \pm 0.10$	[N08]
J100930–002619z0.887	0.887	$19.48 \pm 0.05$	[M07]	J215501–092224z0.081	0.081	$17.98 \pm 0.05$	[J05]
J101033–004724z1.327	1.327	$19.81 \pm 0.05$	[M07]	J215502+135826z3.142	3.142	$19.94 \pm 0.10$	[D03]
J102837–010028z0.632	0.632	$19.95 \pm 0.07$	[D09]	J215502+135826z3.565	3.565	$19.37 \pm 0.15$	[D03]
J102837–010028z0.709	0.709	$20.04 \pm 0.06$	[D09]	J215502+135826z4.212	4.212	$19.61 \pm 0.10$	[D03]
J103744+002809z1.424	1.424	$20.04 \pm 0.12$	[M08]	J221527–161133z3.656	3.656	$19.01 \pm 0.15$	[P07]
J103921–271916z2.139	2.139	$19.55 \pm 0.15$	[S13]	J221527–161133z3.662	3.662	$20.05 \pm 0.15$	[P07]
J103921–271916z2.139	2.139	$19.60 \pm 0.10$	[D09]	J221651–671443z3.368	3.368	$19.80 \pm 0.10$	[P07]
J105440–002048z0.830	0.830	$18.95 \pm 0.18$	[M08]	J233121+003807z1.141	1.141	$20.00 \pm 0.05$	[M07]
J105440–002048z0.951	0.951	$19.28 \pm 0.05$	[M08]	J234403+034226z3.882	3.882	$19.50 \pm 0.10$	[D03]
J110325–264515z1.838	1.838	$19.50 \pm 0.05$	[D03]	J235253–002851z0.873	0.873	$19.18 \pm 0.09$	[M09]
J110736+000328z0.954	0.954	$20.26 \pm 0.14$	[M06]	J235253–002851z1.032	1.032	$19.81 \pm 0.13$	[M09]
J121549–003432z1.554	1.554	$19.56 \pm 0.05$	[M08]	J235253–002851z1.247	1.247	$19.60 \pm 0.24$	[M09]
J121920+063838z0.006	0.006	$19.32 \pm 0.03$	[T05]				

Notes. \*Asterisks mark uncertain column densities for which we assume a flat distribution centred on the listed value and with half width defined by the quoted error.

References: [D03] Dessauges-Zavadsky et al. (2003); [Z04] Zonak et al. (2004); [T05] Tripp et al. (2005); [J05] Jenkins et al. (2005); [P06a] Péroux et al. (2006a); [P06b] Péroux et al. (2006); [P07] Péroux et al. (2007); [M07] Meiring et al. (2007); [M08] Meiring et al. (2008); [N08] Nestor et al. (2008); [M09] Meiring et al. (2009a); [D09] Dessauges-Zavadsky et al. (2009); [L09] Lehner et al. (2009); [T11] Tumlinson et al. (2011); [K12] Kacprzak et al. (2012); [B12] Battisti et al. (2012); [S13] Som et al. (2013); [S15] Som et al. (2015); [P15] Prochaska et al. (2015).



**Figure 1.** Scatter plot of the redshift and hydrogen column density of the LLSs included in this study. The HD-LLS sample is shown with blue squares, while the red circles mark data from the literature.

metallicity. The standard technique followed by many authors is to compute ICs relying on parametric one-dimensional radiative transfer calculations, typically at equilibrium, in which radiation with specified spectral properties strikes the face of a ‘cloud’ that is generally assumed to be a slab of homogeneous medium with specified physical characteristics (e.g. density and metallicity). This problem is very tractable from a theoretical point of view, thanks to publicly available radiative transfer codes. However, in real astrophysical environments, many of the parameters that specify the geometry of the problem, the incident radiation field or the sources of opacity in the radiative transfer equation, are unconstrained. Thus, common practice is to generate large grids of models, and to use observables to constrain the unknown parameters.

Once a parametric grid of models is at hand, one wishes to identify the ‘best’ set of parameters  $\theta$  by comparing observables  $N$  with model predictions  $M$ . In this context,  $N$  is a set of column densities  $N_x$  and associated errors  $\sigma_x$ , while  $M$  is a set of column densities  $\bar{N}_x$  computed from parametric ionization models. Here, the vector index for  $\theta$  runs over the  $D$  parameters that define the dimension of the problem, and the vector indexes for  $M$  and  $N$  run over all the column densities of  $i$ th elements in  $j$ th ionization stages. In the following, we will adopt a Bayesian formalism (see also Cooper et al. 2015; Crighton et al. 2015), with which we can explore the posterior PDFs for parameters of interest (e.g. density and metallicity), after marginalizing over additional nuisance parameters that describe the radiative transfer problem (e.g. dust, local sources, temperature).

More specifically, the joint posterior PDFs of the parameters given the observables and the models is defined by

$$p(\theta|N, M) = \frac{p(\theta|M)p(N|\theta, M)}{p(N|M)}, \quad (1)$$

with  $p(\theta|M)$  the prior on the parameters given the models,  $p(N|\theta, M)$  is the likelihood of the data given the models, and  $p(N|M)$  is the marginal distribution. In this work, the likelihood is defined as product of Gaussian functions

$$p(N|\theta, M) = \prod_{x=1}^{i \times j} \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left(-\frac{(N_x - \bar{N}_x(\theta))^2}{2\sigma_x^2}\right). \quad (2)$$

In presence of upper or lower limits for a given ion, the corresponding term in the product of the right-hand side of equation (2) is replaced by a rescaled cumulative distribution function or

Q-function, respectively. Throughout this work, we will attribute equal probability to the values of unknown parameters (e.g. volume density, metallicity) by means of flat priors. For the redshift and the H I column density, which are measured for each individual system, we assume instead Gaussian priors centred at the observed values. The only exception is for LLSs with saturated Lyman series lines, for which we assume a top-hat function between the minimum and maximum values allowed by observations (for more details see Prochaska et al. 2015, and Table 1). To reconstruct the posterior PDFs for individual systems, we sample the full parameter space using EMCEE (Foreman-Mackey et al. 2013), an affine invariant MCMC ensemble sampler. In Appendix A, we present results of the analysis of mock data to validate our procedure.

We stress that an underlying assumption of this method is that we are comparing model predictions to observations after integrating the ionic column densities over the entire depth of the absorbing cloud. This means that, effectively, we are smoothing possible dishomogeneities in the metal distribution of individual LLSs (e.g. Prochter et al. 2010). Furthermore, the number of transitions included in the analysis of each system varies according to the wavelength range of the spectra, the data quality, and the redshift of the LLS. We refer the readers to Prochaska et al. (2015) for details on individual absorbers, noting that about eight metal transitions are included for a typical LLS.

#### 4 SYSTEMATIC UNCERTAINTIES IN THE IONIZATION CORRECTIONS

In this work, we use the CLOUDY (c13.03; Ferland et al. 2013) radiative transfer code to construct grids of ionization models. Given the limited number of transitions accessible in spectra, it is generally not practical to constrain all the possible free parameters relevant to this radiative transfer problem. Thus, a few simplifying assumptions are often necessary, or at least routinely applied in the literature. Nevertheless, understanding the implications of the assumptions made in the derivation of physical quantities is of clear importance, despite being a computationally intensive task. Subtle systematic errors or any unexplored degeneracy may in fact taint the final results.

In this section, before turning our attention to the statistical analysis of the LLS metallicity and its astrophysical implications, we aim to discuss the robustness of the applied IC by comparing the inferred distributions of metallicity and density under varying model assumptions. Our goal is to provide a simple but quantitative assessment of the degeneracy related to ICs in this type of work. We will start by considering a ‘minimal’ model, that is the model with the least number of free parameters. Next, we will compare the results from this simple but widely used model with results from more complex models, in which additional degrees of freedom are introduced. In practice, it may not always be possible to adopt heavily parametric models, but these different calculations should offer, to first order, an estimate of the systematic uncertainties of our results.

##### 4.1 Minimal model

Our minimal model consists of a static gas slab of constant density  $n_{\text{H}}$ , and thus, we restrict our analysis to a single phase medium. This gas slab is illuminated on one side by the redshift-dependent metagalactic UV background radiation  $J_{\text{uvb}}(z)$  as specified by the Haardt & Madau (2012) model. The cosmic microwave background is also included. The geometry of the problem is specified by the neutral hydrogen column density  $N_{\text{H I}}$ , which in turn defines the depth of the cloud  $\Delta r$ . In the minimal model, we do not include the effects

**Table 2.** Free parameters in the minimal grid of models.

Parameter	Min.	Max.	Step
$\log Z/Z_{\odot}$	-4.0	1.0	0.25
$z$	0.0	4.5	0.25
$\log N_{\text{H I}} (\text{cm}^{-2})$	17.0	20.5	0.25
$\log n_{\text{H}} (\text{cm}^{-3})$	-4.5	0.0	0.25

The columns of the table are: (1) the free parameter as described in the text; (2) the minimum allowed value; (3) the maximum allowed value; (4) the step adopted in the grid.

of grains, and we assume that all the metals are in the gas phase. The relative abundance of each element  $A_i$  is assumed to be consistent with the solar neighbourhood, as compiled in Asplund et al. (2009). Each CLOUDY model is iterated until a converged solution is reached. Table 2 summarizes the parameter space covered by this minimal grid of models. The grid has dimension  $D = 4$ , but only two parameters (density and metallicity) are unconstrained, as both redshift and column density can be directly measured with varying degree of accuracy. Throughout this analysis, we focus on a set of ions that are commonly detected in our sample of LLSs (see e.g. Prochaska et al. 2015).

Before diving into the derivation of the density and metallicity posterior PDFs, we briefly discuss how ICs vary with the free parameters under consideration.<sup>2</sup> As an example, Fig. 2 shows the fraction of carbon in three ionization stages as a function of density, redshift and column density. Well-known trends from photoionization calculations can be seen in this figure.

Considering low ions first (e.g.  $\text{C}^+$ ), one can see that  $X_{\text{C}^+}$  increases towards high column and volume densities with curves of constant ionization fraction that are approximately diagonal in the  $\log n_{\text{H}} - \log N_{\text{H I}}$  plane. Also visible is the effect of the redshift-dependent photoionization rate,  $\Gamma_{\text{H I}}$ , which is encoded in the Haardt & Madau (2012) model. Due to the evolution of the UVB, the lowest neutral fraction can be observed at  $z \sim 2$  for a fixed column density (see right-hand panels), with a rapid increase towards  $z \sim 0$  as  $\Gamma_{\text{H I}}$  plunges. Similar features are also commonly found in the variation of  $X$  with redshift and density for common low-ionization species (e.g. for  $\text{Si}^+$ ,  $\text{O}^0$ ,  $\text{H}^0$ ,  $\text{Al}^+$ ,  $\text{Fe}^+$ ,  $\text{Mg}^+$ ). Fig. 2 also highlights how the fraction of progressively more ionized species shifts with density at constant redshift. At  $z \sim 2.5$ , most of the carbon is singly ionized for  $\log n_{\text{H}} \gtrsim -1.5$ , doubly ionized between  $-3 \lesssim \log n_{\text{H}} \lesssim -1.5$  and triply ionized for  $\log n_{\text{H}} \lesssim -3$ . Thus, carbon (or silicon) are expected to be predominantly ionized within LLSs, yielding strong  $\text{C III}$  and  $\text{Si III}$  absorption as seen in previous work (e.g. Prochaska, O’Meara & Worseck 2010; Ribaldo et al. 2011; Fumagalli et al. 2013, see also Section 5.2). For reference, the mean cosmic density at  $z \sim 2.5$  is  $\sim 8 \times 10^{-6} \text{ cm}^{-3}$ .

With the minimal grid of models in hand, we apply the formalism described in Section 3 to infer the posterior PDFs for density and metallicity. Specifically, we reconstruct the posterior PDFs for each of the 234 LLSs in our sample by running the MCMC code over each set of observations. In principle, we could then assign a unique value of density and metallicity to each systems (e.g. through percentiles of the reconstructed PDFs). However, in some cases, the posterior PDFs show broad and/or bimodal distributions, reflecting the degeneracy between parameters given a limited num-

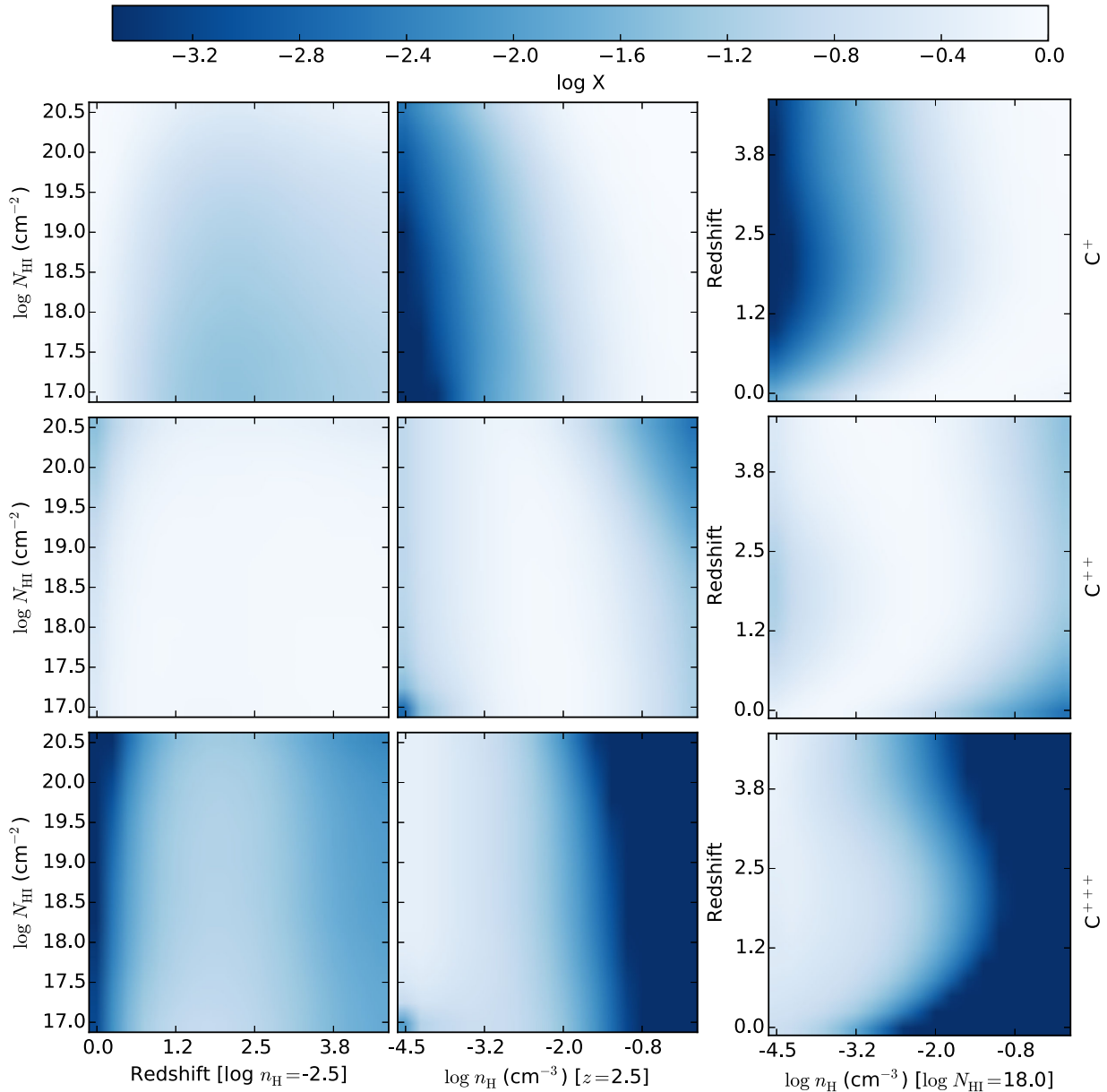
ber of observables. In the following, we therefore prefer to study the statistical properties of the LLS population by exploiting the information contained in the PDFs rather than in ‘best-fitting’ values for individual systems. Comparisons between medians of the posterior PDFs will be used to assess the level of convergence of the different physical parameters with respect to model assumptions, and to evaluate the metal mass density of LLSs (equation 7). To further assess how the sample variance influences the shape of the reconstructed PDFs, we adopt bootstrap techniques. Specifically, we construct 1000 realizations for the PDF of quantity of interest by combining sets of 234 LLSs, drawn from the full sample but allowing for repetitions.

The marginalized PDFs for the density and metallicity are shown in left-most panels of Figs 3 and 4, both for the entire sample (red) and the HD-LLS subsample (blue). From these figures, we infer that, under the assumption of a minimal photoionization model, the density distribution for the HD-LLS subsample is characterized by a well-defined peak between  $-3 \lesssim \log n_{\text{H}} \lesssim -2$ , while the PDF of the full sample exhibits also a tail at higher densities, attributable to the lower redshift and higher column density of the LLSs from the literature (Fig. 1). Similarly, the metallicity distribution for the HD-LLS data set is peaked around  $\log Z/Z_{\odot} \sim -2$ , with broader tails both at high and low metallicity. Again, due to the different nature of the systems from the literature, the entire sample shows a prominent tail towards higher metallicity, possibly with a hint of a second peak around  $\log Z/Z_{\odot} \sim -1$ .

In an attempt to quantify whether this minimal model provides an acceptable description of the data, we examine the average residuals by comparing the observed ions to the model predictions given the median of the PDFs for the free parameters in the model. We also compute the residuals for the third most-deviant ion in each system, to provide further insight into the ability of the model to capture the column density for a wide range of ions. With this exercise, we find that <15 per cent of the systems have either the mean residual or the residual of the third most-deviant ion in excess of three times the observed error. We also examine the joint PDF of density and metallicity to assess how well we can constrain individual parameters with this model. In Fig. 5, we show the mean joint PDF for the HD-LLS subsample, which we construct by combining the joint PDFs for individual systems after normalizing them to the respective medians. This figure shows that, on average, most of the probability is contained within a single region, centred around zero. This means that, on average, both densities and metallicity are characterized by well-defined unimodal PDFs. Furthermore, despite some degree of correlation (see also Cooper et al. 2015), the posterior PDFs are on average well contained within  $\pm 0.2$  dex from the median, indicating that most of the probability is found close to the peak of the distribution. A similar shape is found for the joint PDF for the entire sample (not shown), although the heterogeneous quality of the data from the literature (with some systems having only a couple of measured transitions) broadens the width of the joint PDF. In turn, this means that the large spread observed in Fig. 3, and Fig. 4 arises from intrinsic scatter in the physical properties of the LLS population.

Altogether, we conclude that a very simple photoionization model is able to capture the ion distribution for most of the systems under analysis, although with a few significant outliers. Moreover, high ions such as  $\text{N}^{+4}$  or  $\text{O}^{+5}$ , included in our analysis when measured in the observations, are known to be underproduced in standard photoionization models (e.g. Lehner et al. 2014), and our models are no exception. Despite this good agreement, as we will show in the following via series of model expansions, the reconstructed PDFs

<sup>2</sup> In this work,  $X_{\text{E}j} \equiv (n_{i,j}/n_i)$  defines the fraction of the  $i$ th element ‘E’ that is found in the  $j$ th ionization stage. Thus, as an example,  $X_{\text{H}^0}$  is the neutral fraction of hydrogen.



**Figure 2.** Slices of the fraction of carbon in three ionization stages ( $C^+$  top,  $C^{++}$  middle,  $C^{+++}$  bottom) as predicted by the minimal photoionization model as a function of density, redshift and column density. The ion fractions are shown as a function of redshift and  $N_{\text{HI}}$  for  $n_{\text{H}} = 10^{-2.5} \text{ cm}^{-3}$  (left), of  $N_{\text{HI}}$  and  $n_{\text{H}}$  for  $z = 2.5$  (centre) and of redshift and  $n_{\text{H}}$  for  $\log N_{\text{HI}} = 18$  (right).

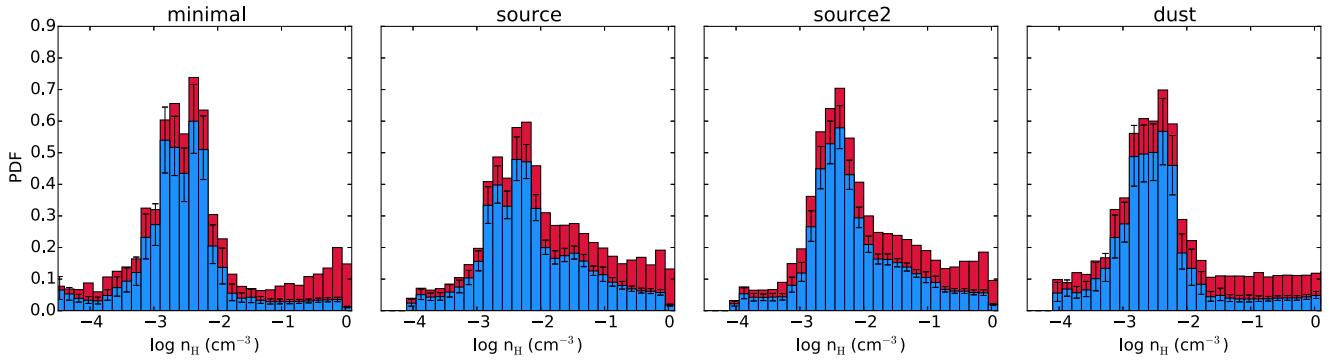
are subject to systematic uncertainties arising from the adopted ionization models.

#### 4.2 Uncertainties in the UVB model and proximity to local sources

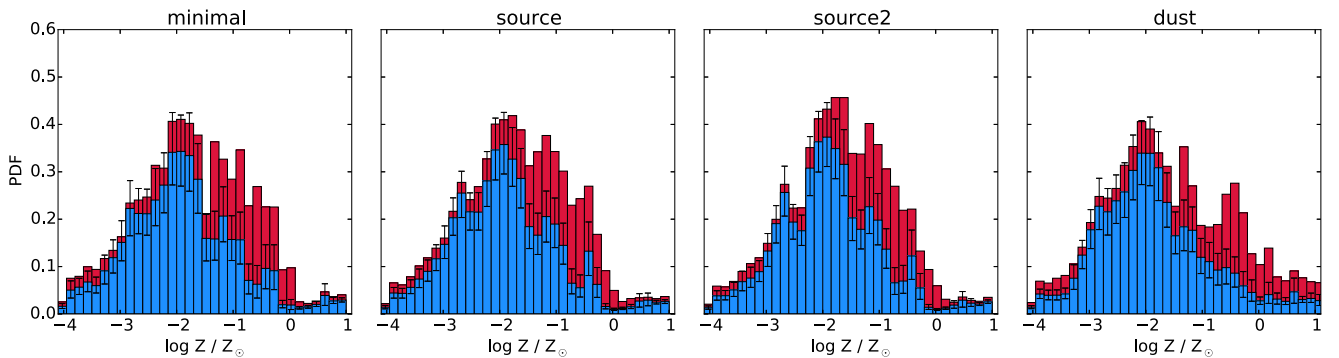
An important assumption of the minimal model is a fixed source of ionizing radiation, which introduces a one-to-one mapping between density and ionization parameter. However, as discussed at length in the literature, both the intrinsic uncertainty in the UVB model (e.g. Faucher-Giguère et al. 2009; Haardt & Madau 2012; Becker & Bolton 2013), and the possible contribution of a local radiation field (e.g. D’Odorico & Petitjean 2001; Dessauges-Zavadsky et al. 2003; Schaye 2006; Meiring et al. 2009b; Nagamine, Choi & Yajima 2010; Fumagalli et al. 2011b) make calculations that rely on a

specific UVB prone to substantial uncertainties (e.g. Agafonova et al. 2005; Fechner 2011; Simcoe 2011; Crighton et al. 2015). For these reasons, parametric models have been devised to allow for variations in the source of radiation.

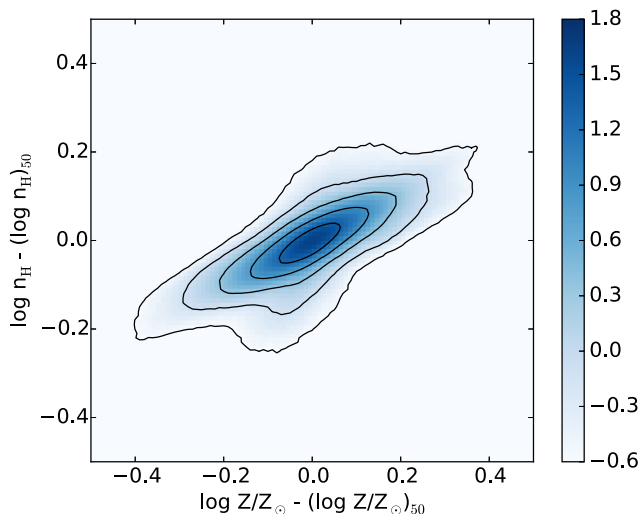
One such parametrization can be found in the recent work by Crighton et al. (2015), who introduce a free parameter  $\alpha_{\text{UV}}$  to tune the hardness of the UVB from a hard (AGN-dominated) to a soft (galaxy-dominated) spectrum. Here, we follow a similar procedure, which we however generalize to allow for a free parametrization in the amplitude of the radiation field, suitable for the treatment of local sources. In the model of Crighton et al. (2015), in fact, the UVB spectrum is renormalized to satisfy independent measurements of the hydrogen and helium photoionization rates within a  $\pm 0.3$  dex interval. More substantial variations in the normalization are, however, especially relevant for LLSs, as many pieces of



**Figure 3.** Posterior PDFs for the density of the entire LLS sample (red, in the background) and the HD-LLS subset (blue, in the foreground), both of which are normalized to the total sample size. Each panel shows the PDF obtained by combining the marginalized posterior PDFs for individual systems under the assumption of different photoionization models, as labelled. Error bars show the 10th and 90th confidence intervals from bootstrapping. Despite non-negligible differences in the shape of these PDFs, typical densities for these LLSs lie in the range  $-3.5 \lesssim \log n_{\text{H}} \lesssim -2$  with broader tails.



**Figure 4.** Same as Fig. 3, but for the posterior PDFs of the metallicity. The metallicity distributions for this sample are characterized by a peak around  $\log Z/Z_{\odot} \sim -2$ , with broader tails and possibly a secondary peak close to  $\log Z/Z_{\odot} \sim -1$ . Appreciable differences among different models can be seen.



**Figure 5.** Joint posterior PDFs for the metallicity and density in the HD-LLS subsample, obtained by combining the PDFs for individual systems relative to the respective medians and under the assumption of a minimal photoionization model. Darker colours (with logarithmic scaling) mark regions of higher probability density in this two-dimensional space.

evidence consistently place optically thick absorbers in proximity to galaxies or even quasars (Faucher-Giguère & Kereš 2011; Fumagalli et al. 2011b, 2013; van de Voort et al. 2012; Prochaska et al. 2013). However, as we will show below, by allowing for a

varying amplitude in the radiation field, we break the one-to-one relation between the physical density and the ionization parameter, thus introducing a degeneracy in the model.

To obtain a more generalize form for the radiation field, we construct a source term by combining three contributions: the intensity from the UVB,  $J_{\text{uvb}}(\nu)$ ; the intensity from local galaxies,  $J_{\text{gal}}(\nu)$ ; and the intensity from a local AGN,  $J_{\text{qso}}(\nu)$ . The combined input spectral energy distribution (SED) therefore becomes  $J(\nu) = J_{\text{uvb}}(\nu) + j_{\text{gal}}J_{\text{gal}}(\nu) + j_{\text{qso}}J_{\text{qso}}(\nu)$ , where  $j_{\text{gal}}$  and  $j_{\text{qso}}$  are free parameters (constrained to be positive). Clearly, many parameters regulate the amplitude of the local radiation field for quasars and galaxies. To minimize the number of free parameters, and given that we are not trying to constrain the astrophysical origin of a radiation field in excess of the UVB, we combine the intrinsic properties of local sources and the escape fraction of ionizing radiation in two ‘phenomenological’ parameters ( $j_{\text{gal}}$  and  $j_{\text{qso}}$ ) that globally describe the effects of local sources, as detailed below.

For  $J_{\text{gal}}$ , we create a STARBURST99 model (v7.0.1; Leitherer et al. 1999) using default input parameters, a continuous star formation rate (SFR)  $\dot{\psi} = 1 M_{\odot} \text{ yr}^{-1}$ , and the Geneva (2012) stellar tracks with no rotation and solar metallicity (Ekström et al. 2012). The normalization coefficient  $j_{\text{gal}}$  is then used to account, altogether, for the intrinsic SFR of the local source, the escape fraction of ionizing radiation  $f_{\text{esc}}$ , the distance between the cloud and the source  $d_{\text{cs}}$ , and a dust extinction  $\kappa$  that does not depend on wavelength. Thus, in our model

$$j_{\text{gal}} = \frac{\dot{\psi}}{1 M_{\odot} \text{ yr}^{-1}} \left( \frac{100 \text{ kpc}}{d_{\text{cs}}} \right)^2 f_{\text{esc}} (1 - \kappa). \quad (3)$$



**Table 3.** Free parameters in the ‘source’ and ‘source2’ grids of models.

Parameter	Min.	Max.	Step
$\log Z/Z_{\odot}$	-4.0	1.0	0.5
$z$	0.0	4.5	0.5
$\log N_{\text{H I}} (\text{cm}^{-2})$	17.0	20.5	0.5
$\log n_{\text{H}} (\text{cm}^{-3})$	-4.0	0.0	0.5
$\log j_{\text{gal}}$	-4.0	2.0	1.0
$\log j_{\text{qso}}$	-6.5	0.5	1.0

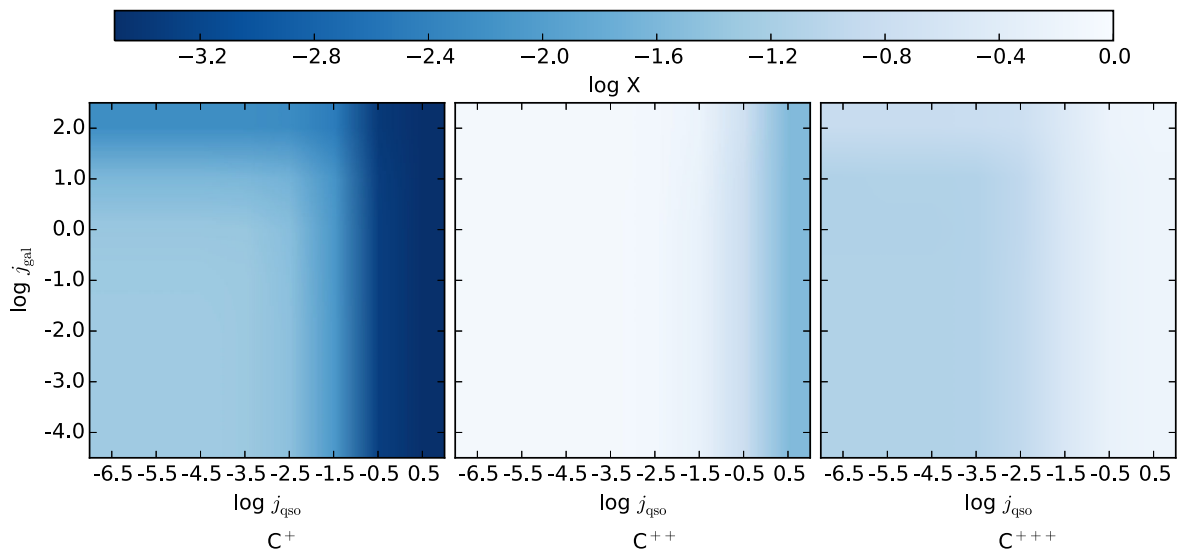
The columns of the table are: (1) the free parameter as described in the text; (2) the minimum allowed value; (3) the maximum allowed value; (4) the step adopted in the grid.

As an example,  $\log j_{\text{gal}} \leq -4$  yields the Haardt & Madau (2012) UVB within  $\sim 1$  per cent at all frequencies, while  $\log j_{\text{gal}} \sim 2$  describes a cloud at  $\sim 30$  kpc from a galaxy with  $\dot{\psi} = 100 M_{\odot} \text{ yr}^{-1}$  and  $f_{\text{esc}} = 0.1$ . For  $J_{\text{qso}}(\nu)$ , instead, we use the mean quasar SED from Richards et al. (2006), which we combine with the other sources with weight  $j_{\text{qso}}$  to account for varying degrees of intrinsic luminosity  $L_{\text{qso, bol}}$ , proximity to the source, and constant dust extinction:

$$j_{\text{qso}} = \frac{L_{\text{qso, bol}}}{4.6 \times 10^{47} \text{ erg s}^{-1}} \left( \frac{100 \text{ kpc}}{d_{\text{cs}}} \right)^2 (1 - \kappa). \quad (4)$$

With this parametrization, the UVB is recovered within 1 per cent at all frequency for  $\log j_{\text{qso}} \leq -6$ , while  $\log j_{\text{qso}} \sim 0.5$  describes a cloud that lies at  $\sim 50$  kpc from an average type 1 quasar. The full parameter space occupied by this grid of models, dubbed ‘source’, is described in Table 3. Finally, to assess the systematic difference arising from different choices of the UVB, we rerun a second grid of source models, labelled ‘source2’, by using the Faucher-Giguère et al. (2009) UVB instead of the Haardt & Madau (2012) model.

Before running the MCMC procedure on the data, in Fig. 6, we provide a visual example on how the ionization stages of common elements are affected by the inclusion of additional sources of radiation. Specifically, the variation of ionization for carbon in  $\text{C}^+$ ,  $\text{C}^{++}$  and  $\text{C}^{+++}$  is shown as a function of  $j_{\text{gal}}$  and  $j_{\text{qso}}$ , for constant redshift ( $z = 2.5$ ), density ( $\log n_{\text{H}} = -2.5$ ), metallicity ( $\log Z/Z_{\odot} = -2.5$ )

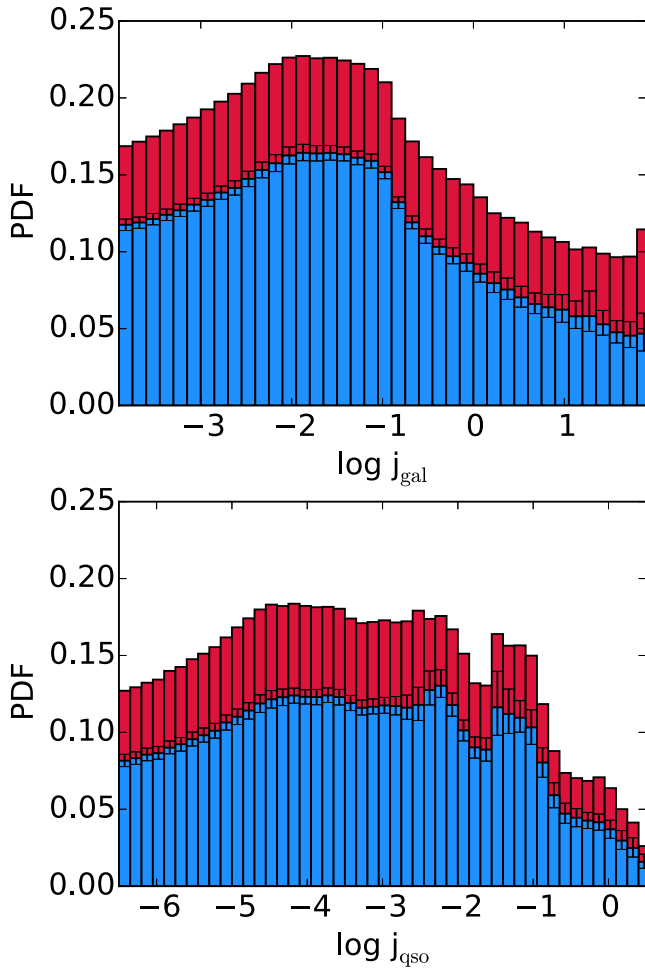


**Figure 6.** From left to right, the variation in the fraction of singly, doubly and triply ionized carbon as a function of additional contribution from galaxies and quasars to the UVB radiation field (see text for details). Progressively higher contributions from local sources reduce the fraction of carbon in low ionization stages, with the harder SEDs of quasars contributing the most to the ionization of  $\text{C}^{+++}$ . Each panel shows a slice in the source grid of models at  $z = 2.5$ ,  $\log N_{\text{H I}} = 18$ ,  $\log n_{\text{H}} = -2.5$  and  $\log Z/Z_{\odot} = -2.5$ .

and column density ( $\log N_{\text{H I}} = 18$ ). As expected, increasing the contribution of local sources increases the ionization parameter at fixed physical density, and a progressively larger fraction of carbon can be found in  $\text{C}^{++}$  and  $\text{C}^{+++}$ . Furthermore, the addition of a harder quasar SED for  $\log j_{\text{qso}} > -1.5$  induces ionization of carbon in higher stages, most notably from  $\text{C}^{++}$  to  $\text{C}^{+++}$  (cf. Simcoe 2011).

When using the source and source2 grid of models to infer the posterior PDFs for the density and metallicity, we find that the metallicity of LLSs, as a population, is not particularly sensitive to variations in the radiation field. Comparing the posterior PDFs derived from the source and source2 models to the one inferred from the minimal model (Fig. 4), one can see that, especially for the HD-LLS subsample, the distributions retain a similar shape. Furthermore, one can also see how the source and source2 models yield a virtually identical PDF for the metallicity of these LLSs. Conversely, substantial differences can be seen for the PDFs of the density (Fig. 3). While the characteristic peak between  $-3 \lesssim \log n_{\text{H}} \lesssim -2$  is retained, appreciable discrepancies can be noted when additional sources are included, particularly with an excess probability for  $-2 \lesssim \log n_{\text{H}} \lesssim -1$  compared to the results from the minimal model. This excess is a consequence of our treatment for the local radiation field, which can only boost the contribution from the UVB, thus skewing the density distribution to higher values.

The same trends emerge when inspecting the medians for the posterior PDFs for individual systems (not shown). Comparing the results of the minimal and source model, a tight correlation is found, with a mean offset of  $\sim 0.01$  dex and dispersion of  $\sim 0.15$  dex. Conversely, for the density, we find both a larger scatter and a systematic offset between the medians, with typical discrepancies of  $0.33 \pm 0.49$  dex. Notably, when we compare medians inferred assuming the Haardt & Madau (2012) UVB model to the ones inferred assuming the Faucher-Giguère et al. (2009) UVB model, we find a tight correlation for both metallicity ( $0.02 \pm 0.11$  dex) and, although with larger scatter, densities ( $0.06 \pm 0.33$  dex). Thus, variations in the shape and amplitude of the UVB among different models are subdominant compared to uncertainties in a local radiation field.

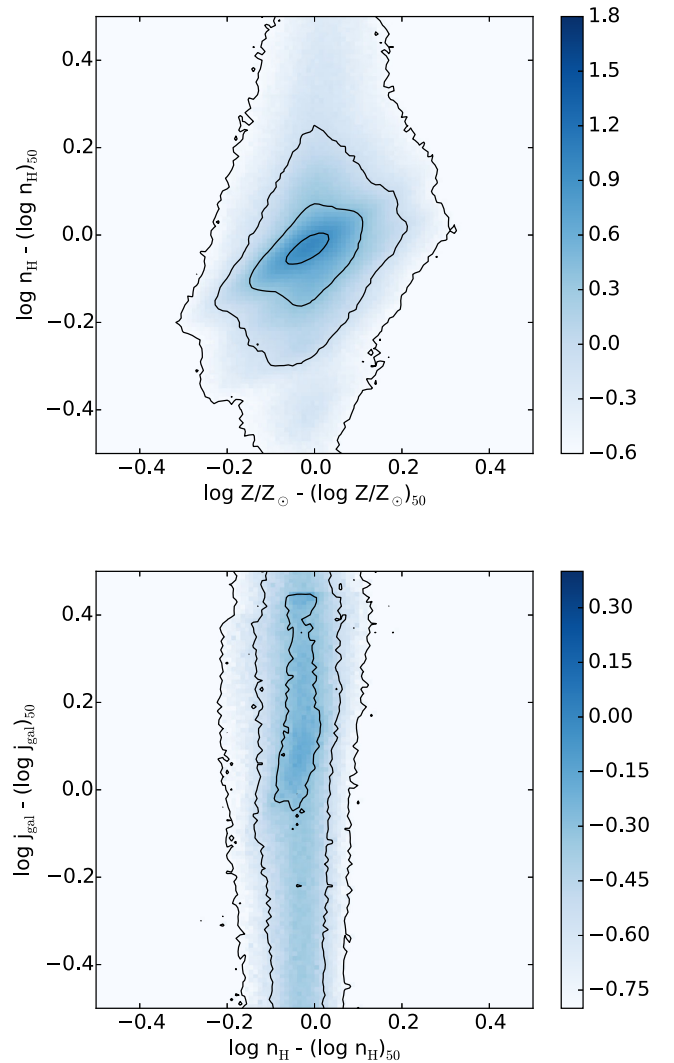


**Figure 7.** Same as Fig. 3, but for the posterior PDFs for the additional contribution of galaxies (top) and quasars (bottom) to the UVB in the source model. The broad PDFs for the population, together with very broad PDFs for individual systems (Fig. 8), suggest that these data do not constrain the spectral shape and amplitude of the radiation field.

The origin of the different behaviour in the density and metallicity PDFs is attributable to the degeneracy between density and intensity of the local radiation field. That is, the available data are able to quite precisely constrain the ionization parameter, and thus the metallicity of LLSs. However, there is not enough information within most of these observations to also constrain the shape and amplitude of the radiation field, as it is clear from the broad posterior PDFs for  $j_{\text{gal}}$  and  $j_{\text{qso}}$  (Fig. 7). Such a broad PDF could in principle be the result of a superposition of many narrow PDFs if LLSs had to be exposed to a diverse range of radiation fields. However, the shape of the mean joint PDF for the density and  $j_{\text{gal}}$  of individual systems (Fig. 8) rather indicates that  $j_{\text{gal}}$  (and  $j_{\text{qso}}$ ) are mostly unconstrained,<sup>3</sup> and thus, we cannot precisely establish the importance of additional sources of radiation with this procedure. In response to a broad PDF for  $j_{\text{gal}}$  and  $j_{\text{qso}}$ , the density PDF broadens (top panel of Fig. 8), often with tails to higher  $n_{\text{H}}$  to compensate for additional sources of ionizing radiation.

We note that, despite the inclusion of two additional parameters, the source and source2 models do not appear to yield significantly

<sup>3</sup> The joint PDF for the density and  $j_{\text{qso}}$  is not shown, but it has a similar shape.



**Figure 8.** Same as Fig. 5, for the joint PDFs of metallicity and density (top) and density and  $j_{\text{gal}}$  (bottom). Due to the weak constraints on the contribution from local sources, the density PDF broadens.

better residuals, which remain comparable to what was found when using the minimal model. Again, we can attribute this result to the ambiguity in separating density and sources of radiation, which shifts the medians from the peaks of the PDFs. Finally, while we should refrain from strong conclusions based on the PDFs for  $j_{\text{gal}}$  and  $j_{\text{qso}}$  given the discussion above, it is worthwhile noting that high values of  $j_{\text{gal}}$  and, especially, of  $j_{\text{qso}}$  have low probability in this sample. Furthermore, there seems to be a preference for a low (perhaps non-zero) contribution from local sources, suggestive that the population of LLSs could be exposed to only modest levels of ambient radiation in excess to the UVB (e.g. Schaye 2006; Prochaska et al. 2013).

### 4.3 Dust depletion

Up to this point, we have considered idealized absorption systems with no dust and a gas-phase abundance pattern equal to the one measured in the solar neighbourhood. Next, we examine, in simple terms, the impact that this assumption has on the metallicity determination for LLSs.

Several authors have addressed the problem of characterizing element by element depletion factors and their variation with physical properties, both in the nearby and in the distant Universe (e.g. Savage & Sembach 1996; Prochaska et al. 2007; Jenkins 2009; Rafelski et al. 2012; De Cia et al. 2013). These studies convincingly show that dust depletes elements on to grains in the denser astrophysical environments. Nevertheless, the complex astrophysics that regulates dust formation or the condensation and evaporation of elements on to and from grains hampers the formulation of a general theory for the depletion factors of all the elements of interest, especially at high redshift and within the poorly explored population of LLSs.

For this reason, in the following, we construct a very simple grid of ‘dust’ models building on the empirical work of Jenkins (2009). Given the many unknowns, and the degeneracy between dust depletion and intrinsic deviations from the assumed solar abundance, we construct a model that relies on a single parameter that regulates both the dust-to-gas ratio at constant metallicity and the element-by-element depletion. Specifically, we expand the minimal model introducing ISM grains as specified by CLOUDY with an abundance relative to solar composition defined by  $Z_{\text{grn}}/Z_{\odot} = \alpha_{\text{dtn}} Z/Z_{\odot}$ , where  $Z/Z_{\odot}$  is the gas-phase metallicity and  $\alpha_{\text{dtn}}$  is a free parameter that specifies the dust-to-metal ratio. It should be noted that the inclusion of grains in CLOUDY affects both the gas thermal state and the opacity. Next, we compute the depletion of relevant elements from the gas-phase relative to their solar abundance following the fitting formulae provided by Jenkins (2009, their table 4 and equation 10), which we generalize to the case of systems with arbitrary metallicity. As described in detail in Jenkins (2009), while the absolute value of the gas-phase abundance of each element is uncertain, it is possible to more accurately establish how rapidly each element is depleted on to dust grains by introducing a depletion strength factor,  $F_*$ , which varies from sightline to sightline.

Following the notation of Jenkins (2009), having defined the depletion from the gas phase of an element  $X$  for a system with intrinsic abundance  $[X/H]_{\text{int}}$

$$[X/H]_{\text{dep}} \equiv \log N_X - \log N_H - (\log N_X - \log N_H)_{\text{int}}, \quad (5)$$

we can write the gas-phase metallicity as

$$[X/H]_{\text{gas}} = [X/H]_{\text{int}} - B_X - A_X(F_* - z_X), \quad (6)$$

where  $B_X$ ,  $A_X$ , and  $z_X$  are locally calibrated coefficients taken from Jenkins (2009) that define  $[X/H]_{\text{dep}}$ . Here, we further expand on the work of Jenkins (2009) in two ways. First, equation (6) returns a non-zero level of depletion even for  $F_* = 0$ . For our purposes, however, we wish to recover a limiting case of no depletion for all elements, and we allow for values  $F_* < 0$ . We note, however, that the limiting case  $[X/H]_{\text{dep}} = 0$  is recovered for different elements by means of different  $F_*$ . Because of the use of a single  $F_*$  parameter, the extrapolation of depletion factors for large negative values of  $F_*$  could yield negative corrections for some elements, which we cap to 0. Similarly, there is no prior reason to impose the condition  $F_* \leq 1$ , which originates from empirical considerations in the analysis of Jenkins (2009). Thus, in our work we allow for cases with  $F_* > 1$ . Indeed, although stronger depletion than observed in the local ISM is probably a rare occurrence for LLSs, it may be plausible for some of the supersolar systems. Secondly, we postulate a connection between  $\alpha_{\text{dtn}}$  and  $F_*$  such that  $\alpha_{\text{dtn}} = F_*$  for  $F_* \geq 0$ , and  $\alpha_{\text{dtn}} = 0$  otherwise.

We emphasize that this choice is rather arbitrary, as the exact scaling between the dust content and the depletion is unexplored for LLSs. Nevertheless, this ansatz lets us specify a simple dust

**Table 4.** Free parameters in the dust grid of models.

Parameter	Min.	Max.	Step
$\log Z/Z_{\odot}$	−4.0	1.1	0.30
$z$	0.0	4.5	0.30
$\log N_{\text{H I}} \text{ (cm}^{-2}\text{)}$	17.0	20.6	0.30
$\log n_{\text{H}} \text{ (cm}^{-3}\text{)}$	−4.0	0.2	0.30
$F_*$	−1.5	1.5	0.50

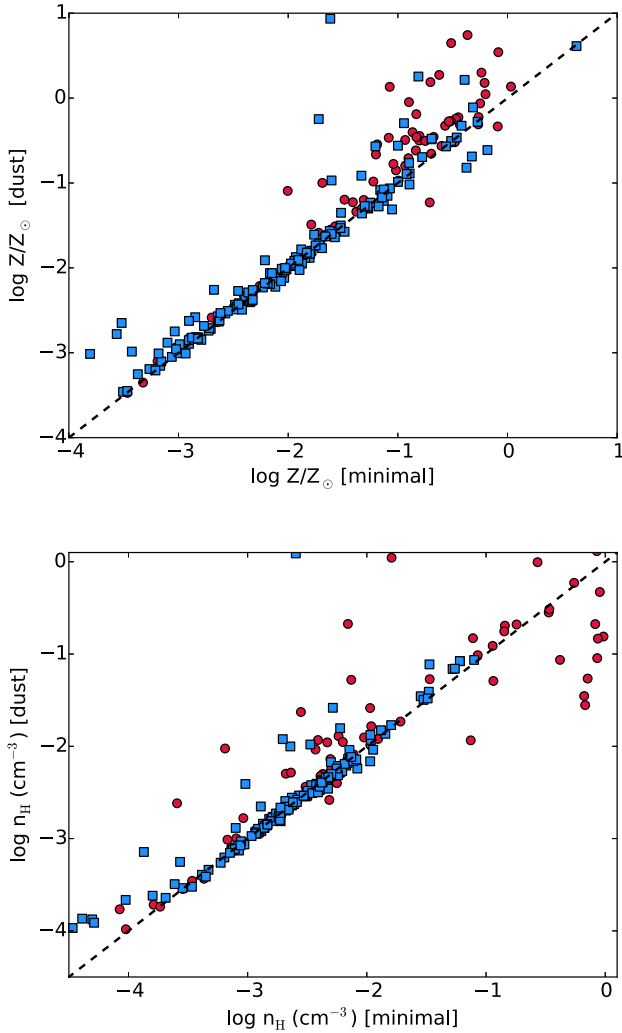
The columns of the table are: (1) the free parameter as described in the text; (2) the minimum allowed value; (3) the maximum allowed value; (4) the step adopted in the grid.

model that, with a single parameter  $F_*$ , provides an expansion of the minimal model with desirable characteristics. For  $F_* > 0$ , dust grains are included in the photoionization modelling with abundance proportional to  $F_*$  and elements are depleted according to equation (6), also allowing for a variable metal-to-dust ratio. For  $F_* = 0$ , dust grains are not included, although residual deviations from the assumed solar abundance pattern are allowed. Finally, for  $F_* < 0$ , the behaviour of the minimal dust-free model is progressively recovered. As previously done for  $j_{\text{qso}}$  and  $j_{\text{gal}}$ , we note that we do not attempt to assign a physical meaning to the precise values of  $F_*$  (see below). A summary of the parameters included in the dust grid of models is provided in Table 4.

Besides the obvious effect of altering the relative ratios of ions when keeping constant all the other parameters of the grid, the ionization stages of the individual elements in the dust grid do not appreciably differ from the trends already discussed for the minimal models (Fig. 2). However, the inclusion of dust shapes the radiation transmitted through the cloud and the gas thermal state. As a consequence, at the corner of the grid where the highest density, metallicity and column densities are found, models with  $F_* \gtrsim 0.5$  develop a molecular phase. In turn, this leads to a reduction of the effective hydrogen column density in the neutral atomic phase when compared to a dust-free model. Throughout our analysis, we account for this effect by considering the output  $\log N_{\text{H I}}$  in the computation of the priors. However, as we will show below, data prefer models with low dust content, and thus, the molecular phase is unimportant for our analysis.

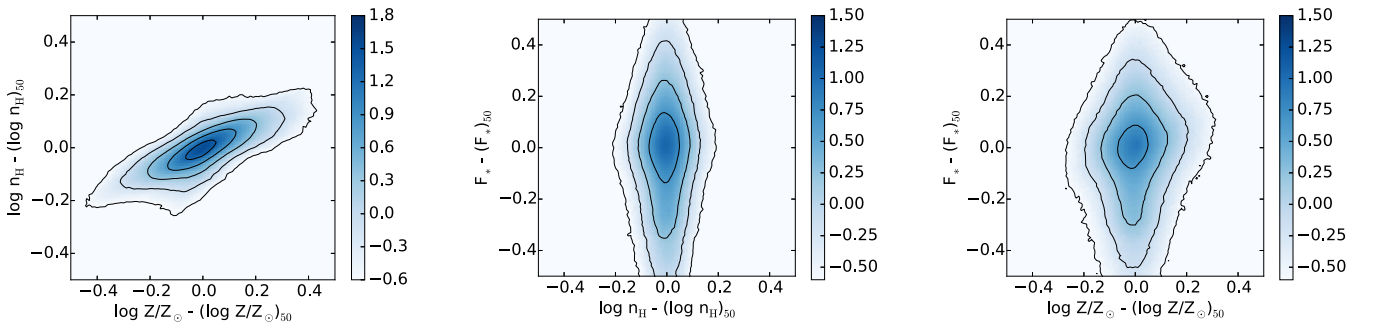
When using the dust grid of models to infer the posterior PDF for density and metallicity, we recover the distributions shown in Fig. 3 and Fig. 4 (right-hand panels). The density distribution appears to be nearly insensitive to the inclusion of  $F_*$  in the dust model, with the exception of the highest density tail for the literature sample. Conversely, the metallicity PDFs show some discrepancies, particularly for  $\log Z/Z_{\odot} \gtrsim -1.5$  and for the literature sample dominated by low-redshift and high-column systems. To better understand the origin of this difference, we inspect the medians (Fig. 9) and shapes (Fig. 10) of the posterior PDFs for the density and metallicity, as well as the posterior PDF for  $F_*$  (Fig. 11).

For the HD-LLS sample, both the median density and metallicity appear to be tightly correlated, with a dispersion of  $\sim 0.3$  dex. For the metallicity, most of this dispersion is driven either by data with  $\log Z/Z_{\odot} > -1$  or  $\log Z/Z_{\odot} \lesssim -3$ . The lower redshift higher column density sample from the literature follows similar trends, with an even more pronounced dispersion in the metallicity for  $\log Z/Z_{\odot} > -1.5$ . Qualitatively, an increasing importance of dust towards lower redshifts is in line with the results from studies of Mg II absorbers (e.g. Ménard et al. 2008). At higher metallicity, despite the small sample size, there is also evidence for a systematic offset, in the direction of having higher metallicity for the dust model. This offset arises from a tentative correlation between  $F_*$

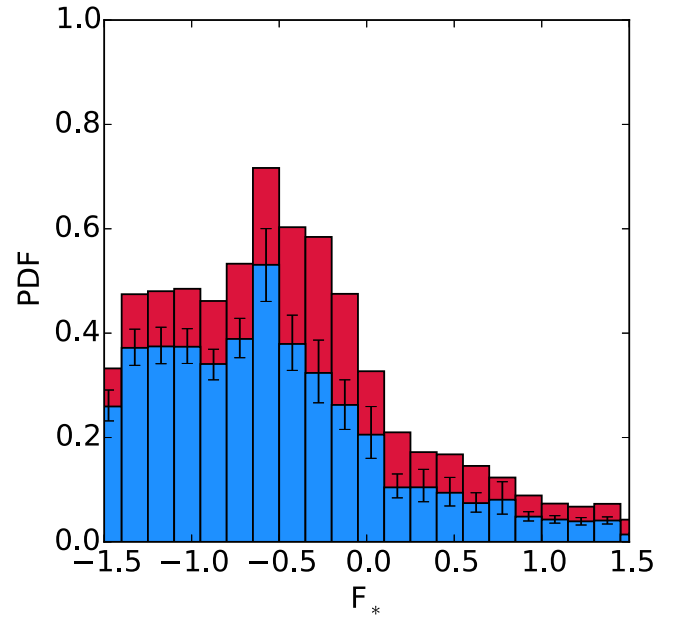


**Figure 9.** Comparison of the median PDFs for metallicity (top) and density (bottom) derived from the minimal and dust models in individual systems. Red circles represent LLSs from the literature, while the blue squares mark LLSs from the HD-LLS sample. The inferred quantities are tightly correlated, with increasing dispersion and systematic offsets towards high metallicity and lower redshifts.

and  $\log Z/Z_*$  (not shown), for which a higher intrinsic metallicity is required to model observations for the gas-phase metal content in the presence of depletion. Regarding the discrepancy at lower metallicity, instead, the deviant points are for LLSs with only one



**Figure 10.** Same as Fig. 5, for the joint PDFs of metallicity and density (left) and density and  $F_*$  (centre) and metallicity and  $F_*$  (right). Density and metallicity in the dust grid of models are well constrained, despite the broader PDFs.



**Figure 11.** Same as Fig. 3, but for the posterior PDF of  $F_*$  in the dust model. The majority of LLSs, especially at  $z > 2$ , are characterized by a low dust content with  $F_* \lesssim 0$ .

or two detected ions. Compared to the minimal model, it appears that values  $F_* > -0.5$  provide a better fit for these detected ions, and in particular for the ubiquitous detection of C IV. Lacking multiple ions in different ionization stages, it is unclear whether this result is physical or whether it arises from a second-order degeneracy among parameters. As this discrepancy is seen in only four systems and does not impact our conclusions, we do not investigate it further.

Differently from the case of the source model, we note that the inclusion of an additional parameter in the grid does not introduce significant degeneracy. The shape of the mean joint PDFs (Fig. 10) reveals that density, metallicity and  $F_*$  are generally well constrained. Due to the simplicity of our dust model, however, it is not surprising that the posterior PDFs for  $F_*$  are quite broad, and that the PDFs for the metallicity broaden compared to what seen in Fig. 5. As before, this figure provides a qualitative way to disentangle the width in the PDF for individual systems ( $\sim 0.2$ – $0.3$  dex) from the scatter within the population when examining the combined posterior PDFs for the entire sample.

Given that the shape of the PDFs for  $F_*$  is generally well constrained, the posterior PDF for  $F_*$  encodes some information on the dust properties of these LLSs, although in a model-dependent way.

**Table 5.** Free parameters in the CIE grid of models.

Parameter	Min.	Max.	Step
$\log Z/Z_{\odot}$	-4.0	1.1	0.30
$z$	0.0	4.5	0.30
$\log N_{\text{H I}} (\text{cm}^{-2})$	17.0	20.6	0.30
$\log n_{\text{H}} (\text{cm}^{-3})$	-4.0	0.2	0.30
$\log T (\text{K})$	4.0	6.1	0.30

The columns of the table are: (1) the free parameter as described in the text; (2) the minimum allowed value; (3) the maximum allowed value; (4) the step adopted in the grid.

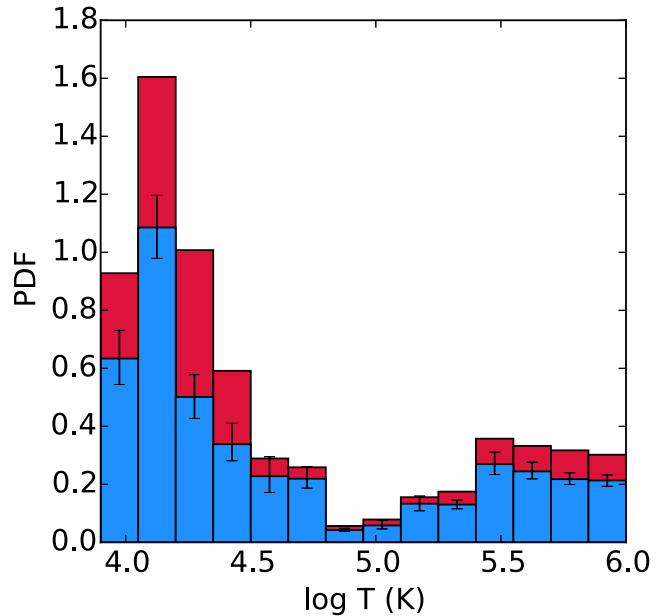
From Fig. 11, we see that most of the probability lies at  $F_{*} < 0$ , which suggests that LLSs typically reside in environments with low dust content. There is also evidence that >50 percent of the probability is contained between  $-1 \lesssim F_{*} \lesssim 0$ . This result stems almost entirely from the fact that, in our model, iron exhibits residual depletion even for  $F_{*} < 0$  (Jenkins 2009). However, the physical interpretation of negative but small  $F_{*}$  is complicated by the degeneracy between iron depletion on to dust grains and its unknown intrinsic abundances relative to  $\alpha$ -elements (cf. Berg et al. 2015). Due to this ambiguity, we cannot make any inference on depletion based on the posterior PDF for  $F_{*}$ . We only note on empirical grounds that a lower than solar iron abundance is preferred by the data, given our simple model. Indeed, inspecting the residuals, we note that the inclusion of  $F_{*}$  now ensures that the observed column densities are reproduced within two times the observational errors for  $\sim 80$  per cent of the LLSs.

#### 4.4 Effects of collisions

Up to this point, we have considered only models in which the gas is predominately photoionized. Indeed, for the majority of LLSs published in the literature, strong absorption from elements which are doubly or triply ionized (e.g. Ribaud et al. 2011; Fumagalli et al. 2013; Lehner et al. 2013; Prochaska et al. 2015) imposes a strong prior on the lack of significant collisional ionization at temperatures  $> 10^5$  K. It is in fact well-known that in collisional ionization equilibrium (CIE) the most abundant elements can be found in neutral or singly/doubly ionized phase only for  $T < 10^5$  K (e.g. Gnat & Sternberg 2007; Gnat & Ferland 2012). Additional constraints on the gas temperature at  $< 10^5$  K in CIE originate from the fact that LLSs, although ionized, contain large amount of neutral hydrogen. Achieving H I column densities as large as  $10^{19}$ - $10^{20}$   $\text{cm}^{-2}$  for temperatures  $\gg 10^5$  K would require extreme column densities ( $> 10^{24}$   $\text{cm}^{-2}$ ) for the total atomic hydrogen.

To seek confirmation of this hypothesis in our own data, we construct a CIE grid of models as an extension of the minimal model, by setting the gas temperature to a constant value over the interval  $10^4$ - $10^6$  K. The parameters of this grid are summarized in Table 5. Differently from the previous calculations, we also impose a maximum column density for the total hydrogen of  $\log N_{\text{H}} = 10^{24}$   $\text{cm}^{-2} \sim 8 \times 10^3 M_{\odot} \text{pc}^{-2}$  to avoid cases in which the desired H I column density is achieved by means of implausibly large columns of highly ionized hydrogen.

When using the CIE grid to model the data, in line with our expectations, we find that observations nearly exclusively prefer models with  $T < 5 \times 10^4$  K (with  $\sim 70$  per cent of the probability), thus recovering the limiting case of gas that is photoionized (Fig. 12). The tail at  $T > 10^5$  K arises from either a tail in the temperature PDF for systems with large residuals (i.e. for which there is no satisfactory model in the grid) or for low-metallicity systems that



**Figure 12.** Same as Fig. 3, but for the posterior PDF of the temperature in the CIE model. The majority of LLSs are characterized by low temperatures, common for photoionized gas.

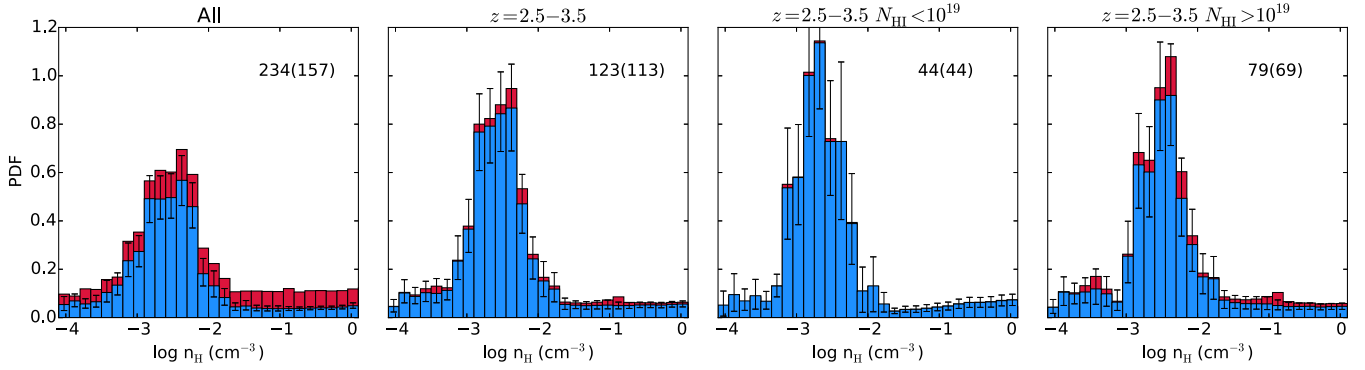
are dominated by non-detections. In the latter case, the temperature PDFs are sometimes double peaked. High-temperature models with very low column densities for most of the ions, particularly singly or double ionized, represent an acceptable solution even if limits or values for lines from triply ionized elements (i.e. C IV and Si IV) are overpredicted. Overall, however, the accepted models from the CIE grid yield worse residuals compared to what found using photoionization models.

While this analysis favours photoionization over collisional ionization at high temperature, we do not exclude the existence of a second gas-phase where collisional ionization is important or even dominant. Indeed, other conditions than those found in photoionized gas are likely required to reproduce the strong O VI absorption seen in many LLSs (e.g. Lehner et al. 2014).

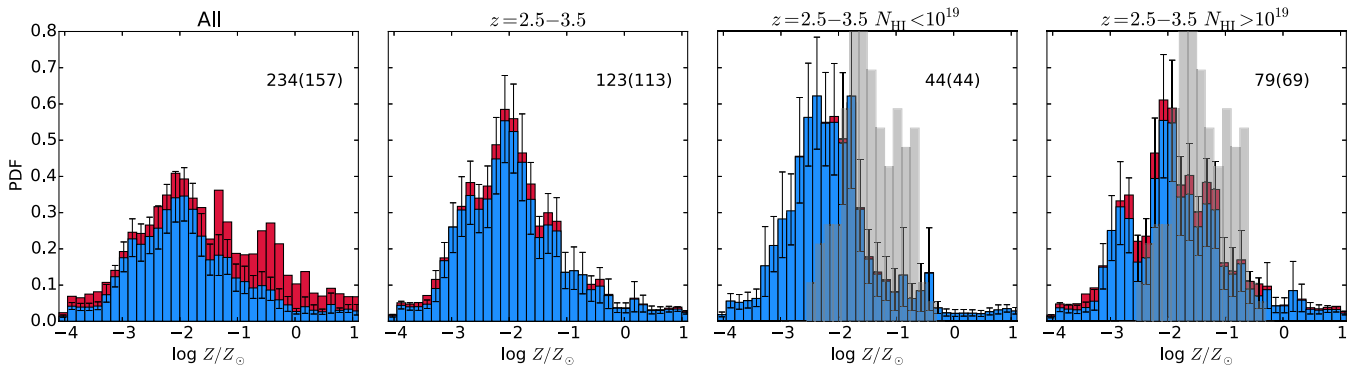
## 5 PHYSICAL PROPERTIES OF LLSS AND ASTROPHYSICAL IMPLICATIONS

In the previous section, we have used Bayesian techniques combined with grids of ionization models to infer the posterior PDFs for the metallicity and density of a sample of 234 LLSs between redshift  $z \sim 0 - 4.4$ , including a homogeneous subset of 157 LLSs from the HD-LLS survey, which form the main statistical sample for this analysis. Through different radiative transfer calculations, we have assessed whether the analysis of multiple ions in individual systems yields robust inference on the physical properties of LLSs. Under the basic assumptions of a single phase medium in ionization equilibrium, we have shown that the inferred PDFs for the LLS metallicity is well converged, while the inferred PDFs for the hydrogen density are less robust, being more sensitive to the assumptions made on the shape and intensity of the radiation field that illuminates the clouds.

Next, we discuss the astrophysical implications of our analysis, focusing on the physical properties of LLSs across cosmic epochs and their connection to accretion and feedback processes. Throughout the remainder of this paper, we use the results derived assuming



**Figure 13.** The posterior PDF for the density of all LLSs included in this study (red) and for a subset from the HD-LLS survey (blue), both of which are normalized to the total sample size. Error bars indicate the 10th and 90th percentile from bootstrapping. From left to right, panels show the PDFs for different cuts in redshift and column density, as labelled. The number of systems included in each bin is shown, with those from the statistical sample in parenthesis. Despite uncertainties in the ionization corrections, the majority of LLSs is characterized by densities between  $10^{-3.5}$ – $10^{-2}$   $\text{cm}^{-3}$ .



**Figure 14.** Same as Fig. 13, but for the PDFs of the metallicity. The metallicity distribution of  $z \sim 2.5$ – $3.5$  DLAs is shown in grey for comparison in the rightmost panels. DLAs are on average more enriched than LLSs, with the lower column density LLSs being the most metal poor.

the dust grid of models, which provide a satisfactory description of the data,<sup>4</sup> as we have shown in Section 4.3.

### 5.1 Redshift evolution and column density dependence

Our best estimate for the posterior PDFs for the density and metallicity inferred from both the total sample and the HD-LLS subsample using the preferred dust model are shown in Figs 13 and 14. The left-most panel in these figures show the distributions from the entire sample. As was clearly shown in Fig. 1, however, the full sample spans a wide range of redshifts and  $N_{\text{HI}}$  column density, but not uniformly. Thus, the reconstructed PDFs carry the imprint of the selection functions for our data. To gain insight into the underlying physical properties of high-redshift LLSs, we therefore restrict our analysis to the HD-LLS sample in the redshift interval  $z \sim 2.5$ – $3.5$  (second panel in Figs 13 and 14), where the data more homogeneously sample the full range of column densities. Given the large sample size, we can further subdivide the sample in two intervals of column densities (third and fourth panels), with a cut at  $\log N_{\text{HI}} = 19$  for comparisons with previous literature on SLLSs (see Section 1).

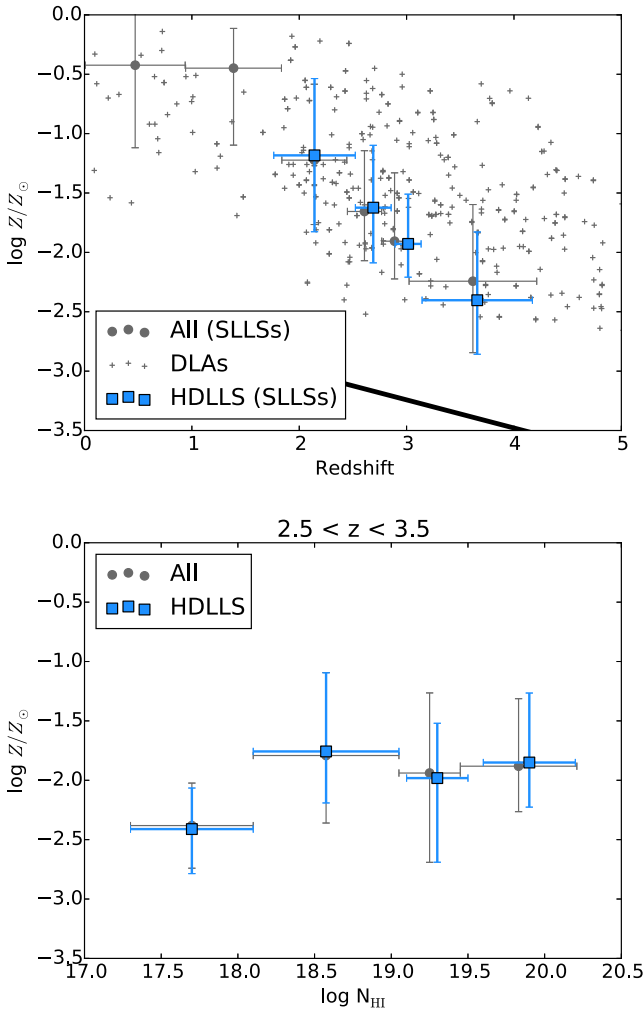
Considering the density PDF first, we find that  $\sim 80$  per cent of the probability is contained within  $10^{-3.5} \leq n_{\text{H}} \leq 10^{-2}$   $\text{cm}^{-3}$ , with a hint of higher density for the SLLS subset. However, due to both

sample variance and the substantial uncertainties in the density determination for individual systems, we regard this difference as marginal given current data. Again, we remark that while the density for the entire population can be constrained, at least loosely, our study confirms a fundamental limitation in the use of photoionization models to infer the density of individual LLSs. More significantly, we conclude that any inference on the sizes of the absorbing clouds relying on comparisons between the column density and the gas physical density are affected by significant systematic uncertainties, typically in the direction of higher density and smaller sizes for clouds that are illuminated by a local radiation field.

Focusing on the metallicity next, LLSs at  $z \sim 2.5$ – $3.5$  are characterized by a unimodal distribution with a peak around  $\log Z/Z_{\odot} \sim -2$  and broad tails towards both high and low metallicity. Thus, the shape of the metallicity distribution of high-redshift LLSs does not exhibit the bimodality that has been reported at lower redshift by Lehner et al. (2013). We note, however, that these authors included many systems with  $\log N_{\text{HI}} < 17.2$  in their analysis, and therefore do not strictly consider only optically-thick LLSs.

From the analysis of the metallicity PDF, we also conclude that LLSs at  $z \sim 2.5$ – $3.5$  are metal poor, with  $\sim 70$  per cent ( $\sim 85$  per cent) of the probability at  $\log Z/Z_{\odot} \leq -1.5$  ( $\log Z/Z_{\odot} \leq -1$ ). These findings extend with a  $\sim 10$  times larger sample some of the results from previous studies based on only a handful of systems (Steidel 1990; Prochaska & Burles 1999; Fumagalli et al. 2013; Cooper et al. 2015). Despite the low metal content, the incidence of very metal poor LLSs ( $\log Z/Z_{\odot} \leq -3$ ) is only of the order of  $\sim 10$  per cent, implying that metals are already widespread

<sup>4</sup> Additional tests based on mock data and the dust grid of models can be found in Appendix A.



**Figure 15.** Top: redshift evolution of the metallicity for all the LLSs with  $\log N_{\text{HI}} \geq 19$  in our sample (grey circles), and for the subset from the HD-LLS survey (blue squares). Data points and their associated error bars represent the median and the 25th/75th percentiles of the composite posterior PDFs in redshift bins that are chosen to contain at least 25 LLSs each. Metallicity measurements for individual DLAs from the literature are also superimposed (crosses). The black line marks the median IGM metallicity (Simcoe 2011). A clear trend with redshift is visible. Bottom: dependency on the metallicity as a function of column density for all the LLSs at  $z \sim 2.5\text{--}3.5$  (grey circles) and the HD-LLS subset (blue squares), showing tentative evidence that lowest column density systems are the most metal poor.

in moderate column density systems at these redshifts. The handful of metal-free gas clouds currently known are thus rare outliers at  $z < 3.6$  (Fumagalli et al. 2011a; Cooper et al. 2015; Prochaska et al. 2015).

When split in bins of column density, a difference between SLLS and the lower column density LLSs can be seen. For LLSs with  $\log N_{\text{HI}} < 19$ ,  $\sim 60$  per cent of the probability is at  $\log Z/Z_{\odot} \leq -2$  while, for SLLSs with  $\log N_{\text{HI}} \geq 19$ , only  $\sim 40$  per cent of the probability is below  $\log Z/Z_{\odot} \leq -2$  (see also Fig. 15). Finally, compared to the coeval population of DLAs with  $\log N_{\text{HI}} \geq 20.3$  from the samples of Rafelski et al. (2012) and Neeleman et al. (2013), we see that LLSs are significantly less enriched than DLAs (see also Cooper et al. 2015; Prochaska et al. 2015). Even when restricting to systems with  $\log N_{\text{HI}} > 19$ , DLAs are still more enriched than,

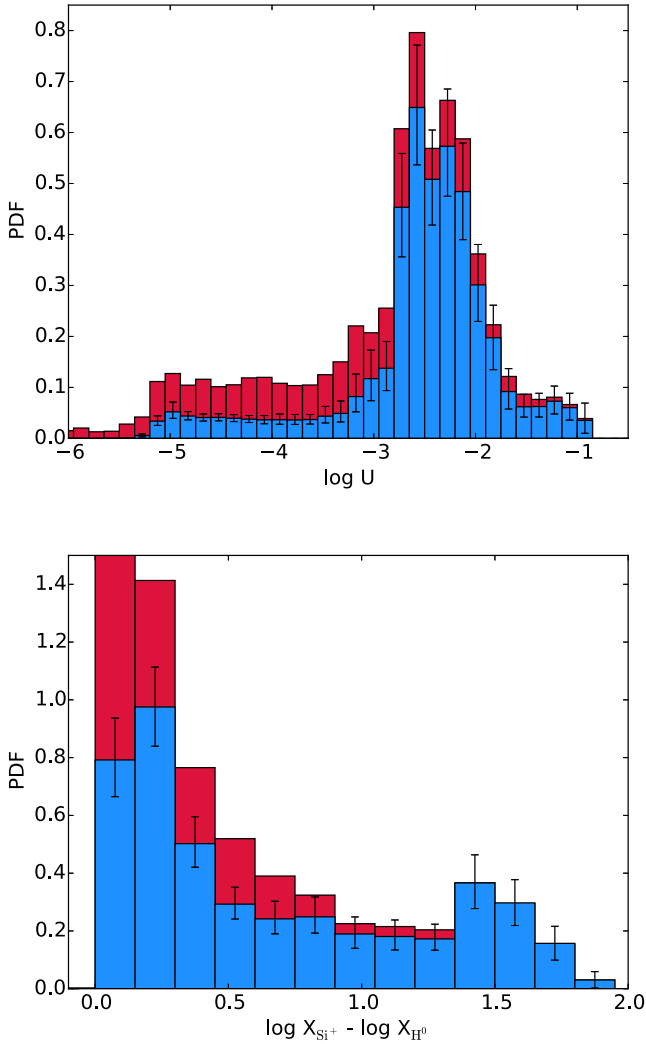
or at least have comparable metallicity to, SLLSs. This result is in contrast with claims according to which SLLSs are more metal rich than DLAs (e.g. Kulkarni et al. 2007; Battisti et al. 2012; Som et al. 2013, 2015). We note, however, that those claims rely on small samples at  $z > 2$ , and are largely based on extrapolations of trends apparent at lower redshift. According to our homogeneous analysis, either the median metallicity of SLLSs crosses the one for DLAs below  $z \sim 2$ , or a reassessment of the data from the literature may be necessary (see Dessauges-Zavadsky et al. 2009, for additional discussion on the topic). Furthermore, as previously noted by other authors (Péroux et al. 2006; Prochaska et al. 2006; Fumagalli et al. 2011a), while the metallicity of DLAs is well described by a Gaussian with mean  $\log Z/Z_{\odot} \sim -1.5$  and width  $\log Z/Z_{\odot} \sim 0.6$  (Rafelski et al. 2012), LLS metallicity spans a range up to  $> 4$  dex.

Additional insight into the redshift evolution of the metal enrichment and its dependence on the  $\text{H I}$  column density can be gained by further studying the median properties of this sample in bins of redshift and column density (Fig. 15). As a function of redshift, LLSs with  $\log N_{\text{HI}} > 19$  appear to evolve rapidly with time, with a median  $\log Z/Z_{\odot} \sim -2.4$  at  $z \sim 3.6$  reaching a median  $\log Z/Z_{\odot} \sim -1.2$  at  $z \sim 2.1$ . Also, the metallicity of LLSs with  $\log N_{\text{HI}} > 19$  appears to evolve more rapidly in comparison to DLAs. In agreement with previous studies (e.g. Péroux et al. 2007), we observe a 10-fold increase in the median metallicity of SLLSs over  $\sim 1.5$  Gyr of cosmic evolution, although with  $\sim 1$  dex scatter about the median relation. This evolution also continues at lower redshift, although we caution that a better understanding of the selection function for the systems from the literature is required before extrapolating this trend to  $z < 2$  using this sample. Such an evolution points towards a very rapid enrichment of the absorbing gas below  $z < 4$  and/or the disappearance of the most metal-poor LLSs which could be ionized or accreted on to galaxies at later times.

Finally, when examining the dependence on column density, the metallicity appears to be only a very weak function of  $\log N_{\text{HI}}$ , although with an apparent drop of  $\sim 0.5$  dex for the lowest column density bin. Thus, the difference between lower column density LLSs and SLLSs seen in Fig. 14 appears to be driven mostly by systems at the lowest column density. This trend suggests a smooth transition between the population of LLSs and the IGM. Thus, while a boundary at  $\log N_{\text{HI}} = 17.2$  is physically motivated given the ionization condition of the gas, the separation between optically thick absorbers that reside in biased environments from those in the IGM may be less clear cut already at  $\log N_{\text{HI}} \lesssim 18$  (see Sect 5.4). Studies based on even larger samples should confirm this conclusion, which remains tentative.

## 5.2 Ionisation corrections

With a model for the ionization properties of LLSs, we now discuss the importance of ICs in deriving the metallicity. To this end, we compute the PDF of the ionization parameter,  $U$ , and of the ICs that are needed for inferring the gas metallicity using  $\text{Si II}$  column densities. Both these quantities are shown in Fig. 16. For completeness, we define the ionization parameter as the dimensionless ratio of the ionizing photon flux to hydrogen density,  $U = \phi/(n_{\text{H}}c)$ , with  $c$  the speed of light. The IC for  $\text{Si II}$  relative to  $\text{H I}$  is instead defined as the logarithmic difference of the fraction of silicon in the first ionization stage and the fraction of atomic hydrogen in the neutral phase. To reconstruct the PDF for  $U$  in the entire sample, we first compute PDFs for individual systems combining the density PDF and the ionizing photon flux from the UVB at the relevant redshifts, and then we co-add the individual PDFs following the



**Figure 16.** Top. The PDF of the ionization parameter for all the LLSs included in this study (red) and for the HD-LLS subsample (blue), computed from bootstrap resampling. Bottom. ICs for Si II relative to H I, computed as the logarithmic difference of the Si<sup>+</sup> and H<sup>0</sup> fraction. The near totality of LLSs is highly ionized and differential ICs cannot be neglected when computing the metallicity.

procedures we have adopted for the density and metallicity case. For the ICs, instead, we extract the ionization fraction of Si<sup>+</sup> and H<sup>0</sup> at each position of the parameter space that has been sampled by the MCMC.

As seen in Fig. 16, the PDF for the ionization parameter shows a clear peak between  $-3 \lesssim \log U \lesssim -2$ , as commonly found for the analysis of individual LLSs (Prochaska & Bures 1999; Fumagalli et al. 2011a; Cooper et al. 2015). This means that, within the HD-LLS sample, there is a >80 per cent probability of finding a LLS that is significantly ionized, with  $\log U \geq -3$ , including a  $\sim 15$  per cent probability of finding a very high ionization for  $\log U \geq -2$ . Thus, the majority of the LLSs at  $z \gtrsim 2$ , including SLLSs, are highly ionized, a conclusion that is in line with more empirical assessments based on ion ratios (Prochaska et al. 2015). At lower redshifts, where the UVB photoionization rate declines, and for the highest column densities that are more common in the literature sample, the ionization parameter progressively decreases below  $\log U \lesssim -3$ , as visible from a long tail in the PDF.

Inspecting the PDF of the ICs for Si II as a metallicity tracer, we note that these are generally small ( $\lesssim 0.5$  dex) for most of the systems. However, the large fraction of ICs above  $\sim 0.2$  already for SLLSs, including a long tail that extends beyond  $\sim 1$  for most of the LLSs, clearly shows that differential ionization effects among elements cannot be neglected when computing the metallicity of optically thick absorbers for all H I column densities (at least at redshifts  $z \sim 2-3$ ).

Finally, since the C III ( $\lambda 977$ ) and Si III ( $\lambda 1206$ ) transitions lie within the Ly  $\alpha$  forest, we do not have clean measurements for these ions, especially at high redshifts (see Prochaska et al. 2015). We can however use the results of the ionization models to verify the hypothesis that most of the carbon (and silicon) in LLSs are doubly ionized (cf. Fig. 2). By computing the fraction of carbon in the singly, doubly and triply ionization stages along the chains, we find  $\log X_{\text{C}^{++}} \gtrsim -0.4$  for the majority of the SLLSs and  $\log X_{\text{C}^{++}} \gtrsim -0.2$  for LLSs with  $\log N_{\text{H}^1} < 19$ . Thus, the majority of the carbon is indeed predicted to be observed as C III. Similar conclusions hold for silicon. We emphasize that, in turn, this result implies that our estimates for the metallicity hinge on ions that trace only a fraction of the total mass in metals.

### 5.3 The cosmic metal budget

With an estimate for the metallicity PDF in a large sample of HI-selected LLSs between  $z \sim 2.5$  and  $3.5$ , we revisit the question of what fraction of the metals ever produced in stars is locked in optically thick absorbers, also comparing to DLAs and the Ly  $\alpha$  forest. Calculations for the metal budget in DLAs and LLSs can be found in the literature

(e.g. Pettini 2006; Prochaska et al. 2006; Bouché et al. 2007; Kulkarni et al. 2007; Péroux et al. 2007; Lehner et al. 2014; Rafelski et al. 2014), but we are now able to fill the gap between  $\log N_{\text{H}^1} \sim 17-19$ , a range of column density that remained largely unexplored by previous studies.

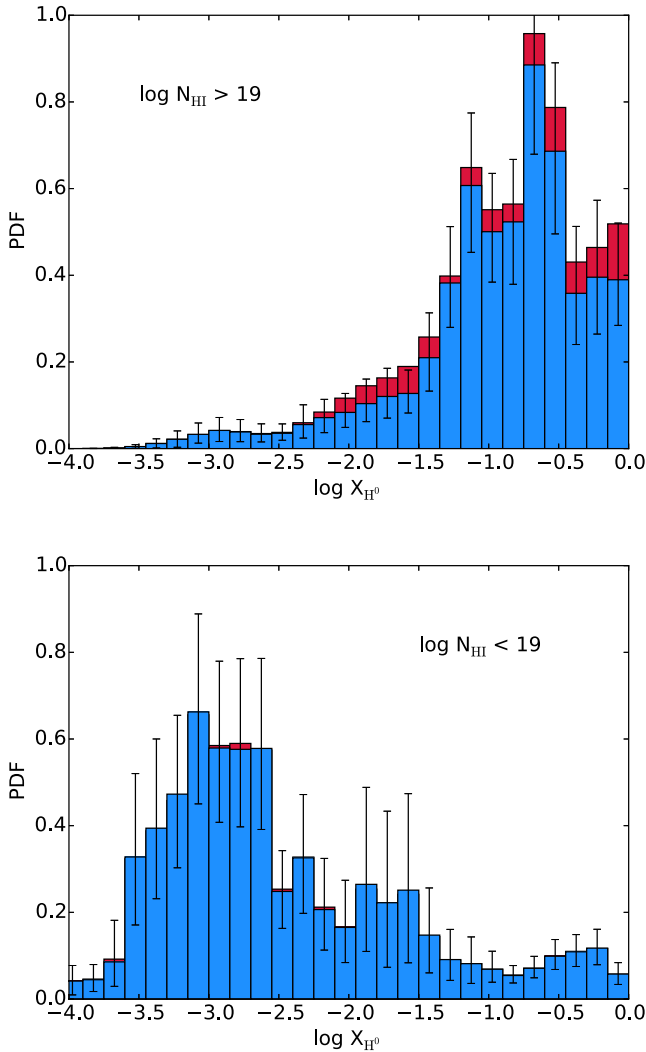
Following the notation in Prochaska et al. (2006), the metal mass density of LLSs,  $\Omega_{\text{m}}^{\text{LLS}}$ , is defined as

$$\Omega_{\text{m}}^{\text{LLS}} = \frac{1.3 m_{\text{p}} H_0}{c \rho_{\text{crit}}} \int_{N_{\text{low}}}^{N_{\text{high}}} \frac{1}{X_{\text{H}^0}} N_{\text{H}^1} f(N_{\text{H}^1}) Z_{\text{LLS}} dN_{\text{H}^1}, \quad (7)$$

with  $X_{\text{H}^0}$  the atomic hydrogen neutral fraction,  $f(N_{\text{H}^1})$  the column density distribution function,  $Z_{\text{LLS}}$  the mass in metals in the LLS gas,  $m_{\text{p}}$  the proton mass and  $\rho_{\text{crit}}$  the critical density at redshift  $z = 0$ . The factor of 1.3 accounts for helium. From equation (7), the first ingredient for computing  $\Omega_{\text{m}}^{\text{LLS}}$  is the metallicity distribution, which is available from the previous section. Next, for the column density distribution, we assume the spline function tabulated in Prochaska et al. (2014), which we evaluate at  $z = 3$ .

For the final ingredient of this calculation, we need to characterize the hydrogen neutral fraction in our sample, which we compute following the same procedures used to derive the PDF for the Si II ICs, i.e. by extracting the hydrogen neutral fractions along the chains that sample the grid parameter space. In Fig. 17, we show the PDFs for SLLSs and for LLSs with  $\log N_{\text{H}^1} < 19$ , restricted to the redshift interval  $z = 2.5-3.5$ . In line with our previous discussion on ionization, SLLSs are generally characterized by neutral fractions  $X_{\text{H}^0} \gtrsim 0.1$ , while LLSs are much more ionized, typically with  $X_{\text{H}^0} \lesssim 0.01$ . As the neutral fraction is a function of  $N_{\text{H}^1}$ , we refine our estimate for  $X_{\text{H}^0}$  with a simple functional form  $\log X_{\text{H}^0} = \alpha \log N_{\text{H}^1} + \beta$ , which is found to be a good description for our data. We adopt linear regression to compute the coefficients  $\alpha$  and  $\beta$  for our statistical sample at  $z = 2.5-3.5$ , also accounting for





**Figure 17.** Same as Fig. 3, but showing the PDF of the hydrogen neutral fraction between  $z = 2.5$ – $3.5$  for the SLLS subsample (top) and the subsample of LLSs with  $\log N_{\text{H I}} < 19$  (bottom).

the width of the  $X_{\text{H}^0}$  PDFs for individual systems. The best-fitting values are  $\alpha = 0.99 \pm 0.06$  and  $\beta = -20.3 \pm 1.2$ , implying that the hydrogen neutral fraction changes by one order of magnitude for every decade of  $\text{H I}$ , and that the transition between neutral and ionized hydrogen is crossed at the SLLS/DLA boundary. This dependence also implies that the total hydrogen column density in LLSs is comparable to the one in SLLSs.

To compute  $\Omega_{\text{m}}^{\text{LLS}}$ , we evaluate equation (7) in bins of column density, chosen to contain  $\gtrsim 15$  systems each. In each bin, we integrate the total hydrogen column density using our fitting formula for  $\log X_{\text{H}^0}$  and weighting by the column density distribution function. We then multiply this integral value by the mean metallicity, in linear space, computed for the LLSs within that bin. To account for sample variance, we repeat this calculation for 5000 samples drawn from our parent distribution allowing for repetitions. The resulting distributions, split for LLSs and SLLSs, are shown in Fig. 18.

For the assumed  $f(N_{\text{H I}})$  and cosmology, LLSs with  $\log N_{\text{H I}} < 19$  and SLLSs, respectively, contain  $\Omega_{\text{H I}} \sim 1.0 \times 10^{-5}$  and  $\Omega_{\text{H I}} \sim 2.2 \times 10^{-4}$ , which is a factor of  $\sim 5$ – $100$  less than the amount of neutral hydrogen locked in the higher column density DLAs, for which  $\Omega_{\text{H I}} \sim 1.0 \times 10^{-3}$ . The difference among these populations

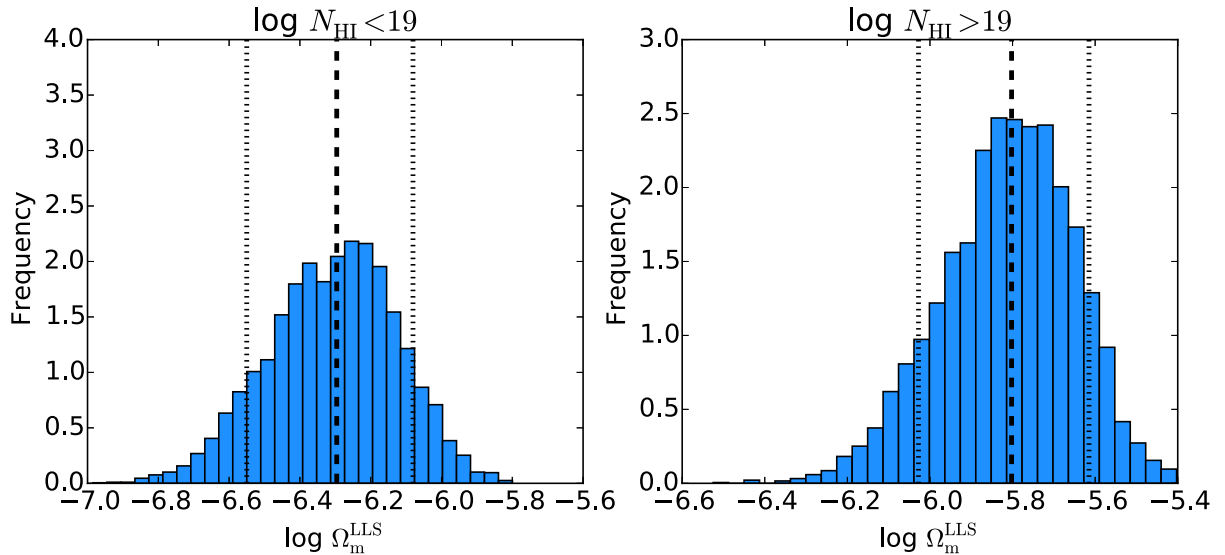
arises mainly from the intrinsic  $\text{H I}$  column density. However, after accounting for ionization corrections and folding in the metallicity distributions, LLSs with  $\log N_{\text{H I}} < 19$  contain  $\Omega_{\text{m}}^{\text{LLS}} = 5.1 \times 10^{-7}$  with a 10th/90th percentile interval of  $(2.8, 8.3) \times 10^{-7}$ . SLLSs, instead, have a median  $\Omega_{\text{m}}^{\text{SLLS}} = 1.6 \times 10^{-6}$  with a 10th/90th percentile interval of  $(0.9, 2.4) \times 10^{-6}$ . Jointly, these optically thick absorbers contain a total mass in metals of  $\Omega_{\text{m}}^{\text{LLS}} \sim 2.1 \times 10^{-6}$ .

According to our estimate, SLLSs contribute about a factor of  $\sim 3$  more than LLSs to the total cosmic metal budget, which is a consequence of having a few very metal rich SLLSs in our sample with no counterparts at  $\log N_{\text{H I}} < 19$ . We emphasize however that, while the extremes of the distribution do not significantly affect the median metallicity discussed in previous sections, the estimate of  $\Omega_{\text{m}}$  depends on the mean (linear) metallicity, which is much more sensitive to the tail of the metallicity PDF. Thus, solar and Super-Solar systems, although a small fraction of the entire sample, contribute significantly to  $\Omega_{\text{m}}$  (Prochaska et al. 2006). To illustrate this point in quantitative terms, we perform a simple idealized experiment, by adding three fake LLSs with  $\log N_{\text{H I}} = 17.5, 18.0, 18.5$  and metallicity  $\log Z/Z_{\odot} = 0.4$  to the sample of 44 LLSs in our statistical sample. With these fake systems, the median metallicity of the ensemble increases by only  $\sim 0.1$  dex. Conversely, the estimate for the total metal content is significantly perturbed by the inclusion of systems with  $\sim 100$  times more metals than the mean population, shifting  $\Omega_{\text{m}}^{\text{LLS}}$  to  $\sim 3.7 \times 10^{-6}$ . This example is clearly based on an arbitrary number of fake systems, but it highlights how sensitive  $\Omega_{\text{m}}^{\text{LLS}}$  is to extremes in the metallicity distribution.

For a similar reason, the uncertainty in the metallicity for individual systems, which is generally computed in logarithmic space, bias the estimate of  $\Omega_{\text{m}}^{\text{LLS}}$  in one direction. To test this effect, we perturb the median metallicity of individual systems with errors drawn from a Gaussian with width 0.2 dex (e.g. Fig. 10), and we repeat the measurements of  $\Omega_{\text{m}}^{\text{LLS}}$  for 5000 realizations. Due to the use of the arithmetic mean, realizations with positive errors weight more than those with negative errors, skewing the distribution for  $\Omega_{\text{m}}^{\text{LLS}}$  towards higher values, by a factor of  $\gtrsim 2 - 3$ . Thus, on the top of the uncertainty in the sample variance captured by Fig. 18, a factor of  $\gtrsim 2$  systematic uncertainties may affect this measurement.

With this uncertainty in mind, we compare our estimate to literature values. Our new measurement for  $\Omega_{\text{m}}^{\text{SLLS}}$  for SLLSs is comparable to the value reported by Prochaska et al. (2006), who quote  $\Omega_{\text{m}}^{\text{LLS}} \sim (2 - 5) \times 10^{-6}$  for SLLSs at  $z \sim 2$ , with their range being dependent on the choice for ionization correction. Similar considerations apply for the estimate in Péroux et al. (2007). For a more detailed comparison, especially for SLLSs, we should consider not only the shape of the metallicity distributions adopted by different authors but also the rapid metallicity evolution shown in Fig. 15. In our estimate, we have restricted to the redshift range  $z \sim 2.5$ – $3.5$  in an attempt to reduce the redshift dependence while retaining a sufficiently large sample size. As  $\Omega_{\text{H I}}$  only mildly evolves with redshift (Péroux et al. 2005), however, a steep evolution in the metallicity for SLLSs likely weights our measurement towards lower redshifts. Accounting for further evolution to  $z \sim 2$ , our determination is likely to be in even better agreement with the estimate of Prochaska et al. (2006) and Péroux et al. (2007).

Compared to the  $z \sim 3$  DLA population, instead, LLSs as a whole account for  $\sim 3$  times the metals in DLAs, according to the recent value of  $\Omega_{\text{m}}^{\text{DLA}} \sim 6.2 \times 10^{-7}$  reported by Rafelski et al. (2014), who revise previous estimates downward by a factor of  $\sim 3$ . The contribution from the Ly  $\alpha$  forest is instead estimated at  $\Omega_{\text{m}}^{\text{Ly}\alpha} \sim 4.6 \times 10^{-6}$ , that is a factor of  $\sim 2$  higher than for LLSs, albeit with substantial uncertainties (Schaye et al. 2003; Bouché



**Figure 18.** Frequency distributions for the cosmic metal content of optically thick absorbers in the HD-LLS sample at  $z = 2.5\text{--}3.5$ . The different contribution of SLLSs (right) and LLSs with  $\log N_{\text{HI}} < 19$  (left) is shown. Once ionization corrections are accounted for, LLSs contribute to  $\sim 15$  per cent of the total metal budget at these redshifts, with large uncertainties.

et al. 2007; Simcoe 2011). Altogether, hydrogen absorbers at  $z \sim 2.5\text{--}3.5$  contain a total amount of metals  $\Omega_{\text{m}}^{\text{ALS}} \sim 7.3 \times 10^{-6}$ . Given a conservative estimate for the metals produced by Lyman break galaxies (LBGs),  $\Omega_{\text{m}}^{\text{LBG}} \sim 1.6 \times 10^{-5}$  (Rafelski et al. 2014), we find that LLSs account for  $\sim 15$  per cent of the metals ever produced by UV-selected galaxies, with the total population of HI absorbers accounting for  $\sim 45$  per cent (cf Bouché et al. 2007). From this analysis, we conclude that LLSs are significant repository of metals at  $z \sim 3$ . It should be noted that in our estimate we are considering only metals that are locked in the main cool gas phase, which gives rise to the bulk of the H I absorption. A second (likely warmer) more ionized phase (e.g. traced by O VI) may account for an even greater fraction of metals (Lehner et al. 2014).

Clearly, this rough budget should be interpreted with caution, as several uncertainties still hamper a precise determination on many quantities that are relevant in this calculation, including the metal yields, the faint end of the galaxy luminosity function, the contribution from an obscured galaxy population, and ionization corrections as a function of column density. Furthermore, we emphasize again that, as we have discussed at length, the LLS population is characterized by an intrinsic scatter in the metallicity that, in turn, is reflected in the scatter for  $\Omega_{\text{m}}^{\text{LLS}}$  in Fig. 18. More significantly, the cosmic metal budget is sensitive to the high end of the metallicity distribution, and thus our estimate based on this sample is likely not to have converged (Prochaska et al. 2006). Refined measurements will require much larger samples or physically motivated models for the metallicity distribution that enters equation (7).

#### 5.4 Optically thick gas and the CGM

We conclude with a discussion of the implications of our analysis for studies of the nature of optically thick absorption line systems and for studies of the physical properties of the CGM at high redshift.

As noted above, the exact shape of the density distribution for LLSs is not well constrained because of a degeneracy with the radiation field. With this caveat in mind, our analysis has nevertheless provided indications that the bulk of LLSs have densities in the range  $n_{\text{H}} \sim 10^{-3.5}\text{--}10^{-2} \text{ cm}^{-3}$ . Thus, for a mean cosmic density

of  $\sim 1.2 \times 10^{-5} \text{ cm}^{-3}$  at  $z \sim 3$ , our work places LLSs in overdense structures, with contrast densities  $\delta \sim 30\text{--}800$ , which are comparable to or greater than the virial densities for most of the systems. In turn, these observations lead to the natural inference that a large fraction of LLSs are associated to galaxy haloes, adding to the many pieces of empirical evidence that suggest a connection between LLSs and the CGM, such as the redshift evolution of the number of LLSs per unit redshift (e.g. Sargent, Steidel & Boksenberg 1989; Fumagalli et al. 2013).

If indeed the link between LLSs and the CGM is established at  $z \sim 2\text{--}3$  by more direct measurements, such as direct imaging of the galaxies giving rise to LLSs or a measurement of the LLS bias (Fumagalli et al. 2014), the physical properties of large samples of LLSs will become some of the most constraining observables for models of galaxy formation. Hydrodynamic simulations consistently predict a connection between LLSs and the halo of galaxies. More specifically, different simulations agree in predicting that the elusive gas accretion on to haloes should be manifest in the form of LLSs absorbers (e.g. Faucher-Giguère & Kereš 2011; Fumagalli et al. 2011b, 2014; van de Voort et al. 2012). However, up until now, most of these predictions have been tested with simple metrics, currently available in observations, such as the covering fraction of optically thick gas around LBGs or quasars (Rudie et al. 2012; Prochaska et al. 2013). Unfortunately, the outcome of these tests have been weakened by the small sample sizes or by the fact that observations probe haloes that are not always representative of the entire galaxy population of interest. Our analysis offers a way to alleviate some of these limitations, by providing new additional metrics that can be used to test predictions of hydrodynamic simulations and, in turn, improve our understanding of the properties of cold gas accretion and feedback around galaxies.

A detailed comparison between our observations and simulations is beyond the scope of this work, but we provide some qualitative considerations of this type of analysis. As an example, one could compare the range of physical densities and hydrogen neutral fractions inferred from data to the predictions of detailed radiative transfer calculations for the denser components of the CGM in hydrodynamic simulations (Faucher-Giguère et al. 2010; Fumagalli

et al. 2011b). A cursory look to published predictions suggests general agreement between these quantities.

Moreover, our measurement of the metal distribution of LLSs opens a new window to quantitatively constrain feedback models, and the interaction between metal-poor cold gas accretion and the ejection of metal-rich gas in galactic winds. Indeed, it is clearly emerging that different implementations of stellar feedback alter the observable properties of the CGM (e.g. Hummels et al. 2013; Liang et al. 2015). Fundamental questions on the reliability of cosmological simulations to predict the transport and mixing of metals on small scales remain, but a quantitative comparison between the observed metallicity PDFs and the results of hydrodynamic simulations will be a simple yet powerful diagnostic of the efficiency of metal ejection and mixing in the haloes of high-redshift galaxies.

While we cannot derive firm conclusions without a proper statistical analysis on large samples of simulated haloes, it is interesting to note that simulations often predict drastic discrepancies for the metal distributions in the CGM. Specifically, if the majority of the LLSs indeed arise in the cold gas near to galaxies, some of the simulations that implement efficient feedback to overcome rapid gas cooling and excessive star formation may fail to reproduce the metallicity PDF (see e.g. Shen et al. 2013, who predict metallicity  $\log Z/Z_{\odot} \sim -1$  for LLSs). Puzzlingly, models with weak feedback implementations that fail instead to reproduce the correct fraction of baryons in stars (Fumagalli et al. 2011b) may more closely reproduce the metallicity PDF for LLSs with a peak around  $\log Z/Z_{\odot} \sim -2$ .

Furthermore, the low metal content of the LLS population at  $z \sim 2.5\text{--}3.5$  poses an additional constraint in the interpretation of the strong equivalent widths in low ionization metal transition near to galaxies (Steidel et al. 2010). Models that are often invoked to explain these observations assert that these lines arise from far-reaching metal-enriched outflows inside the haloes. However, a better understanding of the gas kinematics and of the metal distribution in the wind cold phase becomes critical to assess the relative contribution of column densities and Doppler parameters in shaping the strong (saturated) metal transitions, while keeping the metallicity of LLSs below  $\log Z/Z_{\odot} \sim -1.5$ , in line with our observations.

Finally, we remark that, from a theoretical point of view, these possible tensions motivate ongoing and future efforts to study in detail the coexistence of metal-poor inflows and metal-enriched outflows around high-redshift galaxies. Similarly, from the observational point of view, establishing to what extent LLSs and the halo of galaxies are connected is becoming a necessary task, so as to fully exploit the many diagnostics available in LLSs for studies of accretion and feedback.

## 6 SUMMARY AND CONCLUSIONS

We have presented a detailed study of the physical properties of a sample of 157 optically thick absorption line systems with redshifts  $z \sim 1.8\text{--}4.4$ . This sample has been selected from the HD-LLS survey by Prochaska et al. (2015), which provides us with a representative population of high-redshift LLSs. We have further expanded this statistical sample with 77 additional systems from the literature, with redshifts down to  $z \sim 0$ .

To infer the chemical composition and the physical state of the absorbing gas, we have computed ionization models by means of radiative transfer calculations at equilibrium and for a single gas phase. These calculations are the input of a Bayesian formalism that exploits MCMC techniques to derive the posterior PDF for quantities of interest, such as the gas-phase metallicity, the physical

density, the temperature and the dust content of the absorbing gas. To explore the dependency of our results on the assumptions of the input ionization models, we have computed five different grids of models, varying the shape and intensity of the UVB, and the contribution of local sources, of dust and metal depletion on to grains, and of collisional ionization.

Through comparisons of the PDFs inferred under different model assumptions, we have shown that simple photoionization models provide a good description of the general LLS population, and that the physical properties of LLSs are not extremely sensitive to the assumed ionization corrections. However, the predictions for individual systems, and particularly for their density and size, are more prone to systematic effects attributable to ionization corrections.

Our findings on the physical properties of our statistical sample of LLSs between  $z \sim 2.5\text{--}3.5$  can be summarized in the following way.

(i) LLSs arise from photoionized gas, with temperatures  $T < 5 \times 10^4$  K, and ionization parameters  $U \sim 10^{-3}\text{--}10^{-2}$ . Thus, ionization corrections for hydrogen and metal lines are critical and cannot be neglected when inferring the metallicity of the absorbing gas.

(ii) LLSs have typical densities between  $n_{\text{H}} \sim 10^{-3.5}\text{--}10^{-2}$  cm<sup>-3</sup>. However, the detailed shape of the density PDF is sensitive to the amplitude of the ionization radiation field because of a well-known degeneracy between density and radiation. Indeed, in our analysis, we have found that observations of the most common ions cannot be generally used to robustly constrain the properties of the ionizing radiation field, or the size of the absorbing gas.

(iii) The population of  $z \sim 2.5\text{--}3.5$  LLSs is metal poor, with a peak at  $\log Z/Z_{\odot} \sim -2$ , which is below that observed in higher column-density DLAs with a mean  $\log Z/Z_{\odot} \sim -1.5$ . Further, LLSs appear to contain only modest amounts of dust. The inferred metallicity distribution is very broad, extending over four orders of magnitude. The probability of finding a metallicity  $\log Z/Z_{\odot} \leq -1.5$  is  $\sim 70$  per cent, but the probability of finding very metal poor systems with  $\log Z/Z_{\odot} \leq -3$  is modest, being  $\sim 10$  per cent. The metal content of SLLSs with  $\log N_{\text{H I}} \geq 19$  rapidly evolves with redshift, with a 10-fold increase between  $z \sim 2.1$  and  $z \sim 3.6$ . We have also reported tentative evidence that the lower column density LLSs with  $\log N_{\text{H I}} \lesssim 18.5$  are the least enriched, suggesting a smooth transition with the IGM.

(iv) After accounting for ionization corrections, LLSs with  $\log N_{\text{H I}} < 19$  and SLLSs with  $\log N_{\text{H I}} \geq 19$  jointly contain a total mass in metals of  $\Omega_{\text{m}}^{\text{LLS}} \sim 2.1 \times 10^{-6}$ , which is  $\sim 3$  time more than the amount of metals locked in DLAs. Compared to the metals produced by UV-selected galaxies, LLSs account for  $\sim 15$  per cent of all the metals, although systematic uncertainties as large as a factor of  $\gtrsim 2$  affect this estimate. Moreover,  $\Omega_{\text{m}}^{\text{LLS}}$  is likely not converged in our sample, as rare systems in the tail of the metallicity PDF may contribute significantly to the cosmic metal budget.

Despite our efforts to quantify the extent to which the inferred physical properties of LLSs are robust against systematic uncertainties in the input ionization models, our analysis has not exhausted all possible scenarios, such as the presence of multiple gas phases, a non-constant density profile (e.g. Petitjean, Bergeron & Puget 1992; Ascasibar & Díaz 2010) or non-equilibrium effects (e.g. Oppenheimer & Schaye 2013). Future work should expand on the formalism we have developed to gain an even deeper understanding of the behaviour of ionization corrections in more realistic astrophysical environments.

By characterizing the physical properties of LLSs at  $z > 2$  in a large statistical sample, our work has provided new clues for the origin of LLSs and new empirical constraints for theories of cold accretion and feedback. Given that a significant fraction of LLSs have densities comparable to or higher than the virial densities, the statistical properties of samples of hundreds of optically thick absorbers can now be compared quantitatively to the predictions of numerical simulations for the CGM, providing new metrics to constrain the efficiency of metal ejection and the mixing between enriched gas in the outflows and metal-poor inflows. A simple qualitative comparison already reveals a possible tension. Indeed, successful galaxy formation models that eject baryons from galaxies to avoid an overproduction of stars appear to enrich LLSs in the halo above what is suggested by our observations. Conversely, models with weak feedback that overpredict the fraction of baryons locked in stars may better reproduce the metallicity PDF of LLSs. Future work is now needed to characterize and address this possible discrepancy between theory and observations, and to explore the role of mixing between inflows and outflows in shaping the observed metallicity distribution.

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## APPENDIX A: MOCK TESTS AND METHOD VALIDATION

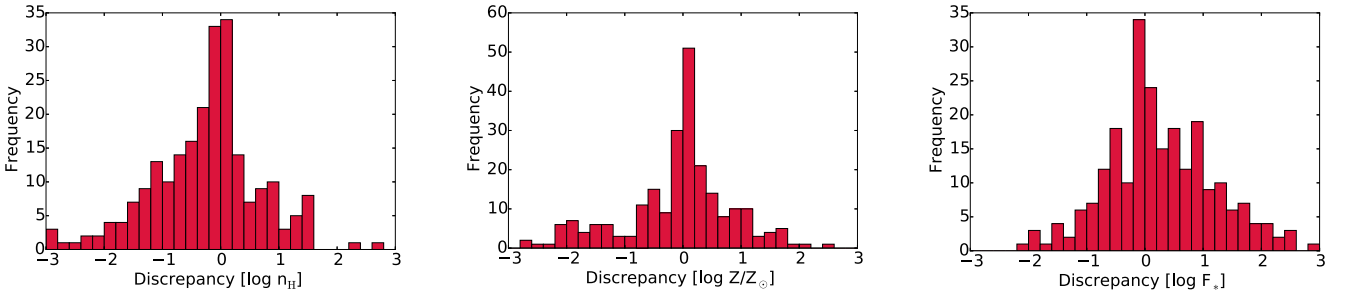
To validate the analysis method and to characterize the performances of the MCMC code in reconstructing the underlying PDFs, we generate sets of mock data that we then process with the same analysis tools used throughout this paper.

As a trivial test to validate the code, we generate a mock set of 234 LLSs with the redshift and density distribution of the observed sample, and with a Gaussian density distribution centred at  $\log n_{\text{H}} = -2.5$  with a width of 0.3 dex, and a metallicity distribution centred at  $\log Z/Z_{\odot} = -2$  with a width of 1 dex. Using the minimal grid, we then generate an idealized mock catalogue of column densities, where each ion is measured with an uncertainty of 0.05 dex. We then perform the MCMC analysis on this data set, successfully recovering the input density and metallicity PDFs for the full sample, as well as a one-to-one relation between the input values and the medians of the metallicity and density PDFs for individual systems.

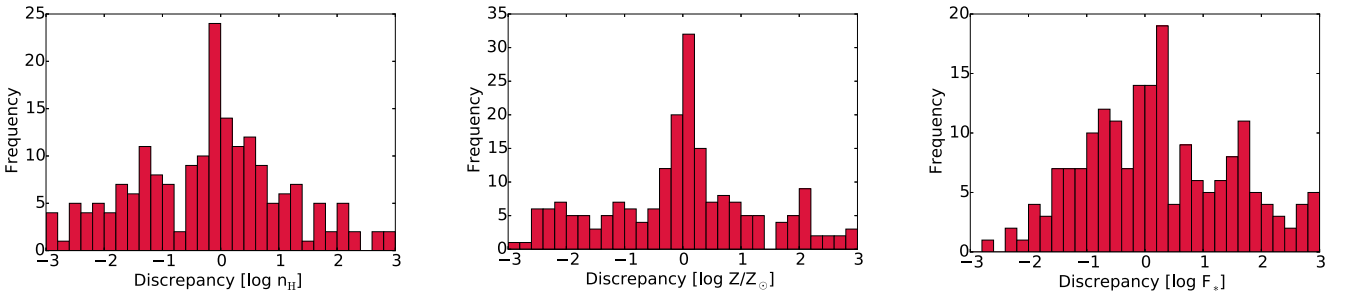
To better understand the performance of the MCMC analysis on more realistic data, we construct a new set of mocks, using the dust grid of models with  $F_{*}$  values distributed as a Gaussian centred at  $F_{*} = 0$  and width 0.6 dex. For this set, we match the data quality to the observed one, including for each simulated LLS only the observed ions in the corresponding real systems, and preserving the associated uncertainties and upper/lower limits. Thus, differently from the previous case, this mock sample is characterized by a variety of data quality, including systems with only a handful of observed ions, larger errors on the column densities or a large number of lower/upper limits. After performing the MCMC analysis, we find a tight correlation between the input quantities and the medians of the posterior PDFs, although with a scatter and the presence of outliers. As a metric of the ability of the MCMC analysis to recover the input data, we compute for each quantity the deviation between the median of the reconstructed PDF and the input value, which we normalize to the a characteristic width of the posterior PDF using the 25th and 75th percentiles.

The histogram of the discrepancies for these mock LLSs is shown in Fig. A1. From this test, we conclude that the posterior PDFs reconstructed with our analysis technique contain the input value within the first and third quartile of the distribution for the majority of the systems, with the near totality being contained by twice the PDF width for individual LLSs. Furthermore, the discrepancies are well centred at zero. However, among the outliers, there is a hint of a small preference for larger  $F_{*}$  and smaller  $n_{\text{H}}$  which we attribute in part to the degeneracy between parameters in the dust model, and to the skewness of the posterior PDFs for the LLSs with least constraining data (e.g. all upper limits).

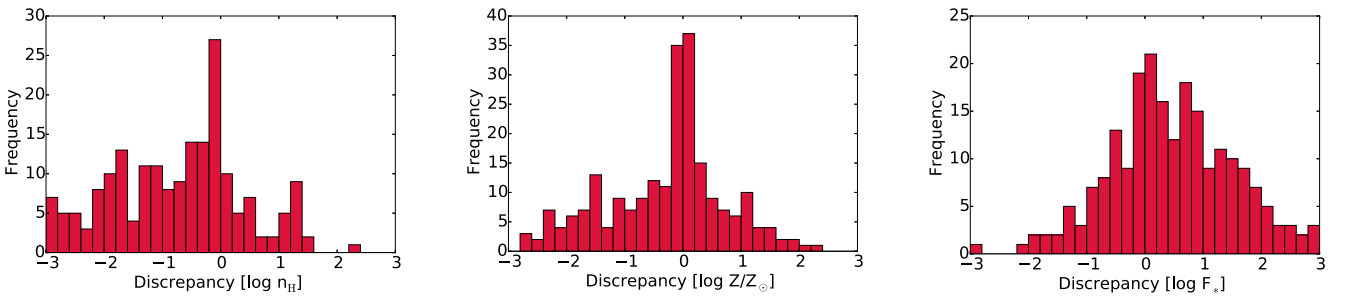
We can also test how robust is the MCMC method to (i) the presence of systematic errors in the ion column densities that are unaccounted for by error bars; (ii) a possible excess of column density in high ionization species (e.g. C IV) from a second phase along the line of sight. For the first test, we add a systematic offset to the ion column densities in the mock sample by drawing from a Gaussian centred at zero and with a width of 0.15. This means that approximately one in three ions for each LLS have their column density offset by  $>0.15$  dex, thus beyond the typical error bars on the column density. After performing the MCMC analysis, despite the addition of systematic errors, we see that the input values are still recovered without large systematic errors (Fig. A2), although, unsurprisingly, the number of outliers increases, particularly between 1–2 times the width of the posterior PDFs. For the second test, we boost the C IV column density by a random amount in the



**Figure A1.** The discrepancy between the input and recovered density (left), metallicity (centre) and  $F_*$  (right) for a mock sample of LLSs that is matched to our observed sample. This discrepancy is quantified as the median of the posterior PDF and the input value, normalized to the PDF width from the 25th and 75th percentiles. For the majority of the mock LLSs, the input values are well within the first and third quartile of the posterior PDF.



**Figure A2.** Same as Fig. A1, but including systematic errors on the ion column densities.



**Figure A3.** Same as Fig. A1, but including additional contribution to the C IV column density from a second, more ionized, gas phase.

interval (0, 0.5) dex. Fig. A3 shows the resulting discrepancy from the analysis of these new mock data. On average, the input PDFs distribution are recovered without significant biases, although there are a handful of individual cases in which the input and output values are in worse agreement, with a hint of systematic effect, which is to be expected given the one-sided perturbation in the data.

In summary, these tests confirm that our MCMC procedure is performing well for the case of idealized mock data. Moreover,

despite discrepancies for individual systems (especially the LLSs with the least constraining data), our analysis appears robust in recovering the underlying PDFs for the LLS population, even in presence of systematic errors on the column densities and a possible second, more ionized, gas phase.

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