Integrating Corporate Ownership and Pension Fund Structures:

A General Equilibrium Approach

M. Shahid Ebrahim, Bangor University, UK.

and

Ike Mathur, Southern Illinois University, Carbondale, IL, USA.

Correspondence Address: Ike Mathur
Professor of Finance
Department of Finance
Southern Illinois University
Carbondale, IL 62901-4626, USA
Tel: 618 581 1613 or 618 288 4025
Fax: 618 453 5626
E-Mail: imathur@business.siu.edu

Abstract

This paper studies pension fund design in the context of investment in the debt and equity of a firm. We employ a general equilibrium framework to demonstrate that: (i) the asset location puzzle is purely a risk neutral phenomenon that disappears with the introduction of sufficient risk aversion; (ii) the inability of policy makers to manage an economy with multiple firms yields a mixed equilibrium, where bonds are observed in both taxable and tax-deferred accounts; and (iii) the pareto-efficient pension plan comprises of a hybrid of defined benefit and defined contribution plans, exploiting the comparative advantages of both.

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Abstract

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I. Introduction

The changing demography of developed nations stemming from an aging population, in conjunction with a shrinking workforce, has contributed to a pension crisis. The ramifications of failing pension systems range from a reduction in the quality of life for the elderly to insolvencies of companies. Many issues surrounding this complex problem remain obscure and inadequately understood.[1]

The complexity of pensions can be attributed to multiple factors ranging from corporate finance, involving tax shelters, to labor economics, involving contracts with insurance like features such as the defined benefit (DB) plan (Bodie, 1990).[2] These complexities provide clues to the crucial role played by: (i) policy makers, who have to use incentives in the form of tax deductions to compel myopic economic agents to save for their retirement (Shiller, 2003); (ii) employers, i.e., firms, who need to judiciously structure their stakes, in the form of debt and equity, to fund their pension plans at levels that do not threaten the financial system of their nation (Allen, 2001); and (iii) employees, i.e., economic agents, who need to appropriately invest for retirement via individual taxable accounts (termed conventional savings accounts, CSAs) and retirement accounts (termed tax deferred accounts, TDAs), referred to as the intertwined issues of asset location and asset allocation (Poterba et al., 2001).[3]

We focus on the perspectives of policy makers and of individuals/ firms as follows. Firstly, policy makers bestow tax incentives on agents, in the earning stage of their life cycle, to help them reach financial independence in their retirement stage. This perspective mitigates the

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1. See Aggarwal and Goodell (2013) for more information on the political economy of pension plans.

2. In general, individuals have the option of saving for retirement through either a defined benefit (DB) plan or a defined contribution (DC) plan. In the DB plan, the payoffs during retirement are fixed and contingent on the pensionable salary (i.e., the average salary during the pre-retirement years along with the years of service). In contrast, the payoffs of a DC plan are contingent on the payoffs of the underlying portfolio and the life span of the individual (Bodie et al., 1988).

3. Asset allocation refers to the distribution of assets in a portfolio, while asset location refers to the appropriate placement of an asset (in either a CSA or a TDA) based on its tax incidence (Poterba et al., 2001).

free-rider problem of asset poor retirees, who would be a burden on society in their old age. Many scholars have taken these tax incentives for granted and have construed pensions as tax shelters, without recognizing the redistribution of the tax burden on earned and investment income (Bankman, 2004).[4]

Secondly, Miller (1977) postulates that in equilibrium: (i) the cost of financing a firm is grossed-up as its tax shield is lost due to the offsetting of the tax benefit of debt at the corporate level by its liability at the personal level; and (ii) the optimal stakeholders, in terms of debt and equity, of an individual firm are indeterminate, while for the aggregate economy they are determined by relative corporate and personal taxes. Thus, Miller concludes that the agents in the aggregate economy cluster as clientele for different securities, such as debt and equity, based on their tax incidence. The Miller study inspired numerous researchers to focus on the issue of tax clienteles of debt and equity. For instance, the financial management perspective of a pension fund's investment policy studied by Black (1980) and Tepper (1981) theorizes that the portfolio of DB pension plans should be composed only of bonds. The rationale behind this argument derives from the tax arbitrage opportunities related to a firm's organizational form (returns on assets held in pension funds are tax deferred), financial policy (interest expense is tax deductible while dividends are not), and investment policy (interest income is fully taxable while some portion of equity returns may be subject to favorable tax rates). This arbitrage can be extended to individuals′ decision choices to allocate bonds in their respective DC−TDAs and equities in their CSAs (Auerbach and King, 1983).

Subsequent research has identified other assets such as (i) high yielding stocks and tax-inefficient equity mutual funds (i.e., those that generate an inordinate amount of short term

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4   Bankman (2004, pp 7-8) contradicts a number of studies on tax-shelters as follows: "*Shelters redistribute the tax burden, lowering the rate on purchasers and requiring higher rates on everyone else. This raises an obvious fairness problem; it may create inefficiencies as well, as the marginal cost of replacing funds exceed the cost of raising funds in the no-shelter world....Shelters threaten to undermine tax compliance.....The sad truth, known to economists, is that all else equal, tax planning is a deadweight loss to the system.*"

capital gains by actively trading their portfolios) as appropriate assets in a TDA, and (ii) municipal bonds and tax-efficient mutual funds as appropriate assets in a CSA (Poterba et al., 2001; Dammon et al., 2004; and Shoven and Sialm, 2004).

Studying pensions from the perspective of individuals/ firms and policy makers (from a micro to a macro level) raises the following intriguing issues. First, to what extent do individuals and firms adhere to the tax clientele hypothesis? Empirical evidence here reveals the stark difference between theory and practice, termed the 'asset location puzzle,' and is attributed to both individuals and firms, who follow inefficient asset location strategies. Agnew et al. (2003), Barber and Odean (2004), and Bergstresser and Poterba (2004) empirically confirm that individual investors place an excessive amount of equity [debt] in their TDAs [CSAs], while Frank (2002) confirms that individual firms do not fully implement the tax arbitrage espoused in theory. They seem to leave around 69% of the "potential tax benefits on the table." [5, 6]

Second, which is more efficient of the two contrasting forms of pension plans and why? This issue is highly debated in the literature, where Bodie et al. (1988) argue that one plan does not dominate the other (in terms of employee welfare), while Kotlikoff (1988) ascribes DB plans to large firms with unionized workforces with significant bargaining power, implying that DB is the better plan for the employee. Given the current trend in pensions towards DC plans (Poterba et al., 2004), the issue that comes to mind is whether this shift is ultimately in the best interest of the beneficiaries of the plan, i.e., the employees. If not, how can it be amended?

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5   The literature here has reconciled this contrasting behavior as follows: common stock investment in TDAs has been attributed to hedging pension liabilities in real terms (Black, 1989); superior investment management ability of funds (Bodie, 1991; Aglietta et al., 2012); tax-inefficiency in the form of a high dividend yield (or excessive short term capital gains in case of mutual funds (Poterba et al., 2001; Dammon et al., 2004; and Shoven and Sialm, 2004)). In contrast, fixed income securities in CSAs have been attributed to precautionary savings (due to restrictions on accessibility of assets in a TDA, Amromin, 2003); liquidity shocks (Dammon et al., 2004); and smoothing the volatility of tax benefits (Garlappi and Huang, 2006).

6   Frank's (2002) result resonates with that of Graham (2000), who finds that a conservative debt ratio with low bankruptcy costs is the norm for large, liquid and profitable companies.

A critical examination of the literature leads to the following realizations: The capital structure irrelevance result of Miller (1977) and the pension fund dedication to bonds advocated in Black (1980) and Tepper (1981) is attributed to a dominance (arbitrage) argument. That is, if the corporation controls both its capital structure and the investment decisions for its pension plan, it is tax efficient to fully fund the plan by issuing corporate debt. In this way, the corporation receives the maximum tax benefit of interest without affecting its net debt. This dominance or arbitrage strategy is rationalized under a linear (i.e. risk-neutral − RN) objective function in a partial equilibrium (PE) framework.[7] This strategy, however, falls apart under a non-linear (i.e., risk-averse) objective function in a General Equilibrium (GE) framework (Varian, 1987).[8] This is because a RN model in a PE setting constrains the inter-temporal marginal rates of substitution (IMRSs) of competing taxable and institutional agents to equal their common discount factor. In other words, the framework does not facilitate the adjustment of the IMRSs to simultaneously own a fraction of a firm and trade-off debt claims against its cash flows. This yields an indeterminate solution in the case of Miller (1977) (where the personal tax liability of debt completely offsets the corporate tax benefit of debt); and an extreme (i.e., corner) solution in the case of Black (1980) and Tepper (1981) (where the net tax advantage of debt still persists, allowing it to be arbitraged away). Recent studies, which assume the exogeneity of stock and bond returns, do not capture their interaction in terms of the capital structure of corporations. Furthermore, assuming the exogeneity of agency costs of debt

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7   PE models illustrate an equilibrium assuming all other parameters remaining constant. That is, they apply the condition *ceteris paribus* to realize equilibrium. The Nobel Prize winning scholar, George Stigler, explains this as follows: "*A partial equilibrium is one which is based on only a restricted range of data, a standard example is price of a single product, the price of all other products being held fixed during the analysis*" (see Jain 2006/7, p. 28).

In contrast to PE models, GE models are more intricate as they encompass an entire economy by simultaneously modelling the asset/ debt markets and the government sector *endogenously*. By avoiding the fundamental errors of PE models, we are able to illustrate that when taxable agents and fiduciaries compete, the tax advantage [disadvantage] of pension 'tax-shelters' can be extricated [subsidized] by the other, by charging a higher [lower] cost of debt.

8   Varian (1987, p. 62) states that: "*..the No-Arbitrage condition only applies directly to linear operations.*"

is in variance with the finance literature, where it is considered endogenous as it impedes the trade-off of debt claims against a project's cash flow in equilibrium (Myers, 1977). Thus, there are few insights in the literature integrating the corporate ownership structure with that of an efficient structure of pension funds in a GE setting.

The purpose of this study is to resolve the above intriguing issues. We study investment in the equity of a firm in a specialized GE environment called the overlapping generations (OLG) model augmented with the principle of rational expectations.[9] We choose GE modeling for its rigor and strong following in the academic and policy communities (Zame, 2007). Our approach is in variance with the pedagogically convenient PE models, stemming from the Marshallian supply and demand framework (also termed as the Marshallian Cross), which employ the Capital Asset Pricing Model (CAPM – Auerbach and King, 1983; An et al. 2013) as explained below. This is because traditional PE models assume that competition is perfect in the sense that no agent possesses market power. This approach, however, fails to incorporate concentrations of corporate control rights (as elaborated below) and so cannot model real world situations.

We model the conflict of interest (agency perspective) between taxable investors and a fiduciary of a pre-funded pension plan, incorporating the impact of an endogenou*s* policy decision (Bankman, 2004).[10] This is an intricate issue; one cannot use the standard mean-variance CAPM framework as differential taxes imply heterogeneous expectations. This issue segregates the efficient frontier for both economic agents, breaching the well-known Two Fund Separation Theorem proposed by Tobin (1958), as illustrated in Figure 1. This result

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9   Maddock and Carter (1982) define rational expectations as "*the application of the principle of rational behavior to the acquisition and processing of information and to the formation of expectations*." This is further discussed by Bray (1992), who classifies a rational expectations equilibrium as "*self-fulfilling*" as economic agents form correct expectations given the pricing model and information, and Sheffrin (1996), who finds the rational expectations hypothesis to be interrelated to the efficient market hypothesis used extensively in financial markets research.

10   Pre-funding increases the rate of savings in an economy. It also reduces the vulnerability of a financial system to demographic shocks and reduces the need for increasing tax rates over an ageing population (Orszag and Stiglitz, 1999).

invalidates the implicit assumption of purely competitive unique equilibrium presumed in the majority of the studies in the Asset Location 'Puzzle', which employ tax arbitrage numerically under a PE framework. Equilibrium is re-established in *four* non-trivial cases under the competition between taxable agents and tax-exempt fiduciary. First, we have two basic equilibria contingent on the type of the firm in the economy as illustrated if Figures 2 and 3 respectively. The first firm is classified as a 'value' firm and prevails in a declining (or mature) sector of the economy, with limited (or no) access to growth options requiring investment. The value firm has ample tangible assets in place, which are collateralized by debt. This makes their stocks behave like volatile financial options in changing economic environments. The second firm is termed as a 'growth' firm and prevails in the sector of the economy, which allows it access to growth options requiring funding. The growth firm finances the investment of its real options by using its operating cash flows or by hoarding cash and in some cases raising debt (see Zhang, 2005).

See Figure 1 below

In addition to the two types of firms we also have two types of clients (the Anti-Tax and Tax Clienteles) as corroborated by both the Asset Location Literature and the Dividend Clientele Literature.[11] Thus, in total we have four solutions as illustrated in Figures 2/ 2' and 3/ 3'. The simultaneous occurrence of the anti-tax and tax clienteles constitutes the Asset Location 'Puzzle'.

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11 It should be noted that the terminology of anti-tax clientele and tax clientele stem from the Dividend Clientele literature in Accounting and Finance and is in variance with that of the Asset Location 'Puzzle' literature in Public Finance. This is because the Asset Location literature terms the high dividend payout stocks (i.e., 'Value' stocks as tax inefficient). Grinstein and Michaely (2005) reports evidence that is directly or indirectly consistent with the anti-tax clientele theory. In contrast, Graham and Kumar (2006) report evidence in support of the tax-clientele hypothesis.

See Figures 2/2' and 3/3' below

Aggregating the two types of firms in an economy, we arrive at either a relatively: (i) flat frontier (in Figure 4) illustrating the "representative" firm in the economy as being 'value' in nature; or (ii) steep frontier (in Figure 4) illustrating that the "representative" firm in the economy as being 'growth' in nature. The ramification of this result determines the efficiency of assets in the Taxable Account (i.e., Conventional Savings Account, CSA) or the Tax Deferred Account (TDA). See Hudson et al. (2013) for more details on the value versus growth phenomenon.

See Figure 4 below

Our approach segregates the supply and demand sides of financing to endogenously determine the unique equilibrium pricing parameters of default-free debt in terms of the tax variables. This helps us evaluate the pricing parameters of equity, again in terms of the tax variables, thereby incorporating the effects of optimal leverage. Risky (defaulting) debt is not considered in the analysis as it is Pareto-inferior to default-free debt. Furthermore, it does not help resolve the asset location 'puzzle,' as it is quasi-equity and leads to a corner solution, where investors invest in either stocks or risky bonds, but not in both. Our assumption of Pareto-inferiority of risky debt is based on Ebrahim and Mathur (2007), who demonstrate that when a durable good [or project] undertaken by a household [or firm] retains some value in the future, then its default-free financing is Pareto-efficient over its defaulting counterpart. Myers (2001, p. 96) supports this assertion as follows: *"Conflicts between debt and equity investors only arise when there is a risk of default. If debt is totally free of default risk, debt holders have no interest in the income, value or risk of the firm. But if there is a chance of default, then shareholders gain at the expense of debt investors."* This result is also corroborated with the

empirical findings of Wald (1999) and Graham (2000), who find that conservative debt ratios are the norm for large, liquid and profitable companies, with low bankruptcy costs.

Thus, given the complexity of the issues as elaborated above, we adopt a novel (but unorthodox) approach described as follows. We model the perspectives of taxable agents and the fiduciary of a pension plan, superimposing the market clearing conditions, the model closing conditions (where we define the payoffs of a DC versus a DB plan), and fiscal policy conditions (where we reinforce the Bankman 2004 perspective) to deter tax avoidance. Our efforts yield three key results. The first is derived explicitly from our model (of a single firm economy), while the remaining are implied by extending it to the multi-firm real world situation. These results are explained below.

First, we attribute the asset location 'puzzle' in the literature to linear (i.e., RN) models in a PE setting. This PE framework yields extreme (corner) solutions in the absence of capital constraints. The use of RN models does not allow for the adjustment of the IMRSs of the taxable and institutional agents as it restricts them to equal their discount factor. This restrictive setting theoretically yields two corner solutions when there is a net tax advantage [disadvantage] to the utilization of debt. A second corner solution not discussed in the literature (i.e., assigning equity [debt] to the TDA [CSA]) is also evaluated in our GE framework, under special tax regimes and the absence of capital constraints. We rationalize that an interior solution, where both the taxable agent and the fiduciary own fractional shares in firms, is not feasible in the framework of risk neutrality as it is restrictive and does not facilitate the adjustment of the intertemporal marginal rate of substitution of the taxable agent and the fiduciary. Introduction of sufficient risk aversion, in contrast, allows for wealth smoothing, where the individual agent and the pension fiduciary adjust their intertemporal marginal rates of substitution to own a fraction of the firm and to trade default-free claims against its cash flow. This wealth smoothing yields conservative debt ratios for the firm and resolves the asset location 'puzzle' under two Pareto-neutral rational expectation equilibria, ensuing from the

various controlling clients of the firm. Our unique solutions for the interest rate, the loan amount and firm prices are ascribed to our non-linear (risk-averse) framework, where the value-additivity, assumed in Miller (1977), fails to hold. Our conservative debt ratio corroborates the empirical observation of Frank (2002) that individual firms do not implement tax arbitrage fully (Black, 1980; and Tepper, 1981). Furthermore, these simultaneously occurring twin results contradict those of Dammon et al. (2004), which are realized separately under a restricted borrowing framework (in the absence of liquidity shocks).[12]

Second, extending the above single-firm analysis to a realistic multi-firm environment still produces distinct bond pricing parameters, both at the firm (micro) level as well as at the aggregate (macro) level. However, the inability of policy makers to steer the economy towards one of the equilibria, where either the taxable investor or the fiduciary is the controlling client of the firm, produces a mixed solution with multiple firms where one set of them are in the first equilibrium, while the remaining are in the second. This explains the empirical 'puzzle' where bonds are observed, on average, in the portfolio of CSAs as well as TDAs (Agnew et al., 2003; Barber and Odean, 2004; Bergstresser and Poterba, 2004; and Zhou, 2009). Nonetheless, this dual equilibrium leads to a dilemma for policy makers, as they either overestimate or underestimate tax rates, leading to accumulating government surpluses or deficits, respectively.

Finally, we demonstrate the Pareto-efficiency of a DB plan over its DC counterpart. A DB plan is endowed with an intergenerational risk sharing advantage. This comparative advantage stems from its intrinsic ability to create welfare-improving securities with agents who are yet to be born (Bodie et al., 1988; and Shiller, 2003). This feature, however, imposes insurmountable impending liabilities on the pension plan, stemming from the obligations to

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12   Dammon et al. (2004, p. 1001) explicitly state "*When investors are prohibited from borrowing,…investors may hold a mix of stocks and bonds in their tax-deferred accounts, but only if they hold an all-equity portfolio in their taxable accounts. Investors may still hold a mix of stocks and (taxable or tax-exempt) bonds in the taxable account, but only if they hold a portfolio composed entirely of taxable bonds in their tax deferred accounts. Investors do not simultaneously hold a mix of stocks and bonds in both the taxable and tax-deferred accounts.*"

meet fixed payments to beneficiaries, especially under increasing longevity, decreasing fertility, and increasing administrative expenses. These looming liabilities constitute an increase in the conflict of interest (or agency issue) between the fiduciary of a DB plan and its beneficiaries. We identify the current shift towards DC plans to the dissipation of the above welfare-enhancing feature of the DB plan in the presence of increasing fragility under the agency cost of liabilities (Myers, 2001). This shift to DC plans is detrimental to the welfare of society for two reasons. First, it forces employees to ultimately bear the investment and mortality risks, i.e., economic agents have to expend personal resources to learn the intricacies of modern financial markets to accumulate a subsistence level of terminal wealth over their life cycle (Samuelson, 1989; and Bodie et al., 1992). Second, it has the capacity of increasing stock market volatility as each cohort redeems its risky DC portfolio on retirement (Rosser, 2005). It is, therefore, imperative to restructure the pension plan to a DB one by reducing its fragility thus minimizing the underlying agency costs of the plan as hypothesized in Myers (2001). This result is in the spirit of Markowitz (1952) as the taxable agent ultimately owns a risky portfolio in his/ her CSA and a riskless (DB) security in the TDA. This reconfiguration of the pension plan should be accompanied by the following: (i) curtailing superfluous regulations, (ii) minimizing the administrative expenses from the employment of institutional accounts through a private/ mutual organization, (iii) using cutting edge information technology to optimally define contributions and benefits by including the payoffs of not only investments but also individual/ cohort/ employer specific factors, (iv) employing innovative financial products and services such as deferred annuities and portfolio hedging techniques to mitigate longevity risk, and (v) increasing the retirement age of employees (James et al., 1999; Shiller, 2003; Angelidis and Tessaromatis, 2010; Horneff et al., 2010, and Cocco and Gomes, 2012).[13]

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13   The literature illustrates an alternative way of reconfiguring pension plans employing survival-contingent investment linked contracts. This, however, suffers from illiquidity (see Horneff et al. 2009).

This paper is structured as follows. Section II presents the theoretical underpinnings of the first model where the taxable investor [pension fiduciary] is the net borrower [lender]. Section III reverses the above principal-agent relationship to evaluate the model solutions (relegating all proofs to the Appendix). Finally, Section IV concludes the study.

II. Model Development

For simplicity and mathematical tractability, we assume an overlapping generational model in which there is no economic growth and no population growth.[14] Each generation consists of n identical agents, who are young in period t, old/ retired in period t+1 and dead in period t+2 and beyond. There is an exogenous entity called the government that appoints a "young person" in each cohort to perform an eleemosynary duty of a fiduciary to manage the generation's pension assets.[15] Each "young" agent is endowed with an amount e_I of the numeraire good. This initial endowment is akin to labor income. A fraction θ of this endowment, i.e., θe_I is allocated to the pension plan, while the residual, after the deductions of taxes, explained below, is retained in the individual account.[16] This fractional contribution of labor income is restricted to a maximum $\theta^*_{Max.}$ by the government as explained in Section II.E. The allocations in the individual and retirement accounts are, respectively, used by the agents and the fiduciary to optimize their two-period utility of wealth by investing in two types of assets in this economy.[17] The first asset is the stock (of a firm or a corporation) that has access

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14   This framework is consistent with Shiller (1999). Economic growth can be incorporated in our model by assuming growth of the project available to the firm and growth of endowments of agents.

15   The role of the fiduciary in our model is similar to that of a session chair (at a conference), who is also a paper presenter. That is, he/ she not only optimizes personal welfare but also that of the group. In doing so, the fiduciary deducts all the administrative charges from the assets of the plan.

16   To keep the model manageable, we opt for a front-loaded plan with no matching contributions (similar to an Individual Retirement Account in the U.S.). This model can be easily extended to one with matching contributions (like a 401(k) or a 403(b) plan in the U.S.) or a back-loaded plan (without matching contributions, like a Roth Individual Retirement plan in the U.S.), without changing the quality of our results.

17   In our framework it is difficult to optimize expected utility of consumption. This is because the fiduciary is performing an eleemosynary duty of serving his/ her cohort by managing their resources without being

to a perpetual project that yields a stream of net operating income (NOI) $\tilde{q}_{t+1}$, and a future (re-sale) price, $\tilde{P}_{t+1}$, such that their growth rates follow an ergodic Markov processes with mean zero.[18, 19]   The second asset is a default-free bond (debt contract) entered by the firm by trading off financial claims on its payoffs.[20]

The government spends an amount G per agent on public goods and imposes flat taxes to recover the same.[21]   The taxes levied consist of (i) a corporate income tax at the rate of τ_C on the operating income of the firm, (iia) an individual tax at the rate of τ_I on the labor income and dividends paid by the firm, and (iib) an effective capital gains tax of $\gamma \tau_I$ on the appreciation of the firm's stock price.[22, 23]   The government also grants a tax deduction at the individual rate of

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compensated for it. We therefore resort to optimization of expected utility of wealth instead of consumption. This approach may seem to be a departure from the norm of optimization over consumption. However, both methodologies are equivalent as consumption ensues from the value of assets, whose payoffs are denominated in the numeraire good.

18   The '~' symbol over a variable indicates that it is stochastic.

19   We also assume that the payoffs and distribution of the NOI ($\tilde{q}_{t+1}$) and future price ($\tilde{P}_{t+1}$) are time-invariant and known to all agents in the economy. Furthermore, the assumption of no economic growth and no population growth implies no appreciation in the expected NOI or the future price of project. That is, $q_t = E_t(\tilde{q}_{t+i+1})$, and $P_t = E_t(\tilde{P}_{t+i+1})$, $\forall$ t and i. This is a special case of asset pricing depicted in the well-known Mehra and Prescott (1985) study, where the stock prices follow a random walk with a mean zero growth rate. To exclude asset bubbles, we need to impose transversality condition. This implies:

$Lim._{T \to \infty} \beta^T E_t(\tilde{P}_T) = 0$, $\forall$ T = t +i +1.

The above condition can be simplified further in the context of Mehra and Prescott (1985) as illustrated in the Proof of our Theorem in the Appendix.

20   We assume that all borrowing in our model occurs through a firm, because, in practice, a firm is able to collateralize its debt with its tangible assets to alleviate the endogenous agency costs (Stulz and Johnson, 1985).

21   For the sake of simplicity, we ignore the evaluation of the optimal amount of public good G* spent by the government, as discussed by Pestieau and Possen (2000). Likewise, we ignore the government smoothing its spending using its own debt, as discussed in Diamond and Geanakoplos (2003).

22   Since the effective tax rate on capital gains is less than that on income/ dividends, capital gains are preferentially taxed. Thus, γ is equivalent to the ratio of the effective capital gains tax rate and the tax on income, i.e., $\gamma = \dfrac{\gamma \tau_I}{\tau_I} \leq 1$. In the context of the asset location literature, γ represents the extent to which investment in equity is efficient in reverse order. That is, 0 and 1, respectively, represent the most and least efficient equity investment. The quality of our results is not affected if dividends are preferentially taxed, as

τ_I on contributions to the pension plan, while taxing the proceeds on retirement at a lower rate of $\alpha\tau_I$.[24, 25] In general, any profits made at the firm level are passed through to the stockholders. Any losses of the firm have to be written off at the corporate level.

The analysis in this section is carried out by modelling the taxable investor [pension fiduciary] as a net borrower [lender] investing in only stocks [stocks and bonds], shown as Model 1 in Figure 5. Here, the taxable agents have a controlling interest in the firm. The scenario is reversed in Model 2 (Figure 5), discussed in the following section, with the taxable investor [fiduciary] as the net lender [borrower] investing in only stocks and bonds [stocks].[26] Here, the fiduciary has a controlling interest in the firm.[27] We then evaluate the solutions after imposing necessary market clearing, model closing and fiscal policy conditions as described in Sections II.C-II.E, respectively.

Figure 5 Here

II.A The Taxable Investors As Net Borrowers (*Agent*)

The goal of each of the n taxable investors is to select the optimal: (i) contribution rate (θ − of initial endowment e_I) to the pension plan; (ii) amount of resources to borrow (Q) through

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in the current U.S. tax regime.

23   Likewise, investors get a tax write-off against personal income for capital losses.

24   The lower marginal tax rate of retirement assets incorporates the perspective of Poterba (2004). That is, the growth generated by assets in a TDA is more valuable than in a CSA.

25   For the sake of simplicity, we ignore bequests and estate taxes.

26   The rationale for segregating the analysis into two models is to incorporate the differential impact of taxes when the institutional investor (pension fiduciary) is the net lender (in Model 1), as opposed to the scenario where he/ she is the net borrower (in Model 2). This is consistent from a microeconomic perspective, where for every net borrower there is a net lender.

27   It should be noted that the anti-tax and tax clienteles illustrated pictorially in Figures 2/3 and 2'/3' imply the possibility of both agents owning equity with one agent being *long* equity and *short* debt, while the other is *long* equity and *long* debt. These two results are consistent with the basic idea of the CAPM. This also implies that the agent, who is short debt (i.e., the net borrower) owns more than 50% of equity of firm and thus has a controlling interest in it.

the firm; and (iii) fractional shares (s) of equity to purchase at a price (P_t-Q) in order to maximize the expected utility of wealth. That is,

$$\textit{Max. } E_t \{U(w_t) + \beta U(\widetilde{w}_{t+1})\}$$

$$(\textit{in } w_t, w_{t+1}, \theta, Q, s)$$

subject to the temporal budget constraints

$$w_t \quad = e_I(1-\tau_I) - \theta e_I(1-\tau_I) - sP_t + sQ = e_I(1-\tau_I)(1-\theta) - s[P_t-Q]_{\text{Corporate Shell}} \tag{1}$$

$$\widetilde{w}_{t+1} = (\widetilde{e}_{t+1})(1-\alpha\tau_I)+s[(\widetilde{q}_{t+1}(1-\tau_C)(1-\tau_I)+\widetilde{P}_{t+1}(1-\gamma\tau_I)+\gamma\tau_I P_t)-Q(1+r(1-\tau_C)(1-\tau_I))]_{\text{Corporate Shell}} \tag{2}[28]$$

Equation (2) can be rewritten in the context of the Mehra-Prescott (1985) in terms of a stochastic growth factor $\widetilde{\lambda}_j$, where $\lambda_{\text{good}} = (1+\delta)$ and $\lambda_{\text{bad}} = (1-\delta)$, as follows.

$$w_{t+1}(\widetilde{\lambda}_j) = (e_{t+1}(\widetilde{\lambda}_j))(1-\alpha\tau_I)+s[(\widetilde{\lambda}_j q_t(1-\tau_C)(1-\tau_I)+\widetilde{\lambda}_j P_t(1-\gamma\tau_I)+\gamma\tau_I P_t)-Q(1+r(1-\tau_C)(1-\tau_I))] \tag{2i}$$

where: $E_t \{\cdot\}$ is the expectation operator at time t,

U($\cdot$) is a strictly concave and twice continuously differentiable (Von Neumann-Morgenstern) utility function of the investor,[29]

w_t and $\widetilde{w}_{t+1}$ are the respective wealth of the investor at times t and (t+1) respectively,[30]

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[28] The net payoffs of a firm consist of the following components:

(i) The inflow component stemming from fractional ownership of the firm times the dividend income (after deducting corporate and personal taxes) added to the liquidating value of the firm (after deducting capital gains tax). The (after-tax) dividend income is evaluated by deducting the interest expense from the NOI and subsequently imposing the dual tax rates as $(\widetilde{q}_{t+1} - Qr)(1-\tau_C)(1-\tau_I)$. The (after-tax) liquidating value of the firm is evaluated by deducting the capital gains tax of $\gamma\tau_I(\widetilde{P}_{t+1}-P_t)$ from the terminal value of the firm $(\widetilde{P}_{t+1})$ as $(\widetilde{P}_{t+1}(1-\gamma\tau_I))+\gamma\tau_I P_t$.

(ii) The outflow component stemming from the payoff of the loan is equal to Q (1+r). It should be noted that the tax advantage of debt is incorporated in (i) above.

[29] We assume that a reduction in risk aversion (to zero) in the limit yields the special case of risk neutrality, which is a linear function.

[30] The initial endowment of the agent (e_I) constitutes the numeraire good in the economy. The remaining variables representing capital resources $(Q/Q', P_t, \widetilde{P}_{t+1}, \widetilde{q}_{t+1})$ or wealth parameters $(w_t, \widetilde{w}_{t+1}, w'_t, \widetilde{w}'_{t+1})$ or

β is the discount factor (assumed to be strictly less than one), i.e., $\beta < 1$.

e_I is the initial endowment (representing labor income),

θ is the rate of contribution (of initial endowment) in the pension plan,

$\tilde{e}_{t+1}$ is the (pre-tax) payoff of the pension plan (i.e. TDA) explained further in the Model
   Closing Condition (Section II.D),

s is the fractional investment (in the firm) by the taxable agent in his/ her CSA,

Q is the amount of capital resources borrowed by the firm,[31]

P_t is the price of the project undertaken by the firm at time t $(P_t=Debt(Q)+Equity(P_t-Q))$,

r is the real interest rate of the default-free bond,

$\tilde{q}_{t+1}$ is the NOI of the project at time (t+1),

$\tilde{P}_{t+1}$ is the future value of the project at time (t+1),

$\tilde{\lambda}_j$ is the stochastic growth factor of the payoffs of the firm in a two (good and bad) state
   world, where $\lambda_{good} = (1+\delta)$, $\lambda_{bad} = (1-\delta)$, and $\bar{\lambda}_j$ = mean growth factor = 1.

δ is the standard deviation of the NOI and Future Price of the Firm.

G is the cost of public good per agent in the economy,

ξ is the income (endowment) multiple promised by the fiduciary of the Defined Benefit
   Plan on retirement as illustrated in Equations (13b) and (13c) in Section II.D,

τ_C is the corporate tax rate,

τ_I is the individual tax rate,

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government spending per agent (G) are denominated in terms of this numeraire good (in *real* terms).

31 This section discusses Model 1, where Q is deemed to be positive reflecting the demand for resources by
    the taxable agent, who is the net borrower. In contrast, Section III.A discusses Model 2, where Q is
    deemed to be opposite to the above case. That is, it reflects a positive demand for resources by the
    fiduciary, who is the net borrower.

ϕ is the ratio of the corporate tax rate (τ_C) to the individual tax rate (τ_I) as defined by
Equation (16) in Section II.E,

γ is the capital gains rate, and

($\alpha \tau_I$) is the tax rate on the payoffs of the pension plan.

The budget constraint for period t (Equation 1) illustrates the residual wealth after contributing to the pension plan and purchasing a fraction s of the stock of the firm at a unit price of (P_t–Q). The budget constraint for period (t+1) (Equation 2) illustrates net payoffs emanating from the pension plan ($\tilde{e}_{t+1}$) along with that from the fraction s of the stock (after corporate and individual taxes).[32]

The Lagrangian L can be written as

$$L = E_t\{[U(w_t) + \beta\, U(\tilde{w}_{t+1})] + \eta_t\, [e_I(1-\tau_I)\,(1-\theta) - s\,(P_t - Q) - w_t]$$

$$+\eta_{\tau+1}\beta[\tilde{e}_{t+1}(1-\alpha\tau_I) + s\,[(\tilde{q}_{t+1}(1-\tau_C)(1-\tau_I)+\tilde{P}_{t+1}(1-\gamma\tau_I)+\gamma\tau_I P_t)-Q(1+r(1-\tau_C)(1-\tau_I))]-\tilde{w}_{t+1}]\}$$

The First Order Necessary Conditions (FONCs) can be stated as follows:

(i) At the optimum, the intertemporal marginal rate of substitution (IMRS$_T$ = $\beta[\dfrac{U'(\tilde{w}_{t+1})}{U'(w_t)}]$) of the taxable investor times the marginal payoffs of the pension plan with respect to the contribution rate (θ) should be at least equal to the unit amount of consumption deferred:

$$[\frac{\beta\,(1-\alpha\tau_I)}{e_I\,(1-\tau_I)}]E_t\{[\frac{U'(\tilde{w}_{t+1})}{U'(w_t)}]\,\frac{d\tilde{e}_{t+1}}{d\theta}\} \geq 1 \qquad\qquad (3)$$

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32   Our model evaluates after-tax payments in real terms. This is in contrast to the real world, where taxes are imposed on nominal payoffs. To simplify this issue one can evaluate the after-tax nominal payoffs and deflate the results with (1+inflation rate) to evaluate real payoffs. This is consistent with Fisher (1977/1930).

The above result is derived under the following assumption: an interior solution yields an equality condition, while a corner solution yields an inequality condition. This rationale is again applied for deriving Equations (4), (5), (8) and (9) as given below.

(ii) At the optimum, the benefit of borrowing a unit amount of the numeraire good is at least equal to its associated cost. This condition incorporates the demand function for a bond described as follows. The $IMRS_T$ of the taxable investor times the compound factor, consisting of one plus the real (after-tax) rate of interest, is at most equal to the unit amount of the numeraire good borrowed:

$$\beta \, E_t\{[\frac{U'(\widetilde{w_{t+1}})}{U'(w_t)}] \, (1+ r(1-\tau_C)(1-\tau_I))\} \leq 1, \; \forall \, s \neq 0 \qquad (4)$$

(iii) At the optimum, the taxable investor will bid for fractional shares of the firm, which yields net benefits at least equal to zero, yielding the optimal price of equity $(P_t{-}Q)$, consistent with Grossman and Shiller (1981), as equal to the expected value of the $IMRS_T$ times the net proceeds of the project after repayment of the bond and payment of appropriate taxes:

$$\beta E_t\{[\frac{U'(\widetilde{w_{t+1}})}{U'(w_t)}][(\widetilde{q_{t+1}}(1-\tau_C)(1-\tau_I)+\widetilde{P_{t+1}}(1-\gamma\tau_I)+\gamma\tau_I P_t)-Q(1+r(1-\tau_C)(1-\tau_I))]\} \geq (P_t{-}Q) \qquad (5)$$

Thus, a unique and constrained maximum of the investor's objective function requires that the following conditions are met: firstly, the deterministic budget constraint at t, as depicted by Equation (1), and the stochastic budget constraint for each state of the economy at (t+1), as depicted by Equation (2), are satisfied; secondly, the simplified FONCs represented by Equations (3), (4) and (5), are satisfied. We note that the second order conditions are automatically satisfied as Chiang (1984) demonstrates that maximization of a strictly concave

and twice continuously differentiable utility function with quasi-convex constraints gives a negative definite bordered Hessian matrix.

II.B The Fiduciary of a Pension Plan as Net Lender (*Principal*)

Similar to the previous case, the goal of the fiduciary is to select the optimal (i) aggregate capital resources (nQ') to lend to the firm, and (ii) fractional amount (ns'$_t$) of equity to purchase at a price (P$_t$ - Q) in order to maximize the expected utility of wealth, contributed by the participants of the pension plan:[33]

$$Max. \ E_t\{V(w'_t) + \beta V(\widetilde{w'}_{t+1})\}$$

$$(in \ w'_t, w'_{t+1}, Q', s')$$

subject to the temporal budget constraints

$$w'_t \quad = e'_t - n \ s' \ [P_t\text{-}Q]_{Corporate \ Shell} - n \ Q' \tag{6}$$

$$\widetilde{w}_{t+1} = n \ s' \ [\widetilde{q}_{t+1}(1\text{-}\tau_C) + \widetilde{P}_{t+1}\text{-}Q(1 + r(1\text{-}\tau_C))]_{Corporate \ Shell} + nQ' \ (1+r) \tag{7}$$

Here too, Equation (7) can be rewritten in the context of the Mehra-Prescott (1985) model in terms of a stochastic growth factor $\widetilde{\lambda}_j$ as follows.

$$w'_{t+1}(\widetilde{\lambda}_j) = n \ s' \ [\widetilde{\lambda}_j \ q_t \ (1\text{-}\tau_C) + \widetilde{\lambda}_j \ P_t - Q(1 + r(1\text{-}\tau_C))]_{Corporate \ Shell} + nQ' \ (1+r) \tag{7i}$$

where: V(·) represents a strictly concave and twice differentiable (Von Neumann − Morganstern) utility function of the fiduciary,

The term e'$_t$ represents aggregate pension assets managed by the fiduciary as explained in the Model Closing Condition (Section II.D), and

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[33]   It should be noted that in our model the beneficiaries of a pension plan may endorse a more conservative (i.e., risk averse) strategy with respect to their pension assets as opposed to their individual assets. This is consistent with empirical literature on trading activity of 401(K) plans versus an individual brokerage account (Agnew et al., 2003; and Barber and Odean, 2004). There is, however, a special case of a DC plan such as a self-directed brokerage account, where an individual may manage the plan at the same level of risk tolerance as his/ her CSA account.

The remaining notations with primes have the same meaning as that in the case of the taxable investor.

The budget constraint for period t (Equation 6) illustrates the residual aggregate wealth (managed by the fiduciary) after purchasing a fraction ns' of stock at a unit price of $(P_t - Q)$ and n bonds at a unit price of Q'. The budget constraint for period (t+1) (Equation 7) illustrates net payoffs emanating from the portfolio of the bond and stock (after corporate tax).

The Lagrangian L' can be written as

$$L' = E_t\{[V(w'_t) + \beta V(\widetilde{w'}_{t+1}))] + \eta'_t [e'_t - n \, s' \, (P_t - Q) - n \, Q' - w'_t]$$

$$+ \eta'_{\tau+1}\beta[\, n \, s' \, (\widetilde{q}_{t+1}(1-\tau_C) + \widetilde{P}_{t+1} - Q(1+ r(1-\tau_C))) + n \, Q' \, (1+ r) - \widetilde{w'}_{t+1}]\}$$

The FONCs can be stated as follows:

(i)   At the optimum, the benefit of lending should at least equal its associated cost. This condition incorporates the supply function for bonds described as follows. The $IMRS_F$ of the fiduciary times the compound factor, consisting of one plus the real (after-tax) rate of interest, is at least equal to the unit amount of the numeraire good loaned:

$$\beta \, E_t\{[\frac{V'(\widetilde{w'}_{t+1})}{V'(w'_t)}] \, (1+r)\} \geq 1, \forall \, (1-s') \neq 0, n \neq 0. \tag{8}$$

(ii)   At the optimum, the fiduciary will bid for the fractional shares of the firm, which yields net benefits at least equal to zero. Thus, the optimal price of equity $(P_t - Q)$ is equal to the expected value of $IMRS_F$ times the net proceeds of the project after repayment of the bond and payment of appropriate taxes (consistent with Grossman and Shiller, 1981):

$$\beta \, E_t\{[\frac{V'(\widetilde{w'}_{t+1})}{V'(w'_t)}] \, [\widetilde{q}_{t+1}(1-\tau_C) + \widetilde{P}_{t+1} - Q \, (1+ r(1-\tau_C))]\} \geq (P_t - Q), \forall \, n \neq 0. \tag{9}$$

Thus, a unique and constrained maximum of the fiduciary's objective function requires that the following conditions are met: firstly, the deterministic budget constraint at t as depicted by Equations (6) and the stochastic budget constraint, for each state of the economy at (t+1), represented by Equation (7), are satisfied; secondly, the simplified versions of the FONCs, as depicted by Equations (8) and (9), are satisfied. The second order conditions for a maximum are automatically satisfied due to the properties of a strictly concave and twice continuously differentiable utility function with quasi-convex constraints (Chiang, 1984).

II.C Market Clearing Conditions

(i)   For the asset (equity) market to be in equilibrium:

The fractional shares of the firm owned in aggregate must sum to one, i.e., $n(s)+ns' = 1$.

Furthermore, pension fiduciaries may not be allowed to short the firm's shares, i.e., $s' \geq 0$.

Finally, in the long run, taxable investors may not permanently go short in the equity market, i.e., $s \geq 0$.   (10)

(ii) For the debt (bond) market to be in equilibrium:

Funds Borrowed at aggregate level (n Q) = Funds Lent at aggregate level (n Q')

That is, $n\,Q = n\,Q' \Rightarrow Q = Q', \forall\, n \neq 0$.   (11)

II.D Model Closing Conditions

(i)   <u>For a Defined-Contribution (DC) Plan:</u>

The aggregate pension contribution of n investors (given by the term $n\,\theta_{DC}e_I(1-\tau_I)$ in Equation (1)) net of all tax benefits (i.e., multiplied by the factor $\frac{1}{(1-\tau_I)}$) yields the initial endowment of the fiduciary.

$(e'_t)_{DC}$ = Initial Aggregate Pension Contribution = $n\,\theta_{DC}e_I(1-\tau_I)\left[\frac{1}{(1-\tau_I)}\right] = n\,\theta_{DC}e_I$ (12a)

The risk averse fiduciary (with the concave utility function $V(\cdot)$) smoothes the wealth from period t to period (t+1). He/ she transmits all fluctuations of the pension portfolio to the beneficiaries. This implies:

$$(\tilde{e}_{t+1})_{DC} = \text{Stochastic Payoff of a DC Plan}$$

$$(\tilde{e}_{t+1})_{DC} = \frac{1}{n} [(\tilde{w'}_{t+1})_{\text{Generation } t} + (w'_{t+1})_{\text{Generation } (t+1)}] - [\text{Administrative Expenses}]_{DC}$$

Our assumption of no economic growth implies $(w'_{t+1})_{\text{Generation } (t+1)} = (w')_{\text{Generation } t} = w'_t$

Furthermore, $(\tilde{w'}_{t+1})_{\text{Generation } t} = \tilde{w'}_{t+1}$

Substituting the values of w'_t and $\tilde{w'}_{t+1}$ from Equations (6) and (7) yields

$$(\tilde{e}_{t+1})_{DC} = \theta_{DC} e_I + \{s'[(\tilde{q}_{t+1} - Qr)(1-\tau_C) + (\tilde{P}_{t+1} - P_t)] + Q r\}_{DC} - [\text{Admin. Exp.}]_{DC} \quad (13a)$$

In the context of the Mehra-Prescott (1985) terminology, $\tilde{e}_{t+1}$ can be rewritten as follows.

$$(e_{t+1}(\tilde{\lambda}_j))_{DC} = \theta_{DC} e_I + \{s'[(\tilde{\lambda}_j q_t - Qr)(1-\tau_C) + (\tilde{\lambda}_j P_t - P_t)] + Q r\} - [\text{Admin. Exp.}]_{DC} \quad (13ai)$$

The first term of the above payoff, i.e., $\theta_{DC} e_I$ may seem a bit odd. However, if it is combined with the third term, i.e., $s'(\tilde{P}_{t+1} - P_t)$, it yields the terminal value of the portfolio of stock and bond originally purchased by the fiduciary, on behalf of a single agent, along with the average residual $(\frac{w'_t}{n})$. Finally, the second and fourth term of the above payoffs, i.e., $[s'(\tilde{q}_{t+1} - Qr)(1-\tau_C) + Q r]$, represent the income, allocated per agent, stemming from the original stock and bond portfolio, respectively.

(ii) <u>For a Defined-Benefit (DB) Plan:</u>

The fiduciary smoothes out the fluctuations of the pension portfolio intergenerationally. This is accomplished by holding a permanent portfolio in an economy where there is no economic growth and no population growth.

$$(e'^{\sim}_{t+1})_{DB} \quad = \quad \text{(Value of Portfolio of DB Plan)}_{t+1} \quad = \quad \text{Residual Stochastic Wealth of}$$

$$\text{Generation (t) + Initial Aggregate Pension Contribution of Generation (t+1)}$$

$$(e'^{\sim}_{t+1})_{DB} \quad = \quad [(w^{\sim}_{t+1})_{\text{Generation(t)}} - n \, ((e_{t+1})_{\text{Generation(t)}})_{DB}] + n \, \theta_{DB} e_I \qquad (12b)$$

$$[(e_{t+1})_{\text{Generation(t)}}]_{DB} = \text{Fixed Payoff of DB Plan } net \text{ of Admin. Expenses (in steady state)}$$

$$= e_I \theta_{DB} + \{[s'(\text{Exp. Dividend Inc.}) + \text{Interest Inc.}]_{DB} - [\text{Admin. Exp.}]_{DB}\}$$

$$= e_I \theta_{DB} + \{[s' \, E_t(q^{\sim}_{t+1} - Qr)(1-\tau_C) + Q \, r]_{DB} - \text{Admin. Exp.}_{DB}\}$$

Employing the time-invariant property of NOI, we get:

$$[(e_{t+1})_{\text{Generation(t)}}]_{DB} = e_I \theta_{DB} + \{[s'(q_t - Qr)(1-\tau_C) + Q \, r]_{DB} - \text{Admin. Exp.}_{DB}\} = \xi e_I, \qquad (13b)$$

$$\text{where: } \xi = \theta_{DB} + \frac{1}{e_I}\{[s'(q_t - Qr)(1-\tau_C) + Q \, r]_{DB} - \text{Admin. Exp.}_{DB}\} \qquad (13c)$$

The above Equations (13b-13c) illustrate that the payoffs of a DB plan are defined in terms of not only the income (initial endowment) but also expected portfolio payoffs. This result is radically different from that observed in practice, where the payoffs are defined strictly in terms of pensionable salary, which is derived by only using the average of that in the pre-retirement years (after incorporating the number of years of service). The result implies that the DB plan promises in practice are perhaps generously determined on an ad hoc basis, which may offer a clue to the burgeoning pension deficits and the current shift from DB plans to DC plans. This crucial issue is revisited in Corollary 2 in Section III.B.

Contrasting the final payoffs from the DC and DB plans as given by Equations (13a) and (13b), we realize that (apart from the administrative expenses) the major difference in the two stems from the absence of the stock appreciation component in the DB plan. This is due to the permanence of the DB portfolio, which stems from the fact that under a

Stationary Rational Expectations Equilibrium, SREE (i.e., with no economic growth), q_t

$= E_t(\tilde{q}_{t+1})$, and $P_t = E_t(\tilde{P}_{t+1})$.

Finally, substituting the value of $(w'_{t+1})_{Generation(t)}$ from Equation (7) in Equation (12b) we

realize:

$$(\tilde{e'}_{t+1})_{DB} = \{[ns'(P_t-Q)+nQ]\}_{DB}+\{ns'[((1-\tau_c)(\tilde{q}_{t+1} - q_t))+(\tilde{P}_{t+1} - P_t)]\}_{DB} \tag{12c}$$

The first term of Equation (12c), i.e., $\{[ns'(P_t-Q) + nQ]\}_{DB}$ represents the permanent

portfolio of a DB Plan. In contrast, the second term represents the impact of stochastic

factors such as the residual income, i.e., $ns'[(1-\tau_c) (\tilde{q}_{t+1}-q_t)]$ along with the fluctuations in

value of stock given by $\{ns'(\tilde{P}_{t+1}-P_t)\}_{DB}$.

II.E Fiscal Policy Conditions (i.e., Conditions Deterring Tax Avoidance)

(i)   Can policy makers steer the equilibrium to either Models 1 or 2, as described below, by

controlling the rate of contribution to a maximum (θ^*)?

For Model 1, the optimal $\theta^*_{Model\ 1}$ is observed as a solution satisfying Equation (3) as an

inequality in an interval $\theta^*_{Model\ 1} \varepsilon (\theta_{Model\ 1-Min.}, \theta_{Model\ 1-Max.})$. In contrast, for Model 2 the

optimal $\theta^*_{Model\ 2}$ (elaborated further in Section III.A) is also observed as a solution of

Equation 3 in the interval $\theta^*_{Model\ 2} \varepsilon (\theta_{Model\ 2-Min.}, \theta_{Model\ 2-Max.})$.

Policy makers can thus allow a contribution rate in a range to discourage or encourage

pensions from acquiring a controlling interest in the equity of a firm (as illustrated in

Model 2 described in Section III.A) as follows:

To Discourage Pensions from Dominating (or taking a Controlling Stake in a Firm):

$$\theta^*_{Model\ 1} \varepsilon (\theta_{Model\ 1-Min.}, \theta_{Model\ 1-Max.}) \tag{14a}$$

The lower and upper limits of θ in Model 1 are derived by respectively restricting the values of w_t and w'_t, for DC and the initial stage of DB, in Equations (1) and (6) to be greater (or equal to) zero.

<u>To Encourage Pensions towards Dominating (or taking a Controlling Stake in a Firm):</u>

$$\theta^*_{Model\ 2} \ \varepsilon \ (\theta_{Model\ 2-Min.}, \ \theta_{Model\ 2-Max.}) \tag{14b}[34]$$

Here too, the lower and upper limits of θ^* in Model 2 are derived by, respectively, restricting the values of w_t and w'_t, for DC and initial stage of DB, in Equations (1a) and (6a) (described in Section III.A) to be greater or equal to zero.

Inability of policy makers to establish disjoint sets of contribution rate θ^* lead to a failure in segregating the two equilibria, stemming from Models 1 and 2. This leads to a mixed equilibrium in a multiple firm economy, where some are in Model 1 equilibrium, and the remaining are in Model 2. This resolves the empirical 'puzzle' (cited in Agnew et al., 2003; Barber and Odean, 2004; and Bergstresser and Poterba, 2004) where bonds are observed on average in both the CSA as well as the TDA, as further explained in Corollary 1 (in Section III.B).

(ii)   The government typically sets the tax rates to restrict agents in the economy from using pension plans as a tax shelter. Thus, the policy makers ensure that net taxes imposed on agents (and the firm) in the economy are adequate enough to cover any tax benefits endowed on agents (or the firm) along with the cost of the public good G, to be delivered per agent, in the economy. That is,

Tax Benefits bestowed on agents of generation (t+1) and firm in period (t+1)

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[34]   In Equations (14a) and (14b) we explicitly assume that $\theta_{Model\ 1-Max.} \leq \theta_{Model\ 2-Min.}$. If this does not hold true then they have to be modified as follows:
<u>To Discourage Pensions from Dominating (or taking a Controlling Stake in a Firm):</u>
$$\theta^*_{Model\ 1} \ \varepsilon \ (\theta_{Model\ 1-Min.}, \ \theta_{Model\ 2-Min.}) \tag{14a'}$$
<u>To Encourage Pensions in Dominating (or taking a Controlling Stake in a Firm):</u>
$$\theta^*_{Model\ 2} \ \varepsilon \ (\theta_{Model\ 1-Max.}, \ \theta_{Model\ 2-Max.}) \tag{14b'}$$

+ Cost of Public Goods to be delivered by the Government (to two overlapping

generations (t) and (t+1) in period (t+1))

 = Exp. Tax Liabilities (on agents of generations t and (t+1)) and the firm in period (t+1)

(15)

Since the above Equation contains two unknown tax parameters, τ_C and τ_I, a further

simplification is needed to solve for optimal taxes. This involves assuming that the tax

rate on corporations τ_C is a linear function of that of individuals τ_I (Gravelle, 2004). That

is,

$$\tau_C = \phi\, \tau_I \qquad\qquad (16)$$

Thus, optimal taxes can be construed as based on the type of equilibria stemming from

Model 1 or Model 2 (described in Section III.A). The government budget constraint

described in Equation (15) can be further simplified for Model 1 as follows:

$$\tau_I^2[((q_t - Q\,r))\,\phi(\alpha\,s' + s)]$$

$$- \tau_I[e_I(1-\theta(2-\alpha))+(\alpha\,Q\,r)+((q_t - Q\,r)(\phi + \alpha\,s' + s))] + 2G = 0 \qquad (17a)[35]$$

Equation (17a) illustrates the Bankman (2004) perspective in the form of a quadratic

equation. That is, the tax incentives on pension contribution and withdrawals are

redistributed on tax rates imposed on earned and investment income. The solution of this

equation, issuing from its least value, yields:

$$(\tau_I)_{\text{Model 1}} = [\frac{-b-\sqrt{(b^2-4ac)}}{2a}],$$

where: $b = - [e_I(1-\theta(2-\alpha))+(\alpha\,Q\,r)+((q_t - Q\,r)(\phi + \alpha\,s' + s))]$,

$\quad$ $a = [(q_t - Q\,r)\phi(\alpha\,s' + s)]$, and

$\quad$ $c = 2G.$ $\qquad\qquad$ (17b)[36,37]

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35   The number 2 multiplied by G in Equation (17a) represents the sum of agents in the overlapping
generations t and t+1.

Thus, fiscal policy is quite intricate as it requires policy makers to steer the economy to a particular equilibria using variables, such as θ^*, τ_I and τ_C described further in Section III.A, and identify the Pareto-optimal choice between a DC and a DB plan based on the information on the exogenous administrative expenses of each plan.

III. MODEL SOLUTIONS

A Stationary Rational Expectations Equilibrium (SREE) is defined as one where all agents in the economy are knowledgeable of the time-invariant payoffs of the pre-tax equity and its probability distribution ($q_t = E_t(\tilde{q}_{t+i+1})$; and $P_t = E_t(\tilde{P}_{t+i+1})$, $\forall$ t and i). This is derived from the interrelationship between an SREE and an Efficient Market Equilibrium, where the movements of stock prices are assumed to follow a random walk which is memory-less (Sheffrin, 1996). This yields four distinct SREE (based on the clientele of the firm and type of plan, assuming competitive markets and no initial capital constraints) for risk-averse agents under default-free bond financing. These equilibria are ranked in a pecking order, based on Pareto-optimality of one over the other, due to the minimization of agency costs of debt incorporating market imperfections such as taxes and administrative expenses under various types of plan.[38] The agents in our economy, thus, opt for Pareto-optimal choices that involve the different clientele of the firm and type of plan. Policy makers may have the capacity to steer the economy towards either one of the equilibria stemming from the clientele of the stock and bond, i.e., Model 1 or Model 2, by resorting to policy variables such as $\theta^*_{Max.}$, τ_I, and τ_C

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36   The variables ϕ and α are defined in Sections II.A and II.E. (pages 23 and 14) respectively as rates of taxation of corporations and pensions in terms of ordinary income (τ_I).

37   It should be noted that $(\tau_I)_{Model\ 1} > 0$ as $a > 0$, $-b > 0$, and $-b > \sqrt{(b^2 - 4ac)}$.

38   It should be noted that the agency costs of debt in our paper are incorporated by modeling the conflict of interest between the taxable entities and the fiduciary of a pension plan. This is consistent with the finance literature as agency costs of debt are considered to be endogenous because they deter the trade-off of debt claims against a project's cash flows in equilibrium (see Myers, 1977).

elaborated further in this section. However, they cannot control the remaining two equilibria stemming from a DC versus a DB plan, which are contingent on the exogenous administrative charges of the respective plans.

We define our solution as follows: if the taxable investors own the firm in its entirety (referred to in the literature as the asset location 'puzzle,' i.e., $s = \frac{1}{n}$ and $s' = 0$), the solution is termed as the first-corner solution. If both types of investors own a fraction of the firm, the solution is called an interior solution ($s \; \varepsilon \; (0, \frac{1}{n})$). Finally, if the pension fiduciary owns the firm in its entirety ($s = 0$ and $s' = \frac{1}{n}$), it is classified as a second-corner solution.

III.A The Necessary Conditions For Model Solutions

The solutions to our model necessitate encapsulating the no-arbitrage conditions on our return distribution in the Lemma given below. These include a basic condition along with that of debt (bond), asset (equity) and fiscal policy conditions. The last three conditions solved simultaneously yield the optimal structure of a firm endogenously.

Lemma

A Stationary Rational Expectations Equilibrium under default-free bond financing involves four distinct solutions. The solution for the different clientele of stocks and bonds, i.e., Model 1 [Model 2] requires satisfaction of the necessary conditions (i), (ii) (iii) and (iv) [(i), (v), (vi) and (vii)] as described below. The equilibria for the different types of plan (DC versus DB) under Model 1 [Model 2] are realized by incorporating the pension payoffs defined in Equations (13a/13b) [Equations (13c/13d) described below] respectively.

(i)   Basic Condition:

The terminal payoffs of the firm (composed of the sum of its NOI plus the liquidating value of its underlying project) are strictly positive even in the worst state of the economy (in the following period). That is, $\text{Min}\ (q_{1j} + P_{1j}) > 0,\ \forall j$.

(ii)   Debt (Bond) Pricing Condition (Interior/ First Corner Solution in Model 1, where the taxable investor is the net borrower) requires equality between the demand and supply functions of bond financing:

$$\beta E_t\{[\frac{U'(\widetilde{w}_{t+1})}{U'(w_t)}](1+ r(1-\tau_C)(1-\tau_I))\} =$$

$$= \beta E_t\{[\frac{V'(\widetilde{w'}_{t+1})}{V'(w'_t)}](1+r)\} = 1,\ \forall\ s\ \varepsilon\ (0,1],\ n \neq 0 \tag{18}$$

(iiia) Asset (Equity) Pricing Condition (Interior Solution in Model 1, where the taxable investor is the net borrower) requires the expected value of the IMRS of each investor times the net proceeds of the project after payment of the loan amount and appropriate taxes to equal the price of equity of the firm:

$$\beta\ E_t\{[\frac{U'(\widetilde{w}_{t+1})}{U'(w_t)}]\ [(\widetilde{q}_{t+1}(1-\tau_C)(1-\tau_I)+\widetilde{P}_{t+1}(1-\gamma\tau_I)+\gamma\tau_I P_t)-Q(1+r(1-\tau_C)(1-\tau_I))]\} =$$

$$= \beta\ E_t\{[\frac{V'(\widetilde{w'}_{t+1})}{V'(w'_t)}][\widetilde{q}_{t+1}(1-\tau_C) +\widetilde{P}_{t+1}-Q\ (1+ r(1-\tau_C))]\} = (P_t-Q),\ \forall\ s\ \varepsilon\ (0,1),\ n \neq 0 \tag{19a}$$

(iiib) Asset (Equity) Pricing Condition (First Corner Solution in Model 1, where the taxable investor is the net borrower) requires:

$$\beta E_t\{[\frac{U'(\widetilde{w_{t+1}})}{U'(w_t)}][(\widetilde{q_{t+1}}(1\text{-}\tau_C)(1\text{-}\tau_I)+\widetilde{P_{t+1}}(1\text{-}\gamma\tau_I)+\gamma\tau_I P_t)\text{-}Q(1+r(1\text{-}\tau_C)(1\text{-}\tau_I))]\} = (P_t\text{-}Q), \ \forall \ s = 1,$$

$$n\neq 0 \tag{19b}$$

(iv) Policy Conditions:

(iva) The optimal contribution rate $\theta^*_{\text{Model 1}}$ of the initial endowment in Model 1 for risk-averse

agents is evaluated as follows:

If $\{[1+ r(1\text{-}\tau_C)(1\text{-}\tau_I)] [\frac{(1\text{-}\tau_I)}{(1\text{-}\alpha\tau_I)}]\} \ < \ 1$, then $\theta^*_{\text{Model 1}} = \theta^*_{\text{Model 1-Max.}}$ (20a)[39]

If $\{[1+ r(1\text{-}\tau_C)(1\text{-}\tau_I)] [\frac{(1\text{-}\tau_I)}{(1\text{-}\alpha\tau_I)}]\} \ = \ 1$, then $\theta^*_{\text{Model 1}} \ \varepsilon \ (\theta^*_{\text{Model 1-Min.}}, \ \theta^*_{\text{Model 1-Max.}})$ (20b)

If $\{[1+ r(1\text{-}\tau_C)(1\text{-}\tau_I)] [\frac{(1\text{-}\tau_I)}{(1\text{-}\alpha\tau_I)}]\} \ > \ 1$, then $\theta^*_{\text{Model 1}} = 0.$ (20c)

Equation (20c) illustrates the case where there is no optimal contribution to the pension

plan, implying that the agents will resort to savings in their CSA (instead of both CSA

and TDA) if they have sufficient aggregate initial capital to purchase the entire firm in an

unleveraged form. Otherwise, there is no feasible equilibrium.

(ivb) The equilibrium in Model 1 is feasible only if the government budget is capped or

bounded by the inequality $b^2 - 4ac \geq 0$, yielding:

$2G \ \leq \ \dfrac{b^2}{4a}$, where a and b are defined in Equation (17b) (20d)

(v) Debt (Bond) Pricing Condition (Interior/ Second Corner Solution in Model 2, where the

fiduciary is the net borrower) requires equality between the supply and demand functions

of bond financing:

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39   In the limit when the coefficient of risk aversion tends to zero, i.e., under risk neutrality, the optimal
      contribution rate is evaluated from Equation (3) (for both Models 1 and 2).

$$\beta E_t\{[\frac{U'(\tilde{w}_{t+1})}{U'(w_t)}][1+ r\,(1-\tau_I)]\} =$$

$$= \beta\, E_t\{[\frac{V'(\tilde{w'}_{t+1})}{V'(w'_t)}][1+ r(1-\tau_C)]\} =1,\ \forall\ s\ \varepsilon\ [0,1),\ n\neq0 \tag{21}$$

It should be noted that the wealth parameters in Model 2, where the fiduciary is the net borrower, are evaluated as follows:

$$w_t = e_I\,(1-\tau_I)(1-\theta) - s[P_t-Q']_{Corporate\ Shell} - Q \tag{1a}$$

$$\tilde{w}_{t+1} = \tilde{e}_{t+1}(1-\alpha\,\tau_I) + s\,[(\tilde{q}_{t+1}(1-\tau_C)(1-\tau_I)+\tilde{P}_{t+1}(1-\gamma\tau_I)+\gamma\tau_I P_t)-Q'(1+r(1-\tau_C)(1-\tau_I))]$$

$$+ Q(1+r(1-\tau_I)) \tag{2a}$$

$$w_{t+1}(\tilde{\lambda}_j) = (e_{t+1}(\tilde{\lambda}_j))(1-\alpha\tau_I)+s[(\tilde{\lambda}_j q_t(1-\tau_C)(1-\tau_I)+\tilde{\lambda}_j P_t(1-\gamma\tau_I)+\gamma\tau_I P_t)-Q'(1+r(1-\tau_C)(1-\tau_I))]$$

$$+ Q(1+r(1-\tau_I)) \tag{2ai}$$

$$w'_t = e'_t - n\,s'\,[P_t - Q']_{Corporate\ Shell} \tag{6a}$$

$$\tilde{w'}_{t+1} = n\,s'\,[\tilde{q}_{t+1}(1-\tau_C) +\tilde{P}_{t+1}-Q'\,(1+ r(1-\tau_C))] \tag{7a}$$

$$w'_{t+1}(\tilde{\lambda}_j) = n\,s'\,[\tilde{\lambda}_j\,q_t\,(1-\tau_C) +\tilde{\lambda}_j\,P_t - Q'(1+ r(1-\tau_C))] \tag{7ai}$$

$$Q = Q',\ \forall\ n\neq0. \tag{11}$$

Furthermore, the Model-Closing Condition defining $(e'_t)_{DC}$ is still given by Equation (12a). However, the conditions defining $\tilde{e}_{t+1}$ and $\tilde{e}_{t+1}$ for the respective DC and DB Plans are evaluated as follows:

$$(\tilde{e}_{t+1})_{DC} = \theta_{DC}\,e_I + [s'[(\tilde{q}_{t+1} - Q\,r)(1-\tau_C)+(\tilde{P}_{t+1}-P_t)]] - Administrative\ Expenses_{DC} \tag{13c}$$

$$(e_{t+1}(\tilde{\lambda}_j))_{DC} = \theta_{DC}e_I +\{s'[(\tilde{\lambda}_j\,q_t - Qr)(1-\tau_C) +P_t\,(\tilde{\lambda}_j-1)]\} - [Admin.\ Exp.]_{DC} \tag{13ci}$$

$$(e_{t+1})_{DB} = ((e_t)_{Generation(t-1)})_{DB} = \theta_{DB}e_I+[s'(q_t - Q\,r)(1-\tau_C)] - Admin.\ Exp._{DB} \tag{13d}$$

$$(\widetilde{e'_{t+1}})_{DB} = \{[ns'(P_t - Q)]\}_{DB} + \{ns' [((1-\tau_c)(\widetilde{q_{t+1}} - q_t)) + (\widetilde{P_{t+1}} - P_t)]\}_{DB} \qquad (12e)$$

Finally, the government budget constraint in Model 2, corresponding to Equation (17a) in Model 1, is evaluated as follows:

$$\tau_I^2 [(q_t - Q\ r)\phi(\alpha\ s' + s)]$$

$$- \tau_I[e_I(1-\theta(2-\alpha)) + (Q\ r) + ((q_t - Q\ r)(\phi + \alpha\ s' + s))] + 2G\ =\ 0 \qquad (17c)$$

$$\Rightarrow (\tau_I)_{Model\ 2}\ =\ [\frac{-b - \sqrt{(b^2 - 4ac)}}{2a}],$$

where: $b = - [e_I(1-\theta(2-\alpha)) + (Q\ r) + ((q_t - Q\ r)(\phi + \alpha\ s' + s))]$,

$a = [(q_t - Q\ r)\ \phi\ (\alpha\ s' + s)]$, and

$c = 2G.$ $\qquad (17d)$

Equation (17c) illustrates that bond income $(Q\ r)$ in Model 2 is fully taxable, in contrast to Model 1, where it is partially taxable, as illustrated in Equation (17a). This result implies that policy makers should levy optimal tax rates τ_I and τ_C contingent on equilibria stemming from Model 1 or Model 2.[40]

(via) <u>Asset (Equity) Pricing Condition</u> (Interior Solution in Model 2, where the fiduciary is the net borrower) requires a condition similar to Equation (19a) above. However, the wealth parameters are described by the above Equations (1a), (2a), (6a) and (7a), respectively.

(vib) <u>Asset (Equity) Pricing Condition</u> (Second Corner Solution in Model 2, where the fiduciary is the net borrower) requires:

$$\beta\ E_t\{[\frac{V'(\widetilde{w_{t+1}})}{V'(w'_t)}][\widetilde{q_{t+1}}(1-\tau_c) + \widetilde{P_{t+1}} - Q\ (1 + r(1-\tau_c))]\}\ = (P_t - Q),\ \forall\ s = 0, n \neq 0 \qquad (22)$$

Here too, the wealth parameters are defined by Equations (6a) and (7a).

(vii) <u>Policy Conditions:</u>

(viia) The optimal contribution rate $\theta^*_{\text{Model 2}}$ of initial endowment in Model 2 for risk-averse

agents is evaluated as follows:

If $\{[1+ r\,(1-\tau_I)]\,[\frac{(1-\tau_I)}{(1-\alpha\tau_I)}]\} \;<\; 1$, then $\theta^*_{\text{Model 2}} = \theta^*_{\text{Model 2-Max.}}$, (20e)

If $\{[1+ r\,(1-\tau_I)]\,[\frac{(1-\tau_I)}{(1-\alpha\tau_I)}]\} = 1$, then $\theta^*_{\text{Model 2}}\; \varepsilon\; (\theta^*_{\text{Model 2-Min.}},\; \theta^*_{\text{Model 2-Max.}})$ (20f)

If $\{[1+ r\,(1-\tau_I)]\,[\frac{(1-\tau_I)}{(1-\alpha\tau_I)}]\} \;>\; 1$, then $\theta^*_{\text{Model 2}} = 0.$ (20g)

Equation (20g) implies that there is no feasible equilibrium.

(viib) The equilibrium in Model 2 is feasible only if the government budget is capped or

bounded by the inequality $b^2 - 4ac \ge 0$, yielding:

$2G \;\le\; \dfrac{b^2}{4a}$, where a and b are defined in Equation (17c) (20h)

Thus, Model 1 [Model 2] is completely solved as the 11 endogenous variables (s, s', Q,

Q', r, P_t, τ_I, θ^*, e'_t, and $(\tilde{e}_{t+1})$) are exactly equal to the number of independent conditions given

by Equations (10), (11), (12a/12c), (13a/13b), (16), (17b), (18), (19a), (20a/20b/20c) [(10),

(11), (12a/12e), (13c/13d), (16), (17d), (21), (19a), (20e/20f/20g)].[41, 42] Our equilibrium yields

the unique value of a firm (P_t) along with the pricing parameters of debt (Q, r) endogenously.

The uniqueness of our result is emphasized by referring to Chiang (1984) as noted at the end of

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41   An interior solution (s ε (0, 1)) necessitates four independent pricing equations, while a corner solution (s = 0 or 1) necessitates only three independent pricing equations.

42   Equations (18/21) and (19a) each consist of two independent equations. Therefore as a whole they include four equations.

our analysis in Sections II.A and II.B. Thus, the equilibrium produces a distinct debt ratio $\frac{Q}{P_t}$

for the representative firm in the economy and is different from the well-known Miller (1977)

invariance result. We attribute our result to our non-linear (risk-averse) framework, where

value-additivity assumed in the linear Miller (1977) model fails to hold (Varian, 1987).

III.B Key Results

Theorem

The asset location 'puzzle' (i.e., a first corner solution, attributed to tax arbitrage in the

finance literature) is purely a partial equilibrium phenomenon, conceived in a risk neutral world

in the absence of capital constraints. The radically opposite solution (i.e., a second corner

solution, not investigated in the literature) is also feasible (under tax regimes compatible with

that of the U.S., in the absence of capital constraints). An interior solution is not feasible in the

framework of risk neutrality, as it is restrictive and does not facilitate the adjustment of the

intertemporal marginal rate of substitution of the taxable agent and fiduciary.

Introduction of sufficient risk aversion, in contrast, allows for wealth smoothing, where

the individual agent and the pension fiduciary adjust their intertemporal marginal rates of

substitution to own a fraction of the firm, where s > 0 and s' > 0, and to trade default-free

claims against it. This interior solution yields conservative debt ratios for the firm and resolves

the 'puzzle' under two Pareto-neutral rational expectation equilibria (ensuing from the various

clients of the firm, stemming from Models 1 and 2). Our distinct solutions for the interest rate,

the loan amount and firm price are attributed to our non-linear risk-averse framework, where

the value-additivity assumed in Miller (1977) fails to hold (Varian, 1987).[43]   Thus, when

taxable agents and fiduciaries compete, the tax advantage [disadvantage] of pension 'tax-

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43   Our conservative debt ratio complements the empirical observation of Frank (2002) that firms do not
      implement the Black (1980) and Tepper (1981) tax arbitrage fully. Finally, our simultaneously occurring
      twin equilibria, comprising of the anti-tax and tax solutions, contradicts the separate results of Dammon et
      al. (2004) conceived under a restricted borrowing framework (in the absence of liquidity shocks).

shelter' acquired [lost] by one agent can be extricated by the other by charging a higher [lower] cost of debt.

Corollary 1

Extending the above single firm analysis to a multi-firm environment still yields unique bond pricing parameters, both at the firm (micro) level as well as the aggregate (macro) level. However, the inability of policy makers to steer the economy towards either the Model 1 or the Model 2 equilibrium yields a mixed equilibrium with multiple firms, where one set of them are in Model 1 equilibrium and the remaining are in Model 2. This resolves the empirical 'puzzle' where bonds are observed, on average, in the portfolio of taxable accounts as well as tax deferred accounts (Agnew et al., 2003; Barber and Odean, 2004; Bergstresser and Poterba 2004; and Zhou, 2009). Nonetheless, this situation leads to a dilemma for policy makers, as they either overestimate or underestimate tax rates (as implied by policy conditions similar to Equations (17a) and (17c) in a growing economy) leading to accumulating government surpluses or deficits.

Corollary 2

The intergenerational risk sharing feature of a DB plan endows it with a comparative advantage over a DC plan enabling it to be Pareto-efficient. This is attributed to the DB plan's intrinsic ability to create welfare-improving securities with agents who are yet to be born (Bodie et al., 1988; and Shiller, 2003). This relative advantage of the DB plan, however, is eroded away with increasing administrative expenses and the inability to optimally define retirement payout policy to resolve the intergenerational conflict of interest. This inability to define an optimal payout stems from the imminent liability to meet fixed payments to beneficiaries, especially under decreasing mortality and fertility. The increasing fragility of DB

plans, stemming from the agency cost of liabilities, explains the current trend towards DC plans.

This shift towards DC plans forces employees to ultimately bear the investment and mortality risks. That is, they have to expend additional resources to learn the intricacies of modern financial markets to accumulate a subsistence level of terminal wealth over their life cycle (Samuelson, 1989; and Bodie et al., 1992). This shift has serious ramifications, as it leads to a loss in the welfare enhancing feature of intergenerational risk sharing and has the capacity of increasing stock market volatility as each cohort redeems its risky DC portfolio on retirement (Rosser, 2005). It is therefore imperative to reform the pension system to a DB one by reducing its fragility. This result ensues from the prognosis of Myers (2001), which recommends undertaking default-free liabilities to the extent that they do not aggravate their underlying agency costs. It is also in the spirit of Markowitz (1952) as it enables an agent to reduce his/ her risk of CSA with riskless payoffs of a TDA. This redesign of the pension architecture should be supplemented by (i) containing the administrative expenses through the use of institutional accounts from a private/ mutual organization and through curtailing superfluous regulations, (ii) employing cutting edge information technology to define contributions and benefits (by incorporating the payoffs of not only investments but also individual/ cohort/ employer specific factors), (iii) being creative in the use of financial products and services such as deferred annuities and hedging portfolio techniques to alleviate longevity risk, and (iv) delaying the retirement age of employees (James et al., 1999; Shiller, 2003; Angelidis and Tessaromatis, 2010; Horneff et al., 2010, and Cocco and Gomes, 2012).

IV. CONCLUSION AND POLICY IMPLICATIONS

The efficient design of pension systems is important to employees, employers and policy makers. We study pension fund design in the context of general equilibrium theory by focusing on investments in the equity of a firm. We model the conflict of interest from an agency

perspective between taxable investors and the fiduciary of a pre-funded pension plan, incorporating the impact of endogenous policy decisions (Bankman, 2004). Our approach segregates the supply and demand sides of financing to endogenously determine the unique equilibrium pricing parameters of default-free debt in terms of the tax variables (Auerbach and King, 1983). This approach allows us to evaluate the pricing parameters of equity, in terms of tax variables, thereby incorporating the effects of optimal leverage.

We derive the following results. First, we demonstrate that the asset location 'puzzle' is purely a partial equilibrium phenomenon, feasible only in the absence of capital constraints. The radically opposite solution (i.e., assigning of equity [debt] to the TDA [CSA]) is also feasible, in U.S. type tax regimes, under the absence of capital constraints. Introduction of sufficient risk aversion, in contrast, allows for wealth smoothing, where the individual agent and the pension fiduciary adjust their intertemporal marginal rates of substitution to own a fraction of the firm and to trade default-free claims against it. This approach yields conservative debt ratios for the firm and resolves the 'puzzle' under two Pareto-neutral rational expectation equilibria ensuing from the various controlling clients of the firm. Our distinct solutions for the interest rate, the loan amount and firm price are attributed to our non-linear, risk-averse framework, where the value-additivity assumed in Miller (1977) fails to hold. Our conservative debt ratio reconciles with the empirical observation of Frank (2002) that firms do not implement tax arbitrage fully (Black, 1980; and Tepper, 1981). Furthermore, these results refute those of Dammon et al. (2004), conceived under a restricted borrowing framework (in the absence of liquidity shocks).

Second, extending the above analysis to the multi-firm environment still yields unique bond pricing parameters both at the firm (micro) level as well as the aggregate (macro) level. However, the inability of policy makers to manoeuvre the economy towards one of the equilibria, where either the taxable investor or the fiduciary is the controlling clientele of the firm, yields a mixed solution with multiple firms, where one set of them are in the first

equilibrium and the remaining are in second. This result resolves the empirical 'puzzle', where bonds are observed, on average, in the portfolio of taxable accounts as well as tax deferred accounts (Agnew et al., 2003; Barber and Odean, 2004; Bergstresser and Poterba, 2004; and Zhou, 2009). Nonetheless, this situation leads to a dilemma for policy makers, as they either overestimate or underestimate tax rates leading to accumulating government surpluses or deficits, respectively.

Finally, the intergenerational risk sharing feature of a DB plan endows it with a comparative advantage over a DC plan, allowing it to Pareto-dominate. This is attributable to its innate ability to create welfare improving securities with agents who are yet to be born (Bodie et al. 1988; and Shiller, 2003). This relative advantage of the DB plan, however, is eroded away with escalating agency costs of pension liabilities under improving longevity, diminishing fertility, and increasing administrative expenses. This increasing fragility of DB plans explains the current shift towards DC plans, compelling employees to ultimately bear the brunt of the investment and mortality risks. That is, they have to expend additional resources to learn the intricacies of modern financial markets to accumulate a subsistence level of terminal wealth over their life cycle (Samuelson, 1989; and Bodie et al., 1992). This shift has serious ramifications, as it leads to a loss in the welfare enhancing feature of intergenerational risk sharing and has the capacity of increasing stock market volatility. It is, therefore, necessary to revamp the pension plan as a DB one, while alleviating its fragility. This reconfiguration of the pension plan is derived from Myers's (2001) premise on the minimization of agency cost of debt and is in the spirit of Markowitz (1952). It should be accompanied by the following: (i) restraining the administrative expenses through the use of institutional accounts from a private/ mutual organization and through curtailing superfluous regulations, (ii) applying cutting edge information technology to define contributions and benefits by incorporating the payoffs of not only investments but also individual/ cohort/ employer specific factors, (iii) utilizing innovations in financial services such as deferred

annuities and hedging techniques to reduce longevity risk, and (iv) extending the retirement age of employees (James et al., 1999; Shiller, 2003; Angelidis and Tessaromatis, 2010; Horneff et al., 2010, and Cocco and Gomes, 2012).

Appendix: Proofs

Proof of Lemma:

(i) This condition is attributed to the fact that real assets, yielding the terminal payoffs of a firm, serve as collateral for the loan. Since these real assets retain some value in the following period, it enables the net borrower to repay the default-free loan with interest in all states of the economy.

(ii) Equation (18) is derived from Equations (4), (8), (10) and (11).

(iiia) Equation (19a) is derived from Equations (5), (9), (10) and (11).

(iiib) Equation (19b) is derived from Equations (5), (10) and (11).

(iva) Substituting the value of $(\tilde{e}_{t+1})_{DC}$ (from Equation (13a)) in Equation (3) we derive:

$$[\frac{\beta\,(1-\alpha\,\tau_I)}{e_I\,(1-\tau_I)}]E_t\{[\frac{U'(\tilde{w}_{t+1})}{U'(w_t)}]\,\frac{d\tilde{e}_{t+1}}{d\theta}\} \geq 1 \text{ (for a corner/ interior solution)}$$

$$\Rightarrow [\frac{\beta\,(1-\alpha\,\tau_I)}{e_I\,(1-\tau_I)}]\,E_t\{[\frac{U'(\tilde{w}_{t+1})}{U'(w_t)}]\,\frac{d[\theta\,e_I + (s'\,((\tilde{q}_{t+1}\, - \,Qr)(1-\tau_C)+(\tilde{P}_{t+1}\, - \,P_t)) + Q\,r)]_{DC}}{d\theta}\} \geq 1$$

$$\Rightarrow [\frac{\beta\,(1-\alpha\,\tau_I)}{e_I\,(1-\tau_I)}]\,E_t\{[\frac{U'(\tilde{w}_{t+1})}{U'(w_t)}]\,e_I\} \geq 1$$

$$\Rightarrow \beta\,[\frac{(1-\alpha\,\tau_I)}{(1-\tau_I)}]E_t\{[\frac{U'(\tilde{w}_{t+1})}{U'(w_t)}]\} \geq 1$$

Substituting the value of expected $IMRS_T$ from Equation (18) we realize:

$$(1+ r(1-\tau_C)(1-\tau_I))\,[\frac{(1-\tau_I)}{(1-\alpha\,\tau_I)}] \leq 1 \tag{23}$$

Thus, when the LHS of Equation (23) is strictly less than 1, we have a corner solution and when the LHS equals 1, we have an interior solution. In contrast, when the LHS is strictly greater than 1, we have no feasible solution involving pensions.

$$\Rightarrow \text{If } (1+ r(1-\tau_C)(1-\tau_I)) \, [\frac{(1-\tau_I)}{(1-\alpha\,\tau_I)}] \; < \; 1, \text{ then } \theta^*_{\text{Model 1}} = \theta^*_{\text{Model 1-Max.}}, \tag{20a}$$

$$\Rightarrow \text{If } \{[1+r(1-\tau_C)(1-\tau_I)][\frac{(1-\tau_I)}{(1-\alpha\tau_I)}]\} = 1, \text{ then } \theta^*_{\text{Model 1}} \; \varepsilon \; (\theta^*_{\text{Model 1-Min.}}, \theta^*_{\text{Model 1-Max.}}) , \tag{20b}$$

$$\Rightarrow \text{If } \{[1+ r(1-\tau_C)(1-\tau_I)] \, [\frac{(1-\tau_I)}{(1-\alpha\tau_I)}]\} > 1, \text{ then } \theta^*_{\text{Model 1}} = 0. \tag{20c}$$

(ivb) Equation (20d) is derived by imposing the basic property of a quadratic equation on Equation (17b).

(v) Equation (21) is the counterpart of Equations (4) and (8) when the net borrower is the pension fiduciary. It is derived by optimizing the two-period expected utility of wealth of both taxable investors and the pension fiduciary using the budget constraints described by Equations (1a), (2a), (6a) and (7a).

Finally, to demonstrate contingency of tax parameters on equilibria stemming from the two models, i.e., $(\tau_I, \tau_C)_{\text{Model 1}} \neq (\tau_I, \tau_C)_{\text{Model 2}}$

This is derived intuitively noting that the first and middle terms of the quadratic equations defining τ_I (namely Equations (17b) and (17d)) are different. This yields different values of τ_I and thus different values of τ_C using Equation (16). (24)

(via) The equity pricing condition is the counterpart of Equation (19a) when the net borrower is the fiduciary. It is derived using the methodology described in (v) above.

(vib) Equation (22) is a subset of the equity pricing condition described in (via) above.

(viia) Equations (20e–20g) are derived using the methodology described in (iva) above.

(viib) Equation (20h) is derived by imposing the basic property of a quadratic equation on Equation (17c). Q.E.D.

Proof of Theorem:

(ia)   The asset location 'puzzle' is rationalized in a PE setting under the assumption of risk neutrality. Optimization in this framework restricts the inter-temporal marginal rates of substitution (IMRSs) of the competing agents. That is, this setting yields a first corner solution (in Model 1), which is treated in the literature as a 'puzzle.' This issue can be explained as follows.

Risk Neutrality implies $U(w_t) = w_t$ and $V(w'_t) = w'_t \Rightarrow U'(w_t) = 1$ and $V'(w'_t) = 1$.

This constrains the IMRSs of both agents to equal the common discount factor as $IMRS_T$

$$= \beta[\frac{U'(\widetilde{w_{t+1}})}{U'(w_t)}] = \beta[\frac{1}{1}] = \beta; \text{ and } IMRS_F = \beta[\frac{V'(\widetilde{w'_{t+1}})}{V'(w'_t)}] = \beta[\frac{1}{1}] = \beta.$$

Thus, $IMRS_T = IMRS_F = \beta$

Model 1 yields a first corner solution (i.e., $s = \frac{1}{n}$ and $s' = 0$) as the only alternative, as the debt pricing condition (Equation 18) and the equity pricing condition (Equation 19a) for an interior solution fail to hold simultaneously. This result is attributed to the inflexibility of risk neutrality to facilitate the adjustment of the IMRSs of taxable agents and pension fiduciary. The remaining variables such as the pricing of debt (in terms of Q and r), equity, and optimal contribution rate are available from the authors in request.

(ib)   Model 2 also yields a contrasting second corner solution ($s = 0$, $s' = \frac{1}{n}$) under risk neutrality. This has not been investigated in the literature. A detailed solution to this too is available from the authors upon request.

(iii)   The asset location 'puzzle' is resolved under risk aversion. This result is attributed to wealth smoothing, where the individual agent and the pension fiduciary adjust their intertemporal marginal rate of substitution to own a fraction of a firm (where s > 0 and s'

> 0) and to trade default free financial claims against it. That is, the framework of risk aversion yields either the interior solution of Model 1 or the second-corner/ interior solution of Model 2, as demonstrated in our Lemma. This is a Stationary Rational Expectation Equilibrium, where either the taxable agent (as in Model 1) or the pension fiduciary (as in Model 2) is the controlling client of the firm, in accordance with the objective of the policy makers. The two equilibria are Pareto-neutral, as market imperfections such as taxes are incorporated differentially by policy makers, as illustrated in Equations (20b) and (20d) respectively. This result yields conservative debt ratios and reconciles with the empirical findings of Frank (2002) that firms do not fully implement the tax arbitrage strategy (Black, 1980; and Tepper, 1981).

Finally, our results for Model 1 contradict those of Dammon et al. (2004) conceived in their restricted borrowing framework (in the absence of liquidity shocks), where the CSA [taxable investor's entire portfolio in our framework] consists of stocks, while the TDA [pension fiduciary's portfolio] contains both stocks and bonds.

Step 1:

Our simplifying assumption of no economic growth and no population growth, leading to no appreciation in the future price of the project (Footnote 19) determines the mixing probabilities of firms in the two regimes.

Ignoring the administrative costs of the two (DC/DB) pension plans and assuming two (good and bad) states of the world in a simplified Mehra and Prescott (1985) world, we can easily write the following for the Tax Regime of Model 1.

$$q_{t+1, g}\, \psi_{11} + q_{t+1, b}\, \psi_{12} = E_t(\tilde{q_{t+1}}) = q_t$$

Furthermore, assuming $q_{t+1, g} = (1+\delta)\, q_t$, $q_{t+1, b} = (1-\delta)\, q_t$, $\psi_{11} = \pi_1$ and $\psi_{12} = (1 - \pi_1)$, we have:

$$(1+\delta) \, q_t \, \pi_1 + (1-\delta) \, q_t \, (1 - \pi_1) = E_t(\tilde{q_{t+1}}) = q_t$$

This yields $\pi_1 = \dfrac{1}{2} = 0.5$ and the transition matrix simplifies to

$$\begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} \tag{25}$$

Likewise, for the Tax Regime of Model 2, we derive similar results.

Step 2:

We simplify the above transversality condition further by emulating the Mehra and Prescott (1985) to state that we need a stable Matrix **A** with elements $a_{ij} = \beta \, \psi_{ij} \, (\lambda_j)^{1-\rho} \, \forall \, i$, $j = 1, 2$. That is, $\text{Lim.}_{T \to \infty} \, \mathbf{A}^T = 0$.

Since $\psi_{ij} = \dfrac{1}{2}$ (from our response to your comment 4) $\forall \, i, j = 1, 2$, we realize the following condition:

$$\dfrac{\beta(1+\delta)}{2} < 1$$

This condition yields $\delta < \dfrac{(2-\beta)}{\beta}$ \tag{26}

Step 3:

We assume $U'(w_{t+1}(\tilde{\lambda}_j))$ and $V'(w'_{t+1}(\tilde{\lambda}_j))$ as the functions $f(w_{t+1}(\tilde{\lambda}_j))$ and $g(w'_{t+1}(\tilde{\lambda}_j))$. We employ the first order approximation of Taylor Series helps us linearize both $U'(\tilde{w}_{t+1})$ and $V'(\tilde{w}_{t+1})$ in terms of the stochastic growth factor $(\tilde{\lambda}_j)$.[44] This enables us to analytically solve the model further and get a deeper understanding of the ramifications our results. We still find it difficult to isolate each of the endogenous variables in a closed form solution. Nonetheless, the model solution is illustrated below for the general case where

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the taxable agents and the fiduciary are represented by the C.R.R.A utility functions with risk aversion levels of ρ_1 and ρ_2 respectively.

Case 1 – Model 1 (DC Pension Plan):

Here, the payoff of the DC Pension plan is defined by Equation (13ai). Employing Taylor's First Approximation, we realize:

$E_t[U'(w_{t+1}(\tilde{\lambda}_j))] = E_t[\ (w_{t+1}(\tilde{\lambda}_j))^{-\rho 1}] \approx (w_{t+1}(\tilde{\lambda}_j))^{-\rho 1}$, $\tilde{\lambda}_j = 1$ and $w_{t+1}(\tilde{\lambda}_j)$ is defined by Equation (2i). $\hspace{2cm}$ (27a)

Similarly, $E_t[V'(w'_{t+1}(\tilde{\lambda}_j))]= E_t[\ (w'_{t+1}(\tilde{\lambda}_j))^{-\rho 2}] \approx (w'_{t+1}(\tilde{\lambda}_j))^{-\rho 2}$, $\tilde{\lambda}_j = 1$ and $w'_{t+1}(\tilde{\lambda}_j)$ is defined by Equation (7i). $\hspace{2cm}$ (27b)

Employing the above results in the Debt Pricing Function for taxable investor and the Fiduciary yields:

$$\left[\frac{w_{t+1}(\tilde{\lambda}_j)}{w_t}\right]^{\rho 1} \approx [\beta(1+r(1-\tau_C)(1-\tau_I)], \text{ and} \hspace{2cm} (28a)$$

$$\left[\frac{w'_{t+1}(\tilde{\lambda}_j)}{w'_t}\right]^{\rho 2} \approx [\beta(1+r)] \hspace{2cm} (28b)$$

Aggregating the initial endowments of n taxable agents and the fiduciary yields the following condition:

$$n\,w_t + w'_t = ne_I\,[1-\tau_I\,(1-\theta)] - n\,P_t \hspace{2cm} (29)$$

The asset pricing functions for the taxable agents and the fiduciary can also be simplified as follows:

$$(P_t - Q)[1+r(1-\tau_C)(1-\tau_I)] \approx \left[q_t(1-\tau_C)(1-\tau_I) + P_t - Q(1+r(1-\tau_C)(1-\tau_I)\right]$$

$$-\left[\frac{\delta^2 \rho_1}{w_{t+1}(\lambda_j)}\right][q_t(1-\tau_C)[(1-\alpha\,\tau_I)-s\tau_I(1-\alpha)]+P_t[(1-\alpha\,\tau_I)+s\,\tau_I(\alpha-\gamma)]]^2[q_t(1-\tau_C)(1-\tau_I)+P_t(1-\gamma\tau_I)], \forall$$

$$s \,\varepsilon\, (0, 1] \tag{30a}$$

$$(P_t - Q)[1+r] \approx \left[q_t(1-\tau_C)+P_t - Q(1+r(1-\tau_C))\right] -\left[\frac{\delta^2 (n(1-s))^2 \rho_2}{w'_{t+1}(\lambda_j)}\right][q_t(1-\tau_C)+P_t]^3 ,$$

$$\forall\, s\,\varepsilon\,[0, 1) \tag{30b}$$

The optimal contribution rate θ^* is still determined by Equations (20a-c).

Case 2 – Model 1 (DB Pension Plan):

In this case, the payoff of the DB Pension plan is defined by Equation (13b).

The equations (27a), (27b), (28a), (28b) and (29) still hold true but for a redefined Equation (2i) incorporating DB Payoffs.

The asset pricing conditions are simplified further as follows:

$$(P_t-Q)[1+r(1-\tau_C)(1-\tau_I)] \approx \left[q_t(1-\tau_C)(1-\tau_I)+P_t-Q(1+r(1-\tau_C)(1-\tau_I))\right]$$

$$-\left[\frac{\delta^2 s^2 \rho_1}{w_{t+1}(\lambda_j)}\right][q_t(1-\tau_C)(1-\tau_I)+P_t(1-\gamma\tau_I)]^3, \;\forall\, s\,\varepsilon\,(0, 1] \tag{30c}$$

$$(P_t-Q)[1+r] \approx \left[q_t(1-\tau_C)+P_t-Q(1+r(1-\tau_C))\right]-\left[\frac{\delta^2 (n(1-s))^2 \rho_2}{w'_{t+1}(\lambda_j)}\right][q_t(1-\tau_C)+P_t]^3 ,$$

$$\forall\, s\,\varepsilon\,[0, 1),\, n \neq 0 \tag{30d}$$

The optimal contribution rate θ^* is still determined by Equations (20a-c).

Case 3 – Model 2 (DC Pension Plan):

Here, the payoff of the DC Pension plan is defined by Equation (13ci).

The Equations (27a), (27b) and (29) still hold true but for initial wealth, final wealth and DC Pension Payoff defined by Equations (1a), (6a) and (13ci).

The Debt Pricing Function for taxable investor and the Fiduciary is given by the following equations:

$$\left[\frac{w_{t+1}(\lambda_j)}{w_t}\right]^{\rho 1} \approx [\beta(1+r(1-\tau_I)], \text{ and}$$ (28c)

$$\left[\frac{w'_{t+1}(\lambda_j)}{w'_t}\right]^{\rho 2} \approx [\beta(1+r(1-\tau_C))]$$ (28d)

The asset pricing functions for the taxable agents and the fiduciary are as follows:

$$(P_t - Q)[1+r(1-\tau_I)] \approx \left[q_t(1-\tau_C)(1-\tau_I) + P_t - Q(1+r(1-\tau_C)(1-\tau_I)\right]$$

$$-\left[\frac{\delta^2 \rho_1}{w_{t+1}(\lambda_j)}\right][q_t(1-\tau_C)[(1-\alpha\,\tau_I)-s\tau_I(1-\alpha)]+P_t[(1-\alpha\,\tau_I)+s\,\tau_I(\alpha-\gamma)]]^2 [q_t(1-\tau_C)(1-\tau_I) + P_t(1-\gamma\tau_I)], \forall$$

$$s \,\varepsilon\, (0, 1]$$ (30e)

$$(P_t - Q)[1+r(1-\tau_C)] \approx \left[q_t(1-\tau_C) + P_t - Q(1+r(1-\tau_C))\right] - \left[\frac{\delta^2 (n(1-s))^2 \rho_2}{w'_{t+1}(\lambda_j)}\right][\ q_t(1-\tau_C) + P_t]^3, \forall\, s\,\varepsilon$$

$$[0, 1), n \neq 0.$$ (30f)

The optimal contribution rate θ^* is determined by Equations (20e-g).

Case 4 – Model 2 (DB Pension Plan):

In this case, the payoff of the DB Pension plan is defined by Equation (13d).

The Equations (27a), (27b), (28c), (28d) and (29) still hold true but for initial wealth, final wealth and DB Pension Payoff defined by Equations (1a), (6a) and (13d).

The asset pricing conditions are simplified as follows:

$$(P_t-Q)[1+r(1-\tau_I)] \approx \left[q_t(1-\tau_C)(1-\tau_I)+P_t-Q(1+r(1-\tau_C)(1-\tau_I)\right]$$

$$-\left[\frac{\delta^2 s^2 \rho_1}{w_{t+1}(\lambda_j)}\right][q_t(1-\tau_C)(1-\tau_I)+ P_t(1-\gamma\tau_I)]^3, \forall\, s\,\varepsilon\,(0, 1]$$ (30g)

$$(P_t-Q)[1+r(1-\tau_C)] \approx \left[q_t(1-\tau_C)+ P_t-Q(1+r(1-\tau_C)) \right] - \left[\frac{\delta^2 \, (n(1-s))^2 \, \rho_2}{w'_{t+1}(\lambda_j)} \right] \left[q_t(1-\tau_C) + P_t \right]^3,$$

$$\forall \, s \, \varepsilon \, [0, 1), \, n \neq 0 \qquad\qquad\qquad (30h)$$

The optimal contribution rate θ^* is still determined by Equations (20e-g). \qquad Q.E.D.

Proof of Corollary 1:

Extending our earlier analysis to a multi-firm economy, we will still realize unique bond pricing parameters both at the firm (micro) level as well as the aggregate (macro) level as the demand for funds (for each firm) by the net borrower will equal the supply of funds (for each firm) by the net lender. Since supply and demand are equal at the micro-level, they will also be equal at the macro-level due to the principle of aggregation.

However, in a multi-firm environment, the intriguing issue is as follows: Do policy makers have the ability to steer the economy towards either the Model 1 or the Model 2 equilibrium by controlling the optimal contribution rates θ^*? In other words, do policy makers have the capacity to integrate corporate and individual taxes to reform the US tax code?

We investigate the above by focusing on the US tax code, where there is no special provision (or allowance) for pension income. That is, pension income flows are taxed as ordinary income, implying $\alpha = 1$. Furthermore, marginal individual tax rate in the middle income bracket is less than or equal to the rate on corporations (see Gravelle, 2004). That is, $\tau_I \leq \tau_C$. This implies $\phi \geq 1$. These variables impact on the optimal contribution rates (θ^*) as discussed below.

<u>For Model 1</u>

Substituting the value of α and Min.(ϕ) in the expression $\left\{ [1+ r(1-\tau_C)(1-\tau_I)] \, [\frac{(1-\tau_I)}{(1-\alpha\tau_I)}] \right\}$ in Equations (20a-20c) we realize $[1+r(1-\tau_I)^2]$.

For a corner solution, i.e., $\theta^*_{\text{Model 1}} = \theta^*_{\text{Model 1-Max.}}$ $[1+ r(1-\tau_I)^2] < 1$, implies $r < 0$.

For an interior solution $[1+ r(1-\tau_I)^2] = 1$. This implies $r = 0$.

Finally for no pension contribution we have $[1+ r(1-\tau_I)^2] > 1$, implying $r > 0$.

For Model 2

Substituting the values of α and Min.(ϕ) in the expression $\{[1+ r (1- \tau_I)] [\frac{(1-\tau_I)}{(1-\alpha \tau_I)}]\}$ in

Equations (20e-20g) we realize $[1+ r(1-\tau_I)]$

For a corner solution, i.e., $\theta^*_{\text{Model 2}} = \theta^*_{\text{Model 2-Max.}}$ $[1+ r(1-\tau_I)] < 1$, implies $r < 0$.

For an interior solution $[1+ r(1-\tau_I)] = 1$. This implies $r = 0$.

Finally for no pension contribution we have $[1+ r(1-\tau_I)] > 1$, implying $r > 0$.

The above implies that in the context of the US, it is difficult for policy makers to steer the equilibrium towards Model 1 or towards Model 2. This is because for both Models 1 and 2 the same condition (i.e., $r < 0$ or $r = 0$) apply. Even though the above results emanate from a strict assumption of no economic growth in our model, it does have empirical credence as the empirical literature demonstrates the existence of both anti-tax clientele (Model 1) as well as the tax-clientele (Model 2) solutions.

The result derived above is consistent with that of Gomes et al. (2006), who find that the optimal contribution rate θ^* varies across households and over life-cycles. This inability of policy makers yields an economy with mixed equilibria such that one set of firms are in Model 1 equilibrium, while the rest are in Model 2. This result raises an interesting issue. That is, since firms in practice can be in different risk categories, this outcome yields multiple default-free rates of return such that bonds exist in both CSA as well as TDA.[45] Thus, our rationale

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45   A second way of demonstrating the inability of segregating firms in the two equilibria is by studying the mixing probabilities of the firms in Models 1 and 2 and their respective transition matrices. The proof of our theorem evaluates them as identical.

with multiple firms is different from that of precautionary savings or liquidity shocks or smoothing taxes (Amromin, 2003; Dammon et al., 2004; and Garlappi and Huang 2006).

<div align="right">Q.E.D.</div>

Proof of Corollary 2:

For a social planner, the key issue is: Which of the two pension schemes (DC or DB) is economically more efficient? To resolve this, we structure a hybrid plan with a fraction ξ in a DC Plan and the remaining $(1-\xi)$ in a DB plan and conduct the optimization process for Models 1 and 2 respectively. We assume negligible administrative expenses in both plans and derive the crucial F.O.N.C. for both Models 1 and 2 as follows:

$$E_t\{U'(w_{t+1}(\widetilde{\lambda}_j)) \frac{de_{t+1}(\widetilde{\lambda}_j)}{d\xi}\} = 0$$

The above equation yields an optimal amount of ξ^*, which is negative. This implies short selling a DC plan and going long more than 100% of a DB plan. Since short selling is not allowed in our framework, optimal $\xi^* = 0$.

Thus, a DB plan is pareto-efficient over a DC one. Intuitively, a DB pension plan, akin to a riskless bond, combined with a risky (taxable) investment yields a result in the spirit of Markowitz (1952).

The essential feature of a DB plan stems from intergenerational risk sharing, which endows it with the ability to create welfare improving securities with economic agents who are yet to be born (Bodie et al., 1988; and Shiller, 2003). This relative advantage of the DB plan, however, is contingent on its ability to meet the promised payments to its beneficiaries in addition to the administrative expenses, described in the context of our model, to include the following: (i) in-house administrative costs, (ii) consultant fees, (iii) costs to conform to

changes in regulations, (iv) investment management fees, and (v) costs to educate participants in the plan (Hustead, 1998; James et al., 1999; and Diamond, 2000).[46]

Extending our model to the real world environment, we incorporate two more risks in the administrative expenses, i.e., residual risk stemming from demographic changes and a decrease in the mortality of the economic agent. These additional residual risks make it difficult to optimally define retirement benefit payout policy in the case of DB plans in a manner compatible with Equations (13b) or (13d) in the current model. If we compound the matter further by adding in structural changes in the economy ensuing from growth or changes in productivity, etc., we then realize that the promises of the DB plan in practice are at best ad hoc. This is because pensionable salary is primarily determined from the average of that in the pre-retirement years after incorporating years of service instead of average salary throughout one's career and expected portfolio payoffs. Failure to define optimal payout policy in itself, in the face of impending liabilities, is a failure to control the intergenerational conflict of interest. If the DB payouts are generous, it will be at the expense of future generations; however, if they are frugal, it will be at the expense of the current generation. Deriving a balance between the generations is not easy in a DB plan. This result explains the current trend in pensions towards DC plans in the context of our model to an increase in agency cost of liabilities ensuing from: (i) increases in administrative expenses, and (ii) residual pricing and economic risks. This result reconciles with the literature, which attributes this trend to (i) a transitory workforce averse to the back-loaded, where older workers accrue more benefits than younger workers, and

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46   The literature reports trade-offs and economies of scale associated with administrative expenses in the two contrasting DB/ DC plans. For instance, DB plans are exposed to inordinately high consultancy (actuary) fees and costs to conform to changes in regulations. In contrast, DC plans are exposed to high investment management fees, especially for retail accounts, and costs of educating participants, due to the intrinsic feature of transferring investment risk to the participants. Therefore, economic agents would select a DB plan if it is sufficiently large, and a DC plan if it is sufficiently small. This is because in a plan with a large number of participants, the economies of scale reduces the administrative expenses per participant significantly for the cohort to benefit from intergenerational risk sharing by increasing their allocation to equities. This result is empirically confirmed by Bajtelsmit and VanDerhei (1997). It is also corroborated by Kotlikoff (1988), who is of the view that DB plans are particularly popular with large firms composed of a unionized work force with significant bargaining power.).

non-portable nature of the DB plan, and (ii) employers, who are apprehensive of their increasing administrative expenses and of their risk exposure stemming largely from decreases in fertility and increases in longevity (Bodie et al., 1988; and VanDerhei and Copeland, 2001). This is also empirically corroborated for employers like the U.S. government (in the case of the Thrift Savings Plan) and State governments (such as Michigan), who have opted for DC plans (James et al., 1999; and Papke, 2004).

This shift towards DC plans forces employees to ultimately bear the investment and mortality risks. That is, they have to expend additional resources to learn the intricacies of modern financial markets to accumulate a subsistence level of terminal wealth over their life cycle (Samuelson, 1989; and Bodie et al., 1992). This shift has serious ramifications, as it leads to a loss in the welfare enhancing feature of intergenerational risk sharing and has the capacity of increasing stock market volatility as each cohort redeems its risky DC portfolio on retirement (Rosser, 2005). It is, therefore, vital to reform the pension plan to a DB one by mitigating its fragility. This result ensues from Myers (2001), which espouses the undertaking of default-free liabilities to minimize agency costs of debt. This restructure of the pension plan should be accompanied by (i) containing the administrative expenses through the use of institutional accounts from a private/ mutual organization and also through curtailing superfluous regulations, (ii) deploying cutting edge information technology to define contributions and benefits by incorporating the payoffs of not only investments but also individual/ cohort/ employer specific factors, (iii) using innovative financial products and services such as deferred annuities and portfolio hedging techniques to alleviate longevity risk, and (iv) extending the retirement age of employees (James et al., 1999; Shiller, 2003; Angelidis and Tessaromatis, 2010; Horneff et al., 2010, and Cocco and Gomes, 2012). The use of an intermediary, such as a private or a mutual organization, would (i) reduce administrative expenses significantly, (ii) increase the portability of the plan, and (iii) curtail the moral hazard stemming from firms raiding pension assets, taking contribution holidays and conveying the

risk of pension deficits to the governmental agencies (such as the Pension Benefit Guaranty Corporation in the U.S. or the Pension Protection Fund in the U.K.), as cited in Sharpe (1976). In addition, the use of modern information technology would help reduce the intergenerational conflict of interest, while an increase in retirement age would increase the pensionable income and thus the quality of life of retirees. Q.E.D.

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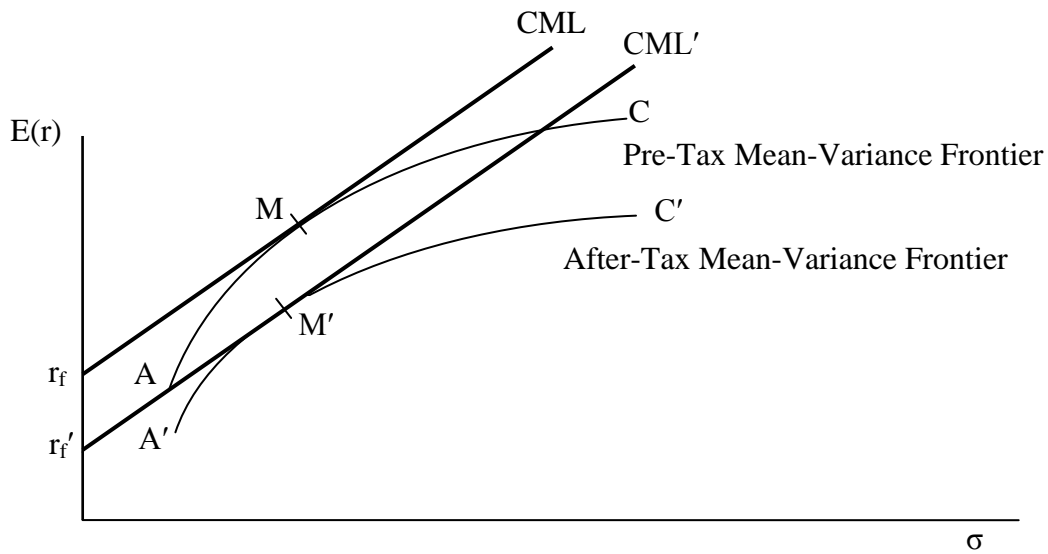
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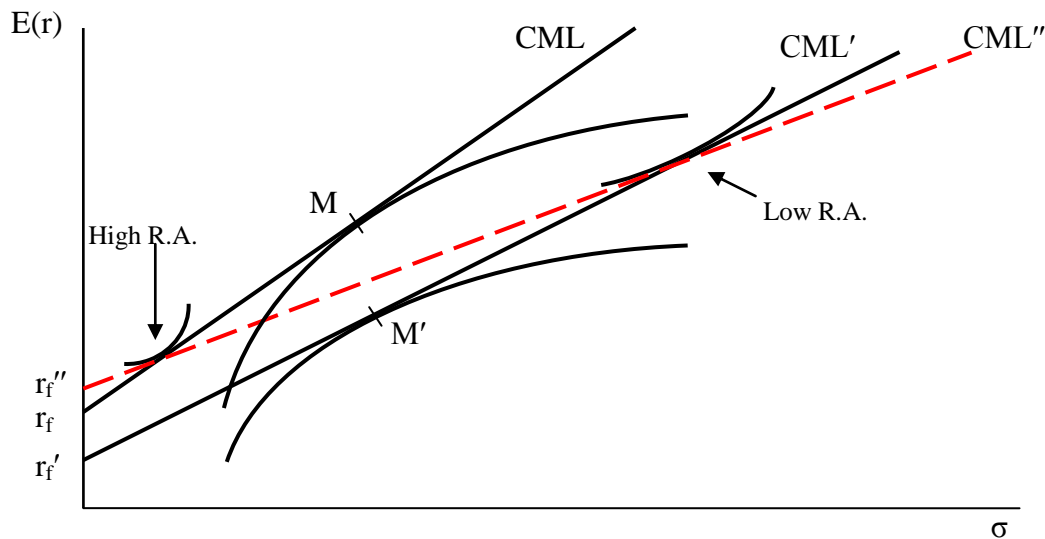

FIGURES

E(r)

CML   CML′

C

Pre-Tax Mean-Variance Frontier

C′

After-Tax Mean-Variance Frontier

M

M'

r_f

A

r_f'

A′

σ

Figure 1: The Breakdown of the Tobin (1958) "Two Fund Separation Theorem"

(Source: Ebrahim et al., 2013)

Figures 2/2' – 3/3' Anti Tax [and Tax] Clienteles of Firms

E(r)

CML   CML′   CML″

M

High R.A.

Low R.A.

M′

r_f''

r_f

r_f'

σ

Figure 2: Model 1 illustrates the Anti-Tax Clientele where the Taxable Investor ("Net Borrower") is a major owner and thus controls the Pure "Value" Oriented Firm.

[Figure 2': Model 2 illustrates the Tax-Clientele where the Tax-Exempt/ Deferred Investor is a major owner and thus controls the Pure "Value" Oriented Firm]   (Source: Ebrahim et al., 2013)
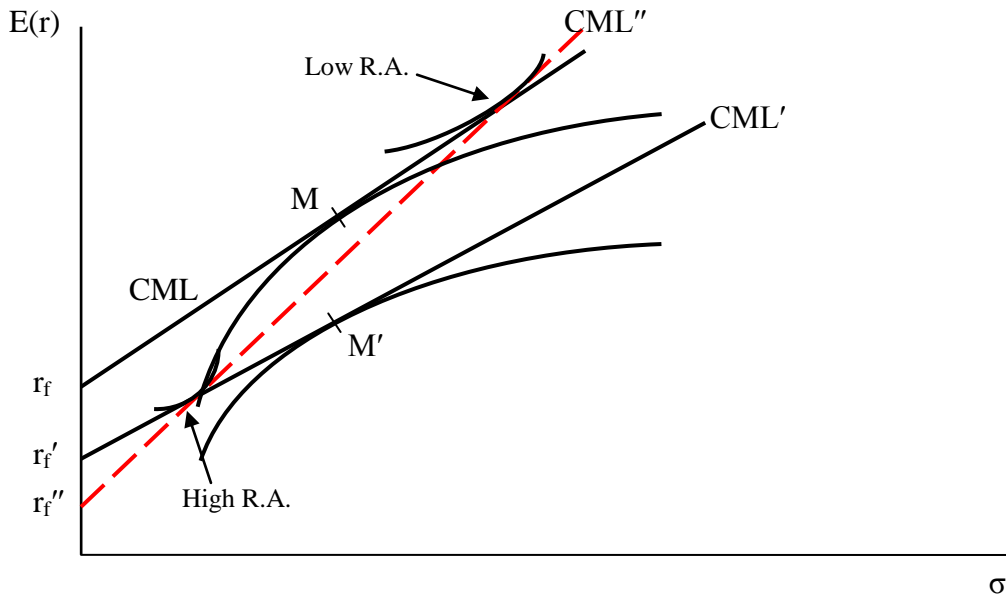
Figure 3: Model 1 illustrates the Anti-Tax Clientele where the Tax-Exempt/ Deferred Investor ("Net Borrower") is a major owner and thus controls the Pure "Growth" Oriented Firm
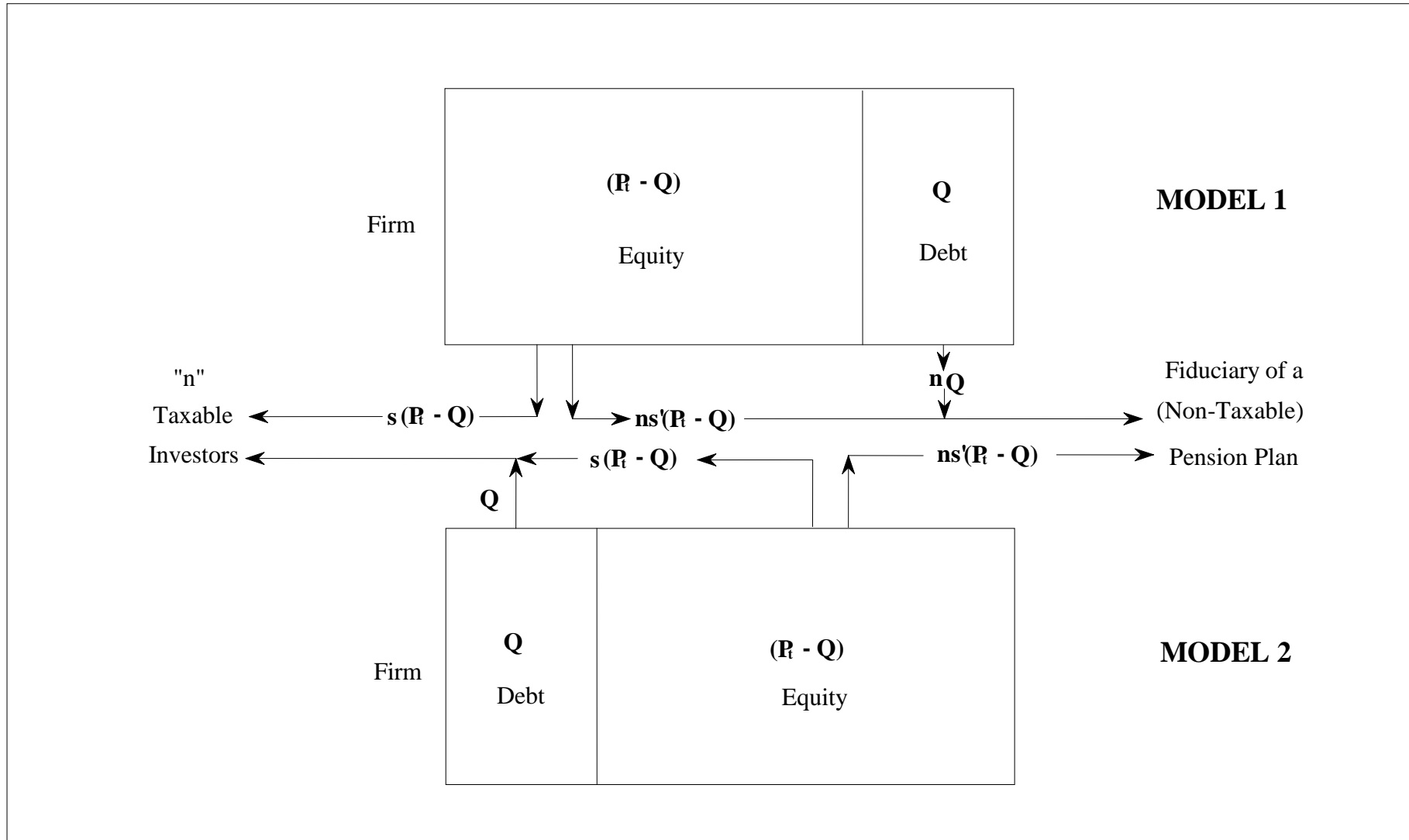
[Figure 3': Model 2 illustrates the Tax-Clientele where the Taxable Investor is a major owner and thus controls the Pure "Growth" Oriented Firm]

(Source: Ebrahim et al., 2013)
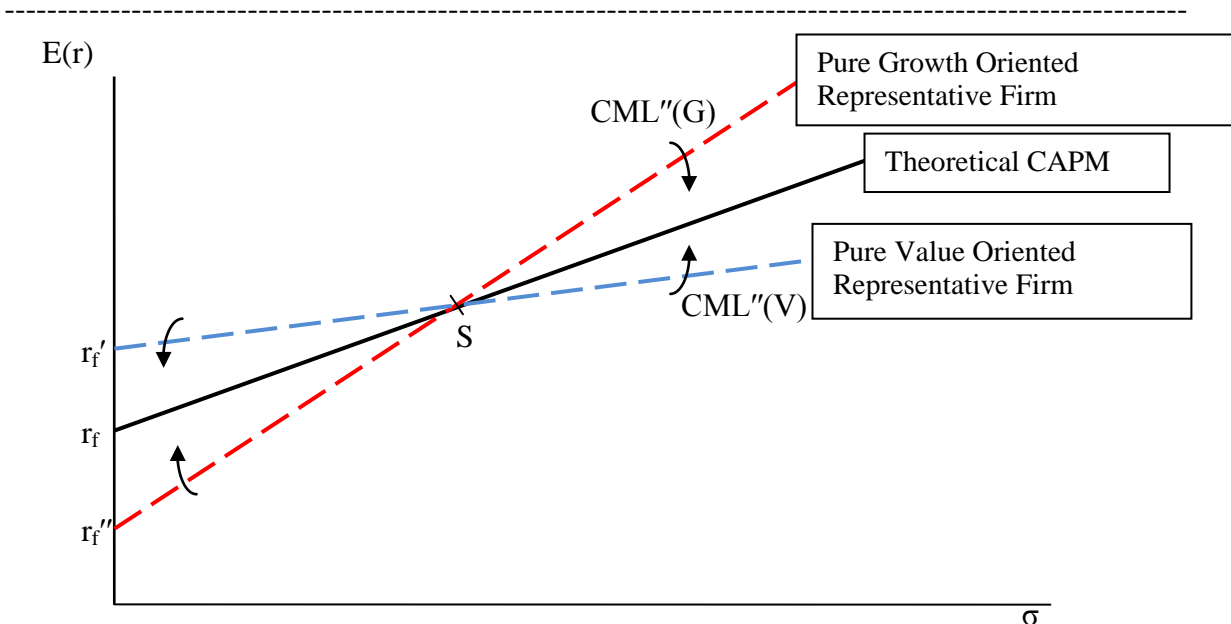
Figure 4: Aggregating the Two Types of Firms Yields Two Frontiers for the 'Representative' Value [Growth] Firm in the Economy.

(Source: Ebrahim et al., 2013)

Figure 5: Two modes of capitalizing a firm