
1 Quantitative Morphology of Bedrock Fault Surfaces and
2 Identification of Paleo-earthquakes

3 He Honglin¹, Wei Zhanyu¹, Alexander Densmore²

4 1 Key Lab. of Active Tectonics and Volcano, Institute of Geology, China Earthquake
5 Administration, Beijing 100029

6 2 Department of Geography and Institute of Hazard, Risk and Resilience, Durham University,
7 Durham DH1 3LE, UK

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10 **Abstract** The quantitative analysis of morphologic characteristics of bedrock fault surfaces
11 may be a useful approach to study faulting history and identify paleo-earthquakes. It is an
12 effective complement to trenching techniques, especially to identify paleo-earthquakes in a
13 bedrock area where trenching technique cannot be applied. In this paper, we calculate the 2D
14 fractal dimension of three bedrock fault surfaces on the Huoshan piedmont fault in the Shanxi
15 Graben, China using the isotropic empirical variogram. We show that the fractal dimension
16 varies systematically with height above the base of the fault surface exposures, indicating a
17 segmentation of the fault surface morphology. We interpret this segmentation as being due to
18 different exposure duration of parallel fault surface bands, caused by periodical earthquakes,
19 and discontinuous weathering. We take the average of fractal dimensions of each band as a
20 characteristic value to describe its surface morphology, which can be used to estimate the
21 exposure duration of the fault surface band and then the occurrence time of the earthquake that
22 exposed the band. Combined with previous trenching results, we fit an empirical relationship
23 between the exposure duration and the morphological characteristic value on the fault: $D =$
24 $0.049 T + 2.246$. The average width of those fault surface bands can also be regarded as an
25 approximate vertical coseismic displacement of characteristic earthquake similar to the
26 Hongdong M8 earthquake of 1303. Based on the segmentation of quantitative morphology of
27 the three fault surfaces on the Huoshan piedmont fault, we identify three earthquake events.
28 The coseismic vertical displacement of the characteristic earthquake on the Huoshan piedmont
29 fault is estimated to be 3-4 m, the average width of these fault surface bands. Gaps with a width
30 of 0.1-0.3 m between two adjacent bands, in which the fractal value increases gradually with
31 fault surface height, are inferred to be caused by weathering between two earthquakes or
32 interseismic slip on the fault.

33 **Keywords: Morphology of bedrock fault surface, Paleo-earthquake, Isotropic empirical**
34 **variogram, Huoshan piedmont fault**

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36 1 Introduction

37 Seismic risk evaluation of active fault zones is mainly based on the seismic records,
38 including both historical and pre-historical earthquakes (Wallace, 1981), a reasonable seismic
39 risk evaluation then mainly relies on the integrity of seismic records (Parsons et al., 2000). Due
40 to the lack of sufficiently long historical or instrumental seismic data sets, paleoseismic
41 investigations aimed to identify paleo-earthquake preserved in the geological and geomorphic
42 records is necessary to prolong seismic records (McCalpin, 2009). Trenching is an important
43 technique widely applied to the paleoseismology and has achieved outstanding results (e.g.
44 Young et al., 2002; Ran et al., 2010 ; Galli et al., 2008), as can often identify past major
45 earthquakes that have ruptured the ground surface at a particular site. However, this method has
46 some weaknesses; for example, interpretation of offset strata and fault-rupture related features is
47 sometimes debatable, and suitable offset materials that can be dated are required to bracket the
48 event times, usually leaving large uncertainties (e.g. Hilley and Young, 2008). Moreover, it can be
49 applied to a fault in bedrock site only in selected cases (Galli and Bosi, 2003; Galli et al., 2006;
50 2012; Galli and Peronace, 2014). Therefore, it is necessary to seek some other techniques to
51 study paleoseismology on faults in bedrock.

52 Bedrock fault scarps may be interpreted as the cumulative result of repeated surface
53 faulting in many active tectonic terrains, and potentially preserve a valuable paleoseismic record
54 (Mayer 1984; Stewart, 1993). In the last decade, bedrock fault scarps have become an attractive
55 complement for paleoseismological studies because the exposure duration of a bedrock fault
56 scarp can be inferred by methods based on the accumulation of cosmogenic nuclides (Zreda
57 and Noller, 1998; Benedetti et al., 2000; Mitchell et al., 2001). Nevertheless, if one want to infer
58 both the age and slip of the last few major earthquakes on the fault using the cosmogenic
59 nuclides technique to a bedrock fault surface, hundreds of samples should be taken
60 continuously on the bedrock fault surface (e.g. in Schlagenhauf A., 2009). This is a very

61 expensive work both in manpower and material.

62 Geologists have noted long before that the features of the bedrock surfaces gradually
63 changed in texture or roughness with exposure duration due to external influences and several
64 geomorphic process (i.e. weathering, karstification, bioerosion) when the fault surface are
65 exposed. As a result, abrupt changes in the features of fault surfaces may appear as parallel
66 bands on the same fault surface exposed in different times. Sharp weathering contrast on fault
67 surface has been used to delimit recent exposure increments through visual observation in field
68 (e.g. Wallace 1984, Stewart, 1993), or through photographic study (Giaccio et al., 2002), but
69 without conclusive results. Recently, terrestrial laser scanning (t-LiDAR) has been widely
70 applied to acquire accurate morphologic features of bedrock fault surfaces in neotectonic and
71 geological earthquake studies (e.g. Sagy et al., 2007; Candela et al., 2009; Brodsky et al, 2011;
72 Renard et al., 2012). Few use of t-LiDAR has been undertaken to characterize the weathered
73 fault surface and identify sequentially exposed bands on fault surfaces (e.g. Wei et al., 2013).
74 Only recently, through t-LiDAR data Wiatr et al. (2015) suggests evidences for repeated faulting
75 of the Pisias fault, Greece, with 30-60 cm of displacement at one site based on the fact that the
76 roughness increases with scarp height on naturally exhumed bedrock fault scarps.

77 Previous studies of bedrock fault surface with t-LiDAR have shown that the morphology of
78 fault surfaces is self-affine and fractal. Fractal dimension is suitable to characterize the
79 roughness in various scale for t-LiDAR data of bedrock surfaces (reviewed in Candela et al.,
80 2009). Our approach applies the fractal analysis on the natural bedrock fault surface to identify
81 possible weathered bands of fault surface which can be related to seismically-exhumed fault
82 surface, and then identify paleoseismic events. Besides identifying differently weathered bands,
83 another important purpose of our research is searching for mathematic model to relate the
84 digital morphologic feature of fault surface to the exposure duration, which has not been done
85 by previous research works. Therefore, it is necessary to quantify the morphology of bedrock
86 fault surface using a special mathematic method fitting to weathering feature. It is the

87 groundwork not only to identify differently weathered bands of paleoseismological interests but
88 also to relate the digital morphologic feature of fault surface to the exposure duration. For this
89 aim, we focus here on the Huoshan piedmont fault (HF), which is an active normal fault
90 extending along the eastern boundary of the Shanxi Graben, China (Figure 1). The M 8.0
91 Hongdong earthquake of 1303 was produced by the fault, and several other destructive
92 earthquakes have been disclosed by trenching (Xu and Deng, 1990; Xu, 2013). Moreover, along
93 the fault zone there are a lot of fault scarps, which supply plentiful samples to be selected to our
94 research.

95 Firstly, we scan three bedrock fault surfaces with a t-LiDAR. Secondly, we describe the fault
96 surface morphology by its fractal dimension as calculated by the isotropic empirical variogram
97 method and a cellular fractal model. We ascribe the characteristic morphologic fractal values of
98 each fault surface band to individual earthquake events, and analyze the relationship between
99 the fault surface morphology and the exposure duration, and further paleoseismologic
100 information recorded in the bedrock fault surface.

101 2 Target fault and 3D data of fault surface

102 2.1 Seismotectonic framework

103 The HF, located on the eastern flank of the central Shanxi Graben at the eastern boundary
104 of the Ordos block, China (Figure 1a, b), is an active boundary fault between the Huoshan
105 Range and its piedmont basin. It extends 116 km to the NNE from Subao town, Hongdong
106 county to Longfeng town, Jiexiu county, and dips to the NW at 65-75° (Xu et al., 2011). In the
107 footwall, the Huoshan Range is an asymmetrical tight anticline with a core of Archean gneiss.
108 Since the Pliocene, this anticline has been tilted along the fault forming a fault block mountain.
109 The hanging wall is filled by sediments that range from Pliocene to Late Pleistocene (Figure 1c).
110 Previous research demonstrated that active faulting occurred from the end of the Pliocene
111 through the Holocene (Xu and Deng, 1990; Zhang et al., 1998; Wen, 2000; Xie et al., 2004). The
112 HF has been identified as the seismogenic fault responsible for the 1303 M 8.0 Hongdong

113 earthquake (Figure 1b), which is the first M 8 earthquake hypothesized from historical
114 descriptions in China (Liu and Meng, 1975; Wang et al., 1996). The HF is marked by
115 well-developed triangular facets and bedrock fault scarps caused by dominantly normal-slip
116 faulting. Trenching investigation on the southern segment of the HF found 4 paleo-earthquakes
117 (Xu and Deng, 1990; Xu, 2013). The oldest one occurred between 28580-26380a BP, whereas,
118 the other three occurred in the Holocene, 4620-5455a BP, 2555-3475a BP and the 1303 M 8.0
119 Hongdong earthquake, showing an average recurrence interval of about 2000a.

120 We chose to focus on three bedrock fault surface exposures near Liwan town (Huozhou city,
121 Figure 1c), which are all located in the epicentral area of the 1303 Hongdong earthquake, and
122 are all carved on Archean gneiss of the Huoshan fault footwall. Therefore, the three fault
123 surfaces have similar faulting histories and are likely to have similar weathering resistance.

124 Field observation also found several weathered horizontal bands on fault surfaces, different
125 morphologic features in different height due to different weathering degree. The fault surface
126 presents smooth with some striations and steps on the bottom due to faulting, rough with some
127 weathered pits and grid fissures on the middle, more rough and more wide cracks due to erosion
128 and plant root growth on the upper, while on the top of the fault scarp covered by weathered
129 debris and shrub.

130 2.2 Scanned fault surface and Data Acquisition

131 We scanned the three fault surfaces (Figure 2) using a t-LiDAR, a Trimble GX 3D (Figure 2,
132 upper of c), which is an automatic survey instrument with high resolution and a 300 m maximum
133 scan radius, and the space between two adjacent scan points ranges from 1.6 mm to 5 mm
134 according to the distance between the scanner and the fault surface from 5 m to 300 m. We
135 scanned only those areas of the fault surface with no vegetation or sediment cover. The scan
136 results are three point clouds in which each scan point is described by its 3D geometric
137 coordinates, and average space between adjacent points is 2 mm across the three point clouds.
138 We interpolated the scan data into a DEM with an equal cell size of 2 mm using natural neighbor

139 method (Figure 2). The lower three of Figure 2 show the morphology of the three scanned fault
140 surface DEMs derived from the point clouds.

141 3 Study methods

142 3.1 Fractal dimension characterizing the surface roughness

143 Assuming that our fault has been exhumed mainly by coseismic slip, the surface observed
144 features are likely produced by the combination of two mechanical processes: faulting abrasion,
145 and post-earthquake weathering and erosion. Faulting abrasion is expected to make the
146 morphology of fault surface more anisotropic, so that the roughness in the direction parallel to
147 slip becomes less than that in the direction perpendicular to slip (Sagy et al., 2007). Conversely,
148 weathering is expected to be a more random process, and makes the fault surface more
149 isotropic and rougher. Previous studies, emphasizing the impact of faulting abrasion on the fault
150 surfaces, often used the power spectral density (Power et al., 1988; Power and Tullis, 1991;
151 Brown, 1987) and standard deviation (Renard et al., 2006; Candela et al., 2009) to describe the
152 roughness based on line profiles parallel or perpendicular to slip. These 1D analysis methods
153 are useful to explore the relation between fault surface morphology and faulting processes
154 (Power et al. 1987; Sagy et al., 2007, 2009; Wei et al., 2010), but not for understanding the
155 effects of weathering on morphologic characteristics, because the weathering is a random and
156 isotropic process on two-dimensional surface. Moreover, t-LiDAR allows genuinely
157 two-dimensional data of fault surface to be gathered, typically in the form of digital images. Such
158 data offer exciting opportunities for addressing issue of spatial variation in a way that is difficult
159 for line transects of surface.

160 Because the fractal dimension can capture the essence of a natural surface roughness
161 (Viewed in Burrough, 1981; Mandelbrot, 1983), it has been widely used in earth sciences for
162 textural analysis of topography and characterizing geological quantities of Earth (Polidori et al.,
163 1991; Klinkenberg and Goodchild, 2002; Sung and Chen, 2004). The surfaces of rocks
164 associated with slip wear, weathering and erosion is similar to the real landscapes which have

165 statistical properties similar to fractional Brownian surface (Brown, 1987), it is possible to
166 describe the roughness of the fault surface using the fractal dimension (e.g., Sage et al., 2007;
167 Candela et al., 2009). Isotropic empirical variogram proposed by Davies and Hall (1999) is an
168 effective method to estimate the fractal dimension of 2D spatial data (Gneiting et al., 2012).
169 Therefore, here we calculate the 2D fractal dimension using a cellular fractal model and the
170 isotropic empirical variogram to quantitatively describe the weathering morphology of fault
171 surface.

172 3.2 Calculation of two-dimensional fractal

173 The isotropic empirical variogram, i.e., the statistical variation of mean differences with
174 distances between two points, is an extension of the Hausdorff dimension in a two-dimensional
175 random field (Davies and Hail , 1999). Taking X as a random process (one dimension series) or
176 random field (two dimensional plane), $\gamma(t) = \text{cov}\{X(t), X(0)\}$ is the covariance of a pair of points
177 with separation distance t . Generally, there is a relationship between the $\gamma(t)$ and the separation
178 distance (t) as follows:

$$179 \quad \gamma(t) \propto c||t||^\alpha \quad (1)$$

180 where α is the fractal index, which is between 0 and 2. There is a linear relationship between the
181 fractal index (α) and fractal dimension (D) as follows:

$$182 \quad D = d+1-\alpha/2 \quad (2)$$

183 where d is the topological dimension of the data field. For random processes d is equal to 1, and
184 for random fields d is 2. The fractal dimension (D) can be calculated by α which is the slope of
185 the best-fitting line based on the double logarithmic linear regression of the isotropic variogram
186 (Equation 1) shown in Figure 3. The cellular fractal model following Sung et al (1998), a moving
187 window operation (Figure 4), is used to calculate the fractal dimension distribution on the fault
188 surface in this paper. The moving window traverse the entire fault surface with the offset for
189 each move in directions of horizontal and vertical to calculate the fractal dimension using the
190 variogram method. The moving window is defined to $N \times N$ (cells of DEM), then the offset for

191 each move is $N/2$ (Figure 4a). The area within each window can be treated as a homogeneous
192 unit and is described by a uniform fractal dimension which can be derived by double logarithmic
193 linear regression of Equation 1. A raster image of fractal dimension is made to display the spatial
194 distribution of roughness on a surface, and the resolution of raster data is $N/2$, the offset of the
195 window (Figure 4b). The surface units within moving windows at the same height likely
196 experienced the similar faulting abrasion and weathering erosion, and their fractal dimension
197 values display a normal distribution. To characterize differences in natural fault surface
198 alteration along the height of the fault surface, we calculated the mean value with 95%
199 confidence interval of the normal population for each horizontal row in raster image of fractal
200 dimension. The average fractal along the surface height is then used to evaluate the disparity of
201 the fault surface topography.

202 3.3 Choice of the moving window

203 The proper size of the window is very important to both the accuracy and the precision of
204 fractal dimension using variogram (Sung et al., 1998). Too small windows cover insufficient data
205 and increase the uncertainty of the fractal dimension estimate; too large windows increase the
206 changes of capturing heterogeneous and multi-fractal characteristics within the window and
207 decrease the spatial resolution of the fractal dimension. Sung et al. (1998) found that the
208 percentage of acceptable fractal dimension estimations for three synthesized surfaces
209 decreases with a decreasing window size. It drops drastically if the window size is smaller than
210 30×30 . They suggested that 30×30 is the smallest data matrix that provides $>80\%$ of the
211 accurate estimate of the surface fractal dimension, and there is little difference in the estimator
212 of fractal dimension when the window size is larger than 60×60 . This technique has been
213 applied for quantifying the heterogeneity of various surfaces, such as sea floor (Wilson et al.,
214 2007), landform (Bi et al., 2012) and bedrock surface (Wiatr et al., 2015). We chose three types
215 of the moving window, 32×32 , 64×64 and 128×128 in grid, to calculate the 2D fractal dimensions
216 of the three bedrock fault surfaces. Because the grid spacing of the morphologic DEM is 2 mm,

217 the sizes of three windows are $64 \times 64 \text{ mm}^2$, $128 \times 128 \text{ mm}^2$ and $256 \times 256 \text{ mm}^2$, respectively.
218 Based on the supposed coseismic vertical displacement of AD 1303 earthquake, these window
219 sizes are significantly smaller than the displacements which control the abrupt changes in the
220 features of fault surfaces.

221 4 Results and interpretation

222 The raster images in figure 5 show the spatial distribution of two-dimensional fractal values
223 on the scanned fault surfaces. Because our primary objective is to reconstruct the past seismic
224 slip history of bedrock fault scarps, we are more interested in the vertical changes (dip changes)
225 of roughness along the fault surface. However, the raster image of fractal show faint variations
226 of the fractal dimension, without a clear trend as a function of the fault height. As a result, it is
227 not easy to identify the presence of bands characterized by different weathering degree along
228 the surface height in such raster images (figure 5). Therefore, averaging the fractal values of
229 each horizontal row (perpendicular to dip) in raster image was performed in order to determine
230 the roughness changes along the height of the fault surface. Through viewing the plots of
231 average fractal against surface height, a stair-like increase can be recognized on the analyzed
232 surface from base to top, with values ranging from 2.2 to 2.7. (i.e., the base of the fault surface
233 is smoother with low fractal value, and the top is rougher with high fractal value). Such changes
234 in roughness along fault surface is similar to the changes in the amount of specific cosmogenic
235 isotopes along seismic exhumed bedrock scarps (e.g. Schlagenhauf et al., 2010), which allow to
236 use the surface roughness to provide earthquake information from bedrock fault scarps.

237 “Stair-like” increase in the surface roughness with increasing scarp height has originally
238 been described by Wallace (1984) and Stewart (1996) for seismic exhumed bedrock fault scarps.
239 These authors first proposed a mechanism that may have produced such roughness
240 fluctuations: Before being exposed above ground level by an earthquake, the bedrock fault
241 surfaces had been smoothed by sliding wear during faulting actives. Faults generally emerge

242 from the ground as smooth, polished planes. Once exposed above ground level by an
243 earthquake, the scarp rock begin to be roughened by the weathering processes which leads to
244 increased rock surface roughness with time (Giaccio et al., 2002; Galli et al., 2010; Wei et al.,
245 2013). One band on a fault surface exhumed by an earthquake experienced the same
246 weathering processes under similar sub-aerial conditions, thus, it would have the same
247 roughness. As bedrock scarps are progressively exhumed by the action of repeated large
248 earthquakes, the roughness along the entire exposed scarp should take a “stair-like” curve
249 made of a series of approximate straight sections separated by sharp discontinuities. The
250 vertical separation between two successive discontinuities provides a measure of the
251 displacements produced by the earthquakes.

252 5 Discussions

253 5.1 Determination of weathering bands along surface

254 Identification of bands is a decisive step as far as the seismic intensity, the seismic cycle
255 and seismic hazard assessment are concerned (Schlagenhauf et al., 2010; Mouslopoulou et al.,
256 2011). Through visual identification in the plots of average fractal versus scarp height (Figure 5),
257 there are three obvious bands for surface A and surface B, and two bands for surface C. In
258 addition to the visual identifications, a statistical analysis was used to validate these bands with
259 different roughness on a scanned fault surface (i.e., Student's *t*-test. See details and results in
260 the Supplementary materials).

261

262 The result of Student's *t*-test, similar to the visual interpretation, show that there are three
263 bands for surface A and B, and two bands for surface C, but with a more robust statistical
264 evidence for band division. For surface A, two discontinuities in surface roughness are located
265 at the height of 0.9–1.2 m, 5.1-5.3 m, respectively; for surface B, two discontinuities are located
266 at the height of 1.5-1.8 m, 5.8-5.9 m, respectively; surface C has only one discontinuity in

267 roughness located at the height of 1.1-1.3 m. We also found some tiny fluctuations in the mean
268 fractal curves (Figure 6) that may be caused by the local slight difference in rock constituents
269 along fault surface or noise data from t-LiDAR measurement. Therefore, we do not think those
270 tiny fluctuations can act as a discontinuity in roughness.

271 According to the band division above, we calculated the mean and the standard deviation of
272 fractal dimension for each surface band, and the results are summarized in table 1 and showed
273 by the red lines in Figure 5. These mean values can be seen as the characteristic fractal
274 dimensions for bedrock fault surface bands, and as a morphologic parameter characterizing the
275 surface roughness to quantify the degree of weathering.

276 **5.2 Paleoseismic events and coseismic slips**

277 Under the hypothesis that the weathered bands of fault scarp are the result of repeated fault
278 slip events, our three bands would indicate three slip events. Historical earthquake analyses
279 and paleoseismic investigations along the HF have indicated that three surface-rupturing
280 earthquakes occurred during the Holocene (Xu and Deng, 1990; Xu, 2013), which is consistent
281 with our paleoseismic result demonstrated via differential weathering on fault surfaces. Therefore,
282 these three bands from bottom to top match the three earthquakes dated 1303, 2555-3475 a BP
283 and 4620-5455 a BP.

284 As the upper and lower extents of the surfaces were not scanned completely, we cannot
285 use these surface segments to estimate the vertical co-seismic displacement. Conversely, the
286 middle segments of surface A and B had been scanned completely, and their width of about 4 m
287 quite possibly represent the dip-slip offset of the penultimate earthquake. Considering the fault
288 dip of 75° , the vertical co-seismic displacement during the penultimate earthquake is 3.8 m.
289 Compared to the vertical co-seismic displacement of the Hongdong earthquake of 1303 (4-5
290 m), the vertical co-seismic displacement of 3.8 m implies that the penultimate earthquake had a
291 similar magnitude with the Hongdong earthquake. Wei et al. (2015) used faulting knickpoints to
292 indicate that the ruptures on the HF obey a characteristic slip model with a similar slip (about 4

293 m) for several successive earthquakes as well. Moreover, between two adjacent weathered
294 bands, there is a narrow gap, the fractal value of which increases gradually with fault surface
295 height. The two possible explanations for the formation of the gap are an inter-seismic creep slip
296 along the fault, or a gradual erosion along the base of the fault scarp for a long time.

297 We made an evolutionary model of fault scarp surface in bedrock (Figure 7), where the fault
298 scarp has been divided into two main sections according to the dip angle and weathering degree.
299 The upper section has seriously been weathered, its dip angle has changed to be lower than the
300 original fault dip angle; the lower section has not been weathered so much and some faulting
301 abrasion feature has been saved, and its dip angle has been kept to be similar to the original
302 fault dip angle. We can identify the weathering band by naked eye according to large scale
303 morphological feature and dip angle variation in the upper section. In the model, there are two
304 surface morphological bands with different weathering degree in the upper section identified by
305 naked eye, and three surface morphological bands in the lower section identified by roughness
306 analysis based on high-resolution DEM measured by t-LiDAR. These five surface morphological
307 bands with different weathering degree might correspond to five earthquake faulting events.

308 **5.3 Weathered characteristics as a function of exposed time of the fault surface**

309 In order to make fractal index a reliable palaeoseismological tool, it should be understand
310 how the fractal indices of fault surface changes over the exposure time, that is, what is the
311 relationship between the fractal index and the exposed time of the fault surface.

312 To build such relationship, we need in advance two variables: one is the fractal index
313 quantifying the degree of weathering, and another is the exposure time . In our case, the
314 youngest event can be dated at 1303 AD by historic documents, and other two
315 paleoearthquakes were dated at 2555-3475a BP and at 4620-5455a BP, respectively, by means
316 of paleoseismological trenching (Xu et al., 1993). In Figure 8 we plotted the characteristic fractal
317 dimensions vs the occurrence times of supposed paleo-earthquakes, i.e. the exposed time of
318 surfaces segments, for surface A, Surface B and the set of studied surfaces. There seem to be

319 an ascending linear trend of fractal value with the exposed time of fault surface (dashed lines in
320 Figure 8).

$$321 \quad D = 0.049 T + 2.246 \quad (3)$$

322 where D is the fractal dimension, and T is the exposed time of fault surface which unit is ka. As
323 we know that the two-dimensional fractal has an upper limiting value of 3, this will gradually
324 approach a steady value over the exposed time when the morphology of bedrock fault surface
325 reaches the equilibrium with the weathering and erosion on surface. As a result, the relationship
326 between the characteristic fractal dimensions and the exposed time should be nonlinear and
327 complicated in a long enough time scale. At present, however, we do not know more information
328 about this relationship. Our result revealed the linear trend between the characteristic fractal
329 dimensions and the exposed time ranging from 0.7 ka to 5 ka on studies fault surfaces
330 (Equation 3). We speculate that the relationship between the characteristic fractal dimensions
331 and the exposed time can be treated as a linear function approximately in a
332 centennial-millennial scale. However, to obtain a more accurate relationship, the changes of
333 fractal dimension over a much larger time scale are necessary. Therefore, it is one of our
334 important research targets in future that the weathering stability and weathering behavior of the
335 various lithologies on the bedrock fault.

336 6 Conclusions

337 The quantitative analyses of bedrock fault surface morphology is an effective method to
338 study faulting history and identify paleo-earthquake. The 2D fractal dimension on a fault surface
339 calculated by isotropic empirical variogram shows vertical segmentation, and the characteristic
340 fractal dimension of each fault surface segment increases step by step from the bottom to the
341 top. This kind of step increase suggests that those fault surfaces are cropped out intermittently
342 likely due to periodic faulting earthquakes. Therefore, the exposure duration or the occurrence
343 time of an earthquake can be inferred by using the characteristic fractal dimension of each fault
344 surface segment, and the vertical co-seismic displacement by using the width of fault surface
345 segment. Based on the quantitative morphologic analyses of the fault (scarp) surfaces on the
346 Huoshan piedmont fault, we indentified three earthquake events, the Hongdong M 8.0

347 earthquake of 1303 and other two previous earthquakes. Combined with the occurrence times
348 of two pre-historical earthquakes estimated by trenching study, we got an empirical relationship
349 between the characteristic fractal dimension and the occurrence time of earthquake
350 displacement of characteristic faulting earthquake on the Huoshan piedmont fault has also been
351 estimated to be 3-4 m. Moreover, 0.1-0.3 m wide gap between two adjacent fault surface
352 segments, which fractal dimensions increase gradually as fault height increases, is produced by
353 erosion between two earthquakes.

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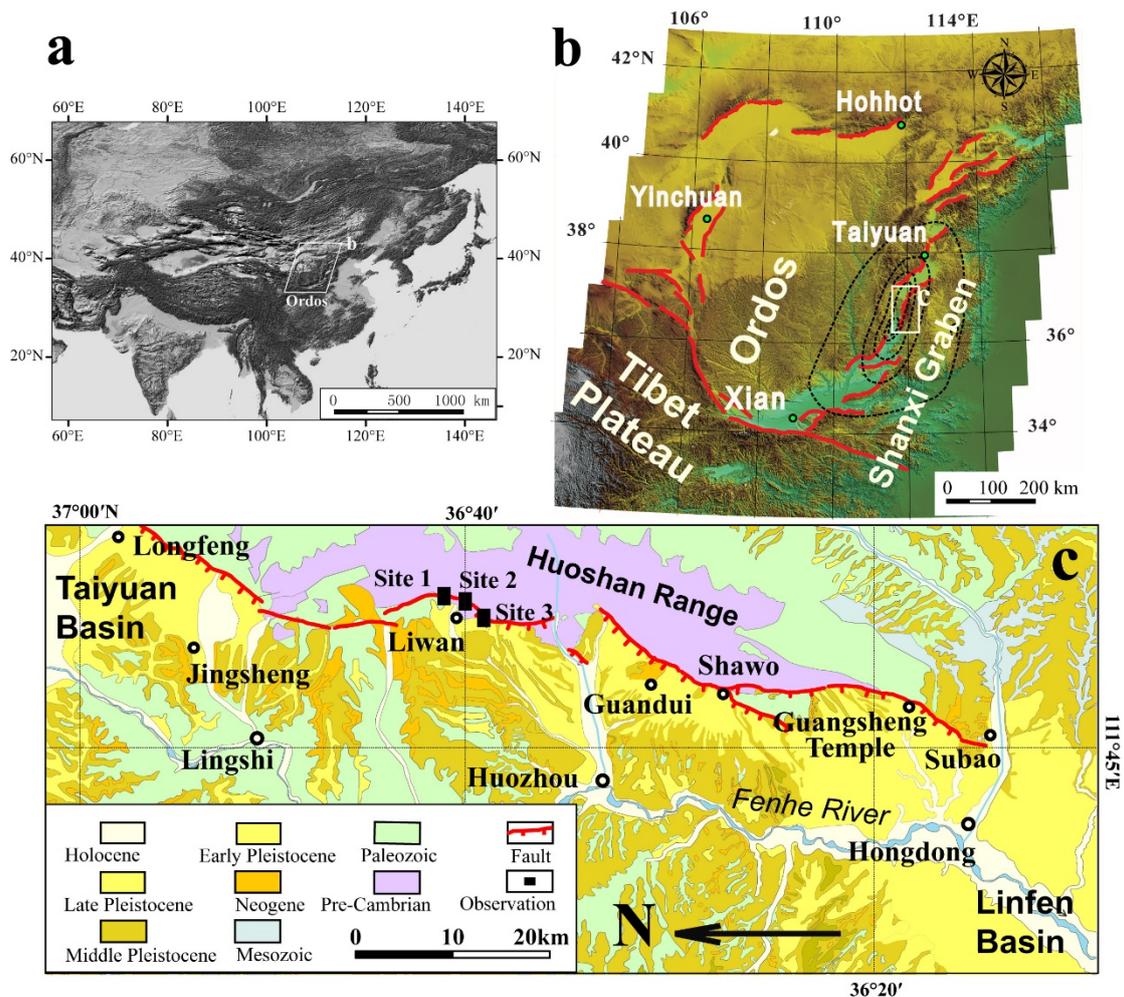
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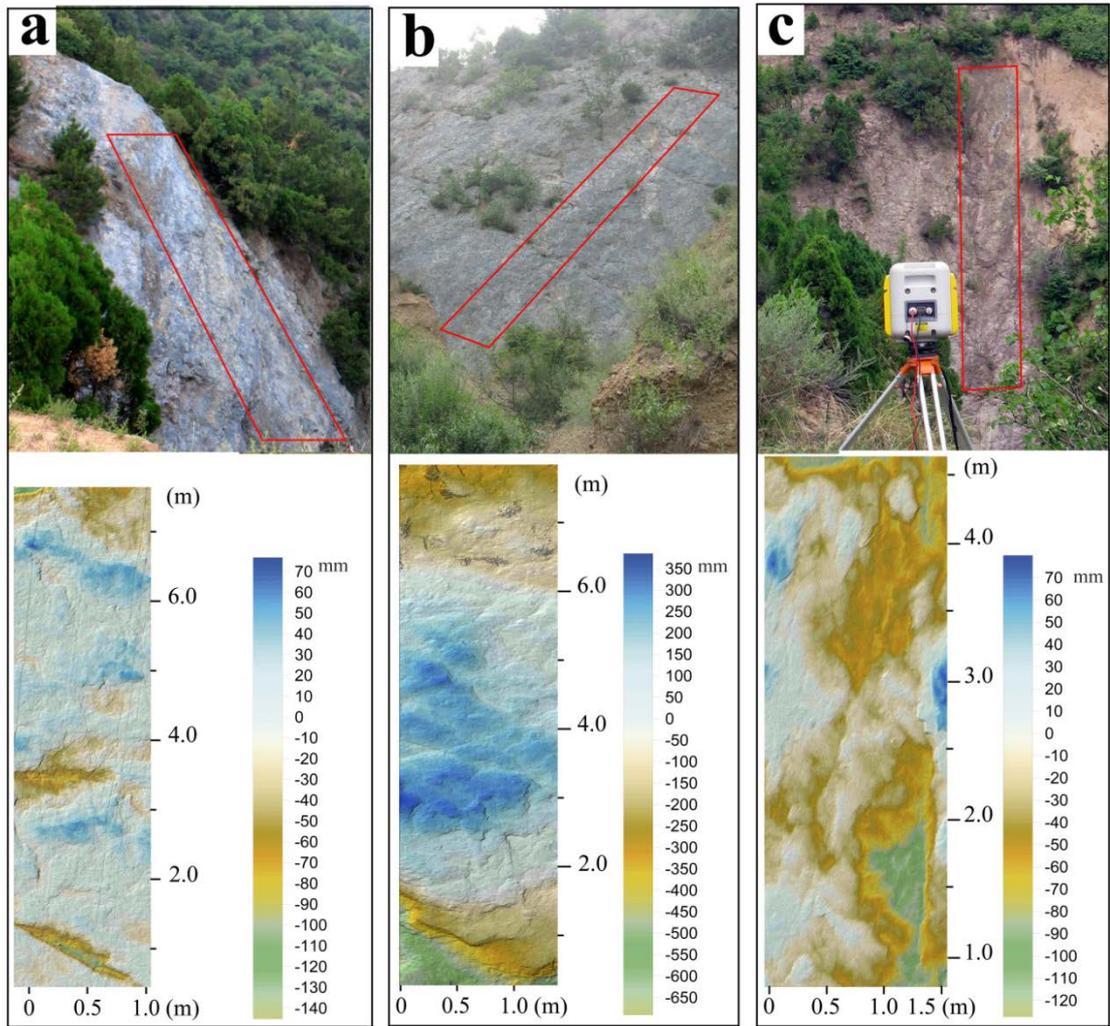
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2 Figure 1 Geologic map of the Huoshan piedmont fault. a, location of the Ordos
 3 block within north-central China. b, Active faults surrounding the Ordos block. The
 4 background is a color shaded-relief view of SRTM elevation data, the red lines are
 5 active faults (from Deng et al., 2007), the black dashed lines are isoseismals of the
 6 Hongdong M 8 earthquake of 1303 (Earthquake Engineering Investigation Institute
 7 of Shanxi Province, 2009), and the white rectangle shows the location of Figure 1c.
 8 c, Geometry of the Huoshan piedmont fault. Black rectangles show the locations of
 9 scanned fault scarps. Other geologic information is shown in the legend.

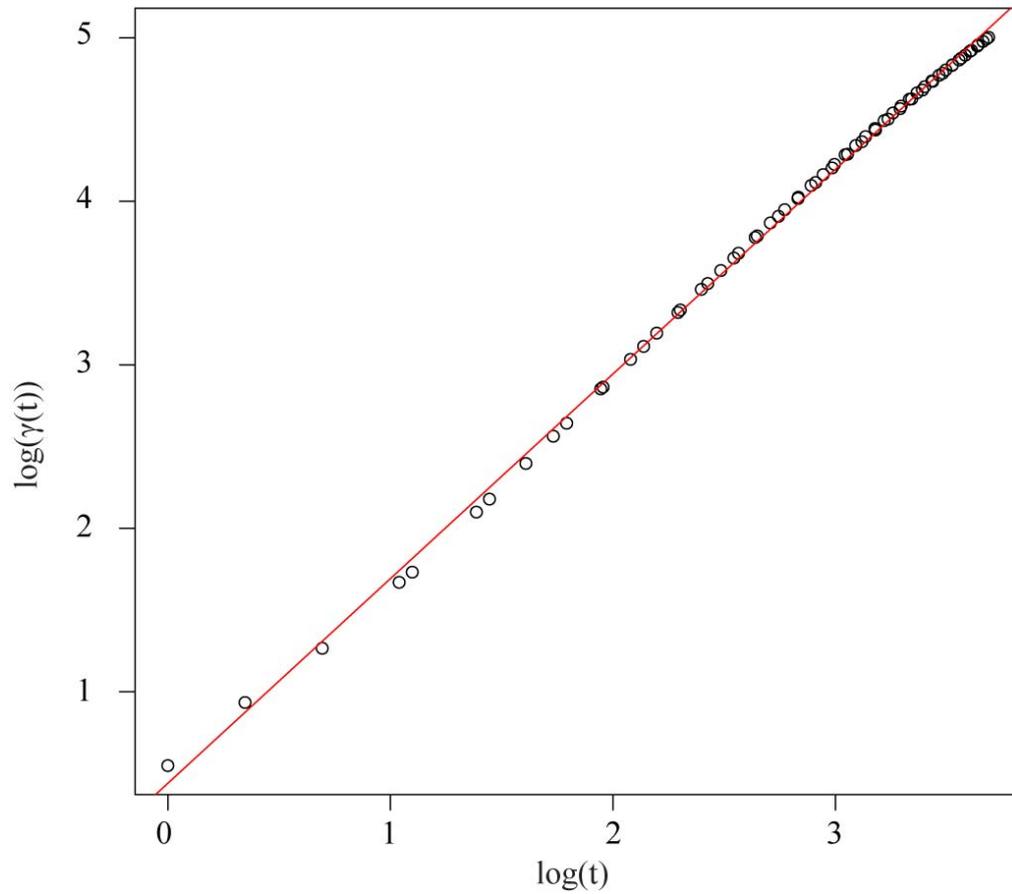
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12 Figure 2 Fault outcrops (upper) and their rendering morphology derived from
 13 scanned point clouds (below). a, b and c are three fault surface outcrops, which
 14 locations are indicated by Site 1, Site 2 and Site 3 in Figure 1c, as well as the scan
 15 locations (red rectangles). Panels show the morphologies of the three fault
 16 surfaces, respectively.

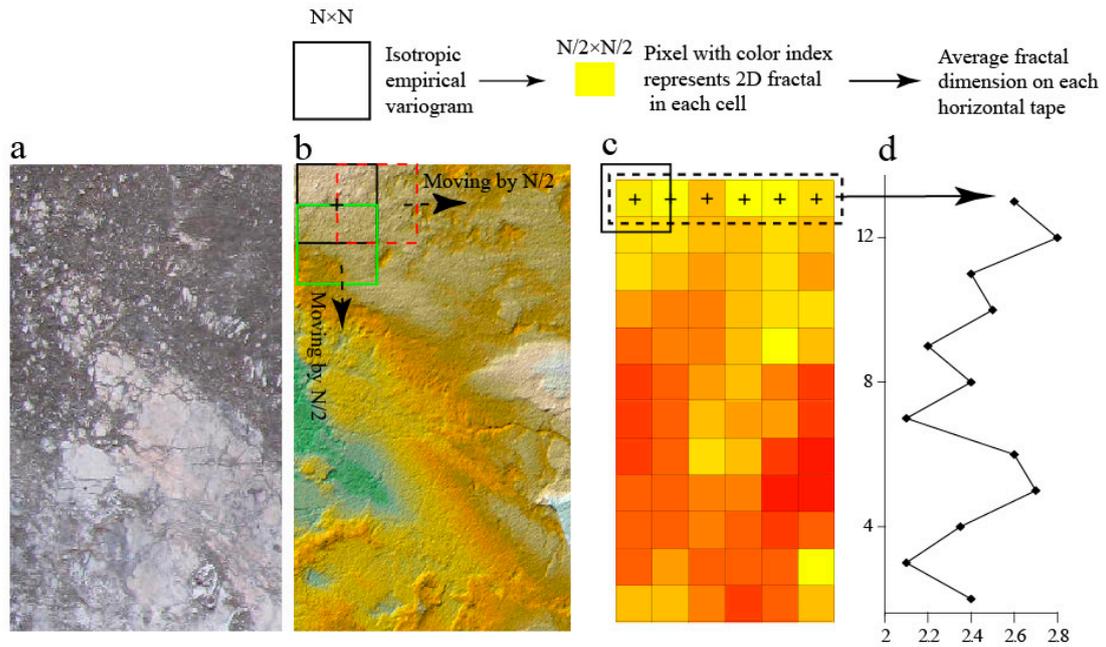
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19 Figure 3 Log-log regressions for the isotropic variogram each dot represents the
20 covariance of points with a certain separation distance t ; red line is the linear
21 fitting of the dots, and its slope is linearly associated with the fractal dimension of
22 2D spatial data.

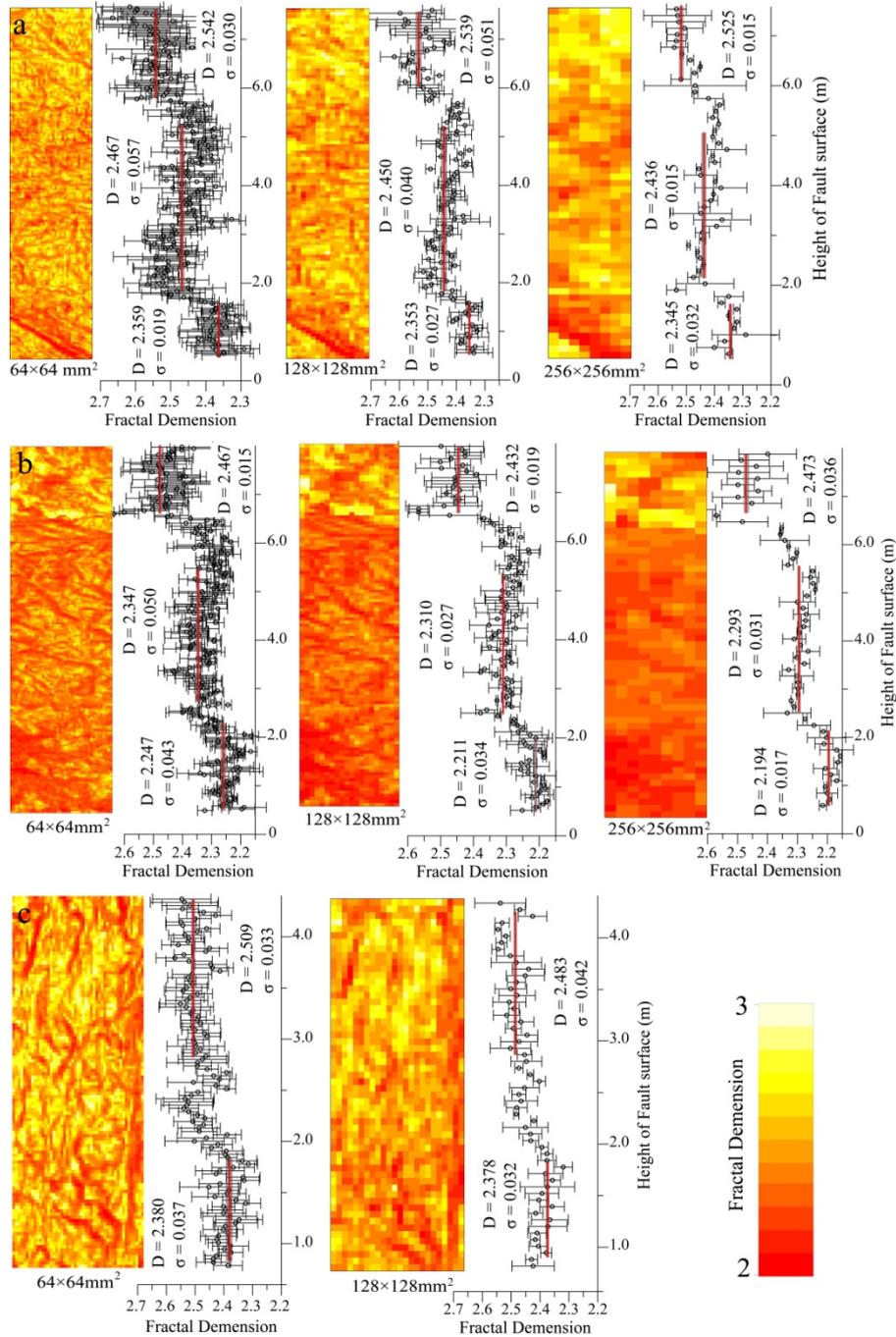
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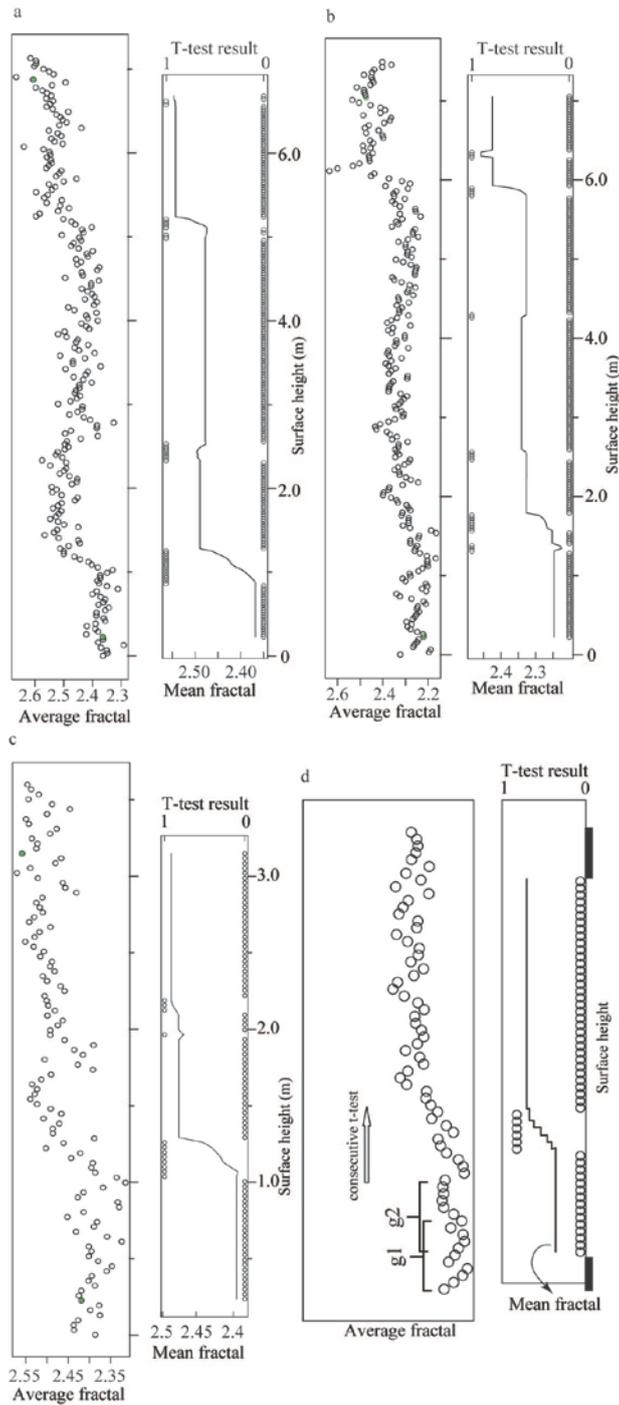
25 Figure 4 Schematic diagram for calculating fractal dimension of fault surface. (a)
 26 Photo of the fault surface; (b) Hillshade image created by the high resolution digital
 27 elevation model of scanned surface; the colored squares stand for the moving
 28 windows with size of $N \times N$ in both directions of horizontal and vertical; (c) shows
 29 image of fractal dimension for fault surface; each pixel stand for the estimator of
 30 fractal dimension for surface cell with size of $N/2 \times N/2$; the yellow-red colors are
 31 index of fractal value; (d) the diagram shows the fractal values distribution of fault
 32 surface along with vertical height; each black point is the average fractal
 33 dimension on each horizontal row.

34



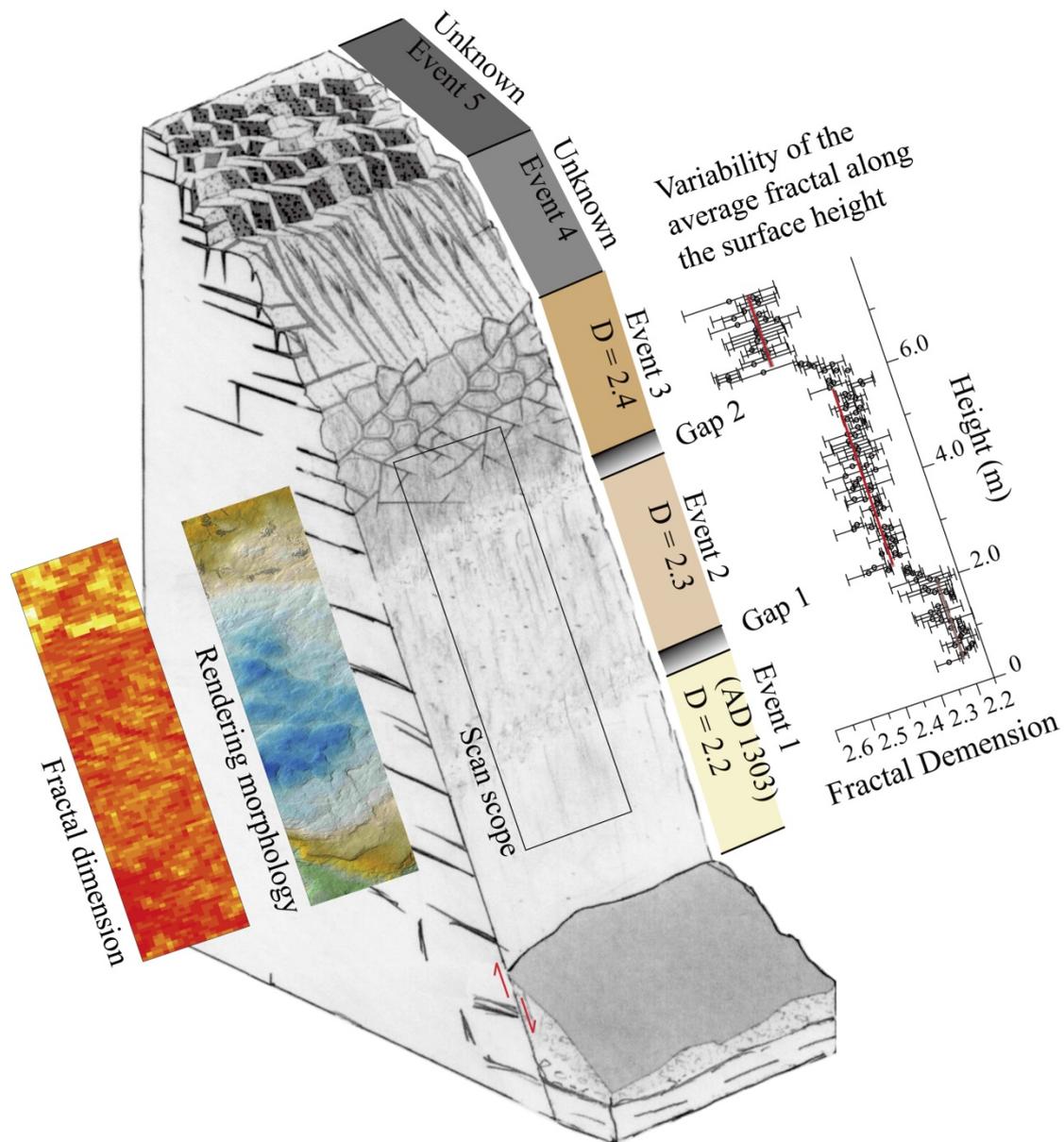
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36 Figure 5 Raster images of fractal dimension and corresponding average fractal
 37 along surface height for fault surfaces. a, b and c correspond to the three scanned
 38 bedrock fault surface. In each panel, the raster images with color range from red to
 39 light yellow (left) show the spatial distribution of fractal dimensions on the fault
 40 surface, while the scatter diagrams (right) show the variability of the average
 41 fractal along the surface height. The raster images are generated by three types of
 42 moving window, 64x64 mm², 128x128 mm² and 256x256 mm² from left to right.
 43 Black dots with error bars are the mean value of each horizontal row in the fractal
 44 raster images and gray error bars represent 95% confidence interval. The red
 45 vertical bars show the average value for each group of samples (see text for
 46 details).



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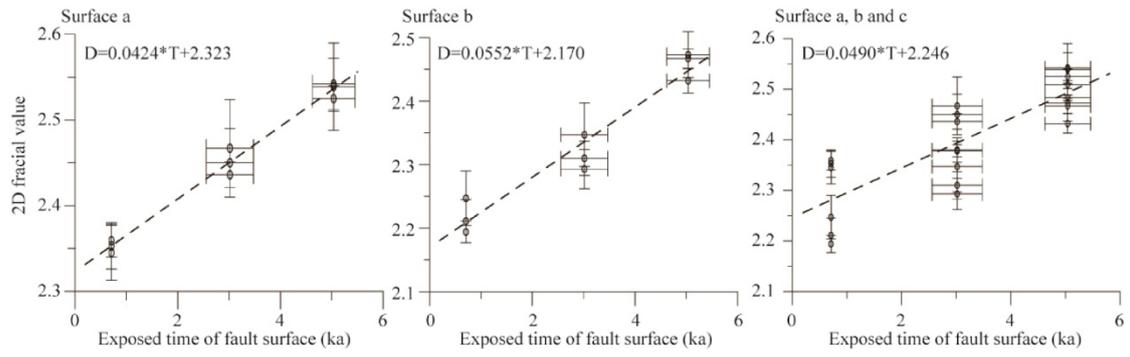
48 Figure 6. Results of determination of weathering bands applying Student's *t*-test.
 49 (a), (b) and (c) show the weathering bands on surface a, surface b and surface c,
 50 respectively, based on the fractal derived by using the moving window of
 51 $64 \times 64 \text{mm}^2$. (d) Shows a simplified sketch of the process using two-sample
 52 Student's *t*-test to quantitatively determine the bands. "g1" and "g2" indicate two
 53 adjacent data sets of n data points, which overlapped each other in a half of data
 54 points ($n/2$), standing for two surface segments. Dots assigned by 0 or 1 on right
 55 plots are the *t*-test result, and detailed description is showed in section 5.1; each
 56 straight segment on the mean fractal curve indicate one weathering band on fault
 57 surface.



58

59 Figure 7 Evolutionary model of fault scarp surface, showing five weathering bands
 60 corresponding to five different exposure times (modified from model of Giaccio et
 61 al, 2002). The two higher bands have conspicuous weathering morphological
 62 feature identified by naked eye easily; while the three lower bands have no
 63 conspicuous weathering morphological feature identified by naked eye, and similar
 64 dip angle as original fault dip angle. The quantitative morphology applied in our
 65 study can identify the three lower bands. The rectangle on the fault scarp surface
 66 shows the scan scope, and the two color rectangles on the left show fractal
 67 dimension and rendering morphology, respectively. The characteristic fractal of the
 68 three lower bands are demonstrated by color bars and scatter diagrams on the
 69 right.

70



71

72 Figure 8 Fitting lines of relationship between 2D fractal dimension and exposure
 73 time. The gray belts show the time spans of paleo-earthquake occurrence. The
 74 three empirical relations, are all fitted based on the characteristic fractal
 75 dimensions derived by using the moving window of $64 \times 64 \text{mm}^2$.

76