

QCD radiative corrections for $h \rightarrow b\bar{b}$ in the standard model dimension-6 effective field theory

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We calculate the $\mathcal{O}(\alpha_s)$ QCD corrections to the inclusive $h \rightarrow b\bar{b}$ decay rate in the dimension-6 standard model effective field theory (SMEFT). The QCD corrections multiplying the dimension-6 Wilson coefficients which alter the $hb\bar{b}$ -vertex at tree-level are proportional to the standard model (SM) ones, so next-to-leading order results can be obtained through a simple rescaling of the tree-level decay rate. On the other hand, contributions from the operators Q_{bG} and Q_{HG} , which alter the $gb\bar{b}$ -vertex and introduce a hgg -vertex respectively, enter at $\mathcal{O}(\alpha_s)$ and induce sizeable corrections which are unrelated to the SM ones and cannot be anticipated through a renormalization-group analysis. We present compact analytic results for these contributions, which we recommend to be included in future phenomenological studies.

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I. INTRODUCTION

The precise determination of the properties of the Higgs-like particle discovered during Run-I of the LHC [1,2] is a main focus of the Run-II physics program. Within the current experimental precision, the coupling strengths of this particle appear to be consistent with the standard model (SM) Higgs boson [3–7]. With both theoretical and experimental developments, a significant improvement in the precision of the determination of the Higgs couplings is expected at the LHC during the high luminosity (HL) phase [8].

The determination of the Higgs couplings which enter both production and decay processes is achieved by performing a (global) fit to $\sigma \cdot \text{BR}(H \rightarrow X)$ data of the observable final states. This is necessary since a direct measurement of the SM Higgs boson width is not feasible at the LHC (or at proposed future colliders) since the expected value of ≈ 4 MeV [9] is orders of magnitude smaller than the mass resolution of the detectors. The importance of a precision measurement of the $h \rightarrow b\bar{b}$ partial width cannot be overstated, since this constitutes the dominant decay channel for the SM Higgs boson ($\approx 60\%$ [9]). Consequently, a modification of the Higgs coupling to b -quarks can have a sizeable impact on the extraction of all other Higgs couplings as the Higgs production rate and all branching fractions are modified by a shift in the total decay width Γ_h .

Unfortunately, precision extractions of the $h \rightarrow b\bar{b}$ partial and total width at the LHC are challenging. A measurement of the $h \rightarrow b\bar{b}$ partial width is ultimately limited at the LHC, since knowledge of both the shape and normalization of the contributing QCD backgrounds must

be known to high accuracy [10,11]. Current projections for a measurement of the $h \rightarrow b\bar{b}$ signal strength indicate that precision $\approx (5-7)\%$ may be achievable in the HL phase of the LHC [8]. An extraction of the total width from experimental data (under minimal assumptions) through interference effects [12–16] is possible, and additional information can also be gained by including LEP data [17]. However, the current constraint from this technique is approximately $\Gamma_h < 5\Gamma_h^{\text{SM}}$ [18,19]. A precise model independent measurement of both these quantities requires input from a dedicated “Higgs machine”.

There are several design proposals for a collider with such capabilities, such as a linear/circular e^+e^- machine [20,21], muon-collider [22,23], or $\gamma\gamma$ machine [24,25]. Although the proposed physics programs are different in each case, the common goal is to achieve $\mathcal{O}(\%)$ level precision on the measurement of Higgs couplings. For example, through a combination of a Higgs recoil measurement and an exclusive measurement of $h \rightarrow VV$, better than 10% precision on the total width can be expected [26]. In the case of the $h \rightarrow b\bar{b}$ branching fraction, better than $\mathcal{O}(\%)$ precision is expected in some cases—see Table 2.3 of [26]. In such a scenario, precision calculations are also required to interpret the data. In this work, we focus on the QCD corrections to the Higgs decay to b -quarks in the framework of the standard model effective field theory (SMEFT).

In the SMEFT framework, the effects of physics beyond the SM are parametrized by a set of nonvanishing Wilson coefficients of higher-dimensional operators. Practically, the usual dimension-4 SM Lagrangian is extended to include operators of mass-dimension $n > 4$, which are constructed from gauge-invariant combinations of SM fields and multiplied by Wilson coefficients of mass dimension $(4 - n)$. These operators effectively describe the interactions of new physics particles with those present

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in the SM, an approach which may be justified if the energy scale of these new particles Λ_{NP} greatly exceeds the electroweak scale. One of the main benefits of this approach is that new physics effects can be characterized without specifying a particular UV complete model of physics beyond-the-SM (BSM). The interpretation of data in terms of nonvanishing Wilson coefficients is therefore performed in a model-independent fashion. In the absence of any direct evidence for new particle states during Run-I of the LHC, we believe this approach to be both justified and well motivated.

During Run-I, the interpretation of Higgs measurements by ATLAS and CMS was generally performed in the “interim” κ and signal strength formalisms—see for example the combined Run-I analysis of CMS and ATLAS data [7]. In Run-II, it is a recommendation of the LHC Higgs Cross Section Working Group to move toward a more general EFT framework (see for instance Sec. 10.4 of [9]). In doing so, it is important to note that the predictions for observables obtained in SMEFT are not unlike those obtained in the SM (or UV-completions of the SM for that matter) where a perturbative expansion has been applied—higher-order corrections reduce the theoretical uncertainty of the predictions of observables. Moreover, the next-to-leading order (NLO) corrections to a given observable typically depend on Wilson coefficients which are not present in the tree-level result. It is therefore important to extend SMEFT analyses to NLO (and beyond), to allow for a more precise determination of Wilson coefficients (and allowed ranges) through a comparison with experimental data, and much recent work has been dedicated to this task for a wide range of processes [27–53].

At present, it is possible to deduce logarithmically enhanced NLO corrections appearing in fixed-order perturbation theory to arbitrary observables using renormalization-group (RG) equations for the Wilson coefficients along with the full one-loop anomalous dimension matrix calculated in [30–32]. However, it is common practice to use RG-improved perturbation theory to absorb such logarithmic corrections into the running of the scale-dependent Wilson coefficients between Λ_{NP} and the scale at which the underlying decay or scattering process takes place (for $h \rightarrow b\bar{b}$ decays this is m_h). This removes large logarithms involving Λ_{NP} from effective theory matrix elements, and allows constraints on Wilson coefficients obtained at experimentally accessible energy scales to be interpreted at the scale Λ_{NP} where the effective interactions are generated. The remaining NLO corrections do not contain large logarithms involving the scale Λ_{NP} , and cannot be deduced from an RG analysis. However, these corrections can still be important numerically because for the interesting region of $\Lambda_{\text{NP}} \sim 1$ TeV the RG-induced logarithms are not dramatically enhanced.

In this work, we extend our previous calculation of such NLO SMEFT corrections arising from four-fermion

contributions and the (presumably) numerically dominant electroweak corrections [45] by computing the $\mathcal{O}(\alpha_s)$ correction to the $h \rightarrow b\bar{b}$ decay rate. We proceed by introducing the relevant details of the SMEFT framework for the $h \rightarrow b\bar{b}$ decay, and discuss the renormalization procedure we adopt for performing SMEFT NLO calculations. We then provide the analytic results, and make recommendations for their use in phenomenological studies.

II. CALCULATIONAL SETUP

A. Preliminaries

In the SMEFT, the usual SM Lagrangian is appended by higher-dimensional operators multiplied by Wilson coefficients. In the current work we are interested in dimension-6 operators, and so use a Lagrangian of the form

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}^{(6)}; \quad \mathcal{L}^{(6)} = \sum_i C_i(\mu) Q_i(\mu). \quad (1)$$

The operators relevant for this work are listed in Table I. Note that in our convention the Wilson coefficients of dimension-6 operators have mass dimension minus two, so that the C_i are suppressed by Λ_{NP}^2 , and that we have rescaled the operator Q_{dG} by a factor of the strong coupling constant g_s with respect to the usual definition. The relevant interaction Lagrangian for Higgs couplings to down-type quarks is

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} = & -[H^\dagger \bar{d}_r [Y_d]_{rs} q_s + \text{H.c.}] \\ & + [C_{dH}^* (H^\dagger H) H^\dagger \bar{d}_r q_s + \text{H.c.}], \end{aligned} \quad (2)$$

where $[Y_d]$ and C_{dH} are complex matrices in flavor space. The Higgs potential is also altered compared to the SM, and requiring that the kinetic terms are canonically normalized leads one to write the Higgs doublet in unitary gauge in the broken phase of the theory as

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ [1 + C_{H,\text{kin}}] h(x) + v_T \end{pmatrix}, \quad (3)$$

TABLE I. A subset of the 59 independent dimension-6 operators built from standard model fields which conserve baryon number relevant for the current calculation, as given in Ref. [54]. The subscripts p, r are flavor indices, and q_p and d_r are left- and right-handed fields, respectively.

$Q_{H\Box}$	$(H^\dagger H) \Box (H^\dagger H)$
Q_{HD}	$(H^\dagger D_\mu H)^* (H^\dagger D_\mu H)$
Q_{dH}	$(H^\dagger H) (\bar{q}_p d_r H)$
Q_{HG}	$H^\dagger H G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\tilde{G}}$	$H^\dagger H \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{dG}	$g_s (\bar{q}_p \sigma^{\mu\nu} T^A d_r) H G_{\mu\nu}^A$

where

$$C_{H,\text{kin}} \equiv \left(C_{H\Box} - \frac{1}{4} C_{HD} \right) v_T^2, \quad (4)$$

and the vacuum expectation value $v_T \approx (\sqrt{2}G_F)^{-\frac{1}{2}}$, with G_F the Fermi constant. It should be noted that although $C_{H,\text{kin}}$ is dimensionless, due to the presence of v_T^2 in its definition, it is still understood to be implicitly suppressed by Λ_{NP}^2 .

Throughout this work, we are concerned with the flavor conserving process $h \rightarrow b\bar{b}$. However, the matrices $[Y_d]$ and C_{dH} appearing in Eq. (2) are in general not simultaneously diagonalizable, leading to flavor violating effects which are not present in the SM. The running of C_{dH} also introduces further flavor violating effects which should be considered [32,55]. We follow the procedure taken in previous work [45], imposing a minimal flavor violation (MFV) scenario [56,57] and setting $V_{tb} = 1$ throughout. The b -quark Yukawa coupling, defined as the coefficient of the $hb\bar{b}$ -vertex in the mass basis of the broken phase of the theory, is therefore related to the physical mass according to

$$y_b = \sqrt{2} \frac{m_b}{v_T} + \frac{v_T^2}{2} C_{bH}^*. \quad (5)$$

With these preliminaries in place, it is straightforward to compute the tree-level amplitude for the process $h \rightarrow b\bar{b}$ as

$$i\mathcal{M}^{\text{tree}} = -i\bar{u}(p_b)(\mathcal{M}_{b,L}^{\text{tree}} P_L + \mathcal{M}_{b,L}^{\text{tree}*} P_R)v(p_{\bar{b}}), \quad (6)$$

where

$$\mathcal{M}_{b,L}^{\text{tree}} = \frac{m_b}{v_T} [1 + C_{H,\text{kin}}] - \frac{v_T^2}{\sqrt{2}} C_{bH}^*. \quad (7)$$

B. Renormalization procedure

The calculation of the QCD corrections to $h \rightarrow b\bar{b}$ in the SMEFT proceeds much the same way as in the SM. In addition to calculating the real and virtual contributions to the NLO matrix elements, which we discuss below, we must also construct a set of counterterms which render the virtual corrections UV finite. There is some subtlety in the construction of such UV counterterms, and we therefore provide details on the renormalization procedure here.¹ In essence, wave function and parameter/mass renormalisation is performed in the on-shell scheme, and Wilson coefficients are renormalized in the $\overline{\text{MS}}$ scheme.

We first discuss the calculation of the renormalization constants for the external fields and parameters/masses. This proceeds as in the SM, and a detailed discussion of this procedure for the SM can be found in [58]. While the full

procedure including also electroweak corrections is rather involved, for the QCD corrections to $h \rightarrow b\bar{b}$ we need only renormalize the b -quark field and mass. We relate renormalized and bare quantities according to

$$b_{L,R}^{(0)} = \sqrt{Z_b^{L,R}} b_{L,R} = \left(1 + \frac{1}{2} \delta Z_b^{L,R} \right) b_{L,R},$$

$$m_b^{(0)} = m_b + \delta m_b, \quad (8)$$

where the superscript (0) labels the bare field or mass. Explicit expressions for the one-loop renormalization constants are obtained from the b -quark two-point function. When computing the renormalization constants, the divergences are regulated by performing the loop integrals in $d = 4 - 2\epsilon$ dimensions. The relevant one-loop renormalization constants are found to be

$$\frac{\delta m_b^{(6)}}{m_b} = -\frac{\alpha_s C_F m_b v_T}{\pi 2\sqrt{2}} \left(3 \frac{C_\epsilon^b}{\hat{\epsilon}} + 1 \right) (C_{bG} + C_{bG}^*),$$

$$\delta Z_b^{(6),L} = \frac{\alpha_s C_F m_b^2 v_T}{\pi 4\sqrt{2}} \left(3 \frac{C_\epsilon^b}{\hat{\epsilon}} + 1 \right) (C_{bG} - 3C_{bG}^*),$$

$$\delta Z_b^{(6),R} = -\frac{\alpha_s C_F m_b^2 v_T}{\pi 4\sqrt{2}} \left(3 \frac{C_\epsilon^b}{\hat{\epsilon}} + 1 \right) (C_{bG} + C_{bG}^*),$$

$$\frac{\delta m_b^{(4)}}{m_b} = -\frac{\alpha_s C_F}{\pi} \left(\frac{3}{4} \frac{C_\epsilon^b}{\hat{\epsilon}} + 1 \right),$$

$$\delta Z_b^{(4),L} = \delta Z_b^{(4),R} = \delta Z_b^{(4),L*} = \delta Z_b^{(4),R*},$$

$$\delta Z_b^{(4)} = 2\delta Z_b^{(4),L} = -\frac{\alpha_s C_F}{\pi} \left(\frac{3}{2} \frac{C_\epsilon^b}{\hat{\epsilon}} + 2 \right), \quad (9)$$

where $N_c = 3$ is the number of colors in QCD, $C_F = (N_c^2 - 1)/(2N_c)$, and

$$C_\epsilon^b = 1 + \epsilon \ln \left[\frac{\mu^2}{m_b^2} \right], \quad \frac{1}{\hat{\epsilon}} \equiv \frac{1}{\epsilon} - \gamma_E + \ln(4\pi). \quad (10)$$

The SM and dimension-6 contributions are distinguished through the superscript (4) and (6) respectively. It is worth noting that we follow the convention of [59] by requiring δZ_b^R to be real.

We must also include counterterms related to operator renormalization. These counterterms are generated from the operators whose Wilson coefficients appear in the tree-level expression in Eq. (7), and are in fact simple to construct using that expression as a starting point. We do this by interpreting those as bare Wilson coefficients, and replace them in the $\overline{\text{MS}}$ scheme by renormalized coefficients according to

$$C_i^{(0)} = C_i(\mu) + \delta C_i(\mu) = C_i(\mu) + \frac{1}{2\hat{\epsilon}} \frac{\dot{C}_i(\mu)}{(4\pi)^2}, \quad (11)$$

¹The reader is directed to a previous publication [45] for a more extensive discussion.

where

$$\frac{\dot{C}_i(\mu)}{(4\pi)^2} \equiv \mu \frac{d}{d\mu} C_i(\mu). \quad (12)$$

Explicit expressions for the δC_i can be obtained from the one-loop anomalous dimension calculation in the unbroken phase of the theory performed in [30–32]. Where necessary, these results are converted into the broken phase using the tree-level SM relations, a procedure which is consistent to $\mathcal{O}(\Lambda_{\text{NP}}^{-2})$. Extracting only the relevant results for the QCD corrections, we find

$$\delta C_{bH} = \frac{\alpha_s C_F}{\pi} \frac{3}{v_T^2 \hat{\epsilon}} \left(2m_b^2 C_{bG} + v_T \left(\sqrt{2} m_b C_{HG} - \frac{v_T}{4} C_{bH} \right) \right). \quad (13)$$

The corresponding counterterm for $C_{H,\text{kin}}$ at $\mathcal{O}(\alpha_s)$ is zero. Moreover, we have omitted a term proportional to $C_{H\bar{G}}$, which contributes to the counterterm amplitude but not to the decay rate at $\mathcal{O}(\alpha_s \Lambda_{\text{NP}}^{-2})$.

Having provided all the necessary renormalization constants, the counterterm amplitude can be constructed, and is generally written as

$$\mathcal{M}^{\text{C.T.}}(h \rightarrow b\bar{b}) = -\bar{u}(p_b)(\delta\mathcal{M}_L P_L + \delta\mathcal{M}_L^* P_R)v(p_{\bar{b}}). \quad (14)$$

The expression for the (real) SM counterterm is

$$\delta\mathcal{M}_L^{(4)} = \frac{m_b}{v_T} \left(\frac{\delta m_b^{(4)}}{m_b} + \delta Z_b^{(4)} \right), \quad (15)$$

and the corresponding dimension-6 counterterm is

$$\begin{aligned} \delta\mathcal{M}_L^{(6)} = & \left(\frac{m_b}{v_T} C_{H,\text{kin}} \right) \left(\frac{\delta m_b^{(4)}}{m_b} + \delta Z_b^{(4)} \right) \\ & - \frac{v_T^2}{\sqrt{2}} (\delta C_{bH}^* + C_{bH}^* \delta Z_b^{(4)}) \\ & + \frac{m_b}{v_T} \left(\frac{\delta m_b^{(6)}}{m_b} + \frac{1}{2} \delta Z_b^{(6),L} + \frac{1}{2} \delta Z_b^{(6),R*} \right). \end{aligned} \quad (16)$$

III. RESULTS FOR DECAY RATE

The differential decay rate to NLO is obtained by evaluating the expression

$$d\Gamma = \frac{d\phi_2}{2m_h} \sum |\mathcal{M}_{h \rightarrow b\bar{b}}|^2 + \frac{d\phi_3}{2m_h} \sum |\mathcal{M}_{h \rightarrow b\bar{b}g}|^2, \quad (17)$$

where $d\phi_i$ is the i -body differential phase-space factor. The UV finite two-body contribution is defined by

$$\mathcal{M}_{h \rightarrow b\bar{b}} = \mathcal{M}^{\text{one-loop}} + \mathcal{M}^{\text{C.T.}} + \mathcal{M}^{\text{tree}}, \quad (18)$$

and the three-body term $\mathcal{M}_{h \rightarrow b\bar{b}g}$ is the real emission amplitude.

To compute real and virtual amplitudes (squared), the SM and the relevant dimension-6 Lagrangian have been implemented in FEYNRULES [60]. The contributing Feynman diagrams are subsequently generated and computed with FEYNARTS [61] and FORMCALC [62]. We show the Feynman diagrams contributing to the real emission amplitude and to the one-loop virtual correction in Fig. 1. We regularize IR divergences which are present individually in both two- and three-body contributions to the decay rate by performing loop integrals and phase-space integrals in $d = 4 - 2\epsilon$ dimensions. It is an important check on our calculation that these IR divergences cancel at the level of the decay rate, while UV divergences are removed by the counterterms. A further check is that we reproduce the known SM results as a part of the full SMEFT calculation.

We write the result for the NLO decay rate in the SMEFT in the form

$$\Gamma = \Gamma^{(4,0)} + \Gamma^{(4,1)} + \Gamma^{(6,0)} + \Gamma^{(6,1)}, \quad (19)$$

where the first superscript differentiates between the SM and dimension-6 contributions, and the second between powers of α_s . We provide results for the decay rate to $\mathcal{O}(\alpha_s)$ as an expansion in Λ_{NP}^{-2} , keeping only the leading terms of $\mathcal{O}(\Lambda_{\text{NP}}^{-2})$. The dimension-6 contributions to observables thus appear through the interference of diagrams containing one dimension-6 operator with purely SM diagrams.²

In writing the results we shall make use of the shorthand notation

$$\beta = \sqrt{1 - \frac{4m_b^2}{m_h^2}}, \quad x = \frac{1-\beta}{1+\beta}, \quad y = \frac{1-\beta^2}{4} = \frac{m_b^2}{m_h^2}. \quad (20)$$

Although the computation is performed with complex Wilson coefficients, the result for the decay rate depends only on the real parts of C_{bH} and C_{bG} . To avoid cluttering the notation we do not write out, for instance, $\text{Re}(C_{bH})$, but this is to be understood in all the equations which follow.

The tree-level results for the decay rate are

$$\begin{aligned} \Gamma^{(4,0)} &= \frac{N_c m_h m_b^2 \beta^3}{8\pi v_T^2}, \\ \Gamma^{(6,0)} &= \left(2C_{H,\text{kin}} - \frac{\sqrt{2}v_T^3}{m_b} C_{bH} \right) \Gamma^{(4,0)}. \end{aligned} \quad (21)$$

²Additional effects which may appear at $\mathcal{O}(\Lambda_{\text{NP}}^{-4})$, through interference of dimension-6 contributions or the introduction of dimension-8 operators should be investigated if evidence for nonvanishing Wilson coefficients is observed at $\mathcal{O}(\Lambda_{\text{NP}}^{-2})$.

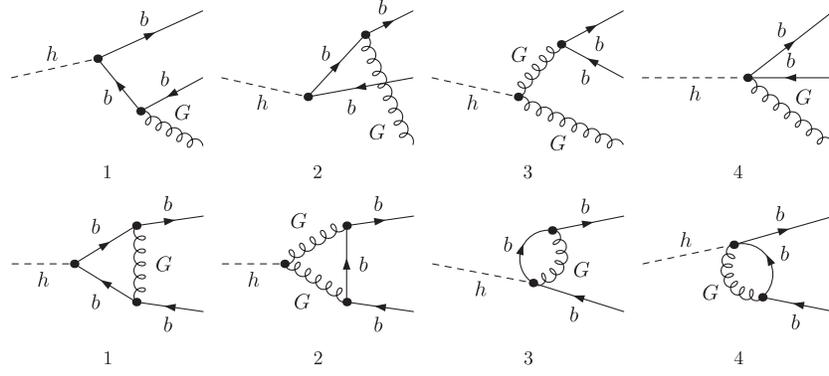


FIG. 1. Feynman diagrams contributing to both real (top) and virtual (bottom) $\mathcal{O}(\alpha_s)$ corrections to the $h \rightarrow b\bar{b}$ decay rate. The real corrections labeled 3 and 4 are generated by Q_{HG} and Q_{bG} operators, respectively. Similarly, the virtual corrections labeled 2 and 3,4 are generated by Q_{HG} and Q_{bG} operators, respectively.

The QCD corrections for Higgs boson decays to massive quarks within the SM have been known for a long time [63–67]. The result is

$$\Gamma^{(4,1)} = \Gamma^{(4,0)} \frac{\alpha_s C_F A(\beta)}{\pi \beta^3}, \quad (22)$$

where the kinematic function $A(\beta)$ is given by

$$A(\beta) = \frac{3\beta}{8}(-1 + 7\beta^2) + \beta^3(3 \ln[y] - 4 \ln[\beta]) + \ln[x] \left\{ \frac{1}{16}(-3 - 34\beta^2 + 13\beta^4) + \beta^2(1 + \beta^2) \left(-\frac{3}{2} \ln[y] + 2 \ln[\beta] \right) \right\} + \beta^2(1 + \beta^2) \left(\frac{3}{2} \ln^2[x] + 2 \text{Li}_2[x] + \text{Li}_2[x^2] \right). \quad (23)$$

The result for the $\mathcal{O}(\alpha_s)$ correction to the decay rate in the SMEFT is a function of the Wilson coefficients $\{C_{bH}, C_{H,\text{kin}}, C_{HG}, C_{bG}\}$. The first two appear at tree-level as in Eq. (21), and the latter two contribute for the first time at $\mathcal{O}(\alpha_s)$. The result is

$$\begin{aligned} \Gamma^{(6,1)} = & C_{bG} \frac{\alpha_s C_F N_c m_h^3 m_b}{\pi 8\sqrt{2}\pi v_T} \left\{ \frac{\beta}{8}(15 + 28\beta^2 - 35\beta^4) - \frac{3}{16}(-5 + 3\beta^2 - 15\beta^4 + 17\beta^6) \ln[x] - 3\beta^3(1 - \beta^2) \ln[y] \right\} \\ & + C_{HG} \frac{\alpha_s C_F N_c m_h^2 m_b \sqrt{y}}{\pi 2\pi} \left\{ \frac{\beta}{8}(15 - 2\pi^2\beta + 23\beta^2) - \frac{3}{4}\beta^2 \ln^2[x] - \frac{3}{2}\beta^3 \ln[y] + \ln[x] \left(\frac{1}{16}(15 + 2\beta^2 + 7\beta^4) + \beta^2 \ln[y] \right) \right. \\ & \left. + 3\beta^2 \left(\text{Li}_2[x] - \frac{1}{2} \text{Li}_2[x^2] \right) \right\} + 2\Gamma^{(4,1)} C_{H,\text{kin}} - C_{bH} \frac{\alpha_s C_F N_c m_h m_b v_T}{\pi 4\sqrt{2}\pi} \left(A(\beta) + \beta^3 - \frac{3}{4}\beta^3 \ln[y] \right) \\ & + \Gamma^{(4,0)} \frac{v_T^3}{\sqrt{2}m_b (4\pi)^2} \dot{C}_{bH} \ln \left[\frac{\mu^2}{m_H^2} \right], \end{aligned} \quad (24)$$

where the $\mathcal{O}(\alpha_s)$ contributions to \dot{C}_{bH} can be deduced from Eq. (13).

The expressions above are a main result of this work. We have written the results such that the explicit powers of m_b correspond to those generated by the Yukawa coupling appearing in $hb\bar{b}$ -vertex, while implicit powers (through, e.g., y) are generated through phase-space factors or through propagators. This will prove convenient when discussing results in the $\overline{\text{MS}}$ scheme for the b -quark mass in the next section. We note that the QCD corrections involving C_{bH} do

not factorize explicitly in the above expression, that is, they are not proportional to the SM ones. This is a consequence of renormalizing the b -quark mass and the Wilson coefficients in different schemes, as we shall see below.

A. Results in the $\overline{\text{MS}}$ scheme and the massless limit

The results presented in the previous section are valid in the on-shell scheme for the masses, and the $\overline{\text{MS}}$ scheme for

the Wilson coefficients. In the presence of a large separation between the scales m_b and m_h , the fixed-order predictions become inappropriate for phenomenology due the appearance of large logarithms of m_h/m_b which deteriorate the convergence of the perturbative series in α_s .

To overcome this, the decay rate predictions can be converted into the $\overline{\text{MS}}$ scheme for the b -quark mass (we refer to such predictions as being in the $\overline{\text{MS}}$ scheme for the decay rate). In this scheme, the dominant large logarithmic corrections are resummed into RG evolution factors relating the value of the running b -quark mass at different energy scales. Mass renormalization in the $\overline{\text{MS}}$ scheme is achieved by dropping the finite contributions to the b -quark mass counterterm in Eq. (9). This implies that the relation between the b -quark pole mass and $\overline{\text{MS}}$ mass is

$$m_b = \bar{m}_b(\mu)(1 - \delta_m(\mu)), \quad (25)$$

where the SM and dimension-6 contributions to $\delta_m(\mu)$ are

$$\begin{aligned} \delta_m^{(4)}(\mu) &= -\frac{\alpha_s C_F}{\pi} \left(1 + \frac{3}{4} \ln \left[\frac{\mu^2}{\bar{m}_b^2} \right] \right), \\ \delta_m^{(6)}(\mu) &= -\frac{\alpha_s C_F}{\pi} \frac{v_T \bar{m}_b}{\sqrt{2}} C_{bG} \left(1 + 3 \ln \left[\frac{\mu^2}{\bar{m}_b^2} \right] \right). \end{aligned} \quad (26)$$

We can then define tree-level results in the $\overline{\text{MS}}$ scheme as³

$$\begin{aligned} \bar{\Gamma}^{(4,0)} &= \frac{N_c m_h \bar{m}_b^2 \beta^3}{8\pi v_T^2}, \\ \bar{\Gamma}^{(6,0)} &= \left(2C_{H,\text{kin}} - \frac{\sqrt{2}v_T^3}{\bar{m}_b} C_{bH} \right) \bar{\Gamma}^{(4,0)}. \end{aligned} \quad (27)$$

The corresponding results for the decay rate in the $\overline{\text{MS}}$ scheme are therefore

$$\begin{aligned} \bar{\Gamma}^{(4,1)} &= \Gamma^{(4,1)} - 2\delta_m^{(4)}(\mu) \bar{\Gamma}^{(4,0)}, \\ \bar{\Gamma}^{(6,1)} &= \Gamma^{(6,1)} - 2\delta_m^{(6)}(\mu) \bar{\Gamma}^{(4,0)} \\ &\quad - 2\delta_m^{(4)}(\mu) \left(2C_{H,\text{kin}} - \frac{v_T^3}{\sqrt{2}\bar{m}_b} C_{bH} \right) \bar{\Gamma}^{(4,0)}, \end{aligned} \quad (28)$$

where it is understood that $m_b \rightarrow \bar{m}_b$ in the first term on the right-hand side of each of the above equations. To obtain the leading-logarithmic (LL) solution for the running mass,

³Note that we have not used the $\overline{\text{MS}}$ mass in the β^3 terms, which are related to phase-space factors for on-shell quark production rather than the Yukawa coupling of the Higgs. If the m_b appearing in β were also converted, one would need to add extra terms to Eq. (28). However, we have checked that the numerical results obtained in that way are nearly identical to those given below, so we have opted for the more streamlined (and physically motivated) version where the phase-space factor is kept in the on-shell scheme.

both SM and dimension-6 corrections to the b -quark mass must be taken into account. By inspecting Eq. (25), the following differential equation should be solved

$$\frac{d\bar{m}_b}{d\ln(\mu)} = -\frac{\alpha_s C_F}{\pi} \frac{3}{2} \bar{m}_b (1 + 2\sqrt{2}v_T \bar{m}_b C_{bG}) + \mathcal{O}(\alpha_s^2). \quad (29)$$

This can be achieved analytically by first finding the LL solution for the running of $C_{bG}(\mu)$, and in addition the running b -quark mass in the SM—which we label as $\bar{m}_b^{(4)}$. In doing so we adopt the following convention for the QCD β -function

$$\frac{d}{d\ln(\mu)} \frac{\alpha_s(\mu)}{\pi} = -2\beta_0 \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 + \dots, \quad (30)$$

where the ellipses refer to higher-order terms in α_s and $\Lambda_{\overline{\text{NP}}}^{-2}$, and $\beta_0 = (11N_c - 2n_f)/12$, with $n_f = 5$ the number of active flavours. A solution for $C_{bG}(\mu)$ can easily be obtained when taking into account only the numerically important self-mixing contribution [32]. This is achieved by solving the equation

$$\frac{dC_{bG}(\mu)}{d\ln(\mu)} = -2 \frac{\alpha_s(\mu)}{\pi} \gamma_C^0 C_{bG}(\mu). \quad (31)$$

We find

$$C_{bG}(\mu) = C_{bG}(\mu_0) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{\gamma_C^0}{\beta_0}}, \quad \gamma_C^0 = -\frac{5C_F - 2N_c}{4}. \quad (32)$$

Note that our result for γ_C^0 differs with respect to that presented in [32], since our operator definition in Table I implies $C_{bG} = \bar{C}_{bG}/g_s$, where \bar{C}_{bG} is the definition used in [32]. Writing

$$\bar{m}_b^{(4)}(\mu) = \bar{m}_b(\mu_0) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{\gamma_m^0}{\beta_0}}, \quad \gamma_m^0 = \frac{3}{4} C_F, \quad (33)$$

a solution for \bar{m}_b to $\mathcal{O}(\Lambda_{\overline{\text{NP}}}^{-2})$ can then be obtained as

$$\begin{aligned} \bar{m}_b(\mu) &= \bar{m}_b(\mu_0) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{\gamma_m^0}{\beta_0}} \left(1 + \frac{2\sqrt{2}v_T}{\gamma_m^0 + \gamma_C^0} \right. \\ &\quad \left. \times [\bar{m}_b^{(4)}(\mu) C_{bG}(\mu) - \bar{m}_b^{(4)}(\mu_0) C_{bG}(\mu_0)] \right). \end{aligned} \quad (34)$$

The motivation for using the $\overline{\text{MS}}$ scheme for the b -quark mass is to resum large logarithms introduced by the scale ratio $m_b/m_h \ll 1$. Under such circumstances, it also makes sense to consider the massless limit of the decay rate, obtained as $\beta \rightarrow 1$, which as we will show below is an

excellent approximation numerically. Keeping the first nonvanishing terms in the SM and for each Wilson coefficient in the SMEFT, we find

$$\begin{aligned}
 \bar{\Gamma}_{\beta \rightarrow 1}^{(4,0)} &= \frac{N_c m_h \bar{m}_b^2}{8\pi v_T^2}, \\
 \bar{\Gamma}_{\beta \rightarrow 1}^{(4,1)} &= \frac{\alpha_s C_F}{\pi} \frac{1}{4} \left(17 + 6 \ln \left[\frac{\mu^2}{m_h^2} \right] \right) \bar{\Gamma}_{\beta \rightarrow 1}^{(4,0)}, \\
 \bar{\Gamma}_{\beta \rightarrow 1}^{(6,0)} &= \left(2C_{H,\text{kin}} - \frac{\sqrt{2}v_T^3}{\bar{m}_b} C_{bH} \right) \bar{\Gamma}_{\beta \rightarrow 1}^{(4,0)}, \\
 \bar{\Gamma}_{\beta \rightarrow 1}^{(6,1)} &= \left(2C_{H,\text{kin}} - \frac{\sqrt{2}v_T^3}{\bar{m}_b} C_{bH} \right) \bar{\Gamma}_{\beta \rightarrow 1}^{(4,1)} \\
 &\quad + \frac{\alpha_s C_F}{\pi} \frac{N_c m_h^3 \bar{m}_b}{8\sqrt{2}\pi v_T} C_{bG} + \frac{\alpha_s C_F}{\pi} \frac{N_c m_h \bar{m}_b^2}{8\pi} C_{HG} \\
 &\quad \times \left(19 - \pi^2 + \ln^2 \left[\frac{\bar{m}_b^2}{m_h^2} \right] + 6 \ln \left[\frac{\mu^2}{m_h^2} \right] \right), \quad (35)
 \end{aligned}$$

where we have used

$$A(\beta \rightarrow 1) = \frac{9}{4} + \frac{3}{2} \ln \left[\frac{m_b^2}{m_h^2} \right]. \quad (36)$$

Furthermore, in the massless limit the dimension-6 corrections from C_{bG} to the running mass in Eq. (34) vanish. It is worth mentioning that even though the phase-space factor multiplying C_{HG} vanishes as $m_b \rightarrow 0$ [because of the factor of \sqrt{y} in Eq. (24)], it is multiplied by a large logarithm in the ratio of m_b/m_h , which is not removed by the conversion to the $\overline{\text{MS}}$ scheme. Finally, in the $\overline{\text{MS}}$ scheme the coefficients C_{bH} and $C_{H,\text{kin}}$, which first appear at tree level, receive NLO QCD corrections proportional to the SM ones, while the coefficients C_{HG} and C_{bG} do not.

To test the validity of the massless approximation, we compare numerically the predictions for the total decay rate in the massless limit with the full result. Specifically, we compare

$$\bar{\Gamma} = \bar{\Gamma}^{(4,0)} + \bar{\Gamma}^{(4,1)} + \bar{\Gamma}^{(6,0)} + \bar{\Gamma}^{(6,1)}, \quad \bar{\Gamma}_{\beta \rightarrow 1}. \quad (37)$$

The following set of inputs are used: $\alpha_s(m_Z) = 0.1184$, $m_Z = 91.1876$ GeV, $\bar{m}_b(\bar{m}_b) = 4.18$ GeV, $m_t = 173.0$ GeV and $m_h = 125.0$ GeV. The replacement $v_T \rightarrow (\sqrt{2}G_F)^{-\frac{1}{2}}$ is made with $G_F = 1.16637 \times 10^{-5}$ GeV⁻² for numerical evaluation, though the results are presented with an explicit factor of $v_T \approx 246$ GeV in order compare their size relative to the electroweak scale. We also introduce the dimensionless Wilson coefficients $\tilde{C} = \Lambda_{\text{NP}}^2 C$, except for $C_{H,\text{kin}}$ where we also extract the factor of v_T^2 and write $\tilde{C}_{H,\text{kin}} = (\Lambda_{\text{NP}}/v_T)^2 C_{H,\text{kin}}$. At the scale $\mu = m_h$, and using Eqs. (30), (32) and (34) to run the various parameters, we obtain

$$\begin{aligned}
 \frac{\bar{\Gamma}}{\text{MeV}} &= \kappa^{\text{QCD}} \left\{ 2.22 \left[1 + 2 \left(\frac{v_T}{\Lambda_{\text{NP}}} \right)^2 \tilde{C}_{H,\text{kin}} \right] \right. \\
 &\quad \left. - 258 \left(\frac{v_T}{\Lambda_{\text{NP}}} \right)^2 \tilde{C}_{bH} \right\} \\
 &\quad + \left(\frac{v_T}{\Lambda_{\text{NP}}} \right)^2 (1.55 \tilde{C}_{bG} + 6.88 \tilde{C}_{HG}) + \dots, \\
 \frac{\bar{\Gamma}_{\beta \rightarrow 1}}{\text{MeV}} &= \kappa_{\beta \rightarrow 1}^{\text{QCD}} \left\{ 2.23 \left[1 + 2 \left(\frac{v_T}{\Lambda_{\text{NP}}} \right)^2 \tilde{C}_{H,\text{kin}} \right] \right. \\
 &\quad \left. - 257 \left(\frac{v_T}{\Lambda_{\text{NP}}} \right)^2 \tilde{C}_{bH} \right\} \\
 &\quad + \left(\frac{v_T}{\Lambda_{\text{NP}}} \right)^2 (1.57 \tilde{C}_{bG} + 6.91 \tilde{C}_{HG}), \quad (38)
 \end{aligned}$$

where we have introduced $\kappa^{\text{QCD}} \approx \kappa_{\beta \rightarrow 1}^{\text{QCD}} \approx 1.20$ to highlight the impact of the QCD corrections on the tree-level result. The ellipses denote terms of $\mathcal{O}(\Lambda_{\text{NP}}^{-4})$ which are generated by the running b -quark mass [see Eq. (34)] and are higher-order in the power counting. The massless limit is therefore found to be an extremely good approximation.

We end this section by commenting on the possible impact of our NLO calculation on global fits to Higgs data. As a concrete example, consider the scenario where a future experiment observes a 5% deviation in the partial width $\Gamma_{h \rightarrow b\bar{b}}$ compared to its SM value. Under the assumption that the Wilson coefficients appearing in Eq. (35) are $\mathcal{O}(1)$, the contributions involving \tilde{C}_{HG} , \tilde{C}_{bG} , $\tilde{C}_{H,\text{kin}}$ can be ignored and only the Wilson coefficient \tilde{C}_{bH} is relevant. The NLO corrections increase the sensitivity to this coefficient by about 20% compared to the LO calculation, and such a measurement on the partial width could be used to probe scales $\Lambda_{\text{NP}} \approx 10$ TeV. However, as discussed in [28], in a broad range of UV complete models it is expected that the Wilson coefficient \tilde{C}_{bH} scales as $\tilde{C}_{bH}^{\text{MFV}} \sim y_b \tilde{C}_{bH}$. In that case, even though the prefactor multiplying \tilde{C}_{HG} is about 45 times smaller than that multiplying \tilde{C}_{bH} , the sensitivity to $\tilde{C}_{bH}^{\text{MFV}}$, \tilde{C}_{HG} , and all the other Wilson coefficients is roughly of the same order since $45y_b \sim 1$, and scales $\Lambda_{\text{NP}} \approx 2$ TeV would be probed. A purely LO calculation would miss the contributions from \tilde{C}_{HG} and \tilde{C}_{bG} entirely.

IV. DISCUSSION AND CONCLUSIONS

As discussed in the Introduction, the Higgs boson couplings are inferred by performing a (global) fit to $\sigma \cdot BR(H \rightarrow X)$ data of the observable final states. Many groups have now performed dedicated analyses and/or global fits to the LHC Higgs coupling data in an EFT framework [27–29, 35, 43, 44, 68–89]. It should be noted that including constraints from precision electroweak and low-energy observables is also important in a global fit to Higgs data at NLO, since operators which contribute to

electroweak precision observables at tree-level can enter the expressions for NLO Higgs decay rates [34].

An important tool used in several of these global fits is the package `εHDECAY` [90,91] which allows the computation of Higgs boson decay widths and branching ratios within the SMEFT (implemented in the SILH basis [92]). This program is an extension of HDECAY [93,94], which computes these observables within the SM⁴. In this case, the expression for the $h \rightarrow b\bar{b}$ EFT decay rate is obtained by applying a scaling of the tree-level prediction. The scaling factor is the same as that used in the SM, and includes the massless QCD corrections up to $\mathcal{O}(\alpha_s^4)$ [95–103]. This is of course a reasonable procedure, since the QCD corrections to the $hb\bar{b}$ -vertex involving C_{bH} and $C_{H,\text{kin}}$ factorize, and the most accurate predictions should be applied wherever possible. In addition to the scaled tree-level results, we suggest to include the $\mathcal{O}(\alpha_s)$ contributions involving C_{bG} and C_{HG} which are currently not included. The relevant results are provided in Eq. (35). It should be noted that the

⁴Numerical values for a range of inputs have been provided by the LHC Higgs Cross Section Working Group [9].

dominant contribution from both of these operators arise from genuine NLO effects, which cannot be predicted by an RG analysis.

Finally, electroweak corrections are known for the $h \rightarrow b\bar{b}$ decay rate in the SM [104–108]. However, unlike the QCD corrections involving C_{bH} and $C_{H,\text{kin}}$, these contributions do not factorize in a straightforward way, and a scaling by known SM corrections is not appropriate. A dedicated calculation of the full NLO electroweak corrections in the SMEFT is in progress. In the meantime, we suggest to include the results involving the one-loop four-fermion and dominant Yukawa corrections which were computed in [45]. In this case, the running of the b -quark mass receives large logarithmic corrections involving the scalar four-fermion operators Q_{qtqb} , and these contributions should be resummed following a similar procedure as was taken for Q_{bG} in this work.

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