In which ways and to what extent do English and Shanghai Students Understand Linear Function?

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Abstract

This study investigates how students in England and Shanghai understand linear function. Understanding is defined theoretically in terms of five hierarchical levels: Dependent Relationship; Connecting Representations; Property Noticing; Object Analysis; and Inventising. A pilot study instrument presented a set of problems to both cohorts, showing that the English cohort generally operated at the levels of Property Noticing and Object Analysis, whereas the Shanghai cohort reached the higher level of Inventising. The main study explored understanding levels and students' errors within each cohort in detail, in order to gain insights into reasons for apparent differences. The instrument used in the main study included two overlapping items, which were the same for both cohorts, while others were pitched at levels of understanding revealed in the pilot. Analysis of students' solutions revealed that the English students' errors were manifested in a lack of basic skills including dealing with negative numbers, while the Shanghai students showed weaknesses in their ability to use graphs. The discussion highlights different views of understanding as a possible background reason for the contrasts observed. Errors and apparent difficulties suggest implications for teaching linear function in each context.

Key words: abstraction, linear function, understanding.

Introduction

In July 2016, the Schools Minister Nick Gibb announced that £41 million of funding would be used to support English primary schools to adopt a 'mastery approach' to teaching mathematics (Department for Education, 2016). The notion of mastery comes from South East Asian countries whose consistent success in the Programme of International Student Assessment (PISA) has been attributed to this method of teaching mathematics (Department for Education, 2014b). Mastery teaching is reported to develop a much deeper understanding of mathematics, procedural fluency and the use of mathematics language (Boylan *et al*., 2016). Implementing the mastery approach has been suggested for all English primary and secondary school classrooms by the National Centre for Excellence in Teaching Mathematics (NCETM). However, Elliott (2014) argues that attempts to emulate the classroom practices of educationally successful areas such as Shanghai are unrealistic, as these so-called effective approaches are rooted in their respective cultures, making them hard to integrate into a Western context. In order to examine how to adapt rather than simply adopt these approaches, it is worth stepping back to examine the existing strengths and weaknesses of students' mathematical understanding in different contexts. There is a lack of research that looks at students' mathematical understanding in the different education contexts of England and Far Eastern countries, one recent exception being Li's (2014) analysis of 12-13 year-old English and Taiwanese students' performance in fraction addition, examining conceptual and procedural knowledge. This study adds to our understanding by exploring how students in Shanghai and England understand linear function and the difficulties that they might face in developing their understanding of this concept further.

Algebra has been regarded as "the most important gatekeeper in mathematics" (Cai, Ng, & Moyer, 2011, p. 26). From the late 1980s, the emphasis of algebra research has moved towards the study of function (Kieran, 2006), the key topic in secondary school mathematics (Brenner *et al*., 1997; Llinares, 2000; Watson, Jones, & Pratt, 2013). Reviewing curricula in England and Shanghai, namely England's Key Stage 4 (KS4) national curriculum (Department for Education, 2014a) and the Shanghai local curriculum (Shanghai City Education Committee, 2004), we found that there are four common types of function covered in the curricula: reciprocal function, linear function, quadratic function and trigonometric function. In this study, we focus on the topic of linear function.

Literature review

Views of understanding in Western and Eastern contexts

Before looking at linear function specifically, we examine how understanding in mathematics is conceptualised in Western and Eastern educational contexts. Learning with understanding has become a crucial issue within the mathematics education community, and numerous mathematics educators have explored definitions of understanding (Hiebert & Carpenter, 1992; Newton, 2000; Sierpinska, 1990; Skemp, 1976). Skemp (1976) defined understanding in terms of two types: instrumental understanding and relational understanding. Instrumental understanding means that one can apply rules, but be unaware of the reason why the rules work. Relational understanding occurs when an individual knows both 'what' and 'why'. Applying these ideas to teaching secondary mathematics, Watson (2003) proposed four forms of understanding based on Skemp's work: instrumental and procedural understanding; contextual understanding; relational understanding; and transformable, generalised and abstract understanding as a higher level of abstraction.

Skemp's work has profoundly influenced Chinese mathematics education (Bao & Zhou, 2009). For example, H. Zhang (2006) proposed three types of understanding: operational understanding; relational understanding; and migratory understanding. The first two are similar to Skemp's instrumental understanding and relational understanding. Migratory understanding refers to the use of existing mathematical methods and ideas in novel situations. Among these different types, not only is there overlap between the Western and Eastern views, but also important differences emerge as well. For example, D. Zhang and Yu (2013) note that in the case of fraction addition, using a visual approach to solve a problem is regarded as understanding in the West, while in China this would not be considered as demonstrating the required level of understanding, due to the longer time spent than using the algebraic approach. D. Zhang and Yu (2013) report an underlying belief among Eastern educationalists that the visual approach facilitates understanding and students should demonstrate understanding without this facility, namely by using approaches that are more abstract. This implies that what counts as understanding differs somewhat between Eastern and Western cultures, advocating visual representations in the West, and abstract symbolic approaches in China.

These differences take place within contrasting education cultures. The dominance of constructivism in Western countries has laid the foundation for a learner-centred teaching approach, in which students construct their knowledge through acculturation and interaction with their teachers and each other. In contrast, Chu and Choi (2011, p. 267) suggest that Chinese culture tends towards a horizontal collectivism where development “focus[es] on close bonding with great influence on attitudes, norms, and behaviours”. This kind of bonding has certainly facilitated whole-classroom instruction within large classes, while a focus on harmony aims to balance opposing views of mathematics, for example “the application of Maths and the formal nature of Maths” (Zheng, 2006, p. 385). A unified syllabus and compulsory textbooks pay great attention to the systematic nature of mathematics and the rigours of knowledge (Xu, 2013). The first aim of mathematical study is to gain the three basics to master the topic: basic knowledge; basic skill; and basic idea and method (Shanghai City Education Committee, 2004, p. 32). That is, learning mathematics includes three aspects: the concept; the skills involved in grasping that concept; and the idea and method linked to other concepts. Thus, while Western mathematics education emphasises students’ thinking, communicating, and their individual learning trajectories, Chinese mathematics education focuses on how to guide students to explore the whole structure of the knowledge. This study probes how these differing views and assumptions of understanding are embodied by students through their different stages of understanding development in the case of linear function.

Views of abstraction

Abstraction describes the processes of emergence which construct students' "new knowledge, taking into account the social, physical, and historical context in which these processes occur" (Dreyfus, 2006, p. 79). Schwarz *et al*. (2009) proposed the Recognizing, Building, and Constructing (RBC) model of abstraction: recognizing previous constructs, building the new construction, and consolidating the new constructions to become part of the knowledge. This theory takes a sociocultural view, where abstraction is considered in the context of mathematical curricula, and their historical and social role. Thus abstraction is not a standard process, but would "strongly depend on the personal history of the participants in the activity of abstraction and on artefacts available to the participants" (Schwarz *et al*. 2009, p. 20). This suggests that the process of abstraction could involve different approaches depending on the context. In this study we drew on this view of abstraction and its implications for how a mathematical topic is presented to students in England and Shanghai in order to design the instruments used in the study.

Sfard (1991) argued that the concept of function has two aspects, operational and structural, in line with the dual nature of mathematical concepts (process and structural). She proposed a three-stage model of concept development: "interiorization, condensation, and reification" (Sfard, 1991, p. 18). In terms of function, engaging with the function machine enables students to acquire variables and formulae via interiorization. At the second stage, students focus on the relationship of input-output rather than actually undertaking the operations. This relationship also contains translations between different representations. These two stages lead to qualitative changes in the last stage, which allow students to probe into certain properties of functions or to solve equations with parameters, referred to as reification.

The two contexts in this study involve differing views of visual versus algebraic understanding. Thus we note here that Breidenbach, Dubinsky, Hawks, and Nichols (1992, p. 279) pointed out that students normally fail to construct processes in their minds for the concept of function and suggested that students should be "de-encapsulating the objects and representing these processes". In terms of process, Schwartz and Yerushalmy (1992, p. 263) argued that symbolic representation could effectively lead students to make sense of the "process" nature of function, while graphical representation would result in the "entity" nature of the function, i.e. the shape. Furthermore, Sfard and Linchevski (1994) argued that function tied the arithmetical processes (primary processes) and formal algebraic manipulations (secondary processes) together, and that both related to relational understanding. Therefore, this study also examines how the different abstract processes towards constructing the concept of function influence students' understanding development.

A general model of understanding function

The development of understanding of function has been modelled in a number of ways proposed by Western and Eastern researchers. Sajka's (2003) model is concerned with the initial conceptualization of function, while models by Hitt (1998), DeMarois and Tall (1996), and Zachariades, Christou, and Papageorgiou (2002) mainly examine how to handle representations. A further model proposed by Ronda (2009) specifically pays attention to one type of representation, the algebraic expression. East Asian researchers Zeng (2002) and Jia (2004) have proposed two models depicting Shanghai secondary school students' cognitive processes. In this study we draw on a general model of understanding function based on these seven models. The general model categorises the growth of students' understanding of linear function into five levels: Dependent Relationship; Connecting Representations; Property Noticing; Object Analysis; and Inventising.

Level 1, Dependent Relationship, involves identifying three main representations for the concept of linear function, namely algebraic expression/equation, graphic representation, and tabular representation. O'Callaghan (1998) provides an example of the development of the dependent relationship in terms of tabular representation, shown in Table 1, which gives the value (V) in dollars of a car in the years (t) after purchase. Students have to find out the dependent relationship between two variables first in order to solve the value of V.

Table 1 An example of Level 1 taken from O'Callaghan (1998)

T	V
0	16800
2	13600
4	10400
6	7200
10	?

Level 2, the Connecting Representations level, involves the ability to translate from one representation to another. For example, when given the algebraic expression $\mathrm{y} = 2\mathrm{x} + 1$, students can draw the graph, which is a straight line, and connect it to a table of ordered pairs such as $(0,1)$ and $(-\frac{1}{2}, 0)$.

At Level 3, the Property Noticing level, students acquire an understanding of properties such as gradient and y-intercept. For example, with the standard form of linear function $\mathrm{y} = \mathrm{ax} + \mathrm{b}$ $(\mathrm{a} \neq 0)$, the y-intercept is the point at which the graph meets the y-axis. At this level, students are required to identify the gradient or y-intercept by rearranging algebraic expressions such as $2x + y + 1 = 0$. In terms of transformation of function, Sfard (1991, p.4) suggested that the transformation, such as symmetry, can be considered as "a static property of geometric form". Therefore, transformation such as translation and symmetry is at this level.

At Level 4, the Object Analysis level, students achieve a structural view of function and regard function as a whole concept. In the case of linear function, students' understanding will move away from looking at coordinates or individual properties, to considering "the entire function", e.g. period (Slavit, 1997, p. 264). For example, students are asked to investigate how the shape of the quadratic graph changes if the values of a, b and c within the algebraic expression of $y = ax^2 + bx + c$ change (Rayner, 2006, p. 379). This requires students to perceive the changing quadratic graphs as a whole rather than a point-to-point view.

At the final Level of Inventising, students have gained a fully structured understanding of function and can link this to other areas of mathematical knowledge. This is illustrated in Figure 1 by an example from the GCSE Mathematics for AQA Higher Student Book (Morrison, Smith, McLean, Horsman, & Asker, 2015, p. 560). Students are asked to work out the equation of a line passing through (2, 3), such that the four lines on the graph form a trapezium with its base passing through the origin.

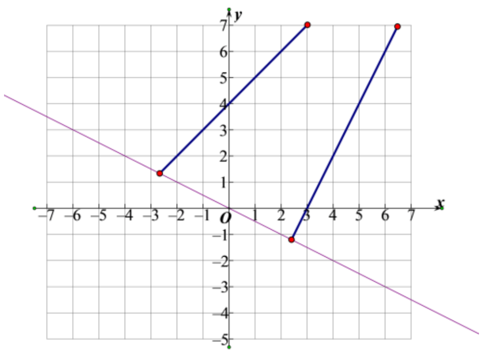

Figure 1. Level 5 example (from Morrison et al., 2015, p. 560)

To conclude, the understanding of linear function starts with identifying the linear relationship between two variables in three ways: algebraic, graphic and tabular. Then, connections are built up among them focusing on two representations: the algebraic expression $y = mx + c$ and the graph plotted in the Cartesian system. Furthermore, the graphic meaning of m and c is regarded as the properties of the linear graph. Once the connections between representations are consolidated further by mastering the properties, understanding moves towards a structural view of linear function and later extends to link with other mathematical knowledge.

Research questions

In this study, we examined the understanding of linear function in English and Shanghai contexts by means of tests based on the above general model of understanding. The pilot study aimed to test the model and establish students' levels of understanding in each context. In the main study, we explored the development of understanding further in each context, probing into the errors shown by each cohort within different levels. Thus our research questions were:

1. Does the general model of understanding function fit with students' understanding development?
2. How well do students understand linear function in the English and Shanghai contexts?
3. What main errors are evident in the understanding levels for each cohort?

Methods

Sample

We considered two main factors in selecting comparable samples of students: the year/grade in which the topic is taught according to the designated regional curriculum, and similar mathematics performance among participating schools (relative to their respective national cohorts). This means that firstly, sampling is content-based in this study instead of grade- or age- focused. Teaching of linear function/graphs takes place in Years 8, 9 and 10 in England and in Grade 8 in Shanghai. Therefore, Year 10 students in England (approx. age 15) were chosen, as they should have learned all of the linear function content required by the KS4 national curriculum. In turn, Grade 8 students in Shanghai (approx. age 14) were selected. It is acknowledged, however, that as a result, there is a one-year age difference between the two samples.

Secondly, the English participants came from three state schools based in the Northeast of England. According to the National Schools League Table for GCSE (The General Certificate of Secondary Education) mathematics in 2012, all three schools were performing within the top 30% level in England. Although there is no league table released in Shanghai, three similar performing (i.e. around the top 30% level) state schools were chosen in accordance with the mathematics league table at district level in the second mock exam test for upper secondary school entrance examination (equivalent to GCSE) in 2012 mathematics. These schools are all located in the Pudong District, the largest district in Shanghai.

In addition, we should note that education systems are influenced by cultural factors (Alexander, 2012), and the focus in England is on individual needs and abilities (Osborn, Broadfoot, McNess, & Raven, 2003). Hence, students are grouped by ability in many subjects; in mathematics, these ability groupings determine whether students are taught at Higher Level or Foundation Level. In contrast, classes in Shanghai are mixed-ability.

Ninety-six English students and 292 Shanghai students from the sample schools participated in a pilot study. Among the 96 English students, 45 students were in the top set of Higher Level students, while 51 students came from the top set of students taking Foundation Level. The sample for the main study included 403 Higher Level students from the English sample schools and 907 students from the Shanghai sample schools. Only Higher Level students were included in the English sample for the main study, due to concerns from the schools involved regarding the difficulty of the tests for the Foundation Level students. It should be borne in mind then that the English sample in the main study relates only to Higher Level students. These were convenience samples in both cases, thus presenting a further restriction, in terms of generalising the findings to all schools in the two regions. Therefore, the intention of this research is to show possible examples of students' understanding of linear function and provide possible suggestions for how the understanding of linear function in each country may be enhanced.

Instrument

Test design

The pilot study. The purpose of the pilot study was to identify the levels of understanding of linear function displayed by the students in each context. The test consisted of nine questions covering all the levels of understanding of linear function from the general model. All of the questions were selected either from standardized tests for GCSE or from the final examinations for Grade 8 in the Pudong District, Shanghai. Appendix I shows the English language version of the nine questions used during the pilot study.

The main study. Based on the pilot study results, the main study tests were designed to probe into the most common errors related to the understanding levels in each context (see the results section of the main study for details). Thus the tests were designed to (1) meet the requirements of each local curriculum instead of just looking at common areas of knowledge as in the pilot study; and (2) fit the format of questions that students are familiar with in each context. Thus, apart from two questions that were the same in both contexts, the remaining questions were not identical in terms of knowledge background, but corresponded to the targeted understanding levels in each context. Furthermore, the curricula and textbooks used in England and Shanghai treat linear graphs differently (Y. Wang *et al*., 2015), and questions were presented in ways which matched how students had learnt the topic. Consequently, one of the two items which were common to both tests, was presented with a graph in the English test (Question 5), and without it in the Shanghai test (Question 2). Thus, with the exception of one question which was identical in both regions (Question 3 in English and Question 1 in

Shanghai test), different questions were applied to the different cohorts in the main study. The aim was to explore in more detail the errors made by each group of students, rather than make a direct comparison between the two cohorts.

For each group, the test featured five questions. Each test used three questions from the pilot study. Table 2 summarises the distribution of questions at each understanding level.

Table 2 Number of questions at each Understanding Level in the main study

	Level 3	Level 4	Level 5
English Higher Level test	3	1	1
Shanghai test	1	2	2

The additional questions (over and above those from the pilot study) came from different sources. In terms of the test for English students, the two additional questions in the main study came from the Higher level textbooks used by the sample schools. The reason for choosing these two new examples was that the types of questions from the corresponding exam-board recommended textbook would resemble their daily class activities. Therefore, students might be more comfortable with the expression of these questions. Appendix II shows the test used for English Higher Level students.

The two additional questions for the Shanghai students were selected from previous assessments instead of textbooks, because the Chinese classroom was normally based on textbooks whose examples or exercises students would be very familiar with. These previous final examinations of Grade 8 pupils were compulsorily used by all state schools in the Pudong District to monitor the progress in the whole district. Question selection was based on two criteria: (1) questions requiring higher levels of understanding (Level 4 and Level 5), and (2) having different mathematics knowledge linked with linear function in terms of Level 5. The reliability and validity of the questions from these previous formal examinations had been previously checked during the usual standardisation process. These examinations, designed by experts, were well regarded by the education authority and schools and considered suitable for meeting the requirements of the curriculum. Appendix III shows the test for Shanghai students in the bilingual language version. We re-emphasise here that the aim in this second stage of the study was not to compare the two cohorts (although on the common questions this was done), but rather to interrogate in more depth the challenges faced by the two different cohorts at different levels of understanding. Thus, different questions were needed for the two cohorts in this second part of the study.

Validity of the tests

Validity is defined as “the extent to which measures and research findings provide accurate representation of the things they are supposed to be describing” (Easterby-Smith, Thorpe, & Jackson, 2012, p. 347). In this study, two types of validity are addressed: construct validity and cultural validity.

Firstly, the theoretical model based on previous research is used to establish levels of students’ understanding. The instruments not only match the curricula in both countries (Y. Wang, 2015), but are also designed to assess particular levels of understanding. Key words were used to establish question levels in a process similar to the corresponding textbook analysis carried out in the larger study of which this is a part (see Y. Wang *et al.*, 2015):

Table 3 Key words at each level

Key words	Understanding Level
Draw	Level 2 Connecting Representations
Intercept, Gradient	Level 3 Property Noticing
Parallel	Level 4 Object Analysis
Linking to other Mathematics areas, e.g. area of triangle	Level 5 Inventising

Cultural validity is a particular concern in cross-cultural research in order to ensure sensitivity to different cultural contexts. Here, cultural factors are embedded in how the instrument fits with both the levels of understanding in each context and their respective curriculum requirements. Firstly, the model of function understanding used to classify students' understanding was developed from models proposed by both Western and Chinese educators. Secondly, the presentation of the topic in the tests was based on worked examples in textbooks studied in the larger project of which the current study is a part. Thirdly, the tests in both the pilot and main study were modified in response to discussion with Heads of Mathematics at the six participant schools, mainly in order to align question wording with normal expressions in each given context.

Test reliability

The pilot study was conducted one academic year before the main study, with the intention of covering the full range of levels of understanding of linear function as modelled above. Cronbach α values for the pilot test (see Appendix I) were 0.85 for the English sample and 0.89 for the Shanghai sample, meeting the criterion of values greater than 0.8 as indicative of reliability (Pallant, 2010). Analysis of the pilot tests indicated what changes were necessary for the main study. As described above, we designed different tests for the English cohort (see Appendix II) and the Shanghai cohort (see Appendix III), with reliability indicated by Cronbach α values of 0.84 and 0.81 for the English Shanghai tests respectively.

Response analysis

Analysis of student performance involved two stages. At the first stage, answers were coded either correct (mark 1) or incorrect (mark 0). Each question including sub-questions in all the tests had a unique correct answer. To receive a score of 1, the answer and the solution process needed to be exactly right. Otherwise, the response was scored as 0 including leaving the answer blank. Thus, students were able to score a maximum of 17 in the pilot test. The main study had a full score of 6 for England and 9 for Shanghai. This generated a percentage correct score for each student in both the pilot and main study tests.

A second stage of analysis in the main study involved categorising the errors in incorrect answers qualitatively. Initially, the incorrect answers were categorised in terms of the similar errors identified. The blank answer was noted as an 'unclear (why no answer)' category. The emerging categories were then reexamined and recombined if deemed reasonable. The analysis finally arrived at 5 categories comprising all of the answers: (1) Correct; (2) Unclear (why no answer); (3) Non-conceptual understanding shown; (4) Partly conceptual understanding, to indicate that students got the part of the understanding right, for example in terms of gradient, students applied $\frac{differences\ betwen\ x}{differences\ between\ y}$ instead of $\frac{differences\ between\ y}{differences\ between\ x}$; and (5) Wrong calculation, to indicate that students understood the concept but due to the incorrect calculation so that they cannot get the right answer. These final categories were then used to calculate the percentages of students making each type of error, thus identifying common categories of errors made by each cohort. In our findings, we present one dominant error in each cohort.

Results

The pilot study

General quantitative results

In general, the Shanghai cohort of students far outperformed the English cohort in the pilot study. Figures 2 and 3 show the distribution of scores for English and Shanghai students. On each graph, the horizontal scale shows the students' scores in the test, and the vertical scale shows the frequency of each score. In the English cohort, the scores were between 1 and 16, while in Shanghai the scores were between 0 and 17. Comparing Figure 2 with Figure 3 reveals that a large percentage of the Shanghai students (41%) achieved full marks of 17.

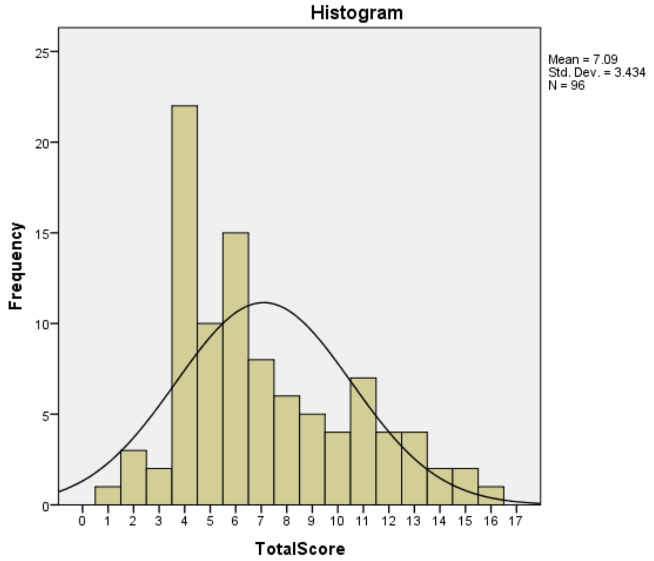

Figure 2. The English students' performance in the pilot test

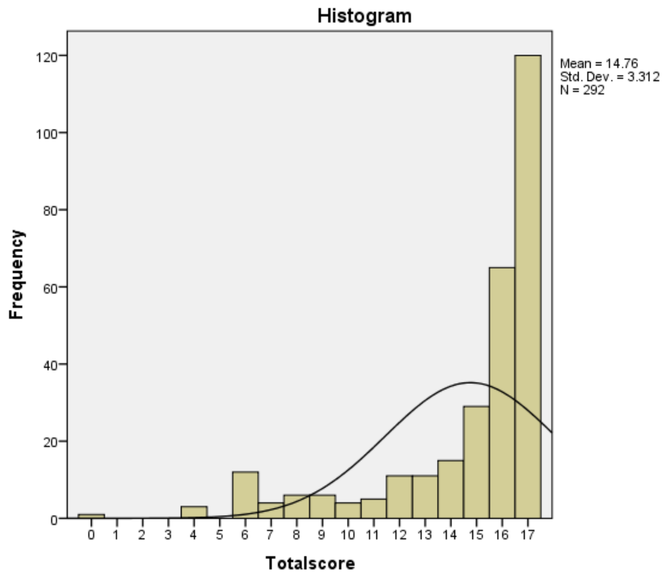

Figure 3. The Shanghai students' performance in the pilot test

The mean score for the Shanghai students (M=14.8, SD=3.3) was much higher than their counterparts in England (M=7.1, SD=3.4). Before checking whether the difference in means was statistically significant, statistical analysis for normality was assessed through the examination of the values of the Kolmogorov-Smirnov (K-S) statistic. For the K-S test a significant result ($\rho < .001$) indicated non-normality. Therefore, the Mann-Whitney U test was used for independent samples. Results revealed that there was a significant difference between England and Shanghai (z (388) = -12.867, p=0.000, two-tailed). The effect size was calculated as $\gamma = 0.65$ which showed a large effect, using Cohen (1988)'s criteria of "0.5 = large effect" (Pallant, 2010, p. 230).

Detailed results

Table 4 summarises the percentage of students who answered each question correctly focusing on a certain level of understanding linear function. In each item, the Shanghai students outperformed the English students. The results also indicated that a majority of the English students were struggling to understand properties such as gradient and intercept (Level 3), while few if any Shanghai students had difficulty with questions at Level 4 or below.

Table 4 Results from the pilot study

Levels of the model	The basic knowledge assessed in each question	Percentage of students answering correctly	
		England	Shanghai
Level 2 Connecting Representations	Question 1a From an algebraic expression to a table	91.7%	96.9%
	Question 1b From tabular to graphic representation	47.9%	79.5%
	Question 2 To generate algebraic expression using two pairs of coordinate (presented by word question)	51%	88.7%
	Average	63.5%	88.4%
Level 3 Property Noticing	Question 3 Intercept from the algebraic expression	20.8%	83.9%
	Question 4a Gradient in a graph (positive)	28.1%	77.7%
	Question 4b Gradient in a graph (negative)	15.6%	76.7%
	Question . 7 Transformation of the graph	34.4%	91.4%
	Average	24.7%	82.4%
Level 4 Object Analysis	Question 5 Parallel and intercept presented in a graphic approach	5.2%	84.2%
	Question 6 Parallel and intercept in an algebraic form	31.3%	91.9%
	Question 8 Parallel and intercept presented in an algebraic form, as intercept has been pointed out	28.1%	94.5%
	Average	21.5%	89.9%

Level 5 Inventising	Question 9 Relating linear function with geometry knowledge	1%	58.2%

At Level 4 in particular, Question 5 and Question 6 used different representations to present the same concepts. The results illustrated that the English students performed very differently when the parallel was presented graphically (No. 5), as opposed to a word problem (No.6). Shanghai students did not show this preference.

Level 3 included two properties, the intercept and gradient. Within the intercept question, only 20.8% of English students could correctly simplify the linear function $y = 2(x-1)+5$ into $y = 2x+3$, and then identify the intercept as 3. Their primary error was classifying 5 as the intercept. The reason might either stem from the students' lack of numeracy skills which would have enabled them to reduce the linear function to the standard form, or that they did not understand the meaning of this property.

In terms of the concept of gradient, the English students performed better with a positive value of gradient than a negative one. The typical English student solution was to draw a right-angle triangle in the graph and then calculate the ratio of the two right-angle sides. This approach conformed to the graphical presentation of gradient in the English textbooks. This solution process, however, differed from their counterparts' method in Shanghai. The Shanghai sample showed that the students achieved far more correct answers for the positive and negative values of gradient using an algebraic method. All of the Shanghai students solved the problem by constructing simultaneous equations, although some of them made a few computing mistakes.

Based on the findings from the pilot study, some modifications to the tests were made to ensure appropriate coverage in terms of levels of difficulty for the students in the main study test. That is, both cohorts would be challenged with questions from Level 3 to Level 5. In terms of the English instrument, the question in Level 5 was modified in line with the type of question and presentation they might be more familiar with. In terms of the Shanghai instrument, Level 4 questions were used to examine the property of increasing and decreasing, which is required by the Shanghai curriculum but not by the English one. The selection of these different questions for the two cohorts considered the width of the curriculum at these understanding levels.

Finally, the validity of the theoretical model of understanding and the questions used were assessed in both contexts. The results shown in Table 4 show a general progression in the levels in terms of difficulty (i.e. percentages of students correctly answering the questions within the levels), except that Level 3 and 4 were difficult to separate for both cohorts with students scoring slightly higher on the Level 4 questions compared to Level 3. This illustrates a potential difficulty with the validity of the questions for these levels, and is an issue that needs to be examined in future research. However, this difficulty did not affect the conclusions drawn for each cohort in the pilot study, as most of the English students' understanding was at Level 2, and almost all the Shanghai students answered the Levels 2, 3 and 4 questions correctly. This issue also did not affect our subsequent focus in the main study on students' errors.

The main study

A related comparative study of textbooks (Y. Wang *et al*., 2015) showed that English textbooks present more graphs to help students understand the topic, suggesting that English students would be more comfortable dealing with questions aided by graphs. Therefore, in the main part of the present study, the questions in the English tests all included a graph, while word problems dominated the Shanghai test. We present the main study results for English Higher Level students (see Table 5) and Shanghai students (see Table 6) separately.

English Higher Level students' understanding

Table 5 shows the percentage distributions of the English Higher Level students' understanding. The English Higher level student test started with the translation from a graphic representation to an algebraic expression. The method used by the English students, however, involved the graphical meaning of gradient rather than the algebraic method to find the value of the gradient. This question was located at Level 3 for the English sample.

Table 5 The English students' performance compared with the pilot study

The model of understanding function	The basic knowledge requirement	Percentage of students answering correctly
Level 3	Question 1 From a graphic representation to an algebraic expression (New question)	44.4%
	Question 2a Gradient (positive)	36.7% (28.1% in pilot)
	Question 2b Gradient(negative)	16.4% (15.6% in pilot)
	Question 5. Transformation	40.4% (34.4% in pilot)
	Average	34.5%
Level 4	Question 3 Parallel and intercept in a word problem	32% (31.3% in pilot)

	Average	32%
Level 5	Question 4 connect to other mathematics knowledge, midpoint (New question)	29%

Looking further at the understanding of gradient, the English students were more successful in identifying a positive gradient in line with the findings of the pilot study. The correct percentage for finding the positive gradient (36.7%, Category 1 Correct) was over double than those discerning the negative one (16.4%). These students who correctly got the positive gradient knew how to calculate the gradient, but were less successful in understanding how to deal with the two conditions: the positive and the negative. In the case of the positive gradient question, four categories of response were found but not the ‘Wrong correction’ Category. 28.6% of students left it blank (Category 2) including one student who commented that they ‘cannot remember how to do it’. The rest of the pupils (34.7%) showed non-conceptual understanding (Category 3) of gradient, with Figure 4 revealing a typical answer. The process and correct answer for gradient BC should be got as $\frac{6-1}{6-4} = 2.5$. In Figure 4, the answer ‘3’ cannot match with operation of the listed coordinates, $(4,1), (5, 3.5), (6,6)$, and the answer of the gradient of AC was left blank. It suggested that the student knew the gradient linked with coordinates, but did not have conceptual understanding of how to. In terms of Category 4 Partly conceptual understanding, 3% of students put the equation for the gradient the opposite way round ($\frac{\Delta x}{\Delta y}$ instead of $\frac{\Delta y}{\Delta x}$).

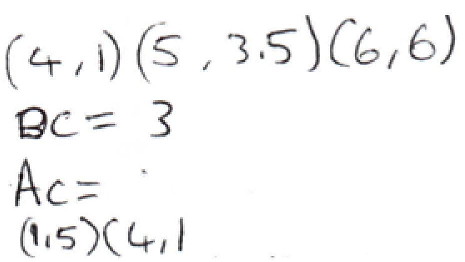

Figure 4. An example showing difficulties in the understanding of gradient

At understanding Level 3, the English students showed their ability to deal with non-routine problems (Question 5). They had not tackled the transformation topic in their school studies yet according to their Heads of Mathematics, but nearly half of the students correctly answered this question. It shows that the students have the ability to reach Level 3 understanding, while they do not master the concept of gradient.

The question at Level 5 (Question 4) required students to make sense of the meaning of midpoint and perpendicular to form the algebraic expression of a new straight line; this question came from their textbook. The results showed that the percentage of students answering correctly rate was 29%, considerably higher than the 1% scoring for the equivalent question in the pilot study, even though it accessed the same level of understanding. It suggests that using students' familiar expressions to design the instrument can get results that are more realistic. Meanwhile, it verified the cultural validity in this study.

In conclusion, one third of the Higher Level students dealt successfully with complex problems and achieved the more abstract levels. The dominant errors shown in the English case was the concept of gradient at Level 3, which was also an important step for successfully solving complex problems at the higher levels of understanding. Once again, as in the pilot study, the students' progression at Level 3 (average 34.5%) and Level 4 (average 32%) could not be clearly distinguished from each other.

Shanghai students' understanding

The Shanghai students demonstrated higher performance in all the common questions. A question related to the global property of monotonicity required by the Shanghai curriculum (Shanghai City Education Committee, 2004) was added to the Shanghai test, and most of the students solved it successfully in the test. Generally speaking the Shanghai students showed few errors in linear function, consistent with the results from the pilot study.

Table 6 reveals their performance in each question. The majority of the Shanghai students could achieve at least an understanding of Level 4 Object Analysis. Almost all of the Shanghai students showed a solid basic understanding in the case of linear function.

Table 6 The Shanghai students' performance compared with the pilot study

The model of understanding function	The basic knowledge required	Percentage of students answering correctly
Level 3	Question.2 Transformation	95.8% (91.4% in pilot)
Level 4	Question 1 Parallel and intercept in word problem	93.2% (84.2% in pilot)
	Question 3 Monotonicity (new question)	88.8%
	Average	91%
Level 5	Question 4 Related with geometry knowledge	60.2% (58.2% in pilot)
	Question 5 Related with algebraic knowledge (new question)	45.4%
	Average	52.8%

Two questions related to Level 5, one linking to the area of a triangle using geometry knowledge, and another linking to the reciprocal function as algebraic knowledge. The students' performance indicated that they had mastered knowledge of linear function itself, but did show some difficulties in linking linear function with other mathematical knowledge, for example the meaning of quadrant and area of triangle in Cartesian system.

In terms of the second question at Level 5, linking with the reciprocal function, the students were not given an existing graph. Most of the students (73.5%) were able to form the correct simultaneous equations in order to calculate points of intersection of the reciprocal function and the linear function. The answer led to two potential coordinates, but with the requirement that the point must be in the third quadrant. Almost all of students who got the wrong answer were unsuccessful only at the last step – picking out the right one between these two points. It is reasonable to assume that none of the students attempted to draw the graph in solving this question, since if they did, they might have been better able to discern the particular coordinates required in the third quadrant. The main errors in their understanding were, therefore, seldom related to the concept of linear function or finding out the intersection for two types of functions. Instead, their primary obstacle was their failure to read the requirements of the question carefully enough, and/or an over-reliance on the algebraic method. This implies that they separated the graphic and symbolic representations, by looking at the coordinates in different quadrants, which were relevant to the Cartesian plane. It also indicated that while the strength of the Shanghai students was their consolidated basic knowledge and procedural understanding, a weakness was a failure to use visual representation or actually draw the graph to help them connect the other knowledge with linear function.

Discussion

In this study, we looked at students' understanding of a particular topic, linear function, in two different educational contexts. Firstly, with respect to Research Question 1, findings from the pilot and main study for both cohorts have shown that the percentages of correct solutions at Level 3 and Level 4 were very close. This may call into question the validity of categorising these two levels, due to the difficulty in separating them. The progress of understanding does not move linearly when it occurs (Newton, 2000), but is spiral (Sierpinska, 1990), or even folding back (Pirie & Kieren, 1994) when building internal representations. Levels of understanding are determined by the quantity of connections from one idea to another, and whether connections are weak or strong (Hiebert & Carpenter, 1992). To verify how separate these two levels needs further research. In the following discussion, we deal with Level 3 and Level 4 together when considering the second and third research questions for the study.

Secondly, with respect to Research Question 2 of how well the Year 10 English Higher Level students and the Grade 8 Shanghai students understood linear function, findings from the pilot study revealed that in general, the English students showed a more varying distribution of understanding levels, while the Shanghai students' performance was more unified at the highest levels of understanding.

Thirdly, with respect to Research Question 3, results from the main study suggested that predominant errors concerned the meaning of gradient, especially involving dealing with negative number for English students, and linking the algebraic method with the graphic representation for Shanghai students (also called the combination of symbolic and graphic, discussed below concerning the holistic view of understanding in China). The essence of Chinese mathematics education emphasises a solid foundation of basic knowledge with proficient numeracy skills before and while learning a new topic (Xu, 2010). The findings from the Shanghai students confirmed their consolidated basic knowledge and skills.

Understanding gradient

In the case of understanding gradient, the Shanghai students showed a more complete understanding of calculating gradients while the English students struggled with it. The underlying reason can be traced back to how students might be taught, since the two cohorts showed contrasting approaches, namely the algebraic approach being used in Shanghai and the graphic approach in England.

The meaning of gradient, especially in the negative case, seemed to be difficult for the English Higher Level students. This may be partly derived from the textbook definition $Gradient = \frac{Differences\ in\ y}{Differences\ in\ x}$, explained as 'along the corridor, and up the stairs'. The meaning of 'differences' does not indicate in which circumstance the gradient would be negative. It was also observed in the broader study from classroom observations in English schools that most of the students went astray due to the meaning of ‘differences’ as interpreted in the rule. For example, two points (-2, 2) and (1, 4) were provided by the teacher to calculate the gradient. The most common incorrect solution for the pupils in the class was $\frac{4-2}{-2-1}$ instead of the correct solution of $\frac{4-2}{1-(-2)}$. The underlying reason may be that students avoid the subtraction of negative numbers.

Algebraic approach and graphic approach in understanding

The particular approaches used at each level suggest that the algebraic method may aid the development of higher levels of understanding in the Shanghai context. The Shanghai students mastered the symbolic method with the generation of simultaneous linear equations to find gradient. This resonates with Li (2014)’s comparative research between Taiwan and England, whose results confirmed the advantages of the symbolic approach used by Taiwanese students in terms of understanding fractions. Preference for the symbolic approach shown in the Shanghai tests helps the Shanghai students achieve the better performance. However, at Level 5, the Shanghai students still relied heavily on the algebraic method in manipulating algebraic expressions, which also turned out to be one of their weaknesses.

In relation to these ideas, it is also worth exploring further the prevailing views towards approaches to understanding mathematics in the two regions. In England, the first overall aim of KS1 to KS4 is to “become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately” (Department for Education, 2013, p. 3). It essentially indicates the importance of conceptual understanding in mathematics. Conceptual understanding refers to “an integrated and functional grasp” of isolated mathematical ideas and methods (Kilpatrick, Swafford, & Findell, 2001, p. 118). Within the English curriculum, the development of understanding mathematics is built on the use of graphical representations. For example, in terms of linear function, both the KS3 and KS4 curricula require students to "find approximate solutions of simultaneous linear equations" by using the graphical representation of linear function (Department for Education, 2013, p. 7; 2014, p. 8). That is, the graphical representation extends the knowledge of linear equation to the new knowledge, such as solving simultaneous linear equations. England's curricula advocate that using visual representations (through the use of graphs) aids the understanding of mathematics knowledge.

Conversely, in the Shanghai case, the different types of equations are the foundations of learning function, developed through algebraic expressions. Here, basic skills, as one of the three basics highlighted in the literature review, consist of (1) calculation, (2) plotting, (3) reasoning; (4) communication including speaking, listening and writing; and (5) data handling including using calculators for the lower secondary school stages (Shanghai City Education Committee, 2004, p. 35). The development of mathematical knowledge means not only the connection of related mathematical concepts, for example between concepts of linear equations $ax + b = 0$ $(a \neq 0)$ and linear function $y = ax + b$ $(a \neq 0)$ through symbolic ways of representation, but also the consistent methods or strategies used among the topics. The development of mathematics understanding towards proficiency or mastery approach is therefore detailed in the Shanghai context.

The argue of mastery or proficiency has been already discussed in the Western context. According to Kilpatrick (2001), Mathematical proficiency has five strands: (1) conceptual understanding; (2) procedural fluency; (3) strategic competence; (4) adaptive reasoning; and (5) productive disposition. The first strand is in line with the views of the English curricula while these five strands also match with the basics in Shanghai. Procedural fluency corresponds to basic skills. Strategic competence and adaptive reasoning are relevant to basic methods, as students have the capability to evaluate the question first, then to identify the appropriate strategy, and finally to defend their solution. The last strand, productive disposition, which describes students' attitudes and beliefs towards and regarding mathematics emphasises individual students' previous experiences in order to shape their own values. The common factor between mathematics proficiency and the basics is that they can be developed together. From this perspective, the view of understanding has broader and deeper elements in Shanghai. It suggests that the Eastern view of understanding, with the explicit emphasis on procedural understanding, might help students attain better mathematical performance, for example in the present case with linear function.

Conclusion

The findings from this study not only provide a better understanding of how students in different educational contexts understand mathematics, but also suggest implications for future practice and studies. Firstly, further research is necessary for how to distinguish Level 4 from Level 3. Secondly, the purely algebraic approach appears to restrict Shanghai students' understanding of the graph, while graphic approach towards gradient did not help students' conceptual understanding. This suggests that teachers could provide both approaches for students to explain the meaning of gradient to enhance their relational understanding with the combination of instrumental understanding. Thirdly, the design of international assessments should consider the cultural validity. In this study, the expression of questions for each cohort heavily influenced the students' performance. The English higher level students demonstrated more understanding at the highest level if the expression of the question was familiar for them. In terms of any assessment of cohorts of students from quite different contexts, the construction of questions should therefore consider ways which are familiar to students using "a wide array of mathematical tasks" as proposed by Cai (1995, p.

106). The further study can be done by exploring other key ideas in Mathematics at secondary level.

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Appendix I The pilot study test

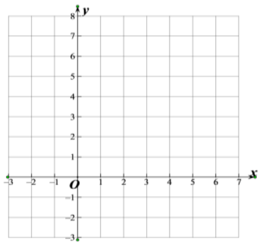
Name: D.O.B. (DD/MM/YY): Circle a or b: a. Male b. Female

1. Complete this table for $y = x + 1$

X	-2		3	6
Y		1		

(4)

On the grid, draw the graph of $y = x + 1$ for x from -2 to 6.



(1)

(Total 5 marks)

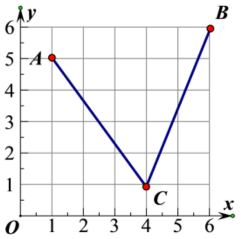
2. A straight line passes through the point (0, 2) and (-2, 0). Find the equation of this line.

(Total 1 mark)

3. Find the intercept of the straight line $y = 2(x - 1) + 5$.

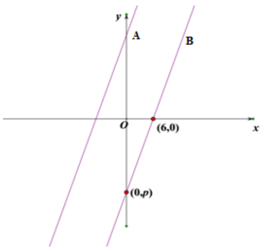
(Total 1 mark)

4. Find the gradients of BC and AC.



(1 for each gradient, Total 2 marks)

5. The diagram shows lines A and B. The equation of the line A is $y = 3x + 5$. The straight line B is parallel to A. Find the value of p.

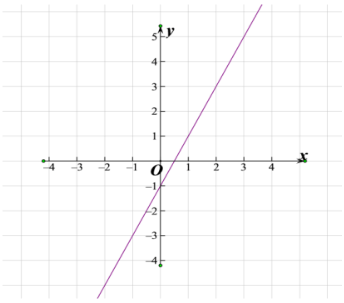


(Total 1 mark)

6. A straight line passes through the point (0, 3) and is parallel to $y = -2x + 1$. Find the equation of this straight line.

(Total 1 mark)

7. A straight line (as seen below) will be translated upward 4 units. Find the equation of the new line.



(Total 1 mark)

8. A straight line is parallel to another straight line $y = 3x + 4$. The intercept of this straight line is 3. Find the equation of this straight line.

(Total 1 mark)

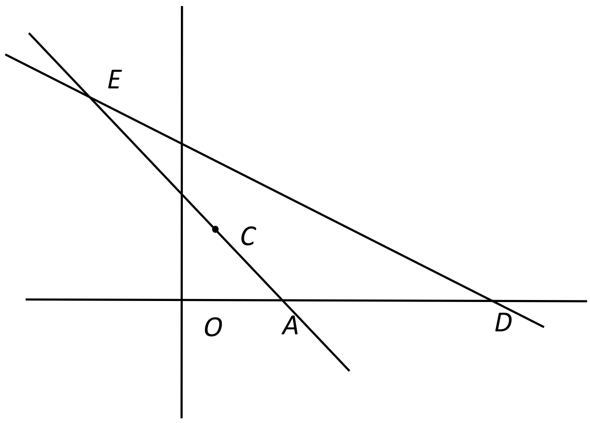
9. A straight line $y = -x + b$ passes through the point C (2, 4) and meets the x-axis at point A. Another straight line DE meets the x-axis at point D (18, 0). The straight lines DE and AC have the point of intersection E. Point E is located at the second quadrant.

1) Find b. (1)

2) Find the coordinate of point A. (1)

3) Find the length of segment DA. (1)

4) If the area of triangle DAE is 72, find the coordinate of point E. (1)

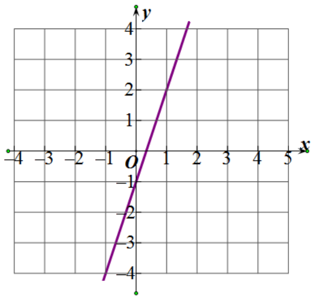


(Total 4 marks)

Appendix II The main study test in England

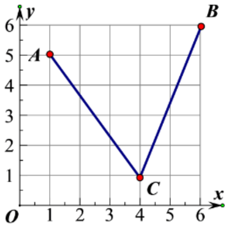
Birthday (DD/MM/YY): **Circle a or b: a Male b Female**

1. Find the equation of the line shown in diagram. Show how you found your answer.



(Total 1 mark)

2. Find the gradients of BC and AC.

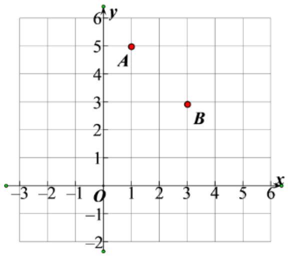


(1 for each gradient, Total 2 marks)

3. A straight line passes through the point (0, 3) and is parallel to $y = -2x + 1$. Find the equation of this straight line.

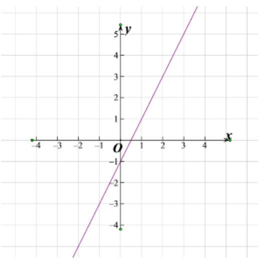
(Total 1 mark)

4. A is the point (1, 5). B is the point (3, 3). Find the equation of the line perpendicular to AB and passing through the midpoint of AB.



(Total 1 mark)

5. A straight line (as seen below) will be translated upward 4 units. Find the equation of the new line.



(Total 1 mark)

Appendix III The main study test in Shanghai

姓名(name):　　　　　　　　班级(class)：　　　　　　　　学号(Enrolled No.)：

性别(Gender)：　　　　　　　　出生年月日(Birth: YY/MM/DD):

1. 一条直线经过点 (0,3) 并且平行于直线 $y = -2x + 1$. 求这条直线的表达式.
(A straight line passes through the point (0, 3) and is parallel to $y = -2x + 1$. Find the equation of this straight line.)

(Total 1 mark)

2. 直线 $y = 2x + 1$向上平移 4 个单位. 求平移后的直线表达式.

(A straight line $y = 2x + 1$ will be translated upward 4 units. Find the equation of the new line.)

(Total 1 mark)

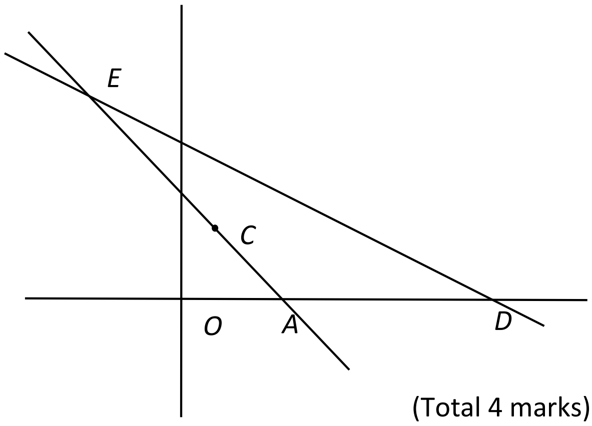
3. 一次函数$y = (k - 1)x + k$中，y随着 x 的增大而减小，求 k 的取值范围.

(Linear function $y = (k - 1)x + k$, when the value of *x* increase, the value of *y* increases as well. Find out the range of *k*.)

(Total 1 mark)

4. 如图，在平面直角坐标系中，直线 AC: $y = -x + b$ 经过点 *C*（2,4），与 *x* 轴相交于点 *A*，直线 *DE* 与 *x* 轴交于点 *D*（18,0），直线 DE 与直线 *AC* 都经过点 *E*，且点 *E* 在第二象限.

（1）求 b; (1)
（2）求点 A 坐标； (1)
（3）求线段 DA 长度； (1)
（4）若△*DAE* 的面积为 72，求点 E 坐标. (1)



(Total 4 marks)

(A straight line $y = -x + b$, passes through the point C (2, 4) and meets the x-axis at point A. Another straight line DE meets the x-axis at point D (18, 0). The straight lines DE and AC have the point of intersection E. Point E is located at the second quadrant.

1) Find *b*.
2) Find the coordinate of point A.
3) Find the length of segment DA.
4) If the area of triangle DAE is 72, find the coordinate of point E.)

5. 已知一次函数 $y = x + 2$与反比例函数 $y = \frac{k}{x}$，其中一次函数 $y = x + 2$的图象经过点 P .

(1) 试确定反比例函数的表达式；(1)

(2) 若点 Q 是上述一次函数与反比例函数图象在第三象限的交点，求点 Q 的坐标．(1)

(Total 2 marks)

(The linear function $y = x + 2$, and the reciprocal function $y = \frac{k}{x}$, the graph of linear function $y = x + 2$ passes by the point P.

(1) Find out the algebraic expression for this reciprocal function;

(2) If the point Q is the intersection of the linear function and reciprocal function at the third quadrant, find out the coordinate of point Q.)