Adolescents, Cognitive Ability, and Minimax Play

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Abstract: We conduct experiments with adolescent participants on repeated fixed play in three different zero-sum games which have mixed strategy minimax solutions. Further, we collect subject information on cognitive abilities and participation rates in competitive activities. We find the adolescents' correspondences with and deviations from minimax play largely consistent with previously and widely studied adult populations. Further, we find strategic sophistication in terms of implementation of the mixed minimax strategy as well as earnings are not correlated with cognitive ability nor experience in competitive situations.

Keywords: Minimax; experiment; adolescent; cognitive abilities

JEL Classification Numbers: C72, C93, D03

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1 Introduction

Mixed strategy minimax solutions are qualitatively consistent with aggregate action frequencies, but at the individual level there is excessive heterogeneity in action frequencies and realized earnings as well as serially correlated action choices. This is the consensus of laboratory and field tests of the repeated play of fixed pairs from populations such as college students (O'Neill, 1987; Rosenthal et al., 2003), professional athletes (Walker and Wooders, 2001; Levitt et al., 2010), experienced poker players (Van Essen and Wooders, 2013), human teams (Okano, 2013), people with schizophrenia (Baek et al., 2013), and primates (Martin et al., 2014). We replicate the O'Neill and Rosenthal et al. (hereafter, RSW) studies using students from a middle school in China. We also augment their designs by collecting data on the participants' cognitive abilities and participation in competitive activities. Our contributions are two-fold. We identify whether the noted consistencies of behavior in repeated zero-sum games with mixed strategy solutions is developed prior or post adolescence. Also, we are the first to examine the correlation between strategic sophistication and cognitive abilities in this environment.

Researchers have increasingly conducted experimental games with children and adolescents to identify when humans develop strategic thinking and some behavioral regularities inconsistent with noncooperative game theory. Brosig-Koch et al. (Forthcoming) find the propensity to use backward induction increases with age. Czermak et al. (2010) compare adolescent and college student study play in normal form games with pure strategy Nash equilibrium. They find that adolescents and college students play Nash equilibrium at the same rate, but adolescents more often best respond to their stated beliefs. A second set of studies, such as Murnighan and Saxon (1998); Harbaugh et al. (2002), have examined bargaining behavior in ultimatum games, and find that younger subjects will make less generous offers which are also more willingly accepted. These results raise the question at what age do pro-social tendencies (Lergetporer et al., 2014) and social preferences (Fehr et al., 2008,

2013) develop. Our study addresses the question, at what age do people develop strategic sophistication in zero-sum games with mixed minimax strategies?

Researchers have also shown increasing interest in understanding the link between strategic sophistication and cognitive abilities. The types of strategic sophistication usually considered are backward induction in extensive form games, steps of iterative dominance in normal form games, and the induction ability to forecast play of opponents and respond optimally. Devetag and Warglien (2003) find positive correlation between levels of iterated dominance and backward induction and subjects' short-term memory capacities. Carpenter et al. (2013) extend these results by using multiple measures of cognitive ability, establishing this correlation also extends to players' ability to model others' sophistication, and make initial steps in establishing a causal relationship by exogenously shocking the cognitive load of subjects. With respect to repeated p-beauty contests, Gill and Prowse (2013) find that those with higher cognitive ability choose numbers closer to equilibrium, converge more frequently to equilibrium play, and earn more than those with lower cognitive ability.

In the context of games with mixed strategy minimax solutions, we propose that strategic sophistication can take two forms corresponding to alternative rationales of the minimax solution in zero-sum games. The original minimax rational is that a player solves the problem, and implements the solution, of choosing a strategy to secure the largest minimum payoff over the set of his opponents' feasible strategies. When the minimax solution is a mixed strategy and the game a repeated one, the behavioral challenge extends to generating a sequence of actions that are realizations from a sequence of identical and independent distributions. In this case we would expect cognitive ability to be positively correlated with the proximity of action frequencies to the minimax implied ones, and serial independence of the action choices. The other rational for minimax play in zero-sum games is the logic of Nash equilibrium where a minimax strategy is the fixed point of the players' best response correspondences. Under this best response rational, strategic sophistication does not necessarily correspond to minimax play, but rather to successfully forecasting opponent play and accordingly best respond. In this case, strategic sophistication would imply higher repeated game payoffs.

Our findings summarized. Aggregate action frequencies are inconsistent with minimax predictions in all three games. With respect to the O'Neill game, we observe aggregate action frequencies similar to those of the original O'Neill study. With respect to the two versions of asymmetric matching pennies of RSW, our data is more systematic in that one player role's action frequencies matches the minimax proportion while the other matches equiprobable play. At the individual pair and player level, we find the same strategic heterogeneity generically found in these studies, as well as serially correlated play. We have a non-result regarding cognitive ability and experience in competitive settings. Cognitive ability and experience in competitive outcomes has no correlation on adherence to minimax predictions, sophistication under the minimax rational, nor subject performance, sophistication under the best response rational.

2 Experimental design

Our experimental design consists of two parts. In the first part, subjects attend a session in which they complete a survey and then play 100 rounds of a zero-sum card game against a fixed opponent. The second part is an non-timed Raven's standard progressive matrices (SPM) test (Raven and De Lemos, 1990), administered two weeks after the first session.

We recruit 128 Chinese students, ages twelve to fifteen, from the Haichang Experimental Middle School in Xiamen, China. In our survey, we collect the average amount of time per week spent playing competitive sports. The school provides the age, gender, and mid-term math exam scores.[1] We also record the subjects' Raven's SPM scores.[2]

We conduct three games sessions: one with 40 subjects for the O'Neill replication, one with 44 subjects for the replication of RSW's deterministic pursue-evade (DPE) game, and one with 42 subjects for the replication of RSW's stochastic pursue-evade (SPE) game.

[1] The average score is 85.14 out of 120, and the scores range from 3 to 120.

[2] The average score is 49.64 out of 60, and the scores range from 27 to 59.

The protocols are the same for each session.[3] Subjects are randomly assigned to pairs and player roles. At a table, each pair sits side-by-side, separated by partition, and opposite a human monitor. Each player is given a set of action cards, and a polystyrene pad in which an endowment of push-pins are lodged. Push-pins are the experimental currency which is exchanged for Renminbi at the end of the experiment. In a stage game, each players selects a card and places it face down in front of the monitor. The monitor then turns over the cards, records the actions, rolls a die when necessary (explained shortly), and executes pin transfers according to the outcome. After the 100 stage games, the monitor pays the subjects according to final number of pins in their pads.[4] Each session concludes within an hour.

In the O'Neill game, each player has four cards: K, 3, 6 and 9.[5] The normal form representation of the zero-sum game is presented, with Player 1's payoff shown, in Table 1. The minimax strategy is the same for each player: play the K, 3, 6 and 9 card with probability 0.4, 0.2, 0.2 and 0.2 respectively.[6] The value of the game is -0.2 for Player 1. Prior to the first stage game, each player is given a forty-five pin endowment. The exchange rate is two pins per Renminbi. The O'Neill game has been adopted in a plethora of studies, with aggregate play robustly aligning to minimax. We chose this game to give adolescent subjects the "best chance" to play minimax.

In the DPE game, the two players, i.e. Pursuer and Evader, can either play Left or Right. If both play Left, the Pursuer captures one pin from the Evader. Similarly if both play Right, the Pursuer captures two pins from the Evader. For the other two action profiles, the Evader avoids losing any pins. The normal form representation of this zero-sum game is presented, with the Pursuer's payoff shown, in Table 1. The symmetric minimax solution is

[3] Please consult the online appendix at http://www.jasonshachat.net/ACMPinstructionsEnglish.pdf, to find instructions in English and Mandarin.

[4] No subjects went bankrupt in the experiments, and only 2 out of 128 subjects had a concluding balance of left less than five pins.

[5] O'Neill (1987) used the following cards: Joker, Ace, 2, and 3.

[6] According to Corollary 3 of Wooders and Shachat (2001), since each player only has two payoff levels in the stage game and the stage game Nash equilibrium is unique, as well as in strictly mixed strategies, the Nash equilibrium of the finitely repeated game is unique and consists of the stage game equilibria played following any history.

Table 1: The normal form of the O'Neill and Pursue-Evade games

O'Neill Game

		Player 2: K	3	6	9
	K	1	−1	−1	−1
Player	3	−1	−1	1	1
1	6	−1	1	−1	1
	9	−1	1	1	−1

Pursue-Evade Game

		Evader: Left	Right
Pursuer	Left	1	0
	Right	0	2

for both players to play Left with probability two-thirds, and the value of the game for the Pursuer is also two-thirds. Prior to the first stage game, the Evader is given a one hundred and thirty-five pin endowment. The exchange rate is three pins per Renminbi.

As the stage game has three payoff levels for each player - the minimax and Nash equilibrium solutions are not invariant to the players' risk attitudes. Correspondingly, RSW introduced the SPE game that only offers two payoff levels to each player in the stage game. The action spaces remain unchange but the winning rule is augmented to make the capture reward-penalty probabilistic. The Pursuer captures a single token when both players play Left and the monitor rolls a 1 or 2 with a six-sided die, or when both play Right and the monitor rolls a 3, 4, 5, or 6. The normal form game remains the same as the DPE, but the payoffs are now the Pursuer's conditional probability of capturing a pin. In the SPE game, Evaders are endowed with forty-five pins, and the exchange rate is one pin per Renminbi.

3 Data analysis

We start with a data visualization of the three games showing how close individual pairs play to the minimax predictions, and the play in the original studies. Figure 1 contains three scatter plots. For each game we plot the joint action frequencies of the adolescent and the original study pairs. The dotted horizontal and vertical lines mark the fifty percent frequencies and the minimax frequencies (0.4 for the ONeill game and 0.67 for DPE and SPE

(a) O'Neill joint King play (b) DPE joint Left play (c) SPE joint Left play

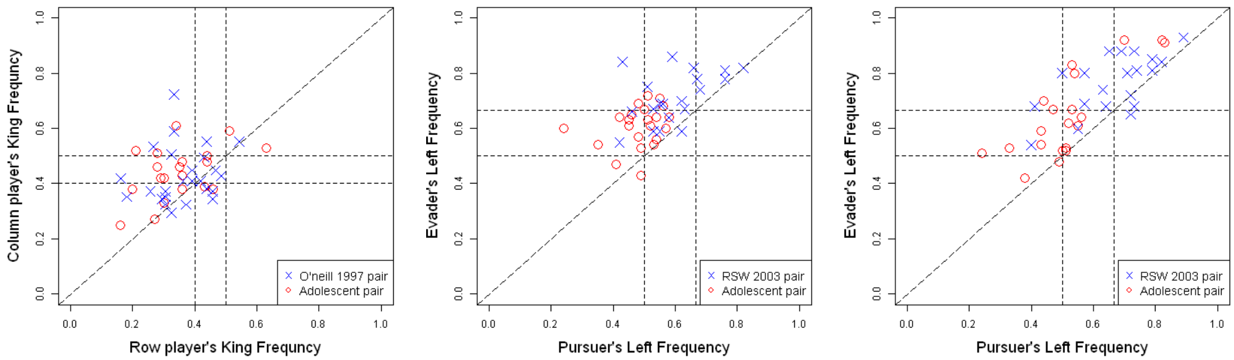

Figure 1: Scatter plots of joint action frequencies for adolescents and original studies

game.) With respect to the O'Neill game,[7] the adolescents data appears to have a similar distribution to the original college student data, and is centered around the minimax profile. In the DPE and SPE games, adolescents play is removed from the minimax strategy. Pursuers playing Left around fifty percent of the time, and the Evaders play a higher frequency than that. Adolescent play also looks different from the original college student play.

3.1 Aggregate play

We quantitatively evaluate how well our aggregated data matches the minimax predicted action frequencies, and the data from the original studies. We assume that action choices follow a player role specific mixed strategy. Under this assumption the sufficient statistics for the mixed strategies and distributions of action profiles are the aggregate frequencies of action choice and joint action play. We present these, along with the minimax predictions and original studies' frequencies, in Panels A and B of Table 2. We reject that the mixed strategies used in our experiments are the minimax ones using χ^2 goodness-of-fit tests on players' actions and the joint action profiles. The one exception is the Evader role of the SPE game. We also reject that adolescents follow the same mixed strategies as the original participants for RSW's two pursue-evade games but fail to do so for the O'Neill game.

Next we ask if play is homogeneous, i.e. if players in the same role follow the same

[7] We "collapse" the action set to King and Numbered card.

strategy. We reject, for all games and all roles, that all players follow the minimax strategy; moreover, we reject there is any common mixed strategy. Panels C of Table 2 presents the details of these individual tests.

Table 2: Aggregate play statistics, hypothesis tests of minimax predictions, and testing homogeneity of adolescents and original play

	O'Neill	DPE	SPE
Panel A: Aggregate action frequencies			
Action set	(King, 3, 6, 9)	(Left, Right)	(Left, Right)
Minimax	(.40, .20, .20, .20)	(.67, .33)	(.67, .33)
Adolescent Player 1/Pursuer	(.35, .23, .21, .21)[m]	(.48, .52)[m,b]	(.51, .49)[m,b]
Original Player 1/Pursuer	(.36, .20, .22, .22) [m]	(.61, .39)[m]	(.67, .33)
Adolescent Player 2/Evader	(.44, .15, .16, .25)[m]	(.60, .40)[m,b]	(.65, .35)[b]
Original Player 2/Evader	(.43, .17, .18, .23)[m]	(.72, .28)[m]	(.76, .24)[m]
Panel B: Aggregate action profile frequencies			
Action profile set	(KK,KN,NK,NN)[a]	(LL,LR, RL, RR)	(LL, LR, RL, RR)
Minimax	(.16, .24, .24, .36)	(.44, .22, .22, .12)	(.44, .22, .22, .12)
Adolescent	(.17, .18, .27, .38)[m,b]	(.31, .17, .29, .22)[m,b]	(.35, .17, .30, .18)[m,b]
Original	(.16, .20, .27, .37)[m]	(.44, .17, .28, .11)[m]	(.52, .15, .24, .09)[m]
Panel C: Heterogeneity - standard deviation of the number of King/Left choices			
Minimax prediction	4.899	4.714	4.714
Adolescent Player 1/Pursuer	11.287[c,d]	7.810[c,d]	13.916[c,d]
Original Player 1/Pursuer	9.817[c,d]	9.521[c,d]	14.790[c,d]
Adolescent Player 2/Evader	9.451[c,d]	7.382[c,d]	15.106[c,d]
Original Player 2/Evader	10.613[c,d]	10.149[c,d]	11.467[c,d]
Panel D: Cross match test for original and adolescent homogeneity			
p-value	0.323	0.222	0.285

[m] Denotes rejection (5% level of significance) of the χ^2 goodness-of-fit test, with the hypothesis the common strategy is minimax.
[b] Denotes rejection (5% level of significance) of the χ^2 goodness-of-fit test, with the hypothesis the Adolescents' and the Original subjects' strategies are the same.
[a] For brevity we treat the set of actions as (K,N) where N is any numbered, i.e. non-King, card. However, full action sets and profiles are used in hypothesis tests.
[c] Denotes rejection (5% level of significance) of the χ^2 test for variance, assuming minimax implied variance.
[d] Denotes rejection (5% level of significance) of the χ^2 test for variance, assuming all players follow the same - but not necessarily minimax - mixed strategy.

Finally, given the strong evidence of heterogeneity, we revisit the question of whether our data and previous data are realizations of the same distribution. We use the cross match test (Rosenbaum, 2005) that allows each pair's strategy profile to vary and for ties on the empirical distribution functions of the two populations.[8] In Panel D of Table 2, we report the p-values of this test and fail to reject, for all three games, the adolescent and original data have the same distribution. But note, while valid, the cross match test lacks power because it does not use all of the sample information.

3.2 Pair and individual level play

Since we reject that the minimax hypothesis holds jointly for all pairs and players, we test which pairs' play is consistent with minimax. We test each pair's action profile with the null hypothesis of minimax play using a χ^2 goodness-of-fit test and a 5% level of significance. We reject, row two of Table 3, joint minimax play for 50% of our O'Neill game pairs and for a large majority of our Pursue-Evade game pairs. Under the Nash equilibrium rational for minimax play in two-person zero-sum games, if one player follows his minimax strategy the other player will be indifferent amongst the strategies in the support of his minimax strategy. Accordingly, we conduct a binomial test for each player's proportion of King/Left play under the null it is chosen according to minimax frequencies. We reject, rows three and four of Table 3, the minimax frequency for the majority of individuals except for Player 2 in the O'Neill game (40%) and Evaders in the DPE game (27%).

We finally test that each action is an independent realization from the minimax mixed strategy. We evaluate each subject's sequence of 100 actions using a nonparametric runs test. The null hypothesis is that every action is an independent realization from a constant distribution. Rows five through nine of Table 3 show that we reject serial independence for at least 40%, 20%, and 33% of the subjects' sequences in the O'Neill, DPE, and SPE games respectively. For each game, the majority of rejections are due to negative serial correlation.

[8] We find such ties in when comparing our data to that of the original studies. This precludes us from using the more commonly adopted Kolmogorov-Smirnov test as the usual p-values are no longer correct with ties and our small sample size.

Table 3: Results summary of pair level hypothesis tests of minimax prediction on joint action profile and King/Left action frequencies; and runs tests for serial independence of actions. We report the rejection percentages of minimax at the 5% level of significance

Hypothesis test	O'Neill	DPE	SPE
χ^2 goodness-of-fit test: action profile is minimax	50%	82%	95%
Binomial test for Player 1/Pursuer minimax play of King/Left	55%	86%	95%
Binomial test for Player 2/Evader minimax play of King/Left	40%	27%	62%
Runs test rejection: negative serial correlation for Player 1/Pursuer	40%	18%	33%
Runs test rejection: positive serial correlation for Player 1/Pursuer	5%	5%	15%
Runs test rejection: negative serial correlation for Player 2/Evader	45%	18%	19%
Runs test rejection: positive serial correlation for Player 2/Evader	0%	0%	10%

3.3 Strategic sophistication and cognitive abilities

Surprisingly neither closeness to minimax play or level of earnings are correlated with cognitive ability or the time spent participating in competitive sports. Our measures of cognitive ability are the Raven's SPM test and midterm Math exam scores; our measure of time engaged in competitive activities is the sum of a subject's reported weekly times playing various sports.

To measure the proximity of a participant's play to minimax, we use the p-value of the χ^2 goodness-of-fit test between his vector of action frequencies and his minimax strategy vector. To assess the correlation between this distance and our cognitive measures we calculate the Spearman rank correlation coefficient, which can capture nonlinear correlations. We report these coefficients in Panel A of Tables 4. No coefficients significantly differ from zero for own and opponent's Raven and math scores, and only one is significant for own sports time. We measure how successful a subject randomizes by using the absolute value of his z-stat from the runs test, noting a larger absolute value equates to more serial dependence. In this case we only find three out of thirty-six Spearman rank correlation coefficients are significant - certainly not enough to make compelling claims.

We finally ask if a participant's performance is correlated with any of our measures. In Panel B of Table 4, we see there is no significant correlation between a player's earnings and

Table 4: Spearman rank correlation coefficients between cognitive measures-sports time and minimax strategy-earnings

Panel A: Spearman rank correlation coefficients for distance from minimax and serial correlation

		χ^2 goodness-of-fit test p-value			Absolute value of the z-stat for runs test		
		O'Neill	DPE	SPE	O'Neill	DPE	SPE
Player 1/ Pursuer	Own Raven score	−0.12	0.14	-0.38	0.27	−0.05	-0.28
	Opponent's Raven score	0.16	0.07	0.11	0.15	0.07	0.21
	Own math score	−0.31	0.29	-0.22	0.29	−0.19	-0.26
	Opponent's math score	−0.13	−0.20	0.25	−0.15	0.08	0.04
	Own sports time	−0.02	−0.04	0.32	−0.14	0.25	0.05
	Opponent's sports time	−0.20	−0.17	0.24	−0.43	0.15	-0.23
Player 2/ Evader	Own Raven score	−0.06	0.17	0.35	−0.24	0.15	-0.17
	Opponent's Raven score	−0.24	0.02	-0.23	−0.24	−0.43	-0.36
	Own math score	0.17	0.14	0.18	−0.67[b]	0.20	-0.10
	Opponent's math score	−0.21	0.11	-0.22	−0.06	−0.49[b]	-0.33
	Own sports time	−0.40[b]	0.17	0.01	0.20	0.54[b]	0.14
	Opponent's sports time	−0.30	0.18	0.10	−0.08	−0.03	-0.36

Panel B: Spearman rank correlation coefficients for Player 1/Pursuer earnings

	O'Neill	DPE	SPE
Own Raven score	0.11	−0.51[b]	-0.21
Opponent's Raven score	−0.22	−0.34	-0.21
Own math score	0.15	−0.52[b]	0.03
Opponent's math score	−0.37	−0.08	-0.17
Own sport time	0.29	−0.22	-0.32
Opponent's sport time	0.23	−0.23	-0.38

[b] Denotes rejection (Denotes significantly different that zero at the 5% level of significance.

own and opponent's Raven and math scores, and sports time. The notable exception is in the DPE game, for which the correlation coefficients between earnings and own Raven and math score is negative; the opposite sign of our conjecture.

4 Discussion

The adolescents in our study have largely reproduced the behavioral consistencies found when adults play zero-sum games with mixed strategy minimax solutions. Thus these behavioral consistencies likely develop prior to adolescence. Further, we are the first to show that cognitive ability is uncorrelated with the strategic skill of implementing a mixed strategy minimax solution, and the ability to detect and exploit deviations of play in this environment. This suggests that identifying or cultivating individuals who will be successful in such situations should not rely upon screening using intelligence tests or other cognitive measures. What personal characteristics lead to success in such settings remains an open question.

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