Models: The Blueprints for Laws

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In this paper the claim that laws of nature are to be understood as claims about what necessarily or reliably happens is disputed. Laws can characterize what happens in a reliable way, but they do not do this easily. We do not have laws for everything occurring in the world, but only for those situations where what happens in nature is represented by a model: models are blueprints for nomological machines, which in turn give rise to laws. An example from economics shows, in particular, how we use—and how we need to use—models to get probabilistic laws.

1. Three Theses About Models And Laws. Margaret Morrison has taught us to think of models as mediators. They mediate between our various parcels of general and specific scientific knowledge and the world that that knowledge is about. Here I want to explain one of the principal mediating roles that models serve. Models show us, I shall argue, where laws of nature come from and how we can produce new ones. This way of putting the claim is tied to the standard, so-called 'empiricist', account of what laws are, the account that tells us that laws describe what regularly and reliably happens. If that is what we mean by a 'law' in science, then laws are few and far between—and that reflects the fact that what they are supposed to represent is scarce. It takes very special arrangements, properly shielded, repeatedly started up, and running without hitch, to give rise to a law; it takes what I call a 'nomological machine'. My claim then is that models serve as blueprints for nomological machines.

There are three separate theses involved in this claim. The first is that

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the general scientific knowledge that we use to construct models is not knowledge of laws. This is a familiar thesis from me. I began my career by arguing that the laws of physics lie (Cartwright 1983). That was on the assumption that what we call *laws* in physics really are laws in the sense I grew up with: claims about what necessarily or reliably happens. Ever since then I have been looking for an alternative philosophical account of laws closer to the way law claims are expressed and more responsive to the way they are used, an account that would give them a more reputable status. I shall not pursue this first thesis now.

The second thesis is my chief focus here. It is hard to get a law in nature. One of the principal functions that models serve is to represent those very special circumstances where laws arise. This is not a new thesis either. I have been building up the case for it already in a number of places (Cartwright 1994, 1995, 1996, 1997). But the focus of my discussion so far has been deterministic and causal laws. Here I shall try to show how we use—and how we need to use—models to get probabilistic laws.

The third thesis is that there are no laws to be represented outside the highly structured arrangements that are well characterized as *nomological machines*. I shall lay out this thesis briefly in §3 and §4 in order to highlight how important models are. Models matter because they represent for us just those peculiar situations where nature is reliable.

2. Where Probabilities Come From. Ian Hacking, in *Logic of Statistical Inference*, taught that probabilities are characterized relative to chance set-ups and do not make sense without them. My discussion is an elaboration of his claim. A chance set-up is a nomological machine for probabilistic laws, and our description of it is a model that works in the same way as a model for deterministic laws (like the Copernican model of the planetary system that gives rise to Kepler's laws). A situation must be like the model both positively and negatively—it must have all the relevant characteristics featured in the model and it must have no significant interventions to prevent it operating as envisaged—before we can expect repeated trials to give rise to events appropriately described by the corresponding probability.

2.1. An Example from Wesley Salmon. I begin with an example familiar to philosophers of science: Wesley Salmon's argument that causes can decrease as well as increase the probability of their effects (Salmon 1971). Salmon considered two causes of a given effect, one highly effective, the other much less so. When the highly effective cause is present, he imagined, the less effective one is absent, and vice versa.

Thus the probability of the effect goes down when the less effective cause is introduced even though it is undoubtedly the cause of the effect in question. That is the story in outline, but exactly what must be the case to guarantee that the probabilities are a) well defined and b) that they fall within appropriate ranges to ensure the desired inequality (Prob (effect/less effective cause) < Prob (effect/¬ less effective cause))?

For that we need an arena—a closed container, then a mechanism for ensuring both that there is a fixed joint probability (or range of fixed probabilities with a fixed probability for mixing) for the presence and absence of the two kinds of causes in the container, and that under these probabilities there is sufficient anticorrelation, given the levels of effectiveness, to guarantee the decrease in probability. There must be no other source of the effect in the container or introduced with either of the elements. There must be nothing present in correlation with either of the causes that annihilates the effect as it is produced. Etc. Etc. Figure 1 is a model of the kind of arrangement that is required: a model for a chance set-up or a probability-generating machine. Salmon himself used radioactive materials as causes, the effect being the presence or absence of a decay product. (Figure 1.) My experiment is designed by Towfic Shomar, who chose different causes to make the design simple.[1] The point is that without an arrangement like the one modeled (or some other very specific arrangement of appropriate design) there are no probabilities to be had; with a sloppy design, or no design at all, Salmon's claims cannot be exemplified.

2.2. How Probability Theory Attaches to the World. Turning from this specific example, we can ask, "In general how do probabilities attach to the world?" The answer is *via models*, just as on my account the laws of quantum mechanics apply to concrete situations, and on Ronald Giere's (1988), those of classical mechanics. Assigning a probability to a situation is like assigning a force function or a Hamiltonian. Set distributions are associated with set descriptions. The distributions listed in the table of contents of Harry E. McAllister's (1975) *Elements of Business and Economic Statistics*, shown in Figure 2, are typical. (Compare, for instance, Kyburg 1969, Mulholland and Jones 1968, or Robinson 1985.) Further familiar distributions appear later: the t-distribution, the Chi-square distribution and the F-distribution.

1. Focusing on the need for a design like Shomar's for Salmon's original choice of radioactive materials shows how odd the so-called quantum probabilities are. They are not real probabilities for events that happen, or happen on 'measurement', for measurement itself is a chance set-up and the probabilities to be expected will depend jointly on the quantum state and the structure of the set-up.

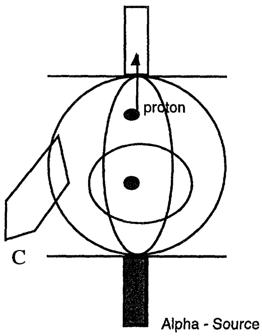

Figure 1A

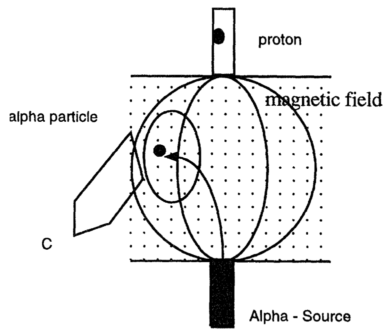

Figure 1B

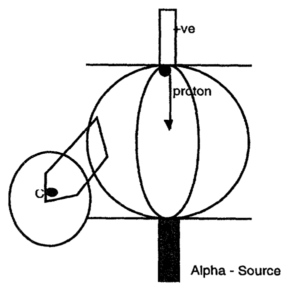

Figure 1C

Figure 1. Consider a radioactive source with a half life such that an alpha particle is on average radiated once every 15 minutes. The source is installed in a cylindrical container opened to a spherical chamber containing a proton. If the radioactive material radiates an alpha particle, the expulsion forces between the alpha particle and the proton push the proton out into the cylindrical box at the other side of the source (see Figure 1A). If the alpha particle is influenced by magnetic field, it will travel through the path toward exit C (as in Figure 1B). Assume that a magnetic field going into the page is turned on in the chamber for 15 minutes and is cut off for 15 minutes. At the moment the magnetic field is off it will cause the upper cylinder to become positivily charged and that will force the proton back into the chamber (as in Figure 1C).

We can assume that the positive charge at the upper cylinder will influence the system for no more than half a minute, allowing the proton influenced by the alpha particle to enter.

So, we have the following probabilities:

Let $c =_{df}$ the presence of an alpha particle,
$e =_{df}$ the proton is forced into the top cylinder,
$m =_{df}$ magnetic field in the chamber.
Then we have
$P(e/c) > P(e/-c)$
$P(e/c)$ is very high (~ 0.9)
$P(c\,\&m) = 0$
$P(e/m)$ is very low (~ 0.1)
So we can conclude that
$P(e/m) < P(e/-m)$,
because $P(e/-m) = P(e/c)$ even though m causes e.

Ch. 6 Probability Distributions
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6.3 The Binomial Probability Distributions
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Figure 2. Harry E. McAllister, *Elements of Business and Economic Statistics*. Wiley, NY: 1975.

As in physics, where the description of a situation that appears in the mediating model must be specially prepared before the situation can be fitted to the theory (e.g., once you call something a harmonic oscillator, then mechanics can get a grip on it), so too in probability theory. As we know, the description of events as *independent* and as *equally likely* or of samples as *random* are key. We can illustrate with the simple binomial distribution, which McAllister describes this way: "A large class of problems applicable to situations in business and economics deals with events that can be classified according to two possible outcomes . . . If, in addition to having two possible outcomes, the outcome of one particular trial is independent of other trials and if the probability . . . is constant for other trials, the binomial probability distribution can be applied" (McAllister 1975, 111).

Again as in physics, in learning probability theory we are taught a handful of typical or paradigmatic examples to which, *ceteris paribus*, the prepared descriptions of the model may be applied (e.g., vibrating strings, pendula, and the modes of electromagnetic fields may all be treated as harmonic oscillators), so probability theory too has its stock examples that show what kinds of more concrete descriptions are likely to support the theoretical descriptions that must be satisfied before the theory can apply. "Uses of the Poisson distribution," McAllister informs us, ". . . include the broad area of theory involving random arrivals such as customers at a drive-in bank, customers at a check-out counter and telephone calls coming into a particular switchboard" (1975, 120). Mulholland and Jones repeat the example of telephone calls in a given period, adding particle emission and the number of minute particles in one milliliter of fluid, as well as a caution: "But there must be a random distribution. If the objects have a tendency to cluster, e.g., larvae eggs, then the Poisson distribution is not applicable" (1968, 167).

The hypergeometric distribution tends to have three kinds of illustrations: defective items (especially in industrial quality control), cards (especially bridge hands), and fish sampling (without replacement of course). And so forth. In each case a given distribution will apply only to situations that have certain very specific—and, generally, highly theoretical—features. Because the requisite features are so theoretical, it is best to supply whatever advice possible about what kinds of situations are likely to possess these features. But these are just rough indications and it is the features themselves that matter: situations that have them—and have no further features that foil them—should give rise to the corresponding probability; and without them, we get no probabilities at all.

2.3. An Economics Example. So far we have looked at the chance set-ups with well-known arrangements of characteristics that feature in probability theory and the corresponding distributions that they give rise to. I would like now to look at an example from an empirical science, in particular economics. Most economic models are geared to produce totally regular behavior, represented, on the standard account, by deterministic laws. My example here is of a model designed to guarantee that a probabilistic law obtains.

The paper we will look at is titled "Loss of Skill during Unemployment and the Persistence of Unemployment Shocks" by Christopher Pissarides (1992). I choose it because, out of a series of employment search models in which the number of jobs available depends on workers' skills and search intensities, Pissarides' is the first to derive results of the kind I shall describe about the probabilities of unemployment in a simple way. The idea investigated in the paper is that loss of skill during unemployment leads to less job creation by employers which leads to continuing unemployment. The method is to produce a model in which

f_t = the probability of a worker getting a job at period t

(i) depends on the probability of getting a job at the previous period (f_{t-1}) if there is skill loss during unemployment—i.e., shows *persistence*; and
(ii) does not depend on f_{t-1} if not.

The model supposes that there is such a probability and puts a number of constraints on it in order to derive a further constraint on its dynamics:

(i) $\partial f_t / \partial f_{t-1} > 0$, given skill loss
(ii) $\partial f_t / \partial f_{t-1} = 0$, given no loss of skill.

The point for us is to notice how finely tuned the details of the model plus the constraints on the probability must be in order to fix even a well-defined constraint on the *dynamics* of f_t, let alone f_t itself.

The model is for two overlapping generations each in the job market for two periods only: workers come in generations, and jobs are available for one period only so that at the end of each period every worker is, at least for the moment, unemployed. 'Short-term unemployed' refers to 'young' workers just entering the job market at a given time with skills acquired through training plus those employed, and thus practising their skills, in the previous period; 'long-term unemployed' refers to those from the older generation who were not employed in the previous period. The probability f_t of a worker getting a job in the between-period search depends critically on x, the number of times a job and worker meet and are matched so that a hire would take place if the job and the worker were both available. By assumption, x at t is a determinate function of the number of jobs available at t (J_t) and the number of workers available at t (2L). Wages in the model are determined by a static Nash bargain which in the situation dictates that the worker and employer share the output equally and guarantees that all matches of available workers and jobs lead to hiring. The central features of the first model are listed in Figure 3. Variations on the basic model that relax the assumptions that all workers search in the same way and thus have the same probability for a job match are developed in later sections of the paper.

The details of the argument that matter to us can be summarized in three steps. (I follow Pissarides' numbering of formulas, but use primes on a number to indicate formulas not in the text but that follow in logical sequence the numbered formula.)

A. A firm's expected profit, π_t, from opening a job at t is
(4) $\pi_t = [1 + f_{t-1} + (1 - f_{t-1})y]\,(L\, f_t / J_t).$
where $y = 1$ represents no skill loss, $y < 1$ the opposite.

Loss-of-skill during unemployment: Model 1

1. Discrete time.
2. Two overlapping generations.
   a. Each of fixed size, $\underline{L}$.
   b. Each generation is in the job market exactly two periods.
3. Each job lasts one period only and must be refilled at the beginning of every period.
4. The number of jobs, J_t, available at beginning of period t is endogenous.
5. Workers in each of their two life periods are either employed or unemployed.
6. a. Output for young workers and old, previously employed workers = 2.
   b. Output for old, previously unemployed workers = 2y, $0 \leq y \leq 1$. ($y < 1$ represents skill loss during unemployment.)
7. Unemployed workers have 0 output, no utility, no income. (This is relevant to calculating wages and profits.)
8. In each period all workers and some jobs are available for matching.
9. Each job must be matched at the beginning of a period to be filled in that period.
10. In each period workers and jobs meet at most one partner.
11. The number of matches between a job and a worker is designated by x, where
    a. x is at least twice differentiable.
    b. $dx > 0$, $d^2x < 0$.
    c. x is homogeneous of degree 1.
    d. $x(0,2L) = x(J_t, 0) = 0$.
    e. $x(J_t,2L) \leq \max(J_t,2L)$.
12. There is a probability that a worker meets a job at the beginning of period t, designated by f_t.
    a. f_t does not depend on what a worker does nor on whether the worker is employed or unemployed.
    b. f_t is a function only of J_t and L.
13. There is a probability that a job meets a worker at the beginning of period t.
    a. This probability is independent of what jobs do.
    b. This probability is a function only of J_t and L.
14. The cost of opening a job and securing the output described in 6 = $1/k$ (whether the job is filled or not).
15. Wages are determined by a Nash bargain.
16. Workers and employers optimize expected utility.

Figure 3.

It is crucial that f_{t-1} appears in this formula. It enters because the expected profit depends on the probability of a job meeting a short- and a long-term unemployed worker, which in turn depends on the number of long-term unemployed workers available and hence on the probability of employment at $t-1$.

B. The number of jobs will adjust so that no firm can make a profit by opening one more, which, given that the cost of opening a job is $1/k$, leads to

(5′) $$\pi_t = 1/k$$

and thus, using (4) to

(7) $$J_t = Lk\,[1 + y + (1 - y)\,f_{t-1}]\,f_t.$$

C. As a part of the previous argument it also follows that $J_t \geq x\,(J_t, 2L)$. In addition

$$f_t = \min\,\{x\,(J_t, 2L), J_t, 2L\}/2L$$

since the number of hires cannot be greater than the number of jobs or workers available nor the number of meetings that take place. Coupling these with the assumption that the homogeneous function x is of degree 1 gives

(8) $$f_t = \min\,\{x\,(J_t/2L, 1), 1\}.$$

When $f_t = 1$—full employment—there are no problems. So consider

(8′) $$f_t = x\,(J_t/2L, 1).$$

To do so, substitute (7) into (8′) to get

(9′) $$f_t = x\,[(k/2)\{1 + y + (1 - y)\,f_{t-1}\}f_t\,,\,1] = x\,(\Phi,1)$$

letting $\Phi =_{df} (k/2)\{1 + y + (1 - y)\,f_{t-1}\}\,f_t$.

We are now in a position to draw the two sought-for conclusions, beginning with the second:

(ii) The case where there is no skill loss during unemployment is represented by $y = 1$. (Short- and long-term workers are equally productive. See Assumption 6, Figure 3.) Then

$$f_t = x\,(k\,f_t\,,\,1),$$

from which we see that f_t does not depend on f_{t-1}. Hence with no skill loss there is no unemployment persistence in this model.

(i) When there is skill loss, $y < 1$. Differentiating (9′) with respect to f_{t-1} in this case gives

(11) $$[1 - \{dx/d\Phi\}\,\{k/2\}\,\{1 + y + (1 - y)\,f_{t-1}\}]\,[\partial f_t/\partial f_{t-1}] = (k/2)(1 - y)f_t\,(dx/d\Phi).$$

Then by the homogeneity of $x\,(\Phi)$,

$$\partial f_t/\partial f_{t-1} \neq 0.$$

"Thus", as Pissarides concludes, "the dynamics of f_t are characterized by persistence" (1992, 1377).

The trick in the derivation is to get f_t to be a determinate function (via x) of a product of f_t and f_{t-1}. x itself is a function of J_t and L. In this model the product form in x is achieved by getting J_t, which is itself determined by profits to be earned from offering a job, to depend on the product of f_t and f_{t-1}. This comes about because J_t depends on the probability of a job being filled by a short- (or long-) term worker, which in turn is equal to the probability of a worker being short- (or long-) term unemployed—into which f_{t-1} enters—times the probability of a short- or long-term worker getting a job, which is indifferently f_t for both.

The derivation of persistence, where it occurs, in the rest of the models in the paper also depends on the fact that the relevant probability analogous to f_t in Model 1 is, through the matching function x, a function of the product of $f_t\, f_{t-1}$. The second model looks to see what happens when the number of jobs is fixed but effort of search varies between the long- and short-term unemployed. In this case the product enters not into the constant factor J of which x is a function, but rather into the second factor, which is not now workers available but units of search effort provided by workers seeking employment. The resulting persistence here is negative ($\delta f_t/\delta f_{t-1} < 0$) which is taken to reflect the process in which low employment increases the numbers of long-term unemployed and thereby lowers the search units supplied, which in turn raises the probability of hire per worker which leads to higher search intensity and thus to more hires.

In the third model, where jobs are again endogenous but firms expect the same profit from the short- and the long-term unemployed, the product appears in the term S_t for search units available and through that in $J_t = k f_t S_t$. Since f_t here is $x\,(J_t, S_t)/S_t$ for x homogeneous of degree 1, it disappears again. The last model, where jobs are endogenous and profits differ, is more complicated. The product appears in S_t and also in J_t, which is no longer a multiple of S_t, so the product ends up in both the numerator and denominator of x. Thus, though the dynamics of f_t are constrained to exhibit persistence, the nature of that persistence is not determinate and could differ depending on what further conditions are added to the model to fix the characteristics of the matching function, x, which is simply hypothesized to exist.

3. Lessons of the Economics Model. I repeat the lesson I wish to draw from looking at Pissarides' search model. Turn again to Figure 3. It takes a lot of assumptions to define this model and, as we have seen, the exact arrangement matters if consequences are to be fixed about whether there is persistence in the dynamics of unemployment probability or not; those arrangements are clearly not enough to fix the exact nature of the persistence, let alone the full probability itself. In Model 1, where job openings are endogenous, the dependence of jobs on workers' histories must be engineered just so, so that J_t will be a function of the product $f_t f_{t-1}$. In Model 2, where the product could not possibly enter through J_t, the facts about how workers search must be aligned just right to get the product into S_t. And so forth.

My claim is that it takes hyperfine-tuning like this to get a probability. Once we review how probabilities are associated with very special kinds of models before they are linked to the world, both in probability theory itself and in empirical theories like physics and economics, we will no longer be tempted to suppose that just any situation can be described by some probability distribution or other. It takes a very special kind of situation with the arrangements set just right—and not interfered with—before a probabilistic law can arise.

As I noted at the beginning, what is special about these situations can be pointed to by labeling them *nomological machines*: they are situations with a fixed arrangement of parts where the abstract notions of *operation*, *repetition*, and *interference* have concrete realizations appropriate to a particular law and where, should they operate repeatedly without interference, the outcome produced would accord with that law. (I discuss nomological machines in more detail in "Ceteris Paribus Laws and Socio-Economic Machines" (Cartwright 1995).)

4. Conclusion. I should like to conclude by pointing out a cherished philosophical thesis that the toy physics model we constructed following Wesley Salmon, the catalogue of models from probability theory, and economics models like the loss-of-skills model I have just described all argue against. The thesis is well expressed by John Stuart Mill:

> The universe, so far as known to us, is so constituted that whatever is true in any one case is true in all cases of a certain description: the only difficulty is to find what description. (1843, vol. 1, 337)

Mill's claim supposes that laws are a dime a dozen: in any situation whatever happens has some description and some law that covers it under that description. But the way in which laws are attached to the world in highly articulated sciences like physics, probability theory, and economics defies this claim. Here I have argued the case with respect to probabilistic laws. Probabilities characterize the outcomes of chance set-ups and chance set-ups are very special kinds of things. What is true of probabilistic laws is analogously true of laws in general: it takes the very special circumstances of a nomological machine before what is true in one case will reliably happen in other cases of some matching description.

REFERENCES

Cartwright, N. (1983), *How the Laws of Physics Lie*. Oxford: Oxford University Press.
———. (1994), "Fundamentalism versus the Patchwork of Laws", *Proceedings of the Aristotelian Society* 94: 279–292.
———. (1995), "Ceteris Paribus Laws and Socio-Economic Machines", *The Monist* 78: 276–294.
———. (1996), 'What is a Causal Structure?', in Vaughn R. McKim and Stephen P. Turner (eds.), *Causality in Crisis? Statistical Methods and the Search for Causal Knowledge in the Social Sciences*. Notre Dame: University of Notre Dame Press, pp. 341–355.
———. (1997), "Where Do Laws of Nature Come From?", *Dialectica* 1 (forthcoming).
Giere, R. (1988), *Explaining Science: A Cognitive Approach*. Chicago: Chicago University Press.
Hacking, I. (1965), *Logic of Statistical Inference*. Cambridge: Cambridge University Press.
Kyburg, H. (1969), *Probability Theory*. Englewood Cliffs, NJ: Prentice Hall.
McAllister, H. E. (1975), *Elements of Business and Economic Statistics*. New York: Wiley.
Mill, J. S. (1843; 1856), *The System of Logic*, 2 vols. London: John W. Parker and Son.
Mulholland, H. and C.R. Jones (1968), *Fundamentals of Statistics*. London: Butterworth.
Pissarides, C. (1992), "Loss of Skill During Unemployment and the Persistence of Unemployment Shocks", *Quarterly Journal of Economics* 107: 1371–1391.
Robinson, E. A. (1985), *Probability Theory and Applications*. Boston: International Human Resources Development Corporation.
Salmon, W.C. (1971), with contributions by Richard C. Jeffrey and James G. Greene, *Statistical Explanation and Statistical Relevance*. Pittsburgh: University of Pittsburgh Press.