New physics in $\Delta\Gamma_d$

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Abstract

The new measurement of the dimuon asymmetry from the D0 collaboration can be interpreted as a sizeable new physics contributions to the decay rate difference $\Delta\Gamma_d$ of neutral B_d mesons. We investigate model independent bounds on this quantity and find that an enhancement of $\Delta\Gamma_d$ of up to 350% compared to its standard model value is currently not excluded by any measurement. We strongly encourage direct experimental investigations of $\Delta\Gamma_d$.

Keywords: B-mixing, New physics

1. Introduction

The experimental measurement of the dimuon asymmetry A_{sl}^b from the D0 collaboration in 2010 and 2011 [1, 2, 3] triggered an enormous amount of interest. If the value of the measured dimuon asymmetry A_{CP} is interpreted solely as a CP violating effect in mixing of neutral B mesons ($a_{sl}^{d,s}$),

$$A_{CP} \propto A_{sl}^b := C_d a_{sl}^d + C_s a_{sl}^s \tag{1}$$

then the experimental number from 2011 [3] deviates by 3.9σ from the standard model [4, 5, 6] predictions given in [7, 8]. One finds, however, that CP violation in mixing cannot be enhanced by any known model of physics beyond the standard model to an extent to explain the large central value of the dimuon asymmetry; such a large enhancement would also violate model independent bounds, see e.g. [9]. In [10] some new standard model sources for the dimuon asymmetry were investigated, which lead to a new interpretation of the measured quantity A_{CP}:

$$A_{CP} \propto A_{sl}^b + C_{\Gamma_d} \frac{\Delta\Gamma_d}{\Gamma_d} + C_{\Gamma_s} \frac{\Delta\Gamma_s}{\Gamma_s} \; . \tag{2}$$

Now the dimuon asymmetry gets also contributions arising from the interference of B decays with and without mixing, besides the semi leptonic asymmetries in the B_d and the B_s systems. These new contributions are proportional to $\Delta\Gamma_d$ and $\Delta\Gamma_s$, albeit the latter ones turn out to be negligible [10].
Very recently the D0 Collaboration presented a new measurement [11] of the coefficients appearing in Eq.(1) and Eq.(2) and more importantly of the inclusive single muon charge asymmetry a_{CP} and the like-sign dimuon asymmetry A_{CP}

$$a_{CP} = -0.032\% \pm 0.042\% \pm 0.061\% \; , \tag{3}$$
$$A_{CP} = -0.235\% \pm 0.064\% \pm 0.055\% \; . \tag{4}$$

This result differs by 2.7 standard deviations from the standard model predictions given in [8]. Moreover, if one assumes the standard model value for $\Delta\Gamma_d/\Gamma_d$ in Eq.(2), then the new measurement corresponds to the following result for CP violation in mixing

$$A_{sl}^b = -0.496\% \pm 0.153\% \pm 0.072\% \; , \tag{5}$$

which differs by 2.8 standard model prediction in [8]. This number is now considerably smaller than the corresponding value presented in [3]

$$A_{sl}^{b,old} = -0.787\% \pm 0.172\% \pm 0.093\% \; . \tag{6}$$

The reason for that large shift of the central value is the neglect of the sizable interference contribution proportional for $\Delta\Gamma_d$ in the interpretation of A_{CP} in [3], see Eq.(2).
Stronger statements can be obtained from the data in [11], if different regions for the muon impact parameter (denoted by the index i) are investigated separately instead of averaging over them, as done to get the values in Eq.(3) and Eq.(4). Now all values for a_{CP}^i and A_{CP}^i differ by 3.6 standard deviations from the standard model expectation. Moreover it is now possible to also extract individual values for a_{sl}^d, a_{sl}^s and $\Delta\Gamma_d$ from the measurements of the a_{CP}^i and A_{CP}^i. One finds [11]:

- Assuming $\Delta\Gamma_d$ is given by its standard model value, the semi leptonic asymmetries are measured to be

$$a_{sl}^d = -0.62\% \pm 0.42\% \,, \tag{7}$$
$$a_{sl}^s = -0.86\% \pm 0.74\% \,. \tag{8}$$

These values differ by 3.4 standard deviations from the standard model.

- Assuming the semi leptonic asymmetries a_{sl}^d and a_{sl}^s are given by their standard model values, then the decay rate difference $\Delta\Gamma_d$ is measured to be

$$\frac{\Delta\Gamma_d}{\Gamma_d} = +2.63\% \pm 0.66\% \,. \tag{9}$$

These value differs by 3.3 standard deviations from the standard model prediction.

- Making none of the above assumptions one gets

$$a_{sl}^d = -0.62\% \pm 0.43\% \,, \tag{10}$$
$$a_{sl}^s = -0.82\% \pm 0.99\% \,, \tag{11}$$
$$\frac{\Delta\Gamma_d}{\Gamma_d} = +0.50\% \pm 1.38\% \,. \tag{12}$$

These three values differ by 3.0 standard deviations from the standard model.

All in all, the new measurement still [11] sees evidence for deviations from the standard model expectations and part of that deviation could root in an anomalous enhancement of the decay rate difference $\Delta\Gamma_d$. Thus, clearly the question arises to what extent $\Delta\Gamma_d$ can be enhanced by beyond standard model effects, without violating other experimental constraints. Possible new physics contributions to the related quantity $\Delta\Gamma_s$ have already been studied in detail in [12] (see also [13, 14, 15]) and they turned out to be strongly constrained (at most +35% of enhancement is realistic) by different observables. In this work we study the maximal size of new physics effects to $\Delta\Gamma_d$. Previous studies of $\Delta\Gamma_d$ can be found e.g. in [16, 17]. In Section 2 we briefly recapitulate the mixing formalism, set our notation and we collect the experimental values and standard model predictions for the mixing quantities.

In Section 3 we re-investigate in detail the standard model structure of $\Delta\Gamma_d$ and we illustrate the principal differences compared to $\Delta\Gamma_s$. In Section 4 we study new physics contributions to $\Delta\Gamma_d$. First we make some general considerations related to a violation of the unitarity of the 3×3 CKM matrix, next we study new physics effects in the dominant tree-level decays and finally we investigate a new $bd\tau\tau$ transition.
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Finally we conclude in Section 6.

2. Mixing formalism

Mass differences and decay rate differences of neutral B-mesons can be expressed to a very high accuracy as (see e.g. [9] for the corrections, which are of the order of five per mille)

$$\Delta M_q = 2|M_{12}^q| \; , \tag{13}$$

$$\Delta\Gamma_q = 2|\Gamma_{12}^q| \cos(\phi_q) \; , \tag{14}$$

with M_{12}^q being the dispersive part of the box diagrams, see Fig. 1, and Γ_{12}^q being the absorptive part. The mixing phase reads $\phi_q = \arg\left(-M_{12}^q/\Gamma_{12}^q\right)$. The dispersive part M_{12}^q is sensitive to off-shell intermediate states; in the case

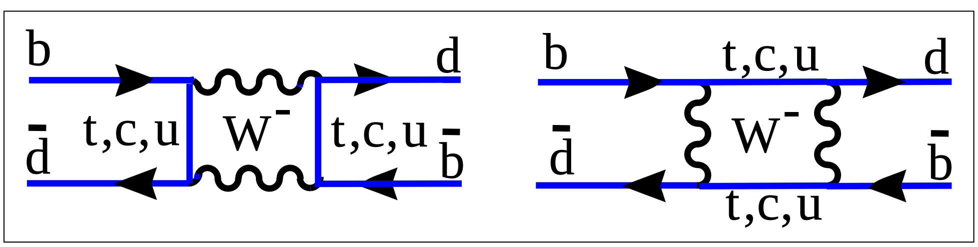

Figure 1: Box diagrams contributing the mixing of B_d-mesons. The diagrams on the l.h.s. can only contribute to M_{12}^q, because the W-bosons are always off-shell, this is also the case for the top-quark contribution of the diagram on the r.h.s. . Contributions to Γ_{12}^q can only arise from the up- and charm-quark on the r.h.s. .

of the neutral B mesons, the largest contribution stems from the virtual top quark in the loop. This part is also very sensitive to hypothetical heavy new physics particles in the loop. Γ_{12}^q is sensitive to on-shell intermediate states; thus only the up- and charm-quark on the r.h.s. of Fig. 1 can contribute. Because of the arising Cabibbo-Kobayashi-Maskawa (CKM) [18, 19] matrix

elements both M_{12}^q and Γ_{12}^q can be complex.
Within the standard model one gets for the mass differences [8, 9]

$$\begin{aligned} \Delta M_d &= 0.543 \pm 0.091 \text{ ps}^{-1} , & (15) \\ \Delta M_s &= 17.3 \pm 2.6 \text{ ps}^{-1} . & (16) \end{aligned}$$

This can be compared with the current experimental numbers taken from HFAG [20] [1].

$$\begin{aligned} \Delta M_d &= 0.510 \pm 0.004 \text{ ps}^{-1} , & (17) \\ \Delta M_s &= 17.69 \pm 0.08 \text{ ps}^{-1} . & (18) \end{aligned}$$

The central values agree very nicely, the experimental errors are, however, much smaller than the theoretical ones, which are dominated by hadronic uncertainties.
The calculation of decay rate differences is more involved, it will be discussed in Section 3. In the standard model one gets the following predictions [8]

$$\begin{aligned} \frac{\Delta\Gamma_d}{\Gamma_d} &= (0.42 \pm 0.08)\% , & (19) \\ \Delta\Gamma_s &= (0.087 \pm 0.021)\text{ps}^{-1} . & (20) \end{aligned}$$

This again can be compared with the current experimental numbers from HFAG [20] [2]

$$\begin{aligned} \frac{\Delta\Gamma_d}{\Gamma_d} &= (1.5 \pm 1.8)\% , & (21) \\ \Delta\Gamma_s &= (0.081 \pm 0.011)\text{ps}^{-1} . & (22) \\ & & (23) \end{aligned}$$

$\Delta\Gamma_s$ agrees perfectly with experiment - deviations are at most at the order of 20%, while in $\Delta\Gamma_d$ still a sizable enhancement cannot be excluded.

[1] Currently the most precise numbers for the mass differences were obtained by the LHCb Collaboration - ΔM_d in [21] and ΔM_s in [22] - the first measurements were done by ARGUS [23] and DELPHI [24] for ΔM_d and by CDF [25] for ΔM_s.

[2] $\Delta\Gamma_s$ was measured by LHCb [26], ATLAS [27], CDF [28] and D0 [29]. The bound on $\Delta\Gamma_d$ was obtained by BaBar [30] and Belle [31] and is now complemented by the value in Eq.(12).

A third class of observables in the mixing system are flavour-specific or semi leptonic CP asymmetries, that describe CP violation in mixing:

$$a_{sl}^q = \left|\frac{\Gamma_{12}^q}{M_{12}^q}\right| \sin\phi_q = \frac{\Delta\Gamma_q}{\Delta M_q}\tan\phi_q \; . \tag{24}$$

The standard model values of these quantities are very small [8]

$$a_{sl}^d = (-4.1 \pm 0.6)\cdot 10^{-4} \; , \tag{25}$$
$$a_{sl}^s = (1.9 \pm 0.3)\cdot 10^{-5} \; . \tag{26}$$

These observables have not been measured yet, there are only bounds available. We do not take the current HFAG [20] value [3] because there the traditional interpretation of the dimuon asymmetry in terms of solely CP violation in mixing was assumed, which seems to be invalid [10]. The semi leptonic asymmetry in the B_d system was studied by BABAR [32] and D0 [33]

$$a_{sl}^d = (+\;6 \pm 17^{+38}_{-23})\cdot 10^{-4} \quad \text{(BABAR)} \; , \tag{27}$$
$$a_{sl}^d = (+68 \pm 45 \pm 14)\cdot 10^{-4} \quad \text{(D0)} \; . \tag{28}$$

In the B_s system it was explored by LHCb [34] and D0 [35]

$$a_{sl}^s = (-\;6 \pm 50 \pm 36)\cdot 10^{-4} \quad \text{(LHCb)} \; , \tag{29}$$
$$a_{sl}^s = (-112 \pm 74 \pm 17)\cdot 10^{-4} \quad \text{(D0)} \; . \tag{30}$$

The numbers for the semileptonic CP asymmetries are now complemented by the values given in Eq.(10) and Eq.(11).
One can also compare the standard model predictions [8] for the mixing phases

$$\phi_d = -4.3^\circ \pm 1.4^\circ = -0.085 \pm 0.025 \; , \tag{31}$$
$$\phi_s = 0.22^\circ \pm 0.06^\circ = 0.0042 \pm 0.0013 \; , \tag{32}$$

with experimental constraints via

$$\phi_q = \arctan\left(a_{sl}^q \frac{\Delta M_q}{\Delta\Gamma_q}\right) \; . \tag{33}$$

[3] $a_{sl}^d = (+7 \pm 27)\cdot 10^{-4}$ and $a_{sl}^s = (-171 \pm 55)\cdot 10^{-4}$.

Currently ϕ_d is not constrained at all[4] from the experimental numbers (because of the large uncertainty in $\Delta\Gamma_d$) and for ϕ_s one gets the following 1-σ range

$$\phi_s \in \left\{ \begin{array}{ll} [-1.04; +0.96] & \text{LHCb} \\ [-1.34; -0.58] & \text{D0} \end{array} \right. . \tag{34}$$

Hence there is still plenty of space in the mixing phases for beyond standard model effects. Following [7] and [36] we parameterise general new physics contribution M_{12}^q and Γ_{12}^q as

$$M_{12}^q = M_{12}^{SM,q} \cdot \Delta_q , \tag{35}$$

$$\Delta_q = |\Delta_q| \cdot e^{i\phi_q^\Delta} , \tag{36}$$

$$\Gamma_{12}^q = \Gamma_{12}^{SM,q} \cdot \tilde{\Delta}_q , \tag{37}$$

$$\tilde{\Delta}_q = |\tilde{\Delta}_q| \cdot e^{-i\tilde{\phi}_q^\Delta} . \tag{38}$$

Thus we get for the observables

$$\Delta M_q = \Delta M_q^{SM} \cdot |\Delta_q| , \tag{39}$$

$$\Delta\Gamma_q = \Delta\Gamma_q^{SM} \cdot |\tilde{\Delta}_q| \cdot \frac{\cos\phi_q}{\cos\phi_q^{SM}} , \tag{40}$$

$$a_{sl}^q = a_{sl}^{SM,q} \cdot \frac{|\tilde{\Delta}_q|}{|\Delta_q|} \cdot \frac{\sin\phi_q}{\sin\phi_q^{SM}} , \tag{41}$$

with the mixing phase

$$\phi_q = \phi_q^{SM} + \phi_q^\Delta + \tilde{\phi}_q^\Delta . \tag{42}$$

As seen above, contributions to ϕ_q from beyond standard model effects, are currently only very weakly constrained if one considers solely $\Delta\Gamma_q$, ΔM_q and a_{sl}^q as observables.

The new physics phase ϕ_q^Δ can also affect other phases that show up in CP violation observables arising from the interference of mixing and decay. Two commonly used notations for these interference phases are β in the decay $B_d \to J/\psi K_S$ and β_s in the decay $B_s \to J/\psi\phi$. Both phases denote the

[4] If one takes the 1-σ range of the D0 value for the semi leptonic asymmetry, then small negative values of ϕ_d are excluded.

ratios of the CKM factors appearing in the $b \to c\bar{c}s$ tree-level decay and in the B_q-mixing amplitude M_{12}^q. We use the definitions from [37] to get the standard model expectations.

$$\beta = \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right] = 25.53^{\circ}{}^{+0.92^\circ}_{-2.10^\circ} = 0.446^{+0.016}_{-0.037}\ , \tag{43}$$

$$\beta_s = \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right] = 1.053^{\circ}{}^{+0.041^\circ}_{-0.037^\circ} = 0.01838^{+0.00072}_{-0.00064}\ . \tag{44}$$

The above definitions are not completely symmetric, β_s has instead been chosen in such way, that it has a positive value in the standard model. Thus $(-\beta_s)$ is the analogue of the well-known angle β from the unitarity triangle. For the numerical values in Eqs. (43),(44) we have used [38], similar results can be found in [39]. Here one should keep in mind that these numbers were obtained under the assumption of a unitary 3×3 CKM matrix; giving up these assumption one can get considerably different values for β and β_s, compared to the ones given in Eq.(43) and Eq.(44).
New physics effects in mixing alter these two phases in the following way

$$2\beta \to 2\beta + \phi_d^\Delta\ , \tag{45}$$

$$-2\beta_s \to -2\beta_s + \phi_s^\Delta\ . \tag{46}$$

As promised, here again the new physics mixing phases ϕ_q^Δ show up. Taking also penguin contributions to the decay $b \to c\bar{c}s$ into account one gets

$$2\beta + \delta_d^{peng,SM} \to 2\beta + \delta_d^{peng,SM} + \phi_d^\Delta + \delta_d^{peng,NP}\ , \tag{47}$$

$$-2\beta_s + \delta_s^{peng,SM} \to -2\beta_s + \delta_s^{peng,SM} + \phi_s^\Delta + \delta_s^{peng,NP}\ . \tag{48}$$

In the standard model penguin contributions are typically expected to be quite small, see e.g. [40, 41, 42, 43] for some related discussions. New physics penguins might, however, give visible effects [36].
HFAG [20] quotes for the directly measured values of β and β_s

$$\beta^{Exp.} = 21.4^\circ \pm 0.8^\circ = 0.374 \pm 0.014\ , \tag{49}$$

$$-2\beta_s^{Exp.} = 2.3^{\circ}{}^{+5.7^\circ}_{-7.4^\circ} = +0.04^{+0.10}_{-0.13}\ . \tag{50}$$

Both numbers are in agreement with the standard model expectations but there is still sizeable room for new physics effects both in the B_d and the

B_s system. For β_s there is even a more precise measurement from LHCb available [26] [5]

$$-2\beta_s^{Exp.} \;=\; 0.6^\circ \pm 4.0^\circ \pm 0.6^\circ = +0.01 \pm 0.07 \pm 0.01 \,, \qquad (51)$$

which is dominated by statistical uncertainties.
Assuming small new physics contributions to tree-level decays one can neglect $\tilde{\phi}_q^\Delta$ as well as $\delta_q^{peng,NP}$. In that case only ϕ_q^Δ appears as a new physics phases in Eq.(42), Eq.(47) and Eq.(48) and its possible value can be strongly constrained, in particular by the measurements of β and β_s. This was the strategy of e.g. [44, 45]. However, in the current work we are exactly interested in the possible size of new physics effects to tree-level decays and thus we do not make the above assumptions.

3. $\Delta\Gamma_d$ within the Standard Model

3.1. Standard model predictions for mixing quantities

Within the framework of the Heavy Quark Expansion (HQE) [46, 47, 48, 49, 50, 51, 52, 53] Γ_{12}^q can be expressed as an expansion in the inverse of the heavy b-quark mass and in the strong coupling:

$$\Gamma_{12}^q \;=\; \left(\frac{\Lambda}{m_b}\right)^3 \left(\Gamma_3^{q,(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{q,(1)} + ...\right) + \left(\frac{\Lambda}{m_b}\right)^4 \left(\Gamma_4^{q,(0)} + ...\right) + \left(\frac{\Lambda}{m_b}\right)^5 \left(\Gamma_5^{q,(0)} + ...\right) + ... \qquad (52)$$

Each term in the above formula with a definite mass dimension, e.g. Γ_3^q, Γ_4^q,... consists of products of perturbative Wilson coefficients and non-perturbative matrix elements of operators with appropriate mass dimensions. The first term, Γ_3, gives rise to dimension six operators, whose matrix elements are parametrised in terms of decay constants and bag parameters, see e.g. the collection of lattice results from the flavour lattice averaging group (FLAG) [54] or some recent determinations in [55, 56] for the decay constants and in [57] for the bag paramters. An update of the QCD-sum rule calculations for the relevant decay constants was given in [58]. The leading perturbative part $\Gamma_3^{q,(0)}$ has already been estimated in [59, 60, 61, 62, 63, 48, 64, 65]. NLO-QCD corrections $\Gamma_3^{q,(1)}$ were calculated in [66] for the case of the B_s-meson and in

[5] In that paper $-2\beta_s^{Exp}$ is denoted as ϕ_s.

[67, 68] for the case of the B_d-meson.
In the sub-leading HQE corrections Γ_4, dimension seven operators appear. Some of them can be rewritten in terms of dimension six operators, others have to be estimated in terms of vacuum insertion approximation. A first step in the non-perturbative determination of the dimension seven operators was given in [69, 70]. The perturbative contributions $\Gamma_4^{q,(0)}$ were first calculated in [71] for the case of the B_s-meson and in [16] for the case of the B_d-meson. Even sub-sub-leading corrections $\Gamma_5^{q,(0)}$ were estimated in [72]. Because of our ignorance concerning the size of the matrix elements of some of the appearing dimension eight operators we will not use these estimates, which resulted in small values.
Including these corrections one gets the following result as a standard model prediction for $\Delta\Gamma_d$. We also show in detail the individual contributions to the theoretical errors in

$$\begin{aligned} \Delta\Gamma_d &= (0.0029 \pm 0.0007)\ \mathrm{ps}^{-1} \\ &= 0.0029\ \mathrm{ps}^{-1} \Big(1 \pm 0.16_{B_{R_2}} \pm 0.14_{f_{B_d}} \pm 0.07_{\gamma} \pm 0.07_{\mu} \pm 0.05_{\tilde{B}_S} \pm 0.04_{B_{R_0}} \\ &\qquad \pm 0.03_{V_{cb}} \pm 0.03_{B} \pm 0.01_{m_b} \pm 0.01_{z} \pm 0.01_{|V_{ub}/V_{cb}|} \\ &\qquad \pm 0.00_{B_{\tilde{R}_3}, \alpha_s, B_{R_1}} \Big) \end{aligned} \tag{53}$$

Within the standard model $\Delta\Gamma_d$ is known with a precision of $\pm 25\%$. The dominant uncertainty comes from matrix elements of dimension 7 operators. The operator denoted by R_2 (see [7] for detailed definitions) gives rise to an uncertainty of $\pm 16\%$. R_0 falls in the same class of operators and it gives an uncertainty of $\pm 4\%$. One should keep in mind that the corresponding bag parameters have been estimated very conservatively by allowing a 50% deviation from the central value of one. Reducing this allowed deviation to 25%, which still looks very reasonable, would reduce also the errors corresponding to R_2 and R_0 by a factor of two. Unfortunately there is currently no non-perturbative determination of these parameters available. A part of a corresponding calculation within QCD sum rules was performed in [69, 70]. Here any progress e.g. a lattice determination or the full QCD sum rule determination would be of outmost importance to reduce the error.
The next-to dominant uncertainty arises from the matrix elements of dimension 6 operators; an overall error of $\pm 14\%$ comes from the decay constant f_{B_d}, $\pm 5\%$ from the bag parameter $\tilde{B}_S$ and $\pm 3\%$ from the bag parameter B. Currently several lattice groups are working on this parameters, so the

corresponding error can be expected to shrink in future.
Number three in the error ranking is the CKM dependence. The uncertainty in γ gives an overall error of $\pm 7\%$, V_{cb} gives $\pm 3\%$ and $|V_{ub}/V_{cb}|$ gives $\pm 1\%$. Here also the errors are shrinking continuously.
Finally the unphysical renormalisation scale (μ) dependence gives rise to an overall uncertainty of about $\pm 7\%$. To reduce this error a NNLO-QCD calculation is necessary. Such an effort would only make sense, if progress on the non-perturbative matrix elements is achieved.
The remaining parametric uncertainties, due to the value of the b-quark mass m_b, of the c-quark mass m_c and of the strong coupling α_s are negligible at the current stage.
All in all the theoretical status of $\Delta\Gamma_d$ is quite advanced and considerable deviations of a future measurement of this quantity from the value in Eq.(53) seems to be a clear indication of new physics. Before investigating the possible space for beyond standard model effects in $\Delta\Gamma_d$, we compare this quantity with $\Delta\Gamma_s$, where the space for new effects is limited.

3.2. Comparison of $\Delta\Gamma_s$ *and* $\Delta\Gamma_d$

The first observation to make is that $\Delta\Gamma_s$ is triggered by the CKM favoured decay $b \to c\bar{c}s$, whose inclusive branching ratios reads $(23.7 \pm 1.3)\%$ [73], while $\Delta\Gamma_d$ is triggered by the CKM suppressed decay $b \to c\bar{c}d$, whose inclusive branching ratios reads $(1.31 \pm 0.07)\%$ [73]. Thus a relative enhancement of $\Gamma(b \to c\bar{c}s)$ by 100% enhances Γ_{tot} by 23.7%, a huge effect, while a relative enhancement of $\Gamma(b \to c\bar{c}d)$ by 100% enhances Γ_{tot} only by 1.31%. Thus a large enhancement of the $b \to c\bar{c}d$ decay rate, will only have a minor effect on the total decay rate and could thus be hidden in the hadronic uncertainties.

Next Γ_{12}^q has three contributions; one with two internal charm quarks (denoted by $\Gamma_{12}^{cc,q}$), one with two internal up quarks (denoted by $\Gamma_{12}^{uu,q}$) and one with an internal up-charm pair (denoted by $\Gamma_{12}^{uc,q}$):

diagrams

$$\Gamma_{12}^q = -\left(\lambda_c^2 \Gamma_{12}^{cc,q} + 2\lambda_c\lambda_u \Gamma_{12}^{uc,q} + \lambda_u^2 \Gamma_{12}^{uu,q}\right) \ , \tag{54}$$

with the CKM structure $\lambda_c = (V_{cq}^* V_{cb})$ and $\lambda_u = (V_{uq}^* V_{ub})$. The minus sign was included to keep the coefficients $\Gamma_{12}^{q_1 q_2, q_3}$ positive. The numerical values - according to the procedure and parameters described in detail in [8] - of the $\Gamma_{12}^{q_1 q_2, q_3}$ read

$$\Gamma_{12}^{uu,d} = 22.5217 \text{ ps}^{-1} , \qquad \Gamma_{12}^{uu,s} = 32.4561 \text{ ps}^{-1} , \tag{55}$$

$$\Gamma_{12}^{cu,d} = 20.7161 \text{ ps}^{-1} , \qquad \Gamma_{12}^{cu,s} = 29.7802 \text{ ps}^{-1} , \tag{56}$$

$$\Gamma_{12}^{cc,d} = 18.8522 \text{ ps}^{-1} . \qquad \Gamma_{12}^{cc,s} = 27.0191 \text{ ps}^{-1} , \tag{57}$$

The results for the B_s mesons are mostly enhanced by the normalisation factor $(f_{B_s} M_{B_s})^2/(f_{B_d} M_{B_d})^2$ compared to the B_d mesons. The three values for the different internal quark content (uu, cu and cc) are quite similar, hence the phase space effects are not very pronounced. In the case of $\Delta\Gamma_s$ almost only $\Gamma_{12}^{cc,s}$ contributes, while in the case of $\Delta\Gamma_d$ a kind of cancellation occurs:

in 10^{-3} ps^{-1}	$-\lambda_c^2 \Gamma_{12}^{cc,q}$	$-2\lambda_c \lambda_u \Gamma_{12}^{uc,q}$	$-\lambda_u^2 \Gamma_{12}^{uu,q}$	Γ_{12}^q
B_d	$1.60 - 0.00i$	$-0.50 + 1.37i$	$-0.25 - 0.21i$	$0.85 + 1.16i$
B_s	$42.81 + 0.00i$	$0.72 - 1.97i$	$-0.02 - 0.02i$	$43.51 - 1.98i$

(58)

This again leads to the fact that a modification in e.g. $b \to c\bar{c}d$ can have a much larger effect in $\Delta\Gamma_d$, compared to the effect of a similar modification in $b \to c\bar{c}s$ in $\Delta\Gamma_s$.

Another way of looking at the mixing systems is the investigation of the ratio Γ_{12}^q / M_{12}^q. In this ratio many of the leading uncertainties cancels, e.g. the factor $(f_{B_q} M_{B_q})^2$, thus one expects - up to different CKM structures - similar results for the B_d and B_s mesons. The three physical mixing observables can be nicely expressed in terms of this ratio:

$$A_{sl}^q = \text{Im}\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) , \tag{59}$$

$$\frac{\Delta\Gamma_q}{\Delta M_q} = -\text{Re}\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) . \tag{60}$$

Moreover the ratio Γ_{12}^q / M_{12}^q can be simplified considerably if the unitarity of the CKM matrix is used, i.e. $\lambda_u + \lambda_c + \lambda_t = 0$

$$-\frac{\Gamma_{12}^q}{M_{12}^q} = \frac{\lambda_c^2 \Gamma_{12}^{cc,q} + 2\lambda_c \lambda_u \Gamma_{12}^{uc,q} + \lambda_u^2 \Gamma_{12}^{uu,q}}{\lambda_t^2 \tilde{M}_{12,q}} \tag{61}$$

$$
\begin{aligned}
&= \frac{\Gamma_{12}^{uu,q}}{\tilde{M}_{12,q}} + 2\frac{\lambda_c}{\lambda_t}\frac{\Gamma_{12}^{uu,q} - \Gamma_{12}^{uc,q}}{\tilde{M}_{12,q}} + \left(\frac{\lambda_c}{\lambda_t}\right)^2 \frac{\Gamma_{12}^{uu,q} - 2\Gamma_{12}^{uc,q} + \Gamma_{12}^{cc,q}}{\tilde{M}_{12,q}} && (62)\\
&= \frac{\Gamma_{12}^{cc,q}}{\tilde{M}_{12,q}} + 2\frac{\lambda_u}{\lambda_t}\frac{\Gamma_{12}^{cc,q} - \Gamma_{12}^{uc,q}}{\tilde{M}_{12,q}} + \left(\frac{\lambda_u}{\lambda_t}\right)^2 \frac{\Gamma_{12}^{cc,q} - 2\Gamma_{12}^{uc,q} + \Gamma_{12}^{uu,q}}{\tilde{M}_{12,q}} && (63)\\
&\approx 10^{-4}\left[(51 \pm 10) - \frac{\lambda_u}{\lambda_t}(10 \pm 2) - \left(\frac{\lambda_u}{\lambda_t}\right)^2 (0.16 \pm 0.03)\right] && (64)
\end{aligned}
$$

The three numerical coefficients in Eq.(64) are almost identical for the B_d and B_s system. The real part of Γ_{12}/M_{12} and thus $\Delta\Gamma_q/\Delta M_q$ is dominated by the first coefficient. An imaginary part can only appear in the two remaining contributions, which are therefore describing the semi leptonic CP asymmetries, whose final sizes are given by the values of the CKM elements. In the B_s system $\lambda_u/\lambda_t = -0.008 + 0.021I$. Hence $\Delta\Gamma_s$ is given to a very good accuracy by the first term only, while a_{sl} is given by the second term and gets a small numerical value. In the B_d system the CKM ratio is larger: $\lambda_u/\lambda_t = -0.033 - 0.439I$. The real part will give a correction to $\Delta\Gamma_d$, while the sizeable imaginary part - it is about a factor of 20 larger than in the B_s-system - gives rise to a semi leptonic CP asymmetry in the B_d sector that is also a factor about 20 larger than the one in the B_s system.

4. New physics effects in $\Delta\Gamma_d$

Next we discuss several model in dependent bounds on hypothetical new physics effects in $\Delta\Gamma_d$.

4.1. Violation of the unitarity of the CKM matrix

To find the maximal size of new physics contributions to $\Delta\Gamma_d$, we start by investigating violations of the unitarity of the CKM matrix, which we parameterise in the following way

$$
\lambda_u + \lambda_c + \lambda_t + \delta_{CKM} = 0 \, . \tag{65}
$$

With that ansatz our expression for Γ_{12}/M_{12} becomes

$$\begin{aligned}
10^4 \frac{\Gamma_{12}^q}{M_{12}^q} \approx & \qquad\qquad (-51 \pm 10) \left(1 + \frac{\delta_{CKM}}{\lambda_t}\right)^2 \\
& + \frac{\lambda_u}{\lambda_t} \quad (10 \pm 2) \left(1 + \frac{\delta_{CKM}}{\lambda_t}\right) \\
& + \left(\frac{\lambda_u}{\lambda_t}\right)^2 (0.16 \pm 0.03) \, .
\end{aligned} \tag{66}$$

To estimate the possible deviations from the standard model expectation we compare the arising CKM factors in the B_d and B_s system.

	B_d	B_s
λ_u	$V_{ud}V_{ub}^* \propto \lambda^{3..4}$	$V_{us}V_{ub}^* \propto \lambda^{4..5}$
λ_c	$V_{cd}V_{cb}^* \propto \lambda^3$	$V_{cs}V_{cb}^* \propto \lambda^2$
λ_t	$V_{td}V_{tb}^* \propto \lambda^3$	$V_{ts}V_{tb}^* \propto \lambda^2$

Fits of flavour observables and electro-weak precision observables within the framework of the standard model with an additional chiral family of fermions (SM4), e.g. [74] have found that $V_{t'd} = \mathcal{O}(\lambda^2)$, $V_{t's} = \mathcal{O}(\lambda^2)$ and $V_{t'b} = \mathcal{O}(\lambda)$ is not yet excluded by any experimental constraint. The observation [75, 76] of the Higgs particle [77, 78, 79] has excluded the possibility of the simple SM4 [80, 81], but one could still think about extended SM4 models, that are not in conflict with the observed Higgs particle. Thus we take the bounds from [74] as a rough estimate for possible violations of the unitarity of the CKM matrix and we get

$$\frac{\delta_{CKM}^d}{\lambda_t^d} = \frac{V_{t'd}V_{t'b}^*}{V_{td}V_{tb}^*} = \mathcal{O}(1) \; , \tag{67}$$

$$\frac{\delta_{CKM}^s}{\lambda_t^s} = \frac{V_{t's}V_{t'b}^*}{V_{ts}V_{tb}^*} = \mathcal{O}(\lambda) \; . \tag{68}$$

Such a violation would lead to an enhancement of the dominant first term in Eq.66) by a factor of 4 for the case of B_d mesons and to an enhancement by a factor of 1.4 for the case of B_s mesons. Depending on the phase of the new contribution this could affect the semi leptonic asymmetries or the ratio $\Delta\Gamma_q/\Delta M_q$. In the latter case one could imagine that ΔM_q is almost unaffected and the enhancement shows up in $\Delta\Gamma_q$ (mostly by a change of the value of the CKM elements V_{cd} and V_{cs}). Thus $\Delta\Gamma_d$ can be enhanced by a factor of about 4, while $\Delta\Gamma_s$ can be enhanced by a factor of about 1.4, which is in agreement with the bounds found in [12].

4.2. NP in tree level decays

Play with $\pm\delta_\Gamma$ deviations in $\Gamma_{cc,uc,uu}$

or

What kind of constraints do we have on the Wilson coefficients $C_{1,2}$?

- *$b \to s\gamma$. The branching ratio is proportional to $|C_7^{eff}|^2$. Comparing current theory predictions and experimental bounds one gets*

$$\delta C_2(M_W) \leq 0.2$$

- *The total inclusive decay rate is largely proportional to $3C_2^2 + 3C_1^2 + 2C_1C_2$. Comparing the total rate with measured lifetimes one finds*

$$\delta C_2(M_W) \leq 0.1$$

- *decay rate differences...*
  $\Gamma_{12}^{old} \propto 3C_1^2 + 2C_1C_2 - C_2^2$
  $\Gamma_{12}^{new} \propto 3/2C_1^2 + C_1C_2 - 3m_c^2/m_b^2C_2^2$
  Looks like C_2 contribution is suppressed!

- *lifetime ratios...*
  Colour allowed $3C_1^2 + 2C_1C_2 + 1/3C_2^2$
  Colour suppressed $C_1^2 + C_2^2$
  For CA again C_2 seems to be suppressed

4.3. New physics effects in $\Delta\Gamma_d$ due to $bd\tau\tau$

Finally we would like to investigate the example of a definite operator, that triggers new physics contributions to $\Delta\Gamma_d$. In that respect we follow closely the work of Bobeth and Haisch [12], who investigated the contributions of $bs\tau\tau$ to $\Delta\Gamma_s$.

We also introduce 10 new operators triggering the $bd\tau\tau$ transition

$$\begin{array}{rclllll} Q_{S,AB} & = & \bar{d} & & P_A b \cdot \bar{\tau} & & P_B \tau \\ Q_{V,AB} & = & \bar{d} & \gamma^\mu & P_A b \cdot \bar{\tau} & \gamma_\mu & P_B \tau \\ Q_{T,A} & = & \bar{d} & \sigma^{\mu\nu} & P_A b \cdot \bar{\tau} & \sigma_{\mu\nu} & P_B \tau \end{array} \tag{69}$$

4.3.1. Direct Constraints

We have the following direct constraints:

- The lifetime ratios of B_s and B_d mesons are expected to be very close to zero [8]

$$\left(\frac{\tau(B_s)}{\tau(B_d)} - 1\right)^{SM} = -0.2\% \pm 0.2\% \; , \tag{70}$$

  which is nicely confirmed by experiment [20]

$$\left(\frac{\tau(B_s)}{\tau(B_d)} - 1\right)^{Exp.} = -0.2\% \pm 0.9\% \; . \tag{71}$$

  Thus one gets the following bound on new physics contributions (Γ_q^{NP}) to the total B_d and B_s decay rates (Γ_q)

$$\frac{\Gamma_d^{NP} - \Gamma_s^{NP}}{\Gamma_d} = 0.0\% \pm 0.9\% \; . \tag{72}$$

  Taking only new physics effects in B_d into accounts one get the following bound for any (also invisible) new physics contribution to B_d decays

$$Br(B_d \to X) < 0.0\% + x \cdot 0.9\% = \begin{cases} 0.9\% & 1-\sigma \\ 1.8\% & 2-\sigma \\ 2.7\% & 3-\sigma \end{cases} \tag{73}$$

  where x denotes the number of standard deviations. We use below the bound $Br(B_d \to X) < 2.5\%$.

- $Br(B_d \to \tau\tau) < 4.1 \cdot 10^{-3}$ BaBar hep-ex/0511015 232M BB - Tim G.: nothing more recent

- $B^+ \to K^+\tau\tau$ PoS ICHEP2010, 234 - still no paper yet!

- No bound for $B^+ \to \pi^+\tau\tau$!

	$Br(B_d \to \tau\tau) < 4.1 \cdot 10^{-3}$	$Br(B_d \to X_d\tau^+\tau^-) < 2.5\%$	$Br(B^+ \to \pi^+\tau^+\tau^-) < 2.5\%$
$\lvert C_S(m_b)\rvert$	< 1.04	< 13.54	< 7.52
$\lvert C_V(m_b)\rvert$	< 2.12	< 6.77	< 7.50
$\lvert C_T(m_b)\rvert$	$-$	< 1.95	< 3.33

(74)

For all this bounds always the dominance of a single new physics operator was assumed. Thus we did not take into account hypothetical cancellations among the new physics contributions. From that we get the following possible enhancements of $\Delta\Gamma_d$

$$\begin{array}{rcl} |\tilde{\Delta}_d|_{S,AB} & < & 1 + (0.41 \pm 0.xx)|C_{S,AB}(m_b)|^2 \approx 1.44 \\ |\tilde{\Delta}_d|_{V,AB} & < & 1 + (0.38 \pm 0.xx)|C_{V,AB}(m_b)|^2 \approx 2.71 \\ |\tilde{\Delta}_d|_{T,AB} & < & 1 + (0.93 \pm 0.xx)|C_{S,AB}(m_b)|^2 \approx 4.5 \end{array} \quad (75)$$

4.3.2. Indirect Constraints

- $Br(b \to d\gamma) = 9.2 \pm 3.0 \cdot 10^{-6}$
- $Br(B_d \to \gamma\gamma) < 3.2 \cdot 10^{-7}$
- $Br(B \to \pi l^+ l^-) < 5.9 \cdot 10^{-8}$
- $Br(B \to \pi e^+ e^-) < 11.0 \cdot 10^{-8}$
- $Br(B \to \pi \mu^+ \mu^-) < 5.0 \cdot 10^{-8}$

	$Br(b \to d\gamma) < 4.1$	$Br(B_d \to \gamma\gamma)$
$\lvert C_S(m_b)\rvert$	$-$	< 3.1
$\lvert C_V(m_b)\rvert$	$-$	< 6.4
$\lvert C_T(m_b)\rvert$	$< 5.8(R)$ $< 2.05(L)$	< 4.2

(76)

4.3.3. Possible size of $\Delta\Gamma_d$

5. Alternatives

What else could affect the dimuon asymmetry? Direct CPV in semi leptonic decays? *Descotes - kamenik* How large can this be?

6. Conclusion

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