Alice and Bob in an Expanding Spacetime

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Abstract

We investigate the teleportation of a qubit from a comoving sender Alice to an inertial receiver Bob in an asymptotically flat Robertson-Walker expanding spacetime. We use scalar or fermionic field modes inside Alice's and Bob's ideal cavities and show the degradation of the teleportation quality, as measured by the fidelity, through a mechanism governed by spacetime expansion. This reduction is demonstrated to increase with the rapidity of the expansion and to be highly sensitive to the coupling of the field to spacetime curvature, becoming considerably stronger as it reduces from conformal to minimal. We explore a perturbative approach in the cosmological parameters to compute the Bogoliubov coefficients in order to evaluate and compare the fidelity degradation of fermionic and scalar fields.

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As quantum technologies progress and cutting-edge experiments incorporate relativistic regimes, studying the effects of gravity and motion on these technologies becomes increasingly more important. Consequently, relativistic aspects of quantum information theory in both flat and curved spacetimes have been extensively studied [1]. A good example is quantum metrology, which uses quantum properties of probes to measure precision observables such as time [2], field strengths [3], gravity waves [4], Unruh temperatures [5], effects of gravity on entanglement using Bose-Einstein condensates [6], Schwarzschild parameters of the Earth [7], etc. Hence there has been an increasing interest in estimating effects of gravity and motion on quantum properties. From the theoretical standpoint, the study of effective models such as quantum field theory on curved spacetime [8] has been revisited with an eye towards both estimating and understanding quantum metrology and quantum information protocols [9].

Quantum teleportation is a fundamental tool for the transmission of quantum information [10],[11] where a sender (Alice) aims to transmit an unknown state to the receiver (Bob) without sending the system itself, which is thus never transported between the two sites during teleportation. However, a small amount of classical information must be telegraphed between the two sites, preventing quantum teleportation from being instantaneous which is in accordance with special relativity. A nice account of relativistic effects on quantum teleportation can be found in [12]. In the seminal article [10], an unknown qubit C is teleported from Alice to Bob using an entangled pair of qubits A and B (Bell state) shared by them. Theg generalization to continuous variable systems such as harmonic oscillators was performed in [13],[14].

Besides the issues of creating and maintaining long distance entanglement, it is a matter of paramount importance in quantum information protocols to understand the interplay between quantum nonlocality and relativistic locality, and spacelike correlations and causality. In [15], the quantum fidelity of a teleported state between two moving cavities in relativistic motion (Alice at rest and Bob uniformly accelerated) was computed in order to study how fidelity is degraded by the Unruh effect. Moreover, the Unruh effect is used in [16] to investigate the teleportation of quantum states when one of the entangled qubits used in the process is under the influence of some external force, which leads to the sudden death of entanglement and the loss of teleportation fidelity. Scalar, fermionic and vector fields in accelerated cavities were discussed in [17].

In the present work we investigate the effect on quantum teleportation of entanglement frame dependence in a general relativistic setting, in the context of an expanding, asymptotically flat Robertson-Walker (RW) spacetime. Assuming that the metric is flat in both the distant past and future, we consider for this purpose the following *gedankenexperiment*: in the distant Minkowskian past, two conformal observers Alice and Bob coincide and share a maximally entangled, EPR Bell state $|\beta\rangle$. It is a Bell state of qubits, encoded in terms of the modes of a real massive scalar field or of a fermionic field. Each observer uses an isolated, non-interacting cavity capable of supporting scalar or fermionic field modes to maintain the qubit in his/her possession. Furthermore, Alice holds an extra cavity with a qubit superposition state that she wants to teleport to Bob within the standard protocol T_0 of Bennett et al. (also called the BBCJPW protocol) [10],

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using the Bell state $|\beta\rangle$ as a quantum channel. Because of the symmetry of (RW) spacetime, Alice and Bob can be defined as conformal observes. Alice remains conformal whereas Bob becomes inertial. If the teleportation protocol is performed when Alice and Bob are in the distant past, then the corresponding fidelity is of course maximal. But instead, assuming that the cavities are capable of supporting the field modes indefinitely, the experiment we describe consists of Alice waiting to perform the teleportation task *only in the distant future, after spacetime expansion saturates.*

We will demonstrate that in this case the fidelity of teleportation reduces from the point of view of Bob as the degree of entanglement of the Bell state $|\beta\rangle$ degrades because of spacetime expansion. We study how these reductions are influenced by the coupling of the scalar field to the scalar curvature of spacetime, and discuss how they are affected by the rapidity of the expansion. We will also extend the results to the case of the teleportation of fermions and explore how the fidelity is affected by the bosonic or fermionic character of the fields.

The period of spacetime expansion affects the degree of entanglement of the quantum channel $|\beta\rangle$ because it implies particle creation: under the above assumptions, Bob perceives Alice receding away from him in his local inertial frame at Hubble speed as the experiment proceeds, and for him a scalar field vaccum state inside Alice's cavity becomes a particle populated, thermal state in the distant future. We proceed to describe and quantify this creation process.

In conformal coordinates the metric reads

$$ds^2 = a^2(\eta)\left(d\eta^2 - dx_1^2 - dx_2^2 - dx_3^2\right)\,, \qquad (1)$$

where a is the scale factor characterizing the expansion and η is conformal time, related to Alice's proper time t by $d\eta = a^{-1}dt$. The dynamics in the spacetime described by Eq. (1) of a real non-interacting scalar field ϕ with mass m coupled to gravity is given by the covariant form of the Klein-Gordon equation

$$(\Box + m^2 + \xi R)\phi = 0, \qquad (2)$$

where ξ is the coupling of the field to the Ricci scalar curvature R and $\Box$ is the covariant D'Alambertian. The cases $\xi = 0$ and $\xi = \frac{1}{6}$ correspond to minimal and conformal coupling to gravity, respectively. Therefore, the field equation (2) in the conformal observer frame $(\eta, \mathbf{x})$ becomes

$$\begin{aligned}\phi''(\eta,\mathbf{x}) + 2\frac{a'(\eta)}{a(\eta)}\partial_\eta\phi(\eta,\mathbf{x}) - \nabla^2\phi(\eta,\mathbf{x})&\\ + a^2(\eta)\left(m^2 + \xi R\right)\phi(\eta,\mathbf{x}) = 0,&\end{aligned} \qquad (3)$$

where the prime denotes derivatives with respect to conformal time. In terms of an auxiliary field variable $\chi(\eta,\mathbf{x}) \equiv a(\eta)\phi(\eta,\mathbf{x})$, the Eq. (3) reduces to

$$\eta^{\mu\nu}\partial_\mu\partial_\nu\chi + a^2(\eta)\left[m^2 + \left(\xi - \frac{1}{6}\right)R(\eta)\right]\chi = 0. \qquad (4)$$

Let as assume a scale factor of the form

$$C(\eta) = a^2(\eta) = 1 + \epsilon(1 + \tanh(\rho\eta)), \qquad (5)$$

where the rate of spacetime expansion is given by the parameter $\rho > 0$ and the total amount of expansion by $\epsilon > 0$. Then $C(\eta)$ conveniently represents a spacetime that undergoes a period of smooth expansion and becomes flat in the distant past ($\eta \to -\infty$) and in the far future ($\eta \to +\infty$), namely

$$\begin{aligned}C(\eta \to -\infty) &\to 1,\\ C(\eta \to +\infty) &\to 1 + 2\epsilon.\end{aligned} \qquad (6)$$

In these two asymptotic regions, the *in region* ($\eta \to -\infty$) and the *out region* ($\eta \to +\infty$), there is a timelike Killing vector field ∂_η such that $\mathcal{L}_{\partial_\eta}u_{\mathbf{k}} = -i\omega_k u_{\mathbf{k}}$ for some $\omega_k > 0$, where $\mathcal{L}$ denotes the Lie derivative. Hence, we can define particle states in each of these regions in terms of positive frequency modes, as well as a vacuum state. Treating $\chi(\eta,\mathbf{x})$ as a field operator, we can express $\chi(\eta,\mathbf{x})$ in the in region (resp., out region) as a combination of positive frequency $u_{\mathbf{k}}^{\text{in}}(\eta,\mathbf{x})$ and negative frequency $u_{\mathbf{k}}^{\text{in}\,*}(\eta,\mathbf{x})$ solutions (resp., $u_{\mathbf{k}}^{\text{out}}(\eta,\mathbf{x})$ and $u_{\mathbf{k}}^{\text{out}\,*}(\eta,\mathbf{x})$) to Eq. (4):

$$\chi^{\text{in}}(\eta,\mathbf{x}) = \int d^3\mathbf{k}\left[a_{\mathbf{k}}^{\text{in}}u_{\mathbf{k}}^{\text{in}}(\eta,\mathbf{x}) + a_{\mathbf{k}}^{\text{in}\,\dagger}u_{\mathbf{k}}^{\text{in}\,*}(\eta,\mathbf{x})\right], \qquad (7)$$

$$\chi^{\text{out}}(\eta,\mathbf{x}) = \int d^3\mathbf{k}\left[a_{\mathbf{k}}^{\text{out}}u_{\mathbf{k}}^{\text{out}}(\eta,\mathbf{x}) + a_{\mathbf{k}}^{\text{out}\,\dagger}u_{\mathbf{k}}^{\text{out}\,*}(\eta,\mathbf{x})\right], \qquad (8)$$

where $a_{\mathbf{k}}^{\text{in}}, a_{\mathbf{k}}^{\text{in}\,\dagger}$ (resp., $a_{\mathbf{k}}^{\text{out}}, a_{\mathbf{k}}^{\text{out}\,\dagger}$) are annihilation/creation operators characterizing the in-vacuum $|0\rangle_{\text{in}}$ (resp. out-vacuum $|0\rangle_{\text{out}}$) state of the field, subject to the usual commutation relations. From Eq. (6) and using that the Ricci scalar $R \to 0$ as $\eta \to \pm\infty$, the mode function solutions to Eq. (4) are

$$u_{\mathbf{k}}^{\text{in}}(\eta,\mathbf{x}) \longrightarrow \frac{1}{(2\pi)^{\frac{3}{2}}\sqrt{2\omega_{\text{in}}}}e^{i(\mathbf{k}\cdot\mathbf{x} - \omega_{\text{in}}\eta)}\,, \qquad (9)$$

$$u_{\mathbf{k}}^{\text{out}}(\eta,\mathbf{x}) \longrightarrow \frac{1}{(2\pi)^{\frac{3}{2}}\sqrt{2\omega_{\text{out}}}}e^{i(\mathbf{k}\cdot\mathbf{x} - \omega_{\text{out}}\eta)}\,, \qquad (10)$$

where

$$\omega_{\text{in}} = \sqrt{k^2 + m^2}\,, \qquad (11)$$

$$\omega_{\text{out}} = \sqrt{k^2 + (1 + 2\epsilon)m^2}\,, \qquad (12)$$

$$\omega_{\pm} = \frac{1}{2}(\omega_{\text{out}} \pm \omega_{\text{in}})\,, \qquad (13)$$

and $k = |\mathbf{k}|$.

The relationship between in and out modes is given by a Bogoliubov transformation

$$a_{\mathbf{k}}^{\text{in}} = \int d^3\mathbf{k} \left[\alpha_k a_{\mathbf{k}}^{\text{out}} - \beta_k a_{-\mathbf{k}}^{\text{out}\,\dagger} \right], \qquad (14)$$

mixing only modes of the same momentum, where the (Bogoliubov) coefficients are determined by the scalar product of mode functions: $\alpha_k = \left(u_{\mathbf{k}}^{\text{out}}, u_{\mathbf{k}}^{\text{in}}\right)$ and $\beta_k = \left(u_{\mathbf{k}}^{\text{out}}, u_{\mathbf{k}}^{\text{in}\,*}\right)$. The corresponding relation between the in and out vacua is

$$|0\rangle_{\text{in}} \equiv \mathcal{N} \exp\left(\tfrac{\beta_k}{\alpha_k} a_{\mathbf{k}}^{\text{out}\,\dagger} a_{-\mathbf{k}}^{\text{out}\,\dagger} \right) |0\rangle_{\text{out}}, \qquad (15)$$

where $\mathcal{N}$ is a normalization factor. It shows that the in-vacuum is a two-mode squeezed out-vacuum state, which implies in particular that the out-region number operator $N_{\mathbf{k}}^{\text{out}} = a_{\mathbf{k}}^{\text{out}\,\dagger} a_{\mathbf{k}}^{\text{out}}$ has a positive in-vacuum expectation value

$$_{\text{in}}\langle 0|N_{\mathbf{k}}^{\text{out}}|0\rangle_{\text{in}} > 0$$

in the Heisenberg picture, which describes the dynamical creation of particles from the vacuum in the conformal frame.

We thus see that, for the inertial observer Bob, an initial vacuum state inside Alice's cavity $|0\rangle_{\text{in}}$ becomes populated with particles in the asymptotic future $|0_\infty\rangle$, which is just the right-hand side of (15) at $\eta \to \infty$. After some algebra in the Eq. (15), we get

$$|0_\infty\rangle = \sqrt{1-\gamma} \sum_{n=0}^{\infty} \gamma^n |n_{\mathbf{k}} n_{-\mathbf{k}}\rangle, \qquad (16)$$

where

$$\gamma \equiv \left| \frac{\beta_k}{\alpha_k} \right|^2. \qquad (17)$$

Similarly, the one-particle excitation in the in-vacuum $|1\rangle_{in}$ evolves as $\eta \to \infty$ into the state $|1_\infty\rangle$ given in terms of out-region Fock states by

$$|1_\infty\rangle = (1-\gamma) \sum_{n=0}^{\infty} \gamma^n \sqrt{n+1} |n+1_{\mathbf{k}} n_{-\mathbf{k}}\rangle. \qquad (18)$$

In what follows, we shall also need the explicit value of γ. To obtain it, begin by noticing that spatial translation invariance allows us to express the mode functions for the field equation in the form

$$u_{\mathbf{k}}(\eta, \mathbf{x}) = (2\pi)^{-\frac{3}{2}} e^{i\mathbf{k}\cdot\mathbf{x}} f_{\mathbf{k}}(\eta), \qquad (19)$$

and thus $f_{\mathbf{k}}(\eta)$ satisfies

$$\left[\eta^{\mu\nu}\partial_\mu\partial_\nu + m^2\right] f_{\mathbf{k}}(\eta) + V(\eta) f_{\mathbf{k}}(\eta) = 0, \qquad (20)$$

where $V(\eta)$ given by

$$V(\eta) \equiv [C(-\infty) - C(\eta)]m^2 - \left(\xi - \frac{1}{6}\right) C(\eta)R(\eta). \quad (21)$$

Following [8],[18], observing that $C(\eta)R(\eta) \to 0$ as $\eta \to \pm\infty$ if $\xi \neq 1/6$ and $C(\eta) \to C(\pm\infty) < \infty$ as $\eta \to \pm\infty$ if $m \neq 0$, we may treat $V(\eta)$ as small to solve Eq.(20) by iteration to the lowest order in $V(\eta)$ in terms of the momentum space propagator. Using (9) as initial condition, the solution $f_{\mathbf{k}}(\eta)$ of (20) is given by the following convolution integral involving the retarded propagator:

$$f_{\mathbf{k}}(\eta) = f_{\mathbf{k}}^{in}(\eta) - \int_{-\infty}^{\infty} G_r(\eta, \eta') V(\eta') f_{\mathbf{k}}(\eta') d\eta', \qquad (22)$$

which satisfies

$$\left[\eta^{\mu\nu}\partial_\mu\partial_\nu + m^2\right] G_r(\eta, \eta') = \delta(\eta - \eta'), \qquad (23)$$

and, in momentum space, reads

$$G_r(\eta, \eta') = \frac{1}{2\pi} \int \frac{e^{-ik_0'(\eta'-\eta)}}{k_0'^2 - \omega_k^2 - i\varepsilon k_0'} dk_0'. \qquad (24)$$

The momentum integral can be performed by closing the integration contour in the upper-half complex momentum plane. However, for the calculation of the Bogoliubov coefficients, it suffices to notice that in the limit $\eta \longrightarrow +\infty$, $f_{\mathbf{k}}(\eta)$ can be written in terms of the in-mode functions as

$$f_{\mathbf{k}}(\eta) \longrightarrow (2\omega)^{-\frac{1}{2}} \left[\alpha_k e^{-i\omega\eta} + \beta_k e^{i\omega\eta}\right], \qquad (25)$$

where $\omega \equiv \omega_{\text{in}}$. Taking (25) in (22) yields the following expressions for the Bogoliubov coefficients:

$$\alpha_k = 1 + \frac{i}{2\omega} \int_{-\infty}^{\infty} e^{i\omega\eta'} V(\eta') f_{\mathbf{k}}(\eta') d\eta',$$
$$\beta_k = -\frac{i}{2\omega} \int_{-\infty}^{\infty} e^{-i\omega\eta'} V(\eta') f_{\mathbf{k}}(\eta') d\eta'. \qquad (26)$$

Assuming the conditions justify a perturbative solution, Eq. (26) allows us to calculate α_k and β_k to lowest order in $V(\eta)$. To leading order $f_{\mathbf{k}}(\eta) \cong f_{\mathbf{k}}^{\text{in}}(\eta)$, which together with Eq. (21) gives

$$\beta_k = \frac{i}{2\omega} \int_{-\infty}^{\infty} e^{-2i\omega\eta'} \Big[m^2\epsilon(1 + \tanh(\rho\eta')) + \left(\xi - \frac{1}{6}\right) C(\eta)R(\eta) \Big] d\eta'. \qquad (27)$$

The integrals in (27) can be easily evaluated in the complex plane by replacing the real variable η by the complex variable z and integrating around a closed counterclockwise rectangular contour Σ with vertices at $-\Lambda$, Λ, $\Lambda+\frac{i\pi}{\rho}$, $-\Lambda+\frac{i\pi}{\rho}$. In the upper horizontal path we use that $\tanh\rho\eta = \tanh\rho\left(\eta+\frac{i\pi}{\rho}\right)$. The two vertical paths parallel to the imaginary axis vanish exponentially as $\Lambda\to\infty$. For the first term in Eq. (27), we have

$$I_1 = \frac{im^2\epsilon}{2\omega}\frac{1}{1-e^{2\pi\omega}}\oint_\Sigma dz e^{-2i\omega z}(1+\tanh(\rho z)), \quad (28)$$

whose integrand has a pole at $z=\frac{i\pi}{2\rho}$. Using Cauchy's integral formula we find

$$I_1 = \frac{m^2\epsilon}{2\omega\rho}\frac{\pi}{\sinh(\frac{\pi\omega}{\rho})}. \quad (29)$$

The second term in Eq. (27) can be evaluated in a similar way. The resulting expression for β_k is:

$$\beta_k \approx \left[\frac{m^2\epsilon}{2\omega\rho} - (6\xi-1)\frac{\omega\epsilon}{\rho}\right]\frac{\pi}{\sinh(\frac{\pi\omega}{\rho})}. \quad (30)$$

The calculation of the Bogoliubov coefficient α_k is analogous and to this order $\alpha_k\approx 1$. Therefore

$$\tfrac{\beta_k}{\alpha_k} \approx \left[\frac{m^2\epsilon}{2\omega\rho} - (6\xi-1)\frac{\omega\epsilon}{\rho}\right]\frac{\pi}{\sinh(\frac{\pi\omega}{\rho})} + O(\epsilon^2), \quad (31)$$

which is the desired value of γ in Eq.(17).

We can now get back to our teleportation experiment and examine it in detail. Suppose that Alice's and Bob's cavities support the orthogonal modes $\mathbf{k}_1$, $\mathbf{k}_2$ of the same frequency given by $|\mathbf{k}_1| = |\mathbf{k}_2| \equiv k$ of the scalar field ϕ, labeled respectively as A_i and B_i, $i=1,2$. Assume that the Bell state $|\beta\rangle$ shared by Alice and Bob in the distant past is the maximally entangled, EPR Bell state

$$|\beta\rangle = \frac{1}{\sqrt{2}}\left(|\mathbf{0}_A\rangle_{\rm in}|\mathbf{0}_B\rangle + |\mathbf{1}_A\rangle_{\rm in}|\mathbf{1}_B\rangle\right). \quad (32)$$

The states $|\mathbf{0}_A\rangle$ and $|\mathbf{1}_A\rangle$ are defined in terms of the dual-rail basis as suggested in [15], $|\mathbf{0}_A\rangle = |1_{A_1}\rangle|0_{A_2}\rangle$, $|\mathbf{1}_A\rangle = |0_{A_1}\rangle|1_{A_2}\rangle$, with similar expression for Bob's cavity (for the inertial observer Bob, the in and out modes are the same and the subscript "in" will henceforth be omitted). Then, it follows from the previous considerations that in the distant future, after spacetime expansion saturates, the state in Alice and Bob's possession will no longer be $|\beta\rangle$; instead, it will be the mixed state

$$\hat{\tau} \equiv {\rm Tr}_{-B_1,-B_2}\left(|\beta_\infty\rangle\langle\beta_\infty|\right) \quad (33)$$

which we obtain by tracing the negative momenta $-\mathbf{k}_1, -\mathbf{k}_2$ degrees of freedom out of Bob's party in $|\beta_\infty\rangle\langle\beta_\infty|$, where

$$|\beta_\infty\rangle = \frac{1}{\sqrt{2}}\left(|\mathbf{0}_A\rangle_\infty|\mathbf{0}_B\rangle + |\mathbf{1}_A\rangle_\infty|\mathbf{1}_B\rangle\right). \quad (34)$$

Here, we defined $|\mathbf{0}_A\rangle_\infty = |1_{A_1}\rangle_\infty|0_{A_2}\rangle_\infty$ and $|\mathbf{1}_A\rangle_\infty = |0_{A_1}\rangle_\infty|1_{A_2}\rangle_\infty$, following the notation introduced in Eqs. (16), (18).

The mixed state $\hat{\tau}$ is still an entangled state and may be used to perform the teleportation of a qubit superposition state from Alice to Bob. However, it is no longer a maximally entangled state. In fact, a direct computation shows that the logarithmic negativity of $\hat{\tau}$ is given in terms of γ by

$$\mathrm{E}_{\mathcal{N}}(\hat{\tau}) = \log_2\left(1+(1-\gamma)^3\left[\sum_{j=0}^{\infty}\gamma^{2j}\sqrt{j+1}\right]^2\right), \quad (35)$$

which attains the upper bound $\mathrm{E}_{\mathcal{N}}=1$ only in the limit $\gamma=0$ when there is no particle creation inside Alice's cavity due to spacetime expansion.

This reduction of the degree of entanglement of the quantum channel results in degradation of the fidelity of teleportation. In order to compute the fidelity, assume that Alice has an additional cavity containing the following qubit superposition state:

$$|\psi\rangle = a|\mathbf{0}\rangle + b|\mathbf{1}\rangle. \quad (36)$$

This state is an unknown to Bob. The full input state is $|\psi_0\rangle_{\rm in} = |\psi\rangle_{\rm in}|\beta\rangle$. If Alice made a Bell measurement on her two qubits *in the distant past* with the result $|\mathbf{i}\rangle\otimes|\mathbf{j}\rangle$, $i,j\in\{0,1\}$, then after Alice's measurement the full state would be projected into

$$|\psi\rangle_{\rm in} = |\mathbf{i}\rangle\otimes|\mathbf{j}\rangle\otimes|\phi_{i,j}\rangle_{\rm in}, \quad (37)$$

where the final state received by Bob would be

$$|\phi_{i,j}\rangle_{\rm in} \equiv x_{ij}|\mathbf{0}\rangle_{\rm in} + y_{ij}|\mathbf{1}\rangle_{\rm in}$$

with four possible conditional state amplitudes given by $(x_{00},y_{00})=(a,b)$, $(x_{01},y_{01})=(b,a)$, $(x_{10},y_{10})=(a,-b)$, $(x_{11},y_{11})=(-b,a)$. However, when Alice makes the Bell measurement in the distant future, the final state received by Bob would be obtained by tracing the $-\mathbf{k}_1,-\mathbf{k}_2$ degrees of freedom out of

$$|\phi_{i,j}\rangle_{\infty\infty}\langle\phi_{i,j}|,$$

where $|\phi_{i,j}\rangle_\infty = x_{ij}|\mathbf{0}\rangle_\infty + y_{ij}|\mathbf{1}\rangle_\infty$. Explicitly, this state is

$$
\begin{aligned}
\rho_{ij}^{\mathbf{k}_1,\mathbf{k}_2} &= \mathrm{Tr}_{-\mathbf{k}_1,-\mathbf{k}_2}\left(|\phi_{i,j}\rangle_{\infty\infty}\langle\phi_{i,j}|\right) = \sum_{n=0}^{\infty} p_n \rho_{ij,n}^{\mathbf{k}_1,\mathbf{k}_2} \\
&= (1-\gamma)^3 \sum_{n=0}^{\infty}\sum_{m=0}^{n} \Big[\gamma^{n-1}\left[(n-m)|x_{ij}|^2 + m|y_{ij}|^2\right] \\
&\times |m,n-m\rangle\langle m,n-m| + \Big(x_{ij}y_{ij}^{*}\gamma^{n}\sqrt{n-m-1} \\
&\times \sqrt{m+1}|m,n-m+1\rangle\langle m+1,n-m| + h.c.\Big)\Big],
\end{aligned} \tag{38}
$$

where

$$
p_0 = 0, \quad p_1 = (1-\gamma)^3, \quad p_n = (1-\gamma)^3\gamma^{n-1}. \tag{39}
$$

The teleportation will be complete only when Bob receives from Alice the classical information $\{i,j\}$ concerning the result of her measurement and applies a unitary transformation to verify the protocol in his local frame. Notice that this is feasible, since the form of our scale factor guarantees that Alice and Bob will remain causally connected as $\eta \to \infty$ (see Fig. 1). From Eq. (38), we obtain the fidelity corresponding to the teleportation

$$
F \equiv \mathrm{Tr}\left(|\psi\rangle\langle\psi|\rho^{\mathbf{k}_1,\mathbf{k}_2}\right) = (1-\gamma)^3 . \tag{40}
$$

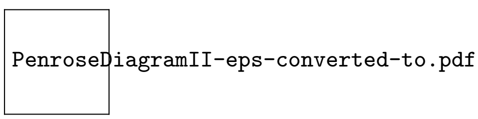

FIG. 1. The scale factor determined by Eq. (5) is such that for every ϵ, ρ, $\ddot{a} < 0$ after a finite η (or t). Thus, although Alice is receding from Bob at Hubble speed, their physical separation will be within the Hubble radius in the distant future. In terms of a Penrose diagram for Bob, we may assume that Alice's worldline will be seen as depicted here.

This result, together with Eq. (31), allows us to determine the influence of the spacetime expansion on the protocol. It suggests that even a small disturbance in conformal symmetry results in degradation of the quality of the teleportation. In fact, it is only under conformal symmetry that there is no particle creation due to spacetime expansion and no response on the logarithmic negativity of the quantum teleportation channel and on the fidelity F. A numerical study of (40) shows that the fidelity of teleportation always degrades due to conformal symmetry breaking (non-zero mass or/and nonconformal coupling).

Moreover for $m \neq 0$, as the coefficient ξ increases from minimal to conformal, the fidelity increases. Concerning the role played by the parameters of the expansion, we find that the rapidity ρ of the expansion leads to a reduction of the fidelity, but only up to a threshold. If the expansion is too fast (large ρ), the fidelity and logarithmic negativity are not affected. Our analysis is summarized in Fig. 2. Because the creation of high-mass and high k modes particles demand a larger amount of energy from spacetime expansion, only low modes significantly contribute to fidelity degradation as shown in Fig. 3.

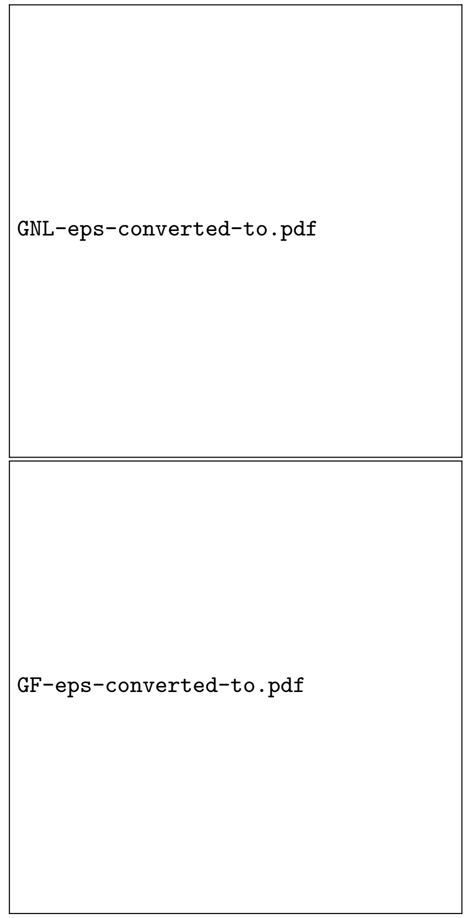

FIG. 2. The logarithmic negativity (top) and the fidelity (bottom) as a function of the expansion parameter ρ for the bosonic field case with minimal coupling (solid line) and conformal coupling (dot line). The dashed line shows the fidelity for the fermionic case. We have fixed $|k| = \mathrm{m} = 1$ and $\epsilon = 0.7$.

Let us extend our considerations to the teleportation of fermionic modes to assess the effect of the Grassman character of the field on the teleportation fidelity. For this purpose consider a Dirac spinless field of mass m. In this case, it can be shown [19] that the relation between the in- and out-region vacua is

$$
|0\rangle_{\mathrm{in}} \equiv \overline{\mathcal{N}} \exp\left(\int d^3\mathbf{k}\, \frac{\beta_k^{(-)*}}{\alpha_k^{(-)*}} X(k) a_{\mathbf{k}}^{\mathrm{out}\,\dagger} b_{-\mathbf{k}}^{\mathrm{out}\,\dagger}\right)|0\rangle_{\mathrm{out}}, \tag{41}
$$

where $\overline{\mathcal{N}}$ is a normalization constant, $a^\dagger, b^\dagger$ denote creation operators for particles and antiparticles (resp.), and $X(k)$ is the polarization tensor given by

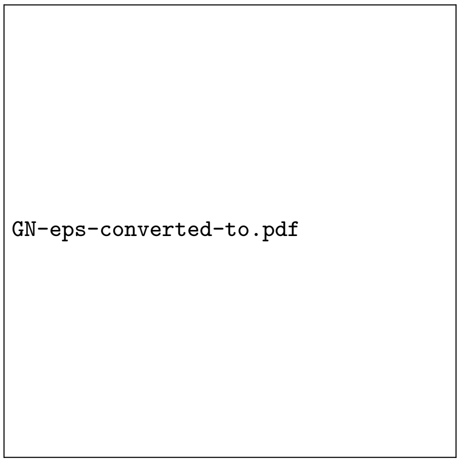

FIG. 3. Fidelity as a function of k for $m = \rho = 1$ and $\epsilon = 0.7$. Dashed line represents the fermionic case, whereas the dot and solid line represent the conformal and minimal coupling, respectively, for the scalar case. Notice that only low modes of the created particles contribute significantly for fidelity degradation

$$X(k) = \frac{\sqrt{1+2\epsilon}\,m}{k}\left(1 - \frac{\omega_{\text{out}}}{\sqrt{1+2\epsilon}\,m}\right). \tag{42}$$

The Bogoliubov coefficients can also be found in a perturbative approach. The result is identical to the scalar case save that $V(\eta)$ is substituted with

$$V_f(\eta) \equiv [C(-\infty) - C(\eta)]m^2 - im\frac{C'(\eta)}{\sqrt{C(\eta)}}. \tag{43}$$

Following the notation used in [19],[20] we redefine $\alpha_k \to \alpha_k^{(-)}$ and $\beta_k \to \beta_k^{(-)}$. The resulting expression for $\frac{\beta_k^{(-)}}{\alpha_k^{(-)}}$ is:

$$\frac{\beta_k^{(-)}}{\alpha_k^{(-)}} \approx \left(\frac{m^2\epsilon}{\omega\rho} - \frac{m\epsilon}{\rho}\right)\frac{\pi}{\sinh(\frac{\pi\omega}{\rho})}. \tag{44}$$

This result coincides to leading order in ϵ in which $\omega_+ = \omega + \frac{m^2\epsilon}{\omega}$ and $\omega_- = \frac{m^2\epsilon}{\omega}$, with γ^- obtained in [20]. After some algebra in (41), we verify that $|0\rangle_{\text{in}}$ evolves in the asymptotic future into the following state in terms of particle and antiparticle modes:

$$|0\rangle_\infty = \overline{\mathcal{N}}\prod_{\mathbf{k}}\left(|0\rangle - \gamma^{-*}|1_{\mathbf{k}}1_{-\mathbf{k}}\rangle\right). \tag{45}$$

Notice that due to the Pauli exclusion principle the number of fermionic excitations is limited to $\{1, 0\}$. Similarly, the 1-particle excitation state in the in-region evolves into

$$|1\rangle_\infty = |1_{\mathbf{k}}0_{-\mathbf{k}}\rangle. \tag{46}$$

Our teleportation experiment can be considered along the same lines as for the scalar field case. A dual rail basis state can be interpreted in terms of particle excitations in one of two possible spatial modes in Alice's or Bob's cavity. Assuming they shared a Bell state of the same form as in Eq. (32) in the distant past, the entangled state $\hat{\tau}$ available to perform the teleportation at $\eta \to \infty$ would result from tracing out the antiparticle degrees of freedom in Alice's part. It would have the same form shown in Eqs. (33), (34) but with the expressions (45) and (46) for the evolved Fock states. The resulting logarithmic negativity and fidelity of teleportation are found to be:

$$\mathrm{E}_{\mathcal{N}} = \log_2\left(1 + \frac{1}{1+|\gamma^- X(k)|^2}\right), \tag{47}$$
$$F = \frac{1}{1+|\gamma^- X(k)|^2}.$$

The numerical analysis of the expression shows that the reduction of the fidelity in the present case is qualitatively similar to the conformally coupled scalar field case. This is expected since the Dirac equation in curved spacetime can be written as a Klein-Gordon-like equation with a non-minimal coupling to gravity and covariant derivatives. This suggests that when it is the mass of the field which is responsible for the conformal symmetry breaking, the fermionic character of the field assumes an important role on how much the expansion influences the quality of quantum teleportation.

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