

Market Structure with Interacting Consumers

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ABSTRACT

Economic theory has developed a typology of markets which depends upon the number of firms which are present. Much of the literature, however, is set in the context of a given market structure, with the consequences of the structure being explored. Considerably less attention is paid to the process by which any particular structure emerges. In this paper, we examine the **process** of how different types of market structure emerge in new product markets, and in particular on markets which are primarily web-based. A wide range of outcome is possible. But the uncertainty of outcome of the evolution of market shares in such markets is based, not on the various strategies of the firms. Instead, it is inherent in the behavioral rule of choice used by consumers. We examine the consequences, for the market structure which emerges, of a realistic behavioral rule for consumer choice in new product markets. The rule has been applied in a range of different empirical contexts. It is essentially based on the model of genetic drift pioneered by Sewall Wright in the inter-war period. We identify the parameter ranges in the model in which the Herfindahl-Hirschman Index is likely to fall within the ranges identified by the US Department of Justice: unconcentrated markets; moderately concentrated markets and highly concentrated markets.

Keywords: Industrial concentration, Neutral selection, Agent based model

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1 Introduction

Economic theory has developed a typology of markets which depends upon the number of firms which are present. We move from perfect competition, to imperfect competition, through oligopoly to monopoly. Much of the literature, however, is set in the context of a given market structure, with the consequences of the structure being explored. Considerably less attention is paid to the process by which any particular structure emerges.

Perhaps the market structure where the evolutionary nature of the outcome figures most strongly is that of oligopoly. It is recognized that outcomes on factors such as pricing and market shares are at their least determinate under the market structure of oligopoly, and, in consequence, the emergent nature of the market structure is often taken into account. Rothschild (1947), for example, introduced the concept of price being the “expression of a strategic policy”. The path-breaking works of Bain (1956) and Labini (1969) emphasized the importance of non-price competition, and the role of both the barriers to and the threats of entry. Caves and Porter (1977) formulated the entry process as “an investment decision made under uncertainty and conjectural”interdependence”. They argued that “the indeterminateness of oligopoly inevitably spills over to the entry process in the form of recognized interdependence between going firms and potential entrants”. Sutton (2002), suggests that “indeterminacy... relating to multiple equilibria is endemic throughout the literature on market structure”.

However, it is possibly in Austrian economics that the importance of the *process* of the evolution of the market is most evident. Hayek (1948) examined the learning and discovery processes of firms in “dynamic competition”. Schumpeter (1942) coined his famous phrase about the “gales of creative destruction”, bringing to the forefront the evolutionary nature of capitalism.

In this paper, we examine the process of how different types of market structure emerge in new product markets, and in particular on markets which are primarily web-based. A fundamental characteristic of new product markets is that, quite literally, they are new, so that consumers find it hard to distinguish between the attributes of the alternatives. In such circumstances, a product which has objectively inferior attributes to a rival might nevertheless obtain a higher market share.

A classic illustration is the clash between Betamax and VHS in the video recorder market in the 1980s. This decade also saw the publication of the paper by Arthur (1989), in which he demonstrate conditions under which one of two competing new technologies would inevitably gain a market share approaching 100 per cent, but that it would not be possible to predict *ex ante* which of the two it would be. In Arthur’s model, a Polya urn problem in non-linear probability theory, consumers adopt one of two alternatives without regard to the attributes of the alternatives.

The question examined by Arthur, namely the consequences for the market structure which emerges when consumers select without regard to the attributes of the alternatives on offer, is also analysed by Kirman (1993). This paper, too, is highly cited in the economics literature. As we show below, it is also a Polya urn model, a genre which has many interesting variants.

Kirman developed his model to explain the outcomes of experiments with a colony of ants visiting two food piles. On leaving the nest, an ant has the choice of two paths along which it can go and forage for food. In the experiments, there is a food pile along each path, and the food piles are adjusted so they remain identical in size regardless of how many grains are removed. However, the ant has no knowledge that it is certain to find food regardless of which of the two paths it takes. Instead, it adopts a copying strategy, based upon the behavior of ants returning to the nest after (successfully) foraging.

Kirman suggests that this model is relevant to the understanding of financial markets, where the principle of “recruitment or contagion” (p. 149) to rival modes of behavior is important. Extensions of the model have subsequently been developed to try and explain fundamental features of asset price changes (Lux, 1998).

Both the Arthur and the Kirman models offer valuable insights into agent choices in circumstances in which it is difficult to distinguish between the attributes of alternatives, and so agents rely on rules of behavior based upon copying the observed behavior of other agents, or “recruitment or contagion”, as Kirman puts it. However, as we discuss in Section 2 below, the models make assumptions which confine their potential applications to quite specific circumstances.

A more general and, in the context of consumer choice in new product markets, more realistic model is ultimately based on the model of genetic drift pioneered by Sewall Wright in the inter-war period (1931). Condensing the model to its essentials, most of the time agents choose amongst not just two, but potentially a large number of alternatives by copying from the choices made by agents who have already made their selections. In addition, occasionally the choice is based on a type of random selection.

This model has been successfully applied in a range of different empirical contexts. The model has had considerable application in anthropology and cultural evolution, where it is often referred to as the “neutral” model of selection. In other words, when agents select between alternatives, they are “neutral” with respect to their attributes. Examples include Neiman (1995), Shennan and Wilkinson (2001), Bentley *et al.* (2004), Mesoudi and Lycett (2009), Bentley *et al.* (2011b), and Bentley and Ormerod (2012).

A wide range of outcomes is possible under reasonable sets of parameter values of the model, and the range is inherent in the behavioral rule of choice used by consumers. A particular interest in the results is the condition under which various ranges of the Herfindahl-Hirschman Index of market

concentration emerge, the index of course being used by many of the regulatory authorities on competition.

Section 2 discusses the behavioral model from an economic perspective. In Section 3, we set out a formal description of the model, and in Section 4 we present the results.

2 The Neutral Model of Selection: The Economic Background

The idea that agents may base their choices not on a comparison of the attributes of the alternatives, as in rational choice theory, but imitate in some way the choices made by other agents has a distinguished pedigree in economics. Alchian (1950) considered uncertainty and economic theory from an evolutionary perspective. As Ormerod (2015) points out, he anticipated by decades many of the insights of the modern mathematical articulation of the theory of evolution.¹

Alchian considered the behavior of firms under uncertainty, which he defines as being characterized by both imperfect foresight and human inability to solve complex problems containing a host of variables even when an optimum is definable. Although he set his argument in the context of firms, he suggests that the argument is readily transferable to consumer behavior. Alchian takes into account that humans are not like other species. We can imagine the future, act with purpose and intent and consciously adapt our behavior.

He postulates that, even in the face of uncertainty, at least a local optimum might be found if firms follow what we would now term a Bayesian learning process. However, for convergence to an equilibrium, he argues that two conditions need to be satisfied. A particular trial strategy must be capable of being deemed a success or failure *ex post*, and the position achieved must be comparable with results of other potential actions. Alchian argues that it is unlikely that such conditions will hold in practice, for the simple reason that the external environment of a firm is not static but changing. Comparability of resulting situations is destroyed by the changing environment.

In such circumstances, Alchian argues that the appropriate rule of behavior is simply to imitate, as far as possible, observed success. He allows for the possibility of innovation, of devising a new strategy yourself, but the basic principle is to observe someone you believe to be doing well and to try and copy them.

Simon, in his seminal paper on behavioral economics, raised fundamental doubts about the ability of agents in general to process information in the way which is implied by rational choice theory. He wrote “the task is to replace the global rationality of economic man with a kind of rational behavior which is compatible with the access to information and the computational

¹The discussion of the Alchian paper is largely based on Ormerod (2015)

capacities that are actually possessed by organisms, including man, in the kinds of environment in which such organisms exist" (Simon, 1955a, p. 99).

In another major paper published in the same year (1955b), Simon identified one such type of behavior, which is also based upon the principle of imitation. The opening sentence of his paper reads: "It is the purpose of this paper to analyse a class of distribution functions that appears in a wide range of empirical data – particularly data describing sociological, biological and economic phenomena. Its appearance is so frequent, and the phenomena in which it appears so diverse, that one is led to the conjecture that if these phenomena have any property in common it can only be similarity in the structure of the underlying probability mechanisms" (p. 425).

In the context of a consumer entering a new product market, he or she chooses amongst the alternatives with a probability equal to the proportions in which they have already been selected by consumers who have already made their choice. This mechanism has subsequently become known as "preferential attachment", following the heavily cited paper by Barabási and Albert (1999), in which they discovered the mechanism independently, apparently unaware of Simon's path breaking article written over forty years previously.

In general, the behavioral choice rule of preferential attachment (PA) leads to highly skewed non-Gaussian outcomes. Such outcomes characterize many social and economic situations. Simon, for example, mentioned distributions of scientists by numbers of papers published; distributions of cities by populations; distributions of incomes by size. Ormerod (2012) cites a number of other, quite disparate right-skewed non-Gaussian outcomes: viewings on YouTube; film producers' earnings; the number of sexual partners people have; the size of price changes in financial assets; crowds at soccer matches; firm sizes; the size and length of economic recessions; the frequency of different types of endgames in chess; sizes of cities; the ratings of American football coaches in *USA Today*; the distribution of £1 million homes across London boroughs; unemployment rates by county in America; deaths in wars; the number of churches per county in William the Conqueror's Domesday Book survey of England in the late eleventh century.

The modelling approach of PA differs from that based upon the concept of rational addiction with preferences which are learned and are intertemporally dependent (Becker and Murphy, 1988). Under preferential attachment, agents are not required to learn preferences over time. Indeed, the preferences of any given agent are not formed over time. At any point in time, an agent makes a choice based simply on the choices made by others.

PA is a powerful model in many contexts, but is not capable of explaining the turnover in rankings which takes place over time. The time scale of this may vary enormously across different contexts, but it is still a key feature. Batty (2006), for example, analyses turnover in the largest cities, in the USA and in the world. With the former, over the 1790–2000 period, 266 cities were at

some stage in the top hundred. From the year 1840, when the number of cities first reached one hundred, only twenty-one remain in the top hundred of 2000. On average, it takes 105 years for 50 per cent of cities to appear or disappear from the top hundred, whilst the average change in rank order for a typical city in each ten-year period is seven ranks. In complete contrast, Ormerod (op.cit.) notes that over the entire period from 1952 to 2006, no fewer than 29,056 songs appeared in the Top 100 chart in the UK. Of these, 5,141 were in the chart for just a single week. Almost exactly a half stayed in for less than a month, so four weeks was the typical life span, as it were, of a song in the Top 100. In contrast, fifty-nine remained popular for more than six months, and one, “My Way” by Frank Sinatra, spent an incredible 122 weeks in the chart.

The cultural evolution literature offers a behavioral rule of choice which is mainly based upon the process of PA. However, with a small probability, an agent can, for example, select an alternative which no-one else has previously selected. Or the agent could make a random choice from a fixed number of alternatives. The inspiration for such models is the work of geneticists such as Sewall Wright. Agents in the model are “neutral” to the attributes of the alternatives. This could be because in a new product market they find it very hard to understand the differences. There may indeed be objective differences between the various offers, but in such numerous, minor and often incomprehensible ways that they exemplify what has come to be called “decision quicksand” (Sela and Berger, 2012) or “decision fatigue” (Baumeister and Tierney, 2011).

A further refinement of the neutral selection model is to introduce a parameter which determines how far back an agent looks when taking into account the previous choices made by other agents (Bentley *et al.*, 2011a, 2014, 2014). The mechanism of PA implicitly assumes that agents take into account *all* previous choices. But, clearly, in practice this will rarely be the case. In teenage music markets, for example, the consumers are usually not interested in anything more than a few weeks, or at most a few months, old. The fact that the Beatles were extremely popular in the 1960s plays no role in the selections which they make today. Gleeson *et al.* (2015) show that the model incorporating the memory time of agents provides an excellent fit to empirical micro-blogging data on hashtag usage. In addition, it is able to predict novel scaling features of the data.

Essentially, then, the neutral model of consumer choice is based upon the principle of preferential attachment, modified by the introduction of a parameter which specifies the previous time period over which choices are counted, and by an “innovation” parameter, which allows various types of random choice to be made.

Before moving to a more formal description of the cultural evolution model, it is useful to conclude this section by a brief comparison with the Arthur and Kirman models mentioned in the section 1 above, both of which are familiar to economists.

All three models are based upon the principle of copying or imitation. However, there are two fundamental differences between the Arthur and Kirman models and the model of cultural evolution. Both the Arthur and the Kirman models involve agents selecting between just two alternatives. In contrast, in the cultural evolution model, agents select from many alternatives. As a special case, the model could contain only two alternatives, but in the general case there are many.

The second main difference is that in the Arthur and Kirman models the agent only takes into account the proportions with which other agents have selected at time $(t - 1)$. In the preferential attachment model, a special case of the cultural evolution model, agents take into account the choices made in all previous time steps. The addition of the memory parameter makes the model completely general in this respect, and therefore non-Markovian. In different contexts, agents will take into account differing amounts of time steps of choices made by other agents. The cultural evolution model is therefore considerably more general.

It is well known that the Arthur model is based on a special case of the Polya urn problem. The Kirman model is simply a variant of the model proposed by Friedman (1949) and which is a well known stochastic process (Feller, 1951; Freedman, 1965).

3 The Formal Model

Consider a model populated initially by N agents. These each select at random one of k_1 products. We assume that each firm produces only one product. Further, there is no re-contracting by agents during any particular solution of the model. The model proceeds in a series of steps. In each step, $n(t)$ new agents enter the model, where the number $n(t)$ is determined by a standard logistic growth function. Logistic growth characterizes the adoption patterns observed in new product markets in general, generating the familiar S-shaped curve of adoption.

With probability $(1 - \mu)$, an agent copies the choice of product from that of an existing agent within the previous m time steps, or else with probability μ , the agent chooses a product at random. In other words, the agent either copies an existing agent from the last m steps, or chooses at random. In keeping with the cultural evolution literature, we refer to the parameter μ as the “innovation” parameter. In this particular context, however, “innovation” refers simply to the act of choosing at random.

We allow the possibility of products being introduced in addition to the k_1 available in the initialisation of the model, so that in all subsequent steps the random choice is made from a total of $k = (k_1 + k_2)$ products.

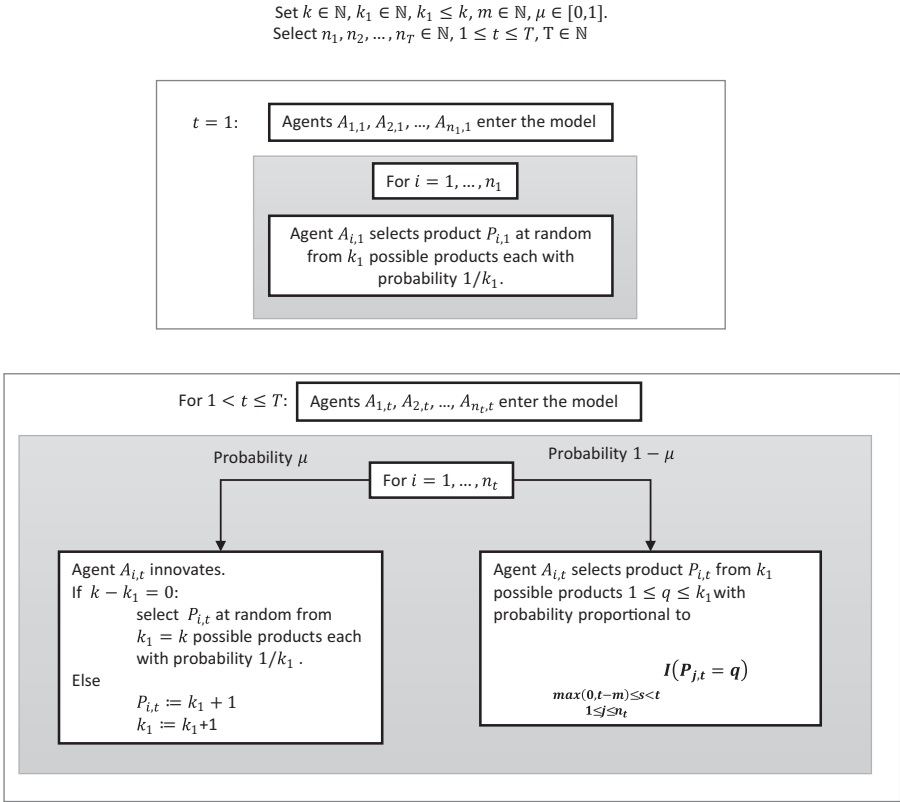


Figure 1: Flow Chart of the Solution Process of the Model.

The “memory” parameter m determines the number of steps of the previous decisions of other agents over which an agent looks when making its decision. The “innovation” parameter μ determines the probability with which an agent takes a random decision rather than replicating one of the decisions previously made by other agents. More formally, the algorithm is described by the flow chart in Figure 1.

4 Results

4.1 Economic Focus and Choice of Parameters

Our focus in the results is the different levels of market concentration which emerge in the solutions of the model. We calculate the Herfindahl-Hirschman Index (HHI) of market concentration. The HHI is calculated by summing the

squares of the individual firms' market shares, and thus gives proportionately greater weight to the larger market shares. The HHI is widely used by competition authorities. The US Department of Justice and the Federal Trade Commission, for example, generally classify markets into three types²:

- Unconcentrated markets: HHI below 0.15
- Moderately Concentrated markets: HHI between 0.15 and 0.25
- Highly Concentrated markets: HHI above 0.25

In terms of the particular sets of results reported here, we set the initial number of agents (n) equal to 100, and the subsequent growth of the population is given by

$$\frac{dP}{dt} = rP \left(1 - \frac{P}{C} \right)$$

where P is population, C is carrying capacity (the total number of agents who eventually buy a product in this market), and r defines the growth rate. We set C equal to 1 million, and $r = 0.001$. The results of the model are robust with respect to the choice of r . Essentially, for any given value of the “innovation” parameter, there is a trade-off between the choice of r and the choice of the memory parameter, m .³ With the values, reported here, carrying capacity is reached after 9546 steps in the solution of the model.

Our main focus is on the market shares recorded for each product at “peak” entry i.e. at the maximum value of dP/dt , which occurs at step 1946, when 250 agents enter the model. Initially, 10 products are available ($k_1 = 10$), and after initial step, agents can choose from a further 10 ($k_2 = 10$), making 20 in total.

Previous analysis with variants of the neutral model similar to this one (Bentley *et al.*, 2011a, 2014, 2015) suggests that the sensitivity of the results with respect to the choice of the memory parameter, m , declines as it increases. So we examine here values of m from 1 through to 10. The same point obtains with the “innovation” parameter, μ . Accordingly, we choose values of μ from zero to 0.02 in steps of 0.0001, making 201 different values in total.

We therefore consider 2010 separate pairs of parameters. For each pair, we obtain 1000 separate solutions of the model.⁴

4.2 Results

The basic features of the results are shown in Figures 2a and 2b. We focus in these initial charts on values of the innovation parameter up to and including

²<http://www.justice.gov/atr/public/guidelines/hmg-2010.html#5c>.

³Results demonstrating this are available on request from the authors.

⁴There is in fact virtually no difference between the results obtained with 1000 solutions and those of only 100 solutions, so we can safely assume that we are reporting the converged properties of the model for both the mean and the variance.

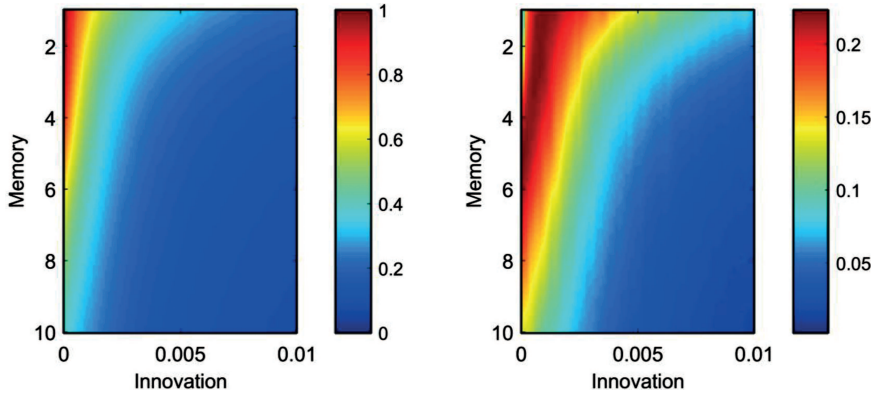


Figure 2: (a) Mean values of the HH Index obtained across 1,000 separate solutions of the model for each pair of the innovation and memory parameters examined. There are 1010 such combinations, with memory taking integer values from 1 through 10, and innovation measured in steps from 0 to 0.01. The Index is evaluated in the step of the model in which the number of agents entering and choosing is at its maximum, the point of inflexion in the logistic growth model. (b) Standard deviation around the mean value in the 1000 separate solutions of the model for each pair of the innovation and memory parameters.

0.01. This is because high levels of concentration are unlikely to be observed for values of $\mu > 0.01$, and so the real interest in the results is for values of the innovation parameter below this level.

Figure 2a shows the mean value of the HHI index across 1000 separate solutions of each of the 1010 pairs of parameters (with μ in the range zero to 0.01), and Figure 2b shows the standard deviation around the mean.

It is known analytically that for values of the memory and innovation parameters of 1 and 0 respectively, in the limit the model generates a “winner take all” solution, where the HH Index takes the value 1 of course. This result is seen in the very top left of Figure 2a, where the memory and innovation parameters take values close to 1 and 0 respectively. It is reflected in the top of left of Figure 2b, where the standard deviation of the index around its mean value in 1,000 solutions is very low.

Figure 2a makes clear that the mean value of market concentration falls as both the memory and innovation parameters increase. However, Figure 2 shows that there is a band of solutions where the outcome is particularly uncertain. We plot here the standard deviation of the H index across each set of 1000 individual solutions. For low values of μ , except for values of both μ and m which are in the immediate neighbourhood of the “winner take all” situation at values of $(0, 1)$, a wide range of outcomes is possible. In such circumstances, even if we knew *ex ante* the precise values of the parameters, predicting the outcome of any single solution would be inherently problematic.

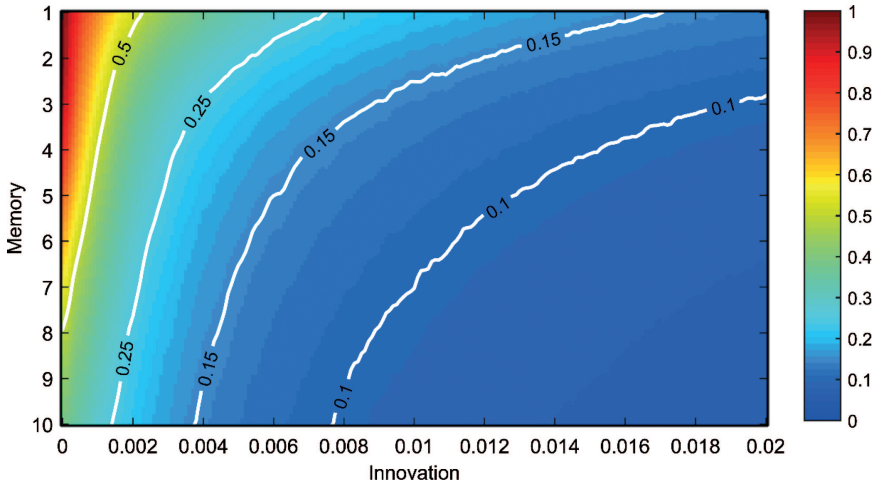


Figure 3: Mean values of the HH Index obtained across 1000 separate solutions of the model for each pair of the innovation and memory parameters examined. There are 2010 such combinations, with memory taking integer values from 1 through 10, and innovation measured in steps from 0 to 0.02. All solutions to the right of the 0.1 contour line have a mean HH Index value of less than 0.1, all between the 0.1 and 0.15 contour lines have a mean value between 0.1 and 0.15 etc.

Further information on the properties of the properties of the model is shown in Figure 3. This reproduces Figure 2a, except that we extend the range of values of the innovation parameter, μ , to 0.02. We also show contour lines for the values of the HHI of 0.1, 0.15, 0.25 and 0.5. For example, all points on the chart to the right of the 0.1 contour line have mean values of the index across 1000 solutions, which are below 0.1, the points between 0.1 and 0.15 have values of the index in this range, and so on.

Figure 3 shows clearly that mean values of the HH Index which are of concern to regulators (i.e. above 0.15) only emerge at low levels of the innovation parameter. In other words, in situations in which social influence on the choice made by individual agents is strong, in which their willingness to act independently is low.

An alternative way of presenting the results with respect to the outcomes for the HHI is to examine the empirical probability of obtaining values of the index within particular ranges of the value of the index. We consider values of the HHI below 0.15, between 0.15 and 0.25, between 0.25 and 1.0, which are plotted in Figures 4a to c below. The charts are constructed in the following way. For any given pair of parameters, we construct the empirical cumulative distribution function based on the 1000 solutions.

Figure 4(a) shows the empirical probability $P(H < 0.15)$ for each memory and innovation pair. Memory clearly plays a large role in market competition.

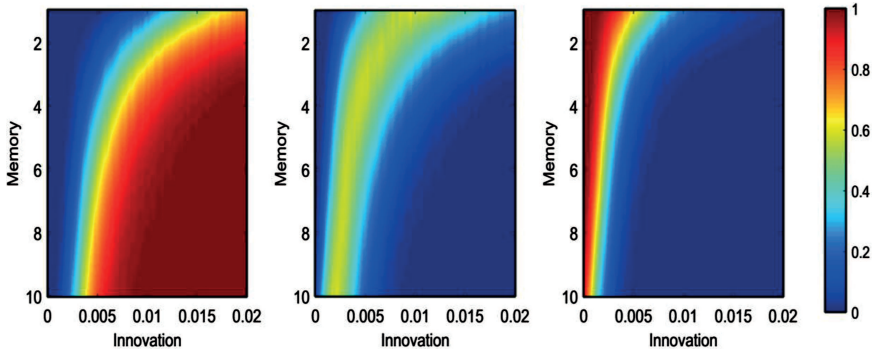


Figure 4: (a) shows an estimate of the frequency with which a value of the HH Index below 0.15 is obtained for each of the parameter pairs, based on 1000 separate solutions of the model for each parameter pair. The Index is evaluated in the step of the model in which the number of agents entering and choosing is at its maximum, the point of inflexion in the logistic growth model. (b) shows the estimate of the frequency of a value of the HH Index between 0.15 and 0.25, and (c) for values above 0.25.

Here we see that, the lower is the memory of the population, the higher innovation needs to be in order to increase the odds of observing a low concentration market.

Figure 4(b) shows the empirical probability $P(0.15 < H < 0.25)$ for each memory and innovation pair. It shows that there is an exponential trade-off between memory and innovation in moderately concentrated markets. The trend displayed indicates that, the higher the memory, the easier it is to jump between low and high concentration markets in low innovation scenarios; in fact, moderately concentrated markets become rarer the higher the population's memory.

Figure 4(c) shows the empirical probability $P(H > 0.25)$ for each memory and innovation pair. That is, the probability of observing a highly concentrated market. For long-memory models, innovation plays a large role in keeping market competition up. We see that the lower the memory, the higher innovation needs to be in order to observe a high concentration market. In fact, the effect of innovation on the concentration index is roughly exponential as a function of memory as noted in 4(b).

We can usefully examine the sensitivity of outcomes to the two behavioral parameters, m and μ , by statistical analysis. A simple linear regression of the mean value of the HH index obtained for each of the 2010 parameter pairs (i.e. using values of μ up to 0.02) against the two parameters is summarized in Table 1.

However, the Ramsey RESET specification test reveals a substantial amount of non-linearity in the relationships. The calculated value of the F statistic to test the null hypothesis of linearity is 2249.6, with (2,2005) degrees of freedom, indicating a very decisive rejection.

Table 1: Results of the simple linear regression of the mean value of the HH Index in each of the 2010 parameter pairs used in the solution of the model on memory and innovation

	Estimate	Std. error	t -value
Intercept	0.4222	0.0054	78.73
Memory	-0.0165	0.0007	-24.36
Innovation	-15.959	0.335	-47.70

Residual standard error: 0.0870.

Multiple R-squared: 0.5884.

Adjusted R-squared: 0.5880.

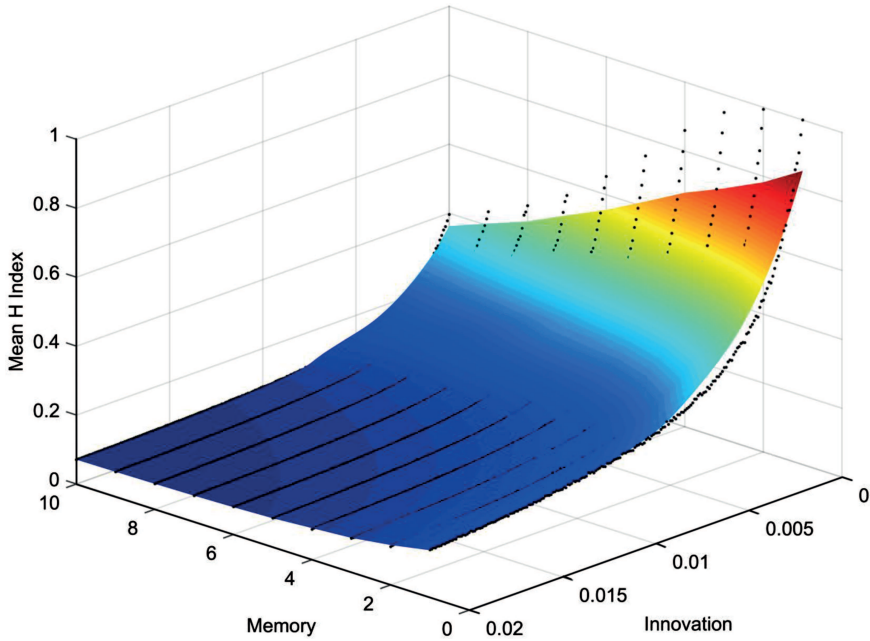


Figure 5: Surface fit using locally weighted non-linear regression of the mean value of the HH Index in each of the 2010 parameter pairs on memory and innovation.

We can readily improve upon the linear by using the general non-linear regression approach of local linear regression (Cleveland and Devlin, 1988). This approach improves on simple polynomial fitting by adjusting the fitted surface locally. Using the Generalised Cross validation criteria for model selection (Golub *et al.*, 1979), we choose the span parameter to be 50 percent. The resulting regression gives a residual standard error of 0.0372, and an adjusted R^2 of 0.9248, using an effective number of parameters of 12. The mean surface of the fit is plotted in Figure 5.

5 Concluding remarks

Oligopolistic outcomes are prevalent in new product markets, especially in markets which are primarily web based. Economic theory is mainly interested in the implications of different types of market structure, such as perfect competition and oligopoly. Our focus here is on the *process* by which different levels of market concentration emerge in new product markets.

In any given market, there will undoubtedly be many reasons why a particular market structure emerges. Here, we offer a general explanation, which is based upon a behavioral rule of consumer choice which is realistic in web based, new product markets.

The amount of information available to consumers in such markets is vast. The various products typically differ in numerous and often incomprehensible ways. We posit that in such markets, consumers essentially choose between alternatives by using a rule which has its foundations in the theory of genetic evolution. In the simplest version of the rule, agents do not select on the basis of the attributes of the different alternatives available to them. In other words, they are “neutral” with respect to these attributes, and the theory is often described as being that of “neutral selection”. Clearly, this is an idealized abstract, but in just the same way, so is the standard theory of consumer choice in the economics textbooks.

An agent selects an alternative with a probability which is equal to the proportion with which the product has been previously selected by other agents. This is the theory advanced by Simon in 1955, and rediscovered independently by Barabási and Albert in 1999, who gave the process the name of “preferential attachment”. We use a rule which generalizes this basic rule in two ways. First, we introduce a memory parameter, which specifies the number of previous periods which the agent takes into account when considering the choices made by others. Second, we introduce an “innovation” parameter, which allows the agent, with a (small) probability defined by this parameter, not to use preferential attachment but instead to select from the available alternatives at random.

The market structure which emerges, and in particular the degree of concentration as measured by the Herfindahl-Hirschman Index, is governed by the values of the memory and innovation parameters. Essentially, the higher the values of these parameters, the less concentrated the market outcome. There are strong non-linearities as the parameters approach their lower bounds of 1 for memory and 0 for innovation. In the neighbourhoods of such values, strong market concentration is very likely to emerge.

The methodology and results we report might well be of more general interest to the agent-based model building community in economics and the wider social sciences. Typically, the results of large numbers of simulations of such models are reported as the averages across the simulations. Here, we present in graphical format the standard deviations around the mean of such simulations.

We also develop “iso-contours” which show the ranges of the two parameters which give mean values of the Herfindahl-Hirschman Index in various ranges, and we illustrate the frequency with which values of the HH Index within certain ranges are obtained for each of the parameter pairs, based on 1000 separate solutions of the model for each parameter pair. We choose the ranges set out by The US Department of Justice and the Federal Trade Commission: Unconcentrated markets: HHI below 0.15; Moderately Concentrated markets: HHI between 0.15 and 0.25; Highly Concentrated markets: HHI above 0.25.

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